PHYSICAL REVIEW C VOLUME 49, NUMBER 3 MARCH 1994

Deformed atomic nuclei with degeneracies of the nucleonic levels higher than 2

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(Received 19 October 1993)

As it is well known, the single-nucleonic levels in a nucleus manifest either the Kramers degeneracy $d = 2$, or, if a nucleus is spherical, a trivial "magnetic" degeneracy $d = 2j+1$. It will be shown using the results of the realistic total nuclear energy calculations that a possibilty of a fourfold degenerate levels exists in a number of $N \sim 136$ isotones due to their high intrinsic symmetry. Those exotic states are predicted to be isomeric; they lie only a few hundreds of keV above the ground state. Other possible nuclear regions where the same mechanism may take place are indicted.

PACS number(s): 21.10.Re, 21.90.+f, 27.90.+b

The classical concept of a nuclear deformation and that of a geometrical form of a nucleus have become important theoretical tools in testing various quantum mechanisms in nuclear structure. For example, if there exists a symmetry of the nuclear field, consisting of its invariance with respect to the $\hat{R}_x \equiv \exp(i\pi\hat{j}_x)$ opertion (rotation of a coordinate frame through an angle of π around an axis, say the $\mathcal{O}_x$ axis), then a conservation of the signature [1] quantum number r results. Such a symmetry implies the existence of two families of rotational bands characterized by $I = 0, 2, 4, \ldots$ and $I = 1, 3, 5, \ldots$ for an even-even nucleus (similarly $I = \frac{1}{2}, \frac{5}{2}, \frac{9}{2}, \ldots$ and $I = \frac{3}{2}, \frac{7}{2}, \frac{11}{2}, \ldots$ for an odd-A case). Conversely, by observing manifestations of those two types of bands in experiment, the existence of the R_x symmetry can be determined.

In this Rapid Communication we suggest the possible existence of highly symmetric shape-isomeric states in several isotones with $N \sim 136$. The underlying symmetry group is the spinor group T_d. It has an attractive feature of implying the existence of three families of nucleonic multiplets: two of them doubly and one *quadruply* degenerate. It is precisely the latter degeneracy which is "unusual" or "exotic" in the nuclear context, the "usual" one, characteristic of deformed nuclear orbitals being the double (Kramers) degeneracy.

To introduce the mathematical arguments let us first present the results of the realistic nuclear total energy calculations performed by using the Strutinsky method [2] with the Woods-Saxon Hamiltonian. The latter has been generalized (with respect to Ref. [3] from which we take the "universal" set of parameters of the potential) to include spherical harmonics $Y_{\lambda\mu}$ with $\lambda = 3, 4, 5, \ldots$ and $-\lambda \le \mu \le +\lambda$ in the nuclear shape definition. The spherical harmonics define the nuclear surface Σ and therefore the underlying geometrical symmetry of the average field Hamiltonian via the standard expansion (for details see, e.g., Ref. [3])

$$\Sigma: \quad R(\vartheta,\varphi) = R_0 c(\{\alpha\}) \left[1 + \sum_{\lambda} \sum_{\mu=-\lambda}^{+\lambda} \alpha_{\lambda\mu} Y^*_{\lambda\mu}(\vartheta,\varphi)\right] . \tag{1}$$

In most of the calculations known in the literature the $\mu \neq 0$ components in the equation of the nuclear surface Σ, for $\lambda \ge 3$, usually have been neglected. Some selected combinations of the ($\lambda = 3,\ \mu \neq 0$) components in the nuclear shapes have been employed [4] in a search for the octupole correlations in the superdeformed nuclei of the Hg-Pb region. Another study of $\mu \neq 0$ effects using a parametrization that involves a nonaxiality parameter and limits the corresponding high-dimensional space to a two-dimensional nonlinear projection has been also presented in [5]. The expansion (1) using the $(\alpha_{20}, \alpha_{3\mu})$ dependence of the total energy surfaces to predict the existence of the $\alpha_{3\mu}$-unstable, superdeformed (SD) isomers was used in [6], while a discussion of similar symmetries in the metallic cluster context can be found in [7]. Here we are going to present for the first time the calculation results for $\lambda \le 5$ with all the μ values included in the total energy cross sections.

Figure 1 illustrates the total energy surface for the $^{222}_{86}\mathrm{Rn}_{136}$ nucleus as a function of the α_{20} and α_{32} deformation parameters, indicating the coexistence of two minima: the ground-state one at $\alpha_{20} \approx 0.16$ and the excited one at $\alpha_{20} \sim 0$ and $\alpha_{32} \simeq 0.15$. The results for $N \sim 134$–138 and the neighboring isotones with $Z \sim 84$–90 are similar. This particular $\alpha_{3\mu\neq 0}$ projection has been selected since the corresponding isomeric minimum lies lowest in energy. To present more precisely the total energy behavior around this interesting minimum several energy cross sections have been done.

Figure 2 presents the cross sections of the total energy surface corresponding to the $\alpha_{3\mu}$, $\alpha_{4\mu}$, and $\alpha_{5\mu}$ dependence, suggesting that the exotic octupole deformation $\alpha_{32} \simeq 0.15$ remains a dominating deformation component in the ^{222}Rn. A similar result remains true also for several neighboring nuclei.

Let us consider first an ideal case of a nucleus with $\alpha_{32} \neq 0$ and all other deformations vanishing (small deviations of α_{20} from zero will imply possible small deviations from the ideal-case prediction). Such a nucleus posesses, as one can easily show, a very high geometrical symmetry composed of 24 symmetry elements. They are, besides the trivial identity operation, three "signature-type" twofold rotations about each of the principal axes, $\hat{R}_x(\pi)$, $\hat{R}_y(\pi)$, and $\hat{R}_z(\pi)$; six fourfold rotary-reflexion

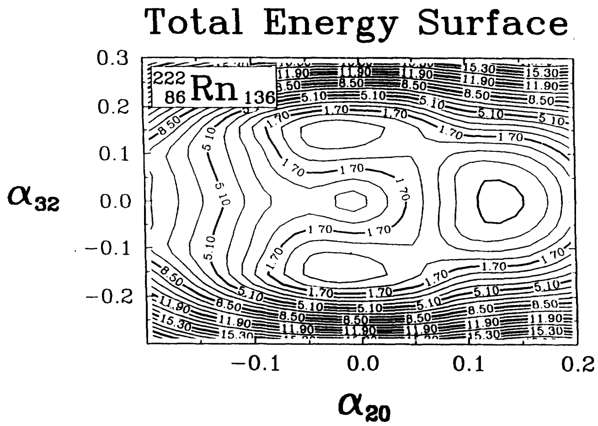

FIG. 1. The total-energy surface calculated for the ^{222}Rn$_{136}$ nucleus in function of the α_{20} and α_{32} deformations. (A comparison of the results in Figs. 3 and 4 indicates that *a priori* the best chances for the $\alpha_{32} \neq 0$ isomeric minimum occur at $Z = 90$ and $N = 136$ and not at $Z = 88$. Indeed, calculations show that the corresponding minima are deeper and the corresponding potential barriers are higher in thorium as compared to radium nuclei. However, those corresponding to the $Z = 90$ case are unstable with respect to the hexadecapole deformation and thus the conditions for the fourfold degeneracy of the nucleonic levels will not be optimal there).

operations along the three principal axes, denoted sometimes $\hat{S}_x(\pi/4)$ and $\hat{S}_x(3\pi/4)$, $\hat{S}_y(\pi/4)$ and $\hat{S}_y(3\pi/4)$, and $\hat{S}_z(\pi/4)$ and $\hat{S}_z(3\pi/4)$; eight threefold symmetry axes passing through the center of the nucleus; and finally six plane reflections.

These symmetry elements form a classical group traditionally denoted T_d. To ensure that the fermion transformation properties are satisfied in the *quantum case*, one usually introduces [8] a special "symmetry" element $\hat{Q}$ such that $\hat{Q}^2 = 1$. With the help of this special element one requires that, e.g., the n-fold symmetry operation $\hat{R}^n(2\pi/n) = \hat{Q}$ and $\hat{R}^{2n}(2\pi/n) = \mathbf{1}$. The corresponding extended group composed of 25 elements is called "fermion T_d" or T_d^D (cf. Ref. [8]). The T_d^D group posesses three irreducible representations (irreps), one four-dimensional one, and two nonequivalent two-dimensional ones.

To illustrate the differences in the intrinsic structure of the nucleonic wave functions corresponding to the three irreducible representations and therefore also three distinct symmetries, let us schematize a construction of the related irreps using the $\{|n\ l\ j\ m\rangle\}$ basis, one of the most standard ones in nuclear structure physics. For that purpose we define a coordinate system in such a way that the z axis coincides with one of the threefold axes and remains at the same time the m-projection axis for the $\{|n\ l\ j\ m\rangle\}$ basis. We see immediately that the corresponding $R'_z(2\pi/3) = \exp[i(2\pi/3)\hat{j}'_z]$ operation reduces to a multiplication by the phase factor $\exp[i(2\pi/3)m]$. After some transformations, and using the above phases, we distinguish three subsets of the whole $\{|n\ l\ j\ m\rangle\}$ ensemble:

$$\{|n\ l\ j = l + \tfrac{1}{2}\ m = \tfrac{1}{2}, \tfrac{5}{2}, \tfrac{7}{2}, \tfrac{11}{2}, \tfrac{13}{2}, \cdots\rangle\}\ , \tag{2}$$

$$\{|n\ l\ j = l - 1/2\ m = \tfrac{1}{2}, \tfrac{5}{2}, \tfrac{7}{2}, \tfrac{11}{2}, \tfrac{13}{2}, \cdots\rangle\}\ , \tag{3}$$

and

$$\{|n\ l\ j\ m = \tfrac{3}{2}, \tfrac{9}{2}, \tfrac{15}{2}, \cdots\rangle\}\ . \tag{4}$$

The ensembles (1) and (2) span the bases of two nonequivalent two-dimensional representations denoted E and E^* while some special combinations of all of them contribute to the G (four-dimensional) representation.

The results of the diagonalization of the deformed Woods-Saxon Hamiltonian are presented in Fig. 3 for the neutrons where the single-particle levels are plotted in function of the α_{32} deformation. A similar illustration for the protons is presented in Fig. 4. Both figures suggest that the isomerism of the α_{32} type illustrated in the present article may repeat itself for the proton-neutron (Z vs N) combinations corresponding to $Z \sim 56$, 70, 90 and similarly $N \sim 70$, 90, 112, 136. In addition, the shell closures at $Z = 50$, 64, 82, and 100 and those at $N = 64$, 82, 100, and 126 are likely to be unstable or susceptible to the α_{32} deformation, thus increasing the chance of observing some manifestations of the T_d^D symmetry in nuclei. The corresponding detailed calculations are in progress.

Figures 3 and 4 illustrate at the same time the content of the irreps of the T_d^D group in the nucleonic orbitals which, at the zero deformation, are labeled using

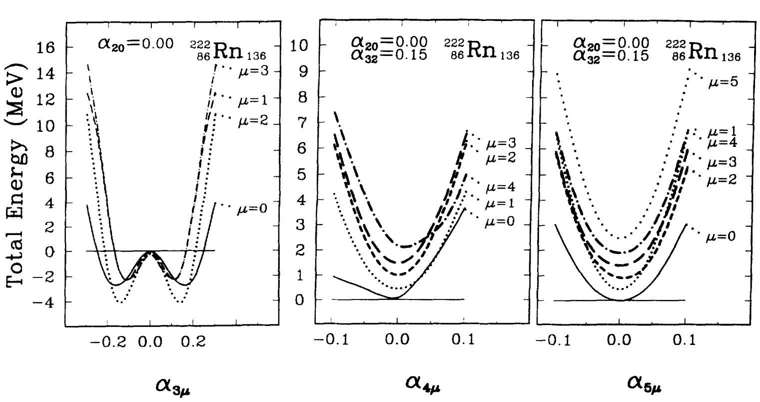

FIG. 2. The total energy cross sections in function of the $\alpha_{3\mu}$, $\alpha_{4\mu}$, and $\alpha_{5\mu}$ deformations at the $\alpha_{32} = 0.15$ local (isomeric) equilibrium characteristic for several isotopes and isotones of the Rn$_{136}$ nucleus. Observe that the isomeric minimum turns out to be stable against any single-multipole distortion. (The couplings between various components have been studied and will be published elsewhere. The preliminary results confirm the tendency to stabilize the isomeric minimum in question.) Calculation results for α_{32} (not displayed) confirm also the stability against this degree of freedom as well.

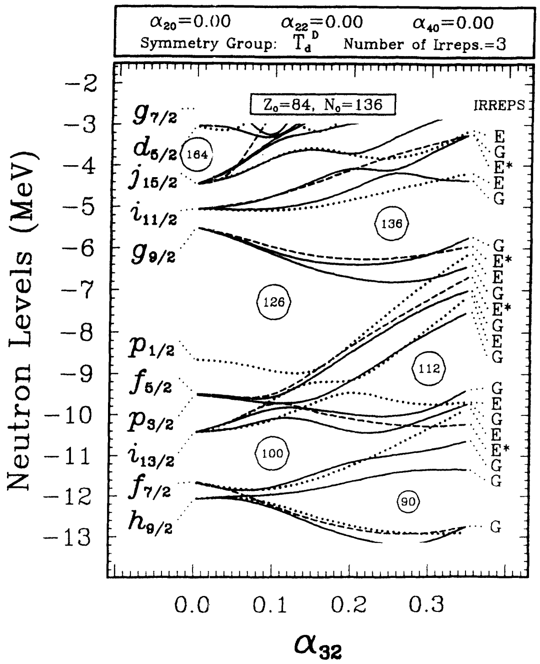

FIG. 3. Neutron single-particle levels in function of the octupole α_{32} deformation. This diagram is illustrative of the isomeric minima like the one in Fig. 1 at $\alpha_{20} \sim 0$ and $\alpha_{32} \neq 0$. The solutions corresponding to the three irreducible representations are marked by G (fourfold degeneracy), solid lines; E (twofold degeneracy), dashed lines; and E^* (a twofold degeneracy related to an irrep nonequivalent to E), dotted lines. Observe the gaps in the spectra corresponding to the strong octupole α_{32} effects at $N = 90$, 100, 112, 126, and 136.

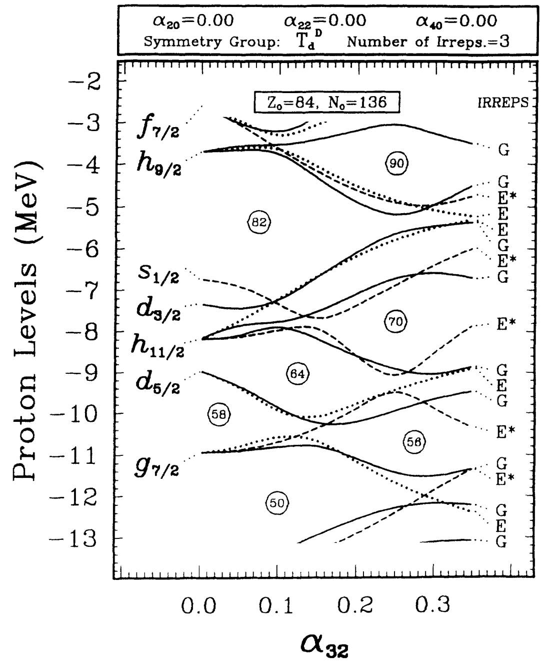

FIG. 4. Similar to Fig. 3 but for the protons. The strong α_{32}-octupole effects are expected at $Z = 56$, 64, 70, and 90.

the traditional spectroscopic notation. One can read, for example, that the $s_{1/2}$ and $p_{1/2}$ levels are related to the E and E^* representations, respectively; the $h_{9/2}$ and $g_{9/2}$ orbitals both contain two members of the G representation (eight states); while the remaining two states belong again to E (in $h_{9/2}$) and E^* (in $g_{9/2}$), etc.

In summary, the existence of an island of low-lying nuclear shape isomers manifesting (at least approximately) the high symmetry of the spinor-T_d group is predicted at $84 \lesssim Z \lesssim 90$, $N \sim 136$ nuclei. Similarly, a possible instability of $^{146}_{64}\mathrm{Gd}_{82}$ with respect to α_{32} exotic octupole deformation (which expresses the spinor-T_d symmetry) is suggested. The isomers of this symmetry should produce (in an ideal case) the single-particle or quasiparticle spectra with an exact fourfold degeneracy.

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