Evidence for phase transitional behavior of even-even nuclei from differential observables

A. Wolf,[1,2,3] R. F. Casten,[1] N. V. Zamfir,[1,3,4] and D. S. Brenner[3]
[1]*Brookhaven National Laboratory, Upton, New York 11973*
[2]*Negev Research Center, Beer-Sheva 84190, Israel*
[3]*Clark University, Worcester, Massachusetts 01610*
[4]*Institute of Atomic Physics, Bucharest Magurele, Romania*
(Received 13 August 1993)

Following recent discussions of critical phase transitional behavior of nuclei, additional data are collected and used to test if there is further evidence supporting such phenomena. Data from nuclear masses, radii, and $E2$ transition rates are used to construct differential observables. Inspection of these for the rare earth region shows that they exhibit the characteristic phase transitional behavior found in condensed matter and thermodynamic systems, namely, nearly constant values before the critical point, and a sharp change in the critical region, followed by a different set of values after the critical point.

PACS number(s): 21.10.−k, 23.20.Js, 05.70.Fh, 21.60.−n

The traditional view of the evolution of structure and collectivity in nuclei envisions a few benchmark nuclei displaying limiting structures or symmetries, such as harmonic vibrator or pure rotor, linked by sequences of nuclei with intermediate or translational structure. This view is reflected in the trends in such observables as $E(2_1^+)$ and $R_{4/2} \equiv E(4_1^+)/E(2_1^+)$ (we speak here always of even-even nuclei). The former drops rather smoothly from values well over 1 MeV near closed shells to rotational values below 100 keV while, in the same span of nuclei, $R_{4/2}$ ranges from "shell-model" values (~ 1.3) through the harmonic vibrator value of 2.0, followed by a gradual increase to the rotor limit of 3.33.

Recently, however, the need to reexamine this picture of a gradual evolution of nuclear structure has been suggested following a new analysis of yrast energies [1]. It was found that almost all nuclei from $Z = 38$–82 with structures between the extreme limits of harmonic vibrator and symmetric rotor (i.e., $2.05 < R_{4/2} < 3.15$) could be described by a single *anharmonic vibrator* (AHV) equation with *constant* anharmonicity; this in spite of the continuously varying internal phonon structure of the 2_1^+ state. Thus even though the nature of the phonons varies, the phonon-phonon interaction remains constant throughout the region. This behavior was illustrated by the evolution of $E(4_1^+)$ against $E(2_1^+)$ as $R_{4/2}$ varies gradually from 2.05 to 3.15: $E(4_1^+)$ is linear with a slope of 2.0 and an intercept of 156(10) keV.

For $R_{4/2}$ values nearing the rotor limit of 3.33 the data can no longer be represented by a linear function. As the rotor limit is approached the slope of $E(4_1^+)$ against $E(2_1^+)$ changes from 2.0 to 3.33. This change in slope occurs over a narrow range of nuclei; in other words the transition from an anharmonic vibrator underlying symmetry to that of a rotor is very rapid. This view differs from the traditional one of a continuous, gradual change in structure as $R_{4/2}$ varies from ~ 2 to ~ 3.33. Now, the range $2.0 \leq R_{4/2} \leq 3.0$ exhibits a persistent AHV structure and the major structural change is squeezed into a narrow range of $R_{4/2}$ values just above 3.0. This rapid change in structure is therefore characteristic of a critical phase transition in which the energies can be described by a power-law expression with a critical exponent and a nuclear order parameter related to the slope of $E(4_1^+)$ with $E(2_1^+)$ or $dE(4_1^+)/dE(2_1^+)$.

This phase transitional description runs counter to prevailing thought concerning the properties of finite-body quantal systems and therefore it is important to test if other nuclear observables exhibit similar behavior. An essential signature of a phase transition (for example, in solid-state or thermodynamic phenomena) is an observable that exhibits one behavior before the critical point, and a rapid change to a different behavior at and after the critical point. Frequently, such observables are constant before the phase transition. The purpose of this paper is therefore to present further evidence relating to phase transitional behavior in nuclei by identifying additional observables that are normally constant but which diverge, or at least show a sharp change, around the transition point. We do not attempt to explain the underlying *reasons* for this behavior but rather to lay out the empirical situation as a necessary preliminary to theoretical studies.

In seeking appropriate observables we note that the essential element of a phase transition region is a rapid *change* in properties. Therefore, the most sensitive observables are those that directly measure these changes, that is, quantities which are in the nature of *derivatives or differentials.* This is the case, for example, in condensed matter systems, where the susceptibility and the specific heat are derivatives of the magnetization and internal energy, respectively. It is therefore the aim of the present work to identify appropriate nuclear differential observables and to collect and present the data relating to them.

In this work we propose three such quantities, each of which is the difference in a particular observable in two adjacent even-even nuclei of mass A and A-2, namely, the change in two-neutron separation energies, S_{2n}, the change in $B(E2:0_1^+ \to 2_1^+)$ transition probabilities, and the change in mean-square charge radius $\delta\langle r^2\rangle$. We define these differential quantities by the equations

$$\delta S_{2n}(A) = S_{2n}(A) - S_{2n}(A-2) \ , \quad (1)$$

$$\delta B(E2) = B(E2:0_1^+ \to 2_1^+)_A - B(E2:0_1^+ \to 2_1^+)_{A-2} \ , \quad (2)$$

$$\delta\langle r^2\rangle = \langle r^2\rangle_A - \langle r^2\rangle_{A-2} \ . \quad (3)$$

All these quantities are differentials of some kind. [Note that while $S_{2n}(A)$ is often obtained, in practice, from binding energies of the nuclei A and A-2, it is a property of an individual nucleus and, hence, it is in fact δS_{2n} that is the differential.] All three observables can be calculated directly from readily available compilations of nuclear data [2–4]. As we shall see momentarily, each is expected to be approximately constant before the structural transition point and each should change rapidly in the transition region if the structural transition indeed behaves like a critical phase transition.

In a region of essentially constant structure, the two-neutron separation energies will be dominated by a constant term, representing the energy required to remove two neutrons from the nucleus A_0+2 where A_0 is the doubly magic closed-shell core, plus a term approximately linear in the number of valence nucleons arising from residual interactions of the valence neutrons. If the basic structure changes, S_{2n} will deviate from this linear behavior. (This is, in fact, implicit in the gauge space plots of Ref. [5].) Hence δS_{2n} should be roughly constant until there is a rapid structural change.

For $B(E2)$ values, it is easy to illustrate the way in which a rapidly changing structure can lead to this characteristic behavior of differential observables. To do this, we take a simple pedogogical model that exploits the analytic properties of symmetries in the interacting boson model (IBA). Suppose that one has a sequence of successive even-even nuclei satisfying the U(5) symmetry (anharmonic vibrator) followed by a sudden transition to SU(3) (rotor). For definitiveness we take the transition to occur at boson number $N_B = 8$ and use the expressions [6]

$$B(E:0_1^+ \to 2_1^+)_{\mathrm{U(5)}} = 5N_B e_B^2 \ , \quad (4)$$

$$B(E2:0_1^+ \to 2_1^+)_{\mathrm{SU(3)}} = (2N_B+3)N_B e_B^2 \ , \quad (5)$$

where N_B is the boson number and e_B the effective charge. The resulting values of $\delta B(E2)$ are shown in Fig. 1 and behave similarly to what is expected in a critical lambda (λ)-type phase transition. Above the critical point ($N_B < 8$) $\delta B(E2)$ is constant. At the critical point it changes suddenly, and below the phase transition it takes on a different (and changing) set of values. The key to obtaining this behavior is a constant underlying structure [in this case, U(5)] before the structural transition, as opposed to a smooth and gradual change in structure from nucleus to nucleus. This is exactly what the data presented in Ref. [1] showed: despite changes in internal phonon structure the underlying AHV symmetry persisted. Since U(5) is the IBA equivalent to an AHV, this calculation provides a nice theoretical analogue to the empirical situation. This discussion in the context of the IBA model recalls previous theoretical treatments of

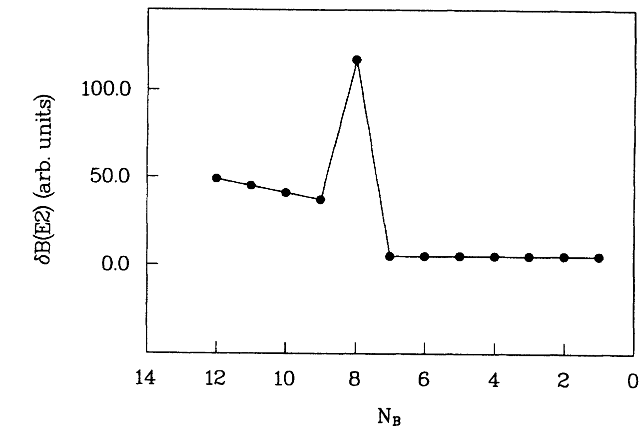

FIG. 1. Calculated differential $B(E2)$ values [Eq. (2)] vs boson number N_B, using IBA analytical expressions [Eqs. (4) and (5)]. The transition between the two limiting symmetries U(5) and SU(3) is assumed here to occur at $N_B = 8$.

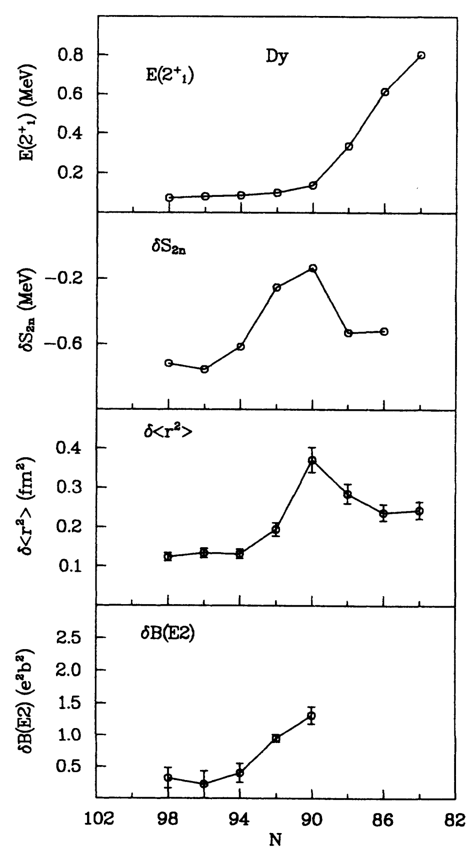

FIG. 2. Experimental values of $E(2_1^+)$, $\delta S_{2n}, \delta\langle r^2\rangle$, and $\delta B(E2)$ [see Eqs. (1)–(3)] vs neutron number N, for Dy isotopes.

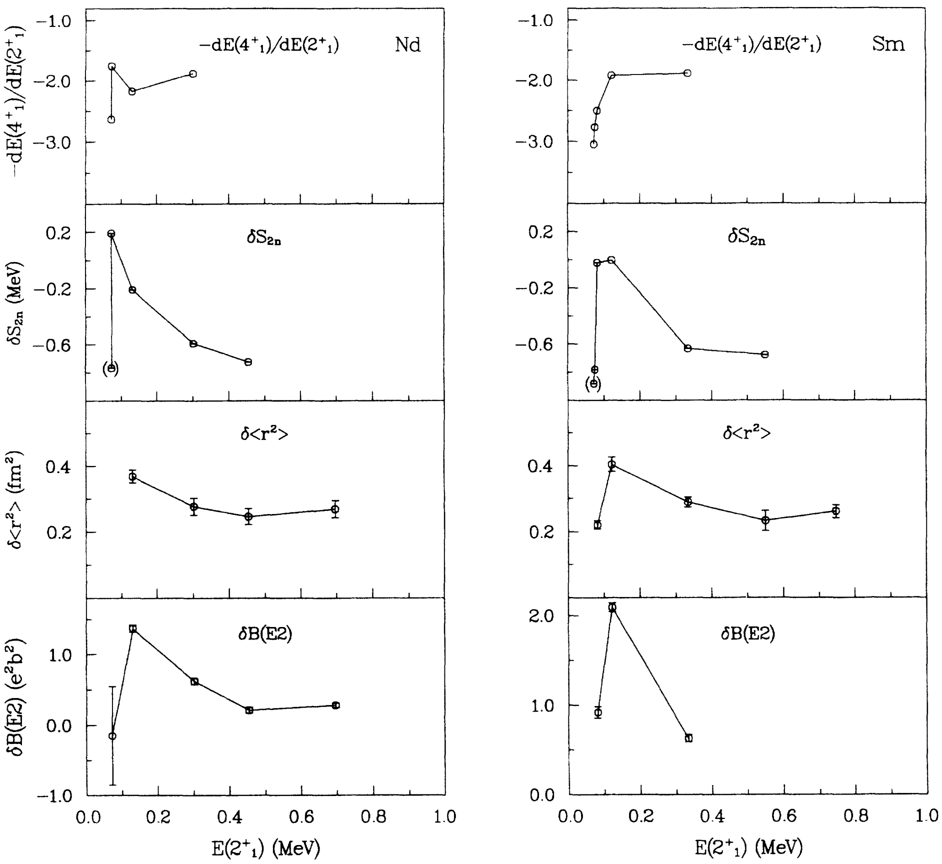

FIG. 3. Experimental values for $-dE(4_1^+)/dE(2_1^+)$, $\delta S_{2n}, \delta\langle r^2\rangle$, and $\delta B(E2)$ (see Eqs. (1)–(3) and Ref. [1]) vs $E(2_1^+)$ for Nd and Sm isotopes with $R_{4/2} \geq 2.0$. Points in parentheses indicate that S_{2n} values were based on systematics in Ref. [4].

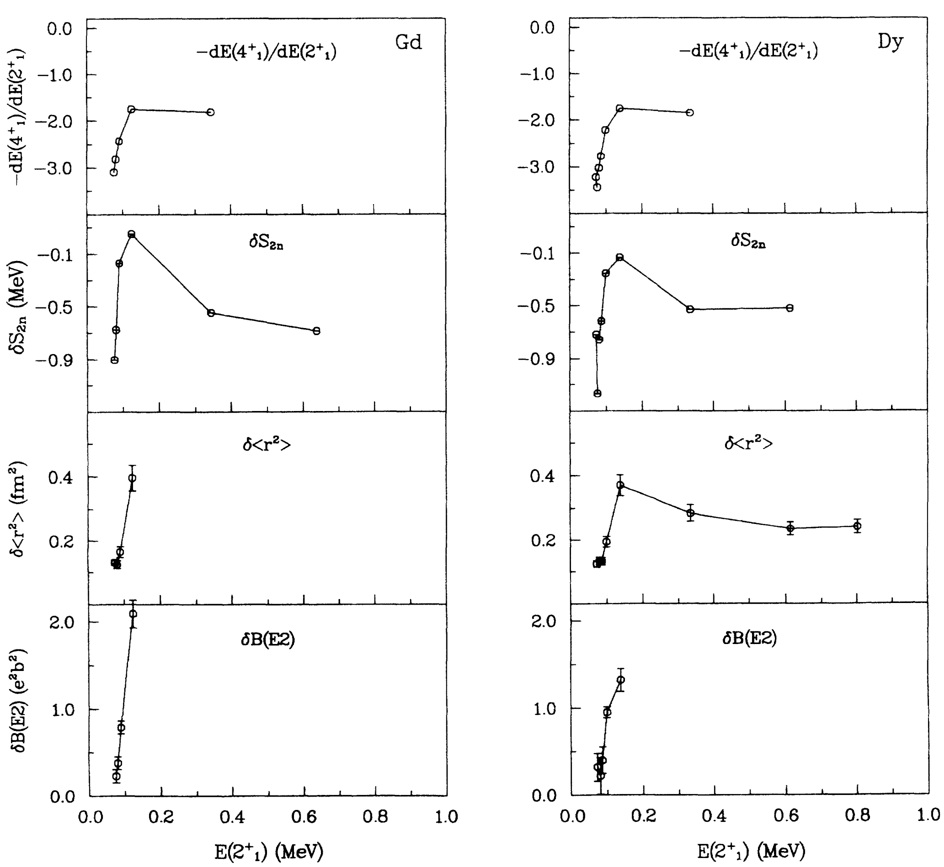

FIG. 4. Same as Fig. 3, for Gd and Dy isotopes.

phase transitional behavior in this model. In particular, it was shown in Ref. [7] that the U(5)→SU(3) transition is a first-order phase transition in the $N_B \to \infty$ limit. Our aim here, of course, is to probe the empirical situation in finite nuclei.

Finally, mean square radii should scale as $A^{2/3}$ barring any shape changes due to structure. In a narrow mass region in heavy nuclei ($A \sim 150$) changes in $A^{2/3}$ are almost exactly linear in A so that $\delta\langle r^2\rangle$ will be roughly constant as long as the nuclear shape is constant. We also note that in the collective model and in the IBA both the $B(E2)$'s and the charge radii are directly related to the nuclear deformation [8,9] and should therefore exhibit a similar behavior. For example, in the collective model $\delta\langle r^2\rangle$ is given by [8]

$$\delta\langle r^2\rangle = \frac{4}{5}R_0^2A^{-1/3} + \frac{3}{4\pi}R_0^2A^{2/3}\delta\langle\beta_2^2\rangle \ . \qquad (6)$$

Since the deformation β_2 is proportional to $[B(E2)]^{1/2}$ in first order, this relation can be approximated

$$\delta\langle r^2\rangle = \frac{4}{5}R_0^2A^{-1/3} + \frac{4\pi}{3R_0^2Z^2e^2}A^{2/3}\delta B(E2) \ . \qquad (7)$$

Hence $\delta\langle r^2\rangle$ should behave similarly to $\delta B(E2)$ for isotopic chains. Indeed, consistency of data from $B(E2)s$ and charge radii, as expected from Eq. (7), has been observed for several isotopic chains (see, for example, Fig. 13 of Ref. [10]).

With this motivation, we can now inspect the data for these differential observables. In this work we focus our discussion on nuclei with $Z = 52$–70, $N = 84$–104 with $R_{4/2} \geq 2.0$, since this is the region where most of the relevant data regarding the proposed phase transition is found. Following the new interpretation proposed in Ref. [1], we expect the observables defined in Eqs. (1)–(3) to be approximately constant for $N = 84$–86, and then to change considerably near $N \sim 88$ and take on a new range of values for $N \geq 90$, reflecting the constancy in structure of the anharmonic vibrator, and the subsequent rapid transition to the rotor.

We first take a traditional look at the data for this region. In Fig. 2, we show the three differentials just discussed along with $E(2_1^+)$ in the Dy isotopes, plotted against the neutron number. The $E(2_1^+)$ plot shows that a *gradual* structural transition of the mean field is taking place between $N = 84$ and 92: $E(2_1^+)$ steadily drops towards asymptotic rotor values. The differentials, however, behave somewhat differently: where sufficient data exist, namely, for $\delta\langle r^2\rangle$ and δS_{2n}, they show rather constant values for $N \leq 88$, a sudden jump at $N = 90$, followed by a transition to a new region of constant values for $N \gtrsim 94$. This behavior is suggestive of the kind of phase transitional behavior discussed in Ref. [1] and the spike at $N = 90$ is exactly the type of behavior illustrated in Fig. 1 for $\delta B(E2)$ values.

It was stressed in Ref. [1], however, that the structural evolution becomes much more evident and transparent if

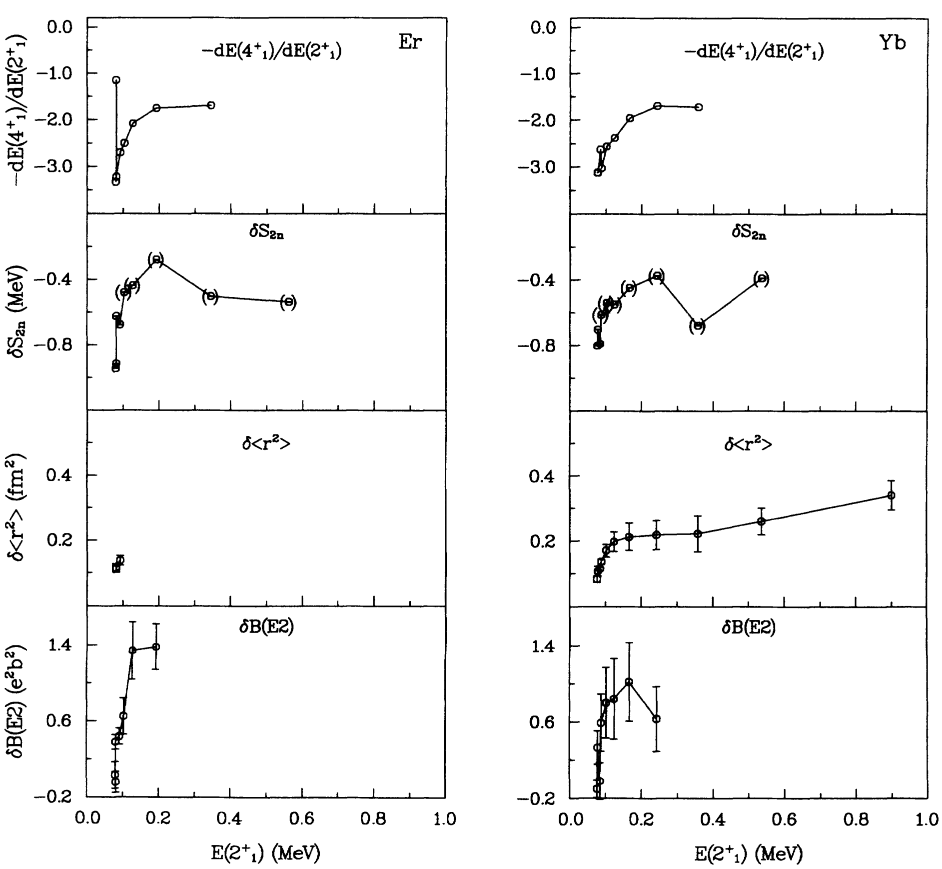

FIG. 5. Same as Fig. 3, for Er and Yb isotopes.

the data are plotted against $E(2_1^+)$ itself rather than N, Z, or A. For example, plots of $E(4_1^+)$ vs $E(2_1^+)$ allow one to see the persistence of a single anharmonic vibrator description, with constant anharmonicity, over broad mass and structural ranges in medium and heavy nuclei [1]. In a sense $E(2_1^+)$ plays the same role in nuclear systems that the temperature plays in condensed matter systems and Ref. [1] showed that the phase transitional behavior commences at a specific 2_1^+ energy, $E(2_1^+)_{\rm crit}$, that is analogous to the critical temperature. This critical value of $E(2_1^+)$ corresponds to $R_{4/2} \sim 3.0$. It would be interesting to pursue a theoretical understanding of this interpretation of $E(2_1^+)$ as a critical parameter.

Our objective here is an empirical one, however, and hence we now present data for all nuclei from Nd to Yb with $82 < N \leq 104$ in terms of plots against $E(2_1^+)$ in Figs. 3–5. In order to connect the current study to the previous one [1], we include in each figure the derivatives $-dE(4_1^+)/dE(2_1^+)$, calculated for the same pairs of isotopes A, $A-2$. Although more experimental data are clearly needed, especially for $\delta B(E2)$ and $\delta\langle r^2\rangle$, the similarity of the individual plots in Figs. 3–5 is quite striking: where the data are sufficient they show a nearly constant value down to $E(2_1^+)$ values of about 0.15–0.20 MeV, and then a rapid deviation from this value as the transition takes place. Such a behavior is characteristic of phase transitions in solid-state or magnetic systems [11]. The phase-transitional behavior is most evident, for example, in the δS_{2n} plots for Gd and Dy and in the $\delta\langle r^2\rangle$ plots for Sm and Dy. Clearly, more extensive data, perhaps obtainable in the future with radioactive beam facilities, would be very useful to complete the mapping of these observables. In particular, a few of the δS_{2n} values (enclosed in parentheses in Figs. 3–5) are obtained from S_{2n} values based on systematics in Ref. [4], and actual measured values would be very useful, such as for Yb where there is an obvious, and probably spurious, break in the systematics at $E(2_1^+)$ of $\sim$ 350 keV.

In Ref. [1], the universality of the phenomenology made it possible to combine the $E(2_1^+)$ data for many elements in single plots against $E(2_1^+)$ and compact global correlations emerged. In the present case, there is considerably more scatter in the results for different elements. Partly, the scatter may simply originate in larger error bars for some of these observables. But, there also seems to be a real difference in behavior for different elements that does not appear in $E(4_1^+)$ vs $E(2_1^+)$ plots. For example, Figs. 3–5 show a more gradual change in values of the differential observables with increasing Z from Nd to Yb, which points to a Z dependence of the phase-transitional mechanism. The sensitivity of different observables to the underlying mean-field structure needs to be understood microscopically. Nevertheless, global plots are useful and revealing in the present case as well. They are shown in Fig. 6. With the exception of a few scattered points, these plots show a nearly constant value for each observable for large $E(2_1^+)$ values, followed, as $E(2_1^+)$ decreases, by a quite rapid deviation from this value, near $E(2_1^+)_{\rm crit}$, and a new behavioral pattern thereafter.

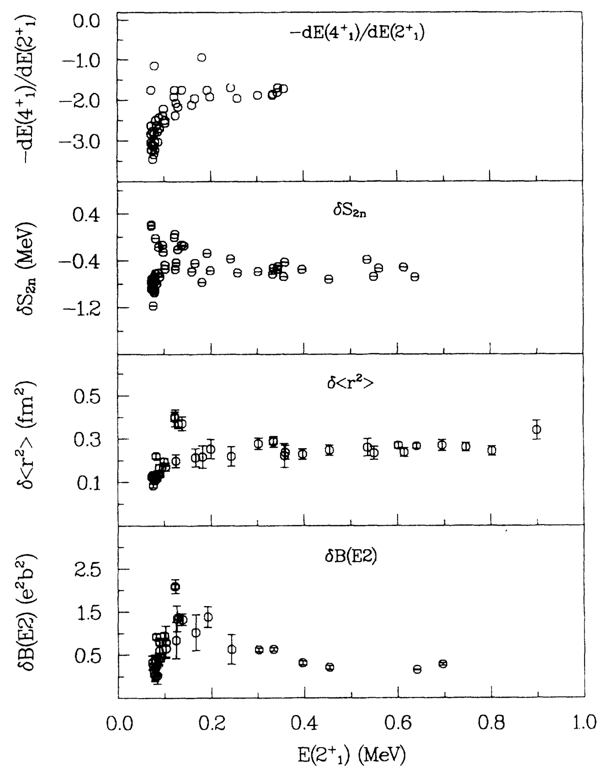

FIG. 6. Same as Fig. 3, showing the combined data for all isotopes with $Z = 52$–70, and $N = 84$–104.

To summarize, we have shown that, by inspecting appropriate nuclear observables that have the character of derivatives or differentials, further evidence for critical phase transitional behavior in nuclei is found. Specifically, we have shown that three nuclear observables, $\delta\langle r^2\rangle$, $\delta B(E2)$, and δS_{2n}, in addition to the previously studied $dE(4_1^+)/dE(2_1^+)$, behave in a similar way, reflecting the constancy in structure of the anharmonic vibrator and then the fast transition to the rotor, which is consistent with a phase transition commencing at $E(2_1^+)_{\rm crit} \sim$ 0.15–0.20 MeV, corresponding to $R_{4/2} \equiv E(4_1^+)/E(2_1^+) \sim 3.00$. These features are evident in traditional plots, for example, against N as in Fig. 2, but, as in Ref. [1], are made much more vivid by plotting against $E(2_1^+)$. These results indicate that finite nuclear matter has some features in common with solid-state and magnetic systems which need to be further investigated. Further systematics work along these lines, and theoretical studies, are needed to explain this analogy and to pursue our understanding of the properties of the finite-body nuclear quantal system.

Research has been supported by the United States Department of Energy under Contract Nos. DE-AC02-76CH00016 and DE-FG02-88ER40417. Discussions with S. Pittel are gratefully acknowledged.

[1] R. F. Casten, N. V. Zamfir, and D. S. Brenner, Phys. Rev. Lett. **71**, 227 (1993).
[2] J. K. Tuli, Nucl. Wallet Cards, 1990.
[3] S. Raman, C. H. Malarkey, W. T. Milner, C. W. Nestor, and P. H. Stelson, At. Data Nucl. Data Tables **36**, 1 (1987).
[4] P. Aufmuth, K. Heilig, and A. Steudel, At. Data Nucl. Data Tables **37**, 455 (1987).
[5] J.-Y. Zhang, Nucl. Phys. **A421**, 353c (1984).
[6] J. N. Ginocchio and P. Van Isacker, Phys. Rev. C **33**, 365 (1986).
[7] A. E. L. Dieperink, O. Scholten, and F. Iachello, Phys. Rev. Lett. **44**, 1747 (1980); A. E. L. Dieperink and O. Scholten, *Interacting Bose-Fermi Systems in Nuclei*, edited by F. Iachello (Plenum, New York, 1981), p. 167.
[8] A. Bohr and B. R. Mottelson, *Nuclear Structure*, Vol. II (Benjamin, Reading, 1975).
[9] F. Iachello, Hyperfine Interact. **15/16**, 11 (1983).
[10] A. C. Mueller, F. Buchinger, W. Klempt, E. W. Otten, R. Neugart, C. Ekström, and J. Heinemeier, Nucl. Phys. **A403**, 234 (1983).
[11] M. F. Collins, *Magnetic Critical Scattering* (Oxford, New York, 1989), pp. 3–15; L. D. Landau and E. M. Lifshitz, *Statistical Physics* (Addison-Wesley, Reading, MA, 1958), pp. 430–445.