

## Modeling complex nuclear spectra: Regularity versus chaos

S. Drożdż,<sup>1,2</sup> S. Nishizaki,<sup>1,3</sup> J. Speth,<sup>1,\*</sup> and J. Wambach<sup>1,4</sup>

<sup>1</sup>*Institut für Kernphysik, Forschungszentrum Jülich, D-52425 Jülich, Germany*

<sup>2</sup>*Institute of Nuclear Physics, PL-31-342 Kraków, Poland*

<sup>3</sup>*College of Humanities and Social Sciences, Iwate University, Ueda 3-18-34, Morioka 020, Japan*

<sup>4</sup>*Department of Physics, University of Illinois at Urbana, Urbana, Illinois 61801*

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A statistical analysis of the spectrum of two-particle–two-hole doorway states in a finite nucleus is performed. On the unperturbed mean-field level sizable attractive correlations are present in such a spectrum. Including particle-hole rescattering effects via the residual interaction introduces repulsive dynamical correlations which generate the fluctuation properties characteristic of the Gaussian orthogonal ensemble. This signals that the underlying dynamics becomes chaotic. This feature turns out to be independent of the detailed form of the residual interaction and hence reflects the generic nature of the fluctuations studied.

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Theoretical models aiming at the quantal description of an excitation and subsequent decay of a collective state in a many-body system are usually based on a division of the full Hilbert space into two sectors  $S_1$  and  $S_2$ , spanned by the vectors  $|1\rangle$  and  $|2\rangle$ , respectively. At the same time, the Hamiltonian is represented as  $\hat{H} = \hat{H}_0 + \hat{V}$ , such that  $\langle 1|\hat{H}_0|1'\rangle = \epsilon_1^0 \delta_{11'}$ ,  $\langle 2|\hat{H}_0|2'\rangle = \epsilon_2^0 \delta_{22'}$ , and  $\langle 1|\hat{H}_0|2\rangle = 0$ . When  $\hat{V}$  is taken into account the above relations no longer hold. By diagonalizing  $\hat{H}$  in the basis  $|2\rangle$  and redefining  $|2\rangle$  one can still have  $\langle 2|\hat{H}|2'\rangle = \epsilon_2 \delta_{22'}$ , however.

A collective state  $|f\rangle$  such as a plasmon or a nuclear vibrational mode is defined as an eigenstate of  $\hat{H}$  in the subspace  $S_1$ :  $|f\rangle = \sum_1 f_1|1\rangle$ . In the full space, including  $S_1$  and  $S_2$ ,  $|f\rangle$  is, however, no longer an eigenstate of  $\hat{H}$  but rather a wave packet which begins to “leak” into the space  $S_2$ . This constitutes a mechanism for dissipation. Thus, for the time-dependent state one has

$$|f(t)\rangle = \sum_1 f_1(t)|1\rangle + \sum_2 f_2(t)|2\rangle \quad [f_2(t=0) = 0] \quad (1)$$

and the Schrödinger equation for  $f_1$  and  $f_2$  reads

$$i \frac{d}{dt} \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} = \begin{bmatrix} H_{11'} & H_{12'} \\ H_{21'} & H_{22'} \end{bmatrix} \begin{bmatrix} f_{1'} \\ f_{2'} \end{bmatrix}. \quad (2)$$

Applying a procedure similar to the Nakajima-Zwanzig projection technique [1], i.e., solving the second of those equations for  $f_2(t)$ , inserting into the first one for  $f_1(t)$ , and assuming that the basis  $|2\rangle$  is already defined such that  $\langle 2|\hat{H}|2'\rangle = \epsilon_2 \delta_{22'}$ , yields

$$i \frac{d}{dt} f_1(t) - \sum_{1'} H_{11'} f_{1'}(t) = \sum_{1'} \int_0^t d\tau f_{1'}(t-\tau) v_{11'}(\tau), \quad (3)$$

where

$$v_{11'}(\tau) = -i \sum_2 H_{12} H_{21'} \exp(-i\epsilon_2 \tau). \quad (4)$$

The right hand side (rhs) in Eq. (3) is a generalized nonlocal collision term which compensates for the formal elimination of the space  $S_2$ . The matrix elements  $H_{12} = \langle 1|\hat{H}|2\rangle$  describe the degree of mixing between  $S_1$  and  $S_2$  after inclusion of  $\hat{V}$ . The internal structure of space  $S_2$  is represented by the eigenvalues  $\epsilon_2$  of the full Hamiltonian  $\hat{H}$  in this space.

In nuclear physics the space  $S_1$  is typically spanned by one-particle–one-hole (1p1h) states generated from the mean-field Hamiltonian  $\hat{H}_0$ . Diagonalizing the full nuclear Hamiltonian in such a basis gives the random phase approximation (RPA) boson excitations. Ideally, the space  $S_2$ , absorbing such an excitation, should contain all possible  $nprh$  states. Then, after diagonalization of  $\hat{H}$ , the space  $S_2$  would reflect the entire complexity of the spectrum of a compound nucleus. This, however, is neither possible for practical reasons (too many states) nor does it seem necessary for physical reasons. The nuclear Hamiltonian involves predominantly two-body interactions and thus couples the 1p1h RPA bosons chiefly to the 2p2h components in the space  $S_2$ . In this way, the 2p2h states play a special role of “doorway states” [2]. For these reasons, the space  $S_2$  is usually specified in terms of the 2p2h states alone [3]. The resulting methods are known as extended RPA and are believed to account for the spreading width of collective modes. Moreover, since the spectrum of such states is already very dense in energy, the interaction in the 2p2h space is usually neglected which corresponds to using the unperturbed 2p2h energies  $\epsilon_2^0$  as  $\epsilon_2$  in Eq. (4).

\*Also at Institut für Theoretische Kernphysik, Universität Bonn, D-53115 Bonn, Germany.

At this point one should recall that “nuclear dissipation is a process occurring in a closed system, and very likely a consequence of chaotic motion” [4]. Evidence supporting the connection between dissipation and chaos comes both from classical [5] and from quantum [6] considerations. In light of the introductory remarks such a link seems indeed to be present. The fluctuation properties of the spectra of compound nuclei are consistent [7] with those of the Gaussian orthogonal ensemble (GOE) of random matrices [8]. The same fluctuation properties are identified theoretically [9] as well as experimentally [10] for those quantum systems whose classical counterparts are chaotic. Similar conclusions can be drawn from the study of open phase space, scattering phenomena [11]. It is thus natural to require that modeling the space  $S_2$  should preserve this fundamental property of the compound nucleus. Because of the special role played by the 2p2h states, the purpose of the present letter is to quantitatively explore the problem of whether and under which conditions already the basis of such states can support the GOE fluctuation characteristics.

In our analysis the mean-field Hamiltonian  $\hat{H}_0$ , generating the single-particle states, is specified in terms of a local Woods-Saxon potential [12] including the Coulomb interaction. As a residual interaction  $V$  we adopt the zero-range Landau-Migdal interaction

$$V(r_1, r_2) = C_0 (f + f' \tau_1 \cdot \tau_2 + g \sigma_1 \cdot \sigma_2 + g' \sigma_1 \cdot \sigma_2 \tau_1 \cdot \tau_2) \times \delta(r_1 - r_2), \quad (5)$$

which provides a good description of low-energy nuclear excitations. In extended RPA calculations with this interaction (including unperturbed 2p2h states only) the basis of single-particle states is typically specified [13] by four major shells, two below and two above the Fermi surface. Consistently, in the present study we use the basis of similar size. We employ the set of empirical Landau-Migdal parameters as given in Ref. [14] ( $f = -0.1$ ,  $f' = 0.6$ ,  $g = 0.15$ , and  $g' = 0.7$  with  $C_0 = 392 \text{ MeV fm}^3$ ) and perform the calculations for  $^{40}\text{Ca}$ .

In spherical nuclei, such as  $^{40}\text{Ca}$ , the total angular momentum  $J$  and parity  $\pi$  are good quantum numbers which determines the selection of the basic 2p2h states, active in the decay of a given RPA boson excitation. All three types of the matrix elements generated by the residual two-body interaction in the basis of 2p2h states are shown diagrammatically in Fig. 1 and the angular momentum coupling scheme is also indicated. The single-

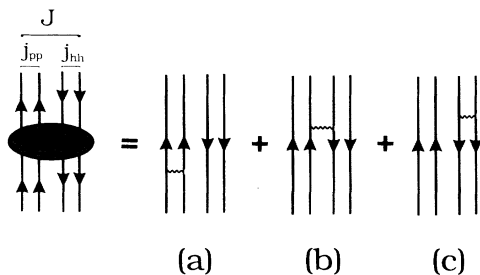


FIG. 1. Diagrammatic representation of the two-body matrix elements in the space of 2p2h excitations with explicit indication of the angular momentum coupling scheme.

particle basis of four major shells in the Woods-Saxon potential allows realistic estimates [13] of global spreading widths because the number of the corresponding 2p2h states is already of the order of  $10^3$ – $10^4$ , depending on the multipolarity of an excitation with the maximum of 11 720 for  $J^\pi = 3^-$ . The matrix of such a size cannot, however, be diagonalized with satisfactory precision. A manageable number (2696) of 2p2h states is found for  $J^\pi = 0^+$  and, therefore, our further discussion will be limited to this multipole. The states are not coupled to the good isospin  $T$  because the Hamiltonian, as specified above, explicitly violates the isospin symmetry. A comparatively weak violation of this symmetry is known [15] to correlate the  $T = 0$  and  $T = 1$  states which allows to combine the relevant spectra for a statistical analysis.

The most obvious measure of spectral fluctuations is the one expressed in terms of the nearest-neighbor spacing (NNS) distribution. It reflects the local correlations between the levels. A standard procedure of analysis is to normalize the spectrum such that the fluctuations on different energy scales are directly comparable. Here we perform the corresponding unfolding [16] by approximating the integrated density of states with polynomials up to the order of 12. This guarantees stability of the result and is equivalent to normalizing the level density to 1. Before unfolding we discard the 50 lowest states in order to eliminate those states which are located far from their main concentration. For the same reason we discard the 50 highest-lying states.

We find three qualitatively different situations as illustrated in Fig. 2. Part (a) shows the NNS distribution for the spectrum of unperturbed 2p2h states coupled to  $J^\pi = 0^+$ . Such a spectrum is not generic and is characteristic for a narrow class of integrable systems involving extra correlations in the Hamiltonian [17]. The pronounced peak for small nearest-neighbor separations illustrates a strong tendency of states for clustering. Actually, in most cases these are even exact degeneracies. They reflect the fact that within the single-particle basis used there are many more two-hole (hh) states than the number of different energies available. Analogous partitioning is even more restrictive on the two-particle (pp) side. Including the interaction in the pp [diagram (a) in Fig. 1] and hh [diagram (c)] channels removes those degeneracies and, as shown in part (b) of Fig. 2, immediately brings the spectrum to the known universality class of generic integrable systems [17] characterized by a completely uncorrelated sequence of eigenenergies. As a consequence, the NNS follows a Poisson distribution. This result essentially does not depend on precise values of the parameters of the residual interaction. Already, a comparatively weak perturbation (for instance 20% of the original strength) produces a similar picture. An important element which makes the spectrum uncorrelated is that, at this level, the ph interaction is still switched off and, therefore, the 2p2h Hilbert space remains a simple product of two subspaces (pp and hh) and the corresponding energies are the sums of the two independent components. In this case, the total angular momentum  $J$  can be written as  $J = J_p + J_h$ , where the pp total angular momentum  $J_p$  and the hh angular momentum  $J_h$  are

constants of the motion. For given  $J$  one thus has a set of independent sequences with good quantum numbers  $J_p$  and  $J_h$  in the  $2p2h$  space. Each such sequence may, of course, already be correlated.

The crucial step which brings us to the situation displayed in part (c) of Fig. 2 is the inclusion of the  $ph$ -type matrix elements, represented by the diagram (b) of Fig. 1. These matrix elements introduce such important correlations that the resulting NNS distribution almost perfectly follows a Wigner distribution, i.e., is consistent with the GOE. In fact, including only this diagram and ignoring the other two [(a) and (c)] gives the same result. Again this feature does not depend significantly on the strength of the interaction parameters. Varying them in a broad range (factors between 1/5 and 2 for instance) leaves the histogram in Fig. 2(c) essentially unchanged even though the energy location of individual levels is very sensitive to detailed values of the interaction parameters. This is another confirmation that the fluctuations, which we are looking at, reflect the generic properties of the system.

The discussion presented above is based on the subspace of  $2p2h$  states coupled to  $J^\pi = 0^+$ . Similar behavior can, however, be expected for the other multipoles. In smaller model spaces where the  $2p2h$  diagonalization can be done reliably for any multipole we see no qualitative

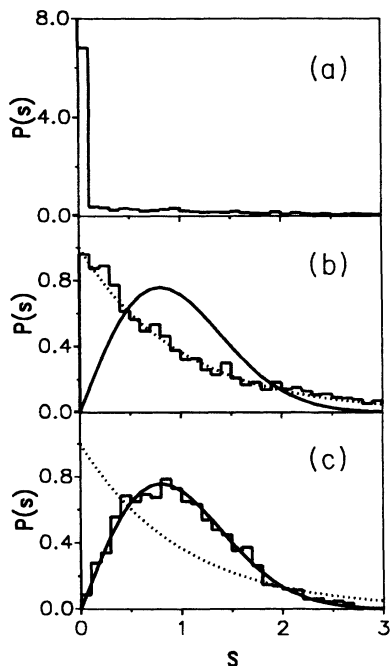


FIG. 2. Nearest-neighbor spacing distributions (histograms) for the sequence of  $2p2h$  states coupled to  $J^\pi = 0^+$  as a function of the normalized relative distance  $s$ . The distribution has been generated in a basis of four major shells (two below the Fermi surface and two above) for a Woods-Saxon potential with parameters corresponding to  $^{40}\text{Ca}$ . Part (a) displays the unperturbed case, (b) corresponds to the results from a diagonalization including diagrams (a) and (c) of Fig. 1, and (c) includes all the diagrams. The residual interaction used is given by Eq. (5). The dotted lines represent the Poisson and the solid lines the Wigner distributions.

difference when performing a similar study. Of course, in the smaller spaces, the mean-square deviations are larger due to poorer statistics.

Because of the similarity of wave functions, the most important doorway states for decay of a typical giant resonance are those  $2p2h$  states which are generated by the single-particle basis of two major mean-field shells on both sides of the Fermi surface. Those states have, therefore, been discussed extensively above. To conclude about the degree of genuine chaoticity of the nuclear Hamiltonian projected onto the full  $2p2h$  space one needs, however, a more restrictive test. The point is that including further single-particle shells in the diagonalization may influence the higher-energy part of the spectrum studied above. Since such an extension cannot be made in practice we go the opposite way. Out of the total of 2696  $J^\pi = 0^+$  states we select a sequence of 400 states starting from 51st state. Such a sequence covers the excitation energies up to about 30 MeV. Concerning the overlap with the high-energy  $2p2h$  states, not taken into account in the diagonalization, we find this choice safe. The resulting NNS distribution and the  $\Delta_3$  statistics [18], the latter being a measure of the rigidity of the spectrum, are shown in Fig. 3, including all diagrams of Fig. 1. A comparison with the corresponding Poisson and GOE predictions [8] is also made. We have verified that using an analogous sequence of states on the unperturbed level reveals a similar clustering as in Fig. 2(a). Furthermore, including diagrams (a) or (c) or both leads to decorrelation seen already for a string of eigenvalues of the same length. The presence of such a transition means that the final good agreement with the GOE does

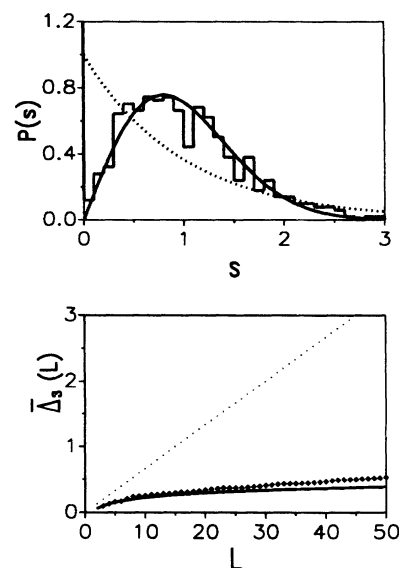


FIG. 3. Nearest-neighbor spacing distribution (histogram in the upper part) and the  $\Delta_3$  statistics (diamonds in the lower part) for a sequence of 400 low-energy states (between 51 and 450) obtained from a diagonalization of the residual interaction Eq. (5) in the basis of  $2p2h$  states for  $J^\pi = 0^+$ . The dotted lines refer to the Poissonian spectrum and the solid lines to GOE predictions.

not involve any kinematical repulsion [19], but is entirely due to the dynamical correlations violating integrability and thus generating chaos.

In conclusion, already on the 2p2h level the spectral fluctuations of the GOE can manifest themselves, provided the residual interaction is taken into account. This means that already on the early stage of nuclear decay the dynamics is associated with fluctuations typical for classically chaotic systems. Most important in this connection are the particle-hole rescattering effects which correlate the states from both sides of the Fermi surface. While the unperturbed 2p2h states certainly provide a reasonable first approximation for the global energy location of the doorway states, they fail completely in the sense of fluctuations. This may have an influence on the fine structure of a resonance. The above aspect of nuclear dynamics should also be kept in mind when addressing the question [20] as to whether the nuclear collective motion is Markovian or not [21]. In classical terms a chaotic system loses memory very fast because of exponential

instabilities. It is thus natural to expect that chaos suppresses the role of the history also on the quantum level. Actually, the integration kernel in Eq. (3) involves a sum of exponents [Eq. (4)] over the whole background spectrum. For a spectrum with a strong tendency to clustering many terms in such a sum may add up constructively and thus amplify the  $\tau$  dependence in Eq. (4). The opposite should apply to the GOE spectrum. In fact, a simple estimate of such effects based on the spectra discussed in this paper shows one order of magnitude reduction of the amplitude of oscillations when going from the case of Fig. 2(a) to Fig. 2(c). This is an important problem [22] which demands a more systematic study.

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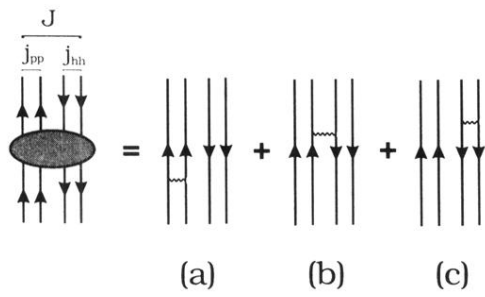


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