Comparison of Gamow-Teller strength in ^{37}Ar and ^{37}K and ^{37}Cl neutrino cross sections

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We compare the Gamow-Teller strength distributions for the $A = 37$ system as measured by the ^{37}Ca β^+ decay experiment with ^{37}Cl (p,n) experiments, considering the effect of suppression of β-delayed protons from the 3.239-MeV state in ^{37}K. We determine a range of values for the suppression of proton emission from this state for which the two measurements of strength are consistent. Recent studies of this state support this suppression and the amount of suppression is within our allowed range. Using the experimentally determined value for the suppression we are able to determine neutrino cross sections on ^{37}Cl from ^{8}B solar neutrinos, supernova neutrinos, and μ^+ decay neutrinos. Our cross section for μ^+ decay neutrinos is found to be 20% larger than the estimates of Bahcall, because of high-lying strength only recently confirmed by the ^{37}Ca β^+ decay experiment.

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I. INTRODUCTION

The $A = 37$ system is unusual because it allows the Gamow-Teller (GT) strength distribution to be measured over a large range of daughter excitation energy in several independent ways. Intermediate energy (p,n) reactions have measured [1,2] the GT strength for ^{37}Cl$\longrightarrow$ ^{37}Ar over roughly 12 MeV of excitation energy in ^{37}Ar. The recent experiment of García *et al.* [3] has looked at the β^+ decay of ^{37}Ca, the isospin mirror of ^{37}Cl, and measured GT strength over 8.5 MeV of excitation energy in ^{37}K. This strength was determined by measuring the β-delayed protons emitted by the excited states of ^{37}K above the threshold for $p+^{36}$Ar. Because the strength distributions from these experiments show significant differences, García *et al.* were led to conclude that charge-exchange reactions were not reliable probes of GT strength. This disagreement is also worrisome because of the resulting ambiguities in determining the nuclear physics uncertainties of the solar neutrino cross section on ^{37}Cl.

In a previous paper [4], we examined the unknowns in the experiment of García *et al.*, reeking a way to reconcile the (p,n) measurements of Rapaport *et al.* [1] with the new decay data. Agreement between the experiments was only possible if one postulated a much larger amount of GT strength in the isobaric analog states (IAS) than is predicted with Wildenthal's *sd*-shell interaction [5,6] (hereafter USD). Such a difference is not likely since USD reproduces the IAS magnetic moment accurately. Another way to reconcile the experiments was to postulate that the proportionality between intensity of β-delayed protons in ^{37}K and the GT matrix elements to these states varied from state to state. One way in which this could happen is if γ decay were able to successfully compete with proton emission in a particular state of ^{37}K. A previous paper concluded with a mention of this possibility, but no systematic study of its consequences.

Recent studies have shown that this competition between γ decay and proton emission is relevant for ^{37}Ca $\longrightarrow$ ^{37}K(3.239 MeV) [7–9]. This $5/2^+$ state in ^{37}K is approximately 1.4 MeV above the threshold for p + ^{36}Ar, and, naively, proton emission is expected to be the dominant decay mechanism. Proton emission definitely dominates the decay of the lowest $5/2^+$ state in ^{37}K at 2.750 MeV, which is even closer to the threshold. It is interesting to ask why the upper $5/2^+$ state is characterized by suppressed proton emission.

In this paper we will examine the consequences and possible causes of this suppression of proton emission. In the next section, we compare the experiments in a fashion similar to Aufderheide *et al.* [4], but taking into account the weak proton emission in the 3.239-MeV state. In the third section, we use USD to examine the effect of mixing the two $5/2^+$ states discussed above and find that the rate of proton emission from the upper mixed state can become much smaller than its rate of γ decay by virtue of this mixing. We also examine the Coulomb force as a possible partial source for this mixing. In the fourth section, we examine how this suppressed proton emission affects the uncertainty in the cross section of ^{37}Cl for solar neutrinos, supernova neutrinos, and neutrinos produced from μ^+ decay. It will be seen that the present data are good enough to constrain the cross sections to

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3σ uncertainties of less than 8%. In the last section we discuss what remains to be done on the problem.

II. COMPARISON OF THE EXPERIMENTS USING A VARIABLE PROTON EMISSION STRENGTH FOR THE ^{37}K(3.239 MeV) LEVEL

In this paper we will be concentrating on the nuclear structure of low-energy states in ^{37}Ar and ^{37}K. These nuclei are isospin mirrors and are the daughter nuclei of the (p,n) experiment and the β^+ decay experiment, respectively. The low-energy spectrum of ^{37}Ar and ^{37}K are compared with the predictions of USD in Table I. As one would expect for isospin mirrors, the spectra are quite similar. The energies of states of the same spin and parity do not differ by more than 200 keV between the two nuclei. The slight differences in the location of levels are probably the result of perturbations due to the Coulomb interaction and Thomas-Ehrman shifts [10,11] for the particle unbound states of ^{37}K. Because USD is based on an optimal fit to all spectra ignoring Coulomb-based differences between nuclei, the theoretical spectrum is the same for ^{37}Ar and ^{37}K. The USD-based spectrum does reproduce the spectrum of each nucleus to within approximately 200 keV, about the same range as Coulomb perturbations.

The $3/2^+$ ground states of ^{37}Cl and ^{37}Ca have allowed Gamow-Teller transitions to the first, second, sixth, seventh, and tenth states listed in Table I. Because García *et al.* were only able to measure β-delayed protons, the GT strength in the first two states of ^{37}K had to be determined in other ways. The GT strength in the ground state can be obtained from the ^{37}Ar electron capture decay rate, assuming isospin invariance. The GT strength in the 1.371-MeV state of ^{37}K was determined by satisfying the expression for the total decay rate of ^{37}Ca. The reader is referred to García's thesis [12] or our previous paper [4] for further details of this procedure.

Figure 1 shows the GT strength distribution up to approximately 4 MeV measured by the ^{37}Cl$(p,n)^{37}$Ar experiments of Rapaport *et al.* [1] and Wells *et al.* [2] and the ^{37}Ca β^+ decay experiment of García *et al.* [3]. What has been plotted from the Rapaport *et al.* and the García *et al.* experiments is the GT strength extracted, whereas for the Wells *et al.* experiment, the raw (p,n) spectrum has been plotted, renormalized so as to fit on the plot, because the data analysis on the Wells *et al.* experiment is not yet complete [2]. As in our previous paper, we work in the "García" units for B(GT), i.e., the sum rule is given by $(g_a/g_v)^2 3(N-Z)$.

TABLE I. $A = 37$ low-energy spectra. Low-energy spectrum of ^{37}Ar and ^{37}K, compared with the spectrum predicted by USD [5,13]. All energies are in MeV. The states of negative parity are not predicted by USD, since it is a $0\hbar\omega$ shell model. The $p + {}^{36}$Ar threshold in ^{37}K is also shown. All other thresholds for ^{37}Ar and ^{37}K are much higher energies (at least 8 MeV) and are thus not relevant for this study.

J^π	^{37}Ar	^{37}K	USD
$3/2^+$	0.000	0.000	0.000
$1/2^+$	1.410	1.371	1.586
$7/2^-$	1.611	1.379	
$p+{}^{36}$Ar		1.88	
$7/2^+$	2.217	2.278	2.127
$3/2^-$	2.491	2.173	
$5/2^+$	2.796	2.750	2.884
$5/2^+$	3.171	3.239	3.112
$9/2^-$	3.185	3.270	
$5/2^-$	3.274	3.083	
$3/2^+$	3.602	3.622	3.539
$3/2^+$	3.934	3.840	

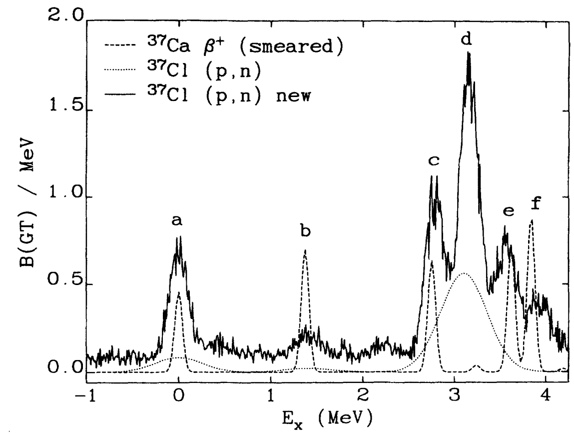

FIG. 1. Comparison of GT strength measured in ^{37}Cl$(p,n)^{37}$Ar and ^{37}Caβ^+ decay experiments. The decay strengths have been given a much larger width than the actual resolution of the experiment, in order to facilitate comparison. For the first (p,n) experiment, the extracted GT strength has been plotted. For the newest (p,n) experiment, the raw spectrum has been plotted, after having been renormalized for comparison with the other curves. To obtain the original raw spectrum in counts, multiply this curve by 118.5.

The first (p,n) experiment [1] was not able to resolve peaks *c*, *d*, *e*, and *f* and thus placed all of the GT strength in one large peak near the centroid of this group. Poor resolution also compromised the ability of this experiment to determine the strength in the 1.410-MeV state; only an upper limit of $B_{1.410}(\mathrm{GT}) \leq 1.406 \times 10^{-2}$ was determined. The newest (p,n) experiment shows much better resolution; all six of the lowest-energy peaks seen by García *et al.* are discernible in the raw (p,n) spectrum. The differences between the new (p,n) and the β^+ decay in location of the peaks is a result of isospin asymmetry and can be understood by comparing the excitation energies for the levels in ^{37}Ar and ^{37}K in Table I.

If one compares the size of the peaks in the raw (p,n) experiment with the strength seen in the decay experiment, it can be seen that there is a fairly good correspondence, except for peaks *b* and *d*. Peak *b* (*d*) in the (p,n) experiment appears to show less (more) strength than seen in the β^+ decay. Recall that the strength in peak *b* had to be determined indirectly by García *et al.* If an error had been made in determining the rest of the GT strength, the strength inferred in peak *b* would be in error. The lack of strength in peak *d* in the García *et al.* data is of more concern, since it was "directly" measured. The state at 3.239 MeV in ^{37}K can decay either by proton emission into ^{36}Ar, or by γ emission into a lower-lying state of ^{37}K. The latter case could not be seen in the García *et al.* experiment because of the large γ-ray background. However, if γ emission did dominate proton emission, it would explain the paucity of counts seen at 3.239 MeV in the decay experiment.

When we consider the competition between γ decay and proton decay of the levels fed by GT transitions, we can write a quantitative expression with a proportionality factor to relate the intensity of the proton emission to B(GT) for the β decay feeding the level, but the factor will depend on both the proton width and the γ width for that level. The amount of GT strength from ^{37}Ca to the ith state of ^{37}K will be given by

$$B_i(\mathrm{GT}) = \mathcal{C}_i\left\{1+\frac{1}{\eta_i}\right\}\Gamma_p(i)\ , \qquad (1)$$

where $\Gamma_{\gamma,p}(i)$ are the γ and proton widths, respectively, of the ith state of ^{37}K, $\eta_i = \Gamma_p(i)/\Gamma_\gamma(i)$, and $\mathcal{C}_i$ is the constant of proportionality for this state. If the proton emission is actually much weaker than the γ emission from a state, Eq. (1) indicates that the GT strength will be much larger than one would expect from the β-delayed proton emission. The enhancement factor for the transition would be $(1+1/\eta_i)$.

The data treatment by García *et al.* is the $\eta_i \gg 1$ case. As $\eta_i \longrightarrow 0$, the amount of GT strength inferred in peak d by the decay experiment increases. At the same time, the amount of strength inferred in peak b will decrease, as the total decay rate of ^{37}Ca is saturated by the new strength inferred in peak d. Figure 2 shows how the strength inferred in peaks b and d varies with the logarithm of the ratio of $\Gamma_p(3.239)/\Gamma_\gamma(3.239)$. To calculate these strengths, we have enhanced $B_{3.239}$(GT) by the factor $(1+1/\eta)$ discussed above and used the total decay rate of ^{37}Ca to compute $B_{1.371}$(GT), in exactly the same manner as our previous paper [4].

The trends in $B_{1.371}$(GT) and $B_{3.239}$(GT) just described are evident in Fig. 2. For $\Gamma_p/\Gamma_\gamma \leq 1/73.5$, the GT strength in the 3.239-MeV state is so large that the strength in the 1.371-MeV state must be negative in order to satisfy the total decay rate. Thus, there is a lower limit to how weak the proton emission from the 3.239-MeV state can be:

$$\frac{\Gamma_p(3.239)}{\Gamma_\gamma(3.239)} \geq \frac{1}{73.5}\ . \qquad (2)$$

For each state, there is uncertainty in the amount of strength due to lack of knowledge concerning the Coulomb mixing of the Fermi strength from the IAS to a neighboring state 40 keV away. This difficulty has been discussed in detail by García [12] and in a previous paper [4]. Here, we have plotted the two extremes: no mixing and full mixing. The true answer will lie somewhere within this range.

As noted earlier, the Rapaport *et al.* experiment was able to set only an upper limit on the GT strength in the 1.410-MeV state of ^{37}Ar: $B_{1.410}(\mathrm{GT}) \leq 1.406\times10^{-2}$.

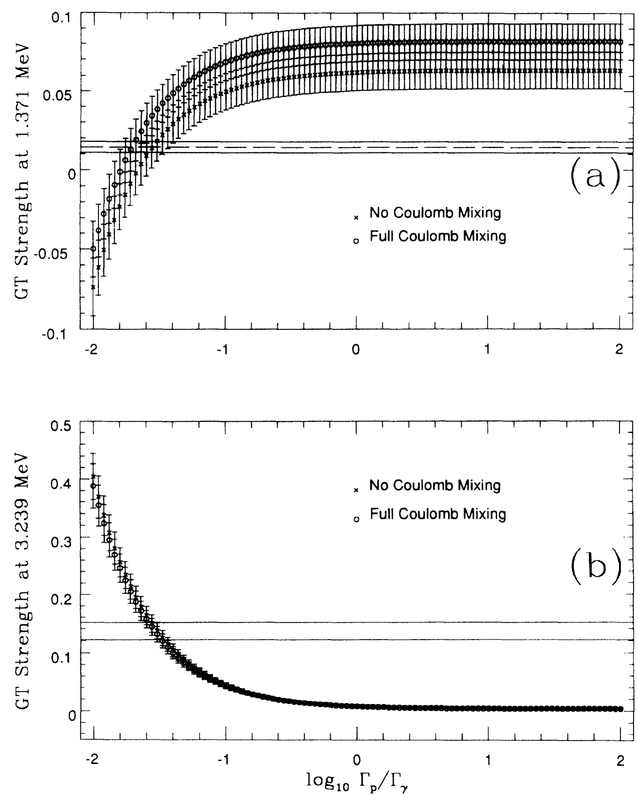

FIG. 2. GT strength in the (a) 1.371-MeV and (b) 3.239-MeV states of ^{37}K as a function of Γ_p/Γ_γ, inferred using the García data. The two extremes of Coulomb mixing of the Fermi strength are plotted in each case. The dashed horizontal line in (a) is the upper limit of the GT strength set by the first (p,n) experiment. The solid horizontal lines in (a) and (b) show the limits for the amount of GT strength in the (a) 1.410-MeV and (b) 3.171-MeV states of ^{37}Ar, inferred from the new (p,n) data.

This limit is plotted in Fig. 2(a). For $\Gamma_p/\Gamma_\gamma \geq 1/26.9$, the amount of GT strength inferred from the García *et al.* data exceeds this upper limit. This value thus provides an upper limit for Γ_p/Γ_γ. If we put the two limits together, we obtain the following range for the ratio:

$$\frac{1}{73.5} \leq \frac{\Gamma_p(3.239)}{\Gamma_\gamma(3.239)} \leq \frac{1}{26.9}\ . \qquad (3)$$

It is possible to tighten this limit by using the new (p,n) data. Because these states are at fairly low excitation energy, it is possible to obtain a fairly good estimate of the strength in each state simply by comparing the integral under each peak with the integral under the ground-state peak, a. The GT strength in this state is known [13] to be $B_{0.000}(\mathrm{GT})=(4.83\pm0.14)\times10^{-2}$. Table II lists the resulting estimates for the strengths in peaks b, c, and d. For all four peaks, a least-squares fit of the data (with a background subtracted) to a Gaussian was obtained, and the integral under the Gaussian is compared in order to estimate GT strength in each peak. The uncertainties in the strength come from the uncertainty in the ground-state strength, and the uncertainty

TABLE II. GT strengths estimated from a new (p,n) experiment. Peak labels correspond to those in Fig. 1. Energies are in MeV.

Peak	Energy	B(GT)
a	0.000	$(4.90\pm0.15)\times10^{-2}$
b	1.410	$(1.44\pm0.35)\times10^{-2}$
c	2.796	$(7.74\pm1.02)\times10^{-2}$
d	3.171	$(1.36\pm0.15)\times10^{-1}$

in the amplitude of each Gaussian fit.

For peak c, García *et al.* obtained $B_{2.750}(\mathrm{GT}) = (6.90 \pm 0.40) \times 10^{-2}$ and $(6.62 \pm 0.38) \times 10^{-2}$ for the unmixed and mixed cases, respectively. The estimate from the (p,n) data, $(7.74 \pm 1.02) \times 10^{-2}$, is consistent with these values. Figure 2 shows the ranges allowed for the GT strength in peaks b and d, from this (p,n) data as were listed in Table II. If one requires that the amount of GT strength measured by the experiments in these states be consistent, one obtains more limits for Γ_p/Γ_γ. For peak b the limit is

$$\frac{1}{64.7} \leq \frac{\Gamma_p(3.239)}{\Gamma_\gamma(3.239)} \leq \frac{1}{24.3} . \tag{4}$$

For peak d the limit is

$$\frac{1}{42.7} \leq \frac{\Gamma_p(3.239)}{\Gamma_\gamma(3.239)} \leq \frac{1}{26.6} . \tag{5}$$

All three limits can be combined into a single limit:

$$\frac{1}{42.7} \leq \frac{\Gamma_p(3.239)}{\Gamma_\gamma(3.239)} \leq \frac{1}{26.9} . \tag{6}$$

In making these comparisons between the GT strength in corresponding states of ^{37}K and ^{37}Ar, we have assumed that the distribution of strength is exactly the same, i.e., isospin invariance holds. However, comparison of the spectra of ^{37}K and ^{37}Ar in Table I indicate that there is some isospin asymmetry between the nuclei. Thus, the GT strength distributions may be slightly different. The fact that it is relatively easy to combine the experiments consistently may be an indication that these asymmetries are relatively small. There is nothing more which can be said at present experimentally. Ormand and Brown [14] examined this question to some extent using isospin-nonconserving Hamiltonians to fit the location of isobaric analog states of nuclei in several oscillator shells, including the *sd* shell. Their paper did not show calculations of spectra for ^{37}K and ^{37}Ar, but Table 4.1 of García's thesis quotes a private communication from Brown in which such an isospin-nonconserving Hamiltonian was used to compute these spectra.

In Table III, we have reproduced the relevant results from this table. It can be seen that the calculations predict little asymmetry for peaks a, b, c, but a much larger asymmetry in peak d. However, the calculated GT strengths listed do not agree very well with the strengths measured and discussed above. Also, it can be seen that the shifts in the energy levels are not very well predicted by the calculations either. Thus, it is not clear how accurate the calculations of the alterations in GT strength for these peaks will be. If we accept these calculations, the limits obtained using peak d are not trustworthy, since the strengths at 3.171 MeV in ^{37}Ar and 3.239 MeV in ^{37}K cannot be directly compared. However, the limits obtained from peak b will still be trustworthy. Thus, the lower limit in Γ_p/Γ_γ will be smaller than the factor of 1/42.1. From peak b, we have a lower limit of 1/64.7. Because of the uncertainty in how firm the tighter limit is, in what follows we will use both lower limits:

TABLE III. Isospin asymmetry between ^{37}Ar and ^{37}K. Comparison of calculation using an Ormand-Brown isospin-nonconserving Hamiltonian [14], as shown in García's thesis, with experimentally known shifts in energy levels between ^{37}Ar and ^{37}K. The first two columns identify the states, by their location in Fig. 1 and their spin parity. The next three columns show the theoretical calculations obtained using the Ormand-Brown isospin-nonconserving Hamiltonian [14]. $\Delta B(\mathrm{GT})/B(\mathrm{GT})=\{B(\mathrm{GT})(^{37}\mathrm{K})-B(\mathrm{GT})(^{37}\mathrm{Ar})\}/\{\frac{1}{2}[B(\mathrm{GT})(^{37}\mathrm{K})+B(\mathrm{GT})(^{37}\mathrm{Ar})]\}$. $\Delta E_x = E_x(^{37}\mathrm{K}) - E_x(^{37}\mathrm{Ar})$.

		Theory			Expt.
Peak	J^π	$B(\mathrm{GT})(^{37}\mathrm{K})$	$\Delta B(\mathrm{GT})/B(\mathrm{GT})$	ΔE_x	ΔE_x
a	$3/2^+$	0.062	+0.033	0.000	0.000
b	$1/2^+$	0.026	−0.038	+0.002	−0.039
c	$5/2^+$	0.188	−0.166	+0.046	−0.046
d	$5/2^+$	0.066	+0.514	−0.091	+0.068

$$\frac{1}{64.7}\left(\frac{1}{42.7}\right) \leq \frac{\Gamma_p(3.239)}{\Gamma_\gamma(3.239)} \leq \frac{1}{26.9} . \tag{7}$$

Recently Iliadis *et al.* [9] have measured the resonance strength for the 3.239-MeV state of ^{37}K, allowing them to determine $\Gamma_p/\Gamma_\gamma = 1/40$. This value is within our tighter limit, indicating that the GT strength for this state is fairly isospin invariant.

The calculations presented here have used a slightly different value for the amount of Gamow-Teller strength in the ^{37}Cl(g.s.) to ^{37}Ar(g.s.) transition than was used in our previous paper or in García *et al.* The value which García *et al.* cite is $B(\mathrm{GT}) = (4.83 \pm 0.14) \times 10^{-2}$, listing Brown and Wildenthal [13] as their source. Brown and Wildenthal [13] listed a $\log ft$ value of 5.106 ± 0.001 for the transition, citing the compilation of Endt and van der Leun [15]. But in this compilation, the $\log ft$ value is listed as 5.10 ± 0.02, resulting in $B_{0.000}(\mathrm{GT}) = (4.90 \pm 0.01) \times 10^{-2}$, which is approximately 2% larger than the García *et al.* value. In this paper we will use Endt and van der Leun's value, with a 3% uncertainty, to include isospin asymmetry uncertainties.

III. MIXING OF THE 5/2$^+$ DOUBLET AS A SOURCE OF THE VARIABLE PROTON EMISSION STRENGTH

As can be seen in Table II, both ^{37}Ar and ^{37}K exhibit a doublet of $5/2^+$ states at excitations of approximately

2.8 MeV and 3.2 MeV. These states correspond to peaks c and d, respectively, and are seen in the USD spectrum. However, it is not clear how accurately the USD wave functions for these states model the actual nuclear wave functions. It may be that some mixing of the theoretical states could more accurately represent the actual properties of these states and not be discernible from the spectrum. Such mixing could alter the proton and γ emission widths. Here we will examine such a possibility by mixing the theoretical states produced by the USD interaction. Let $|c\rangle$ represent the theoretical, unmixed, wave function for the state corresponding to peak d. The mixed states will be given by

$$|c'\rangle = \alpha|d\rangle + \beta|c\rangle , \tag{8}$$

$$|d'\rangle = \alpha|c\rangle - \beta|d\rangle , \tag{9}$$

where

$$\beta = (1-\alpha^2)^{1/2} , \tag{10}$$

for normalization. The energy of these mixed states is shown in Fig. 3(a). For small values of α, $|c'\rangle$ and $|d'\rangle$ have energies close to $|c\rangle$ and $|d\rangle$, respectively. As α approaches 1, the identity of the states is interchanged.

The width for the kth state of ^{37}K as observed by proton emission is given by

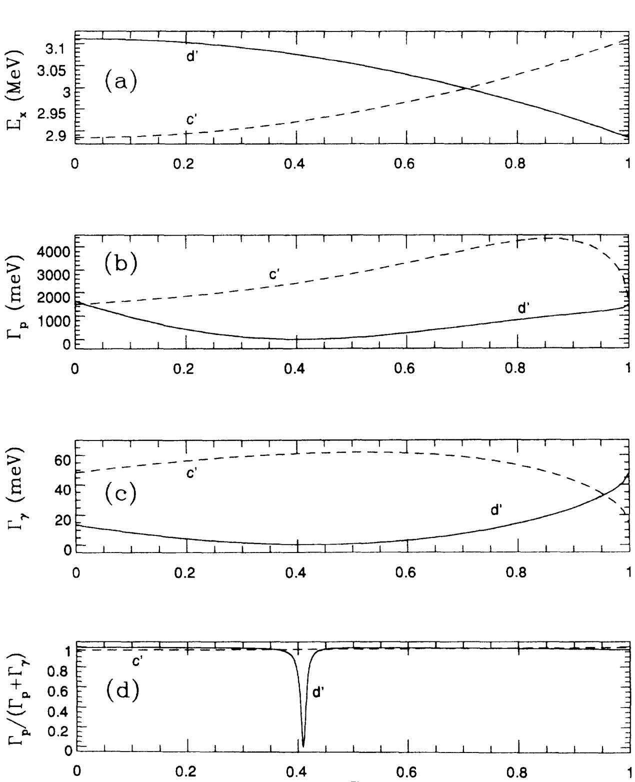

FIG. 3. The effect of mixing theoretical states corresponding to peaks c and d on (a) energy of states, (b) proton emission width, (c) $M1\gamma$ emission width, and (d) ratio of Γ_p to total theoretical width.

$$\Gamma_p(k) = 2P_k\gamma_k^2 = 2P_k\frac{3\hbar^2}{2\mu a^2}\theta_0^2 S_k , \tag{11}$$

where P_k is the proton penetrability, γ_k is the reduced width amplitude for proton emission, μ is the reduced mass of the system, a is the channel radius of the reaction given by $1.2(1^{1/3}+36^{1/3})$, the dimensionless reduced width amplitude, $\theta_0 \approx 0.6$, and S_k is the square of the proton spectroscopic amplitude of this state, here calculated using the CRUNCHER shell model code [16]. Figure 3(b) shows the proton emission widths of the mixed states. It can be seen that for α near 0.41, the proton width of $|d'\rangle$ becomes very small. In fact, for $\alpha \approx 0.408$, the proton width becomes zero. For $|c'\rangle$, the proton emission width remains large. These are exactly the effects seen in experiment. It is also interesting to note that the energies of the two states are not significantly perturbed for this amount of mixing. Figure 3(d) again shows the decrease in Γ_p, as a fraction of the total theoretical width.

In our estimates of widths due to electromagnetic decay, we will only consider magnetic dipole ($M1$) transitions. $E1$ and $E2$ transitions are neglected because these are relatively weak, which is consistent with the expectation that the giant dipole resonance is likely to remove $E1$ strength from low-lying states such as the ones of interest here. The $M1$ γ emission width can be written as

$$\Gamma_\gamma(k) = 0.924E_\gamma^3 B(M1,k) , \tag{12}$$

where E_γ is the energy of the emitted γ ray, $B(M1,k) \equiv \langle J'||M1||J_k\rangle^2/(2J_k+1)$, using the Brink convention for the reduced matrix element, and Γ_γ is in units of meV. Figure 3(c) shows the γ widths for $|c'\rangle$ and $|d'\rangle$. The γ emission width for $|d'\rangle$ exhibits a minimum near $\alpha \approx 0.42$ of 0.28 meV. Table IV summarizes what is known about the γ widths of peaks c and d in ^{37}Ar and ^{37}K. It can be seen that the agreement between theory and experiment is poor, especially for the mixed states of peak d. For peak d, the experimental γ widths lie between the $\alpha = 0$ and $\alpha = 0.408$ cases, indicating that perhaps some mixing of the theoretical states is preferable to none. For peak c, in ^{37}Ar both theoretical widths are too large, but the unmixed case is closer to the measured width. In ^{37}K, both theoretical widths for c are too small, but the mixed case is closer to the experimental limit. It would seem that the details of the wave functions being examined

TABLE IV. Comparison of experimental measurements of $M1$ widths with unmixed and mixed theoretical calculations in ^{37}Ar and ^{37}K. All widths are given in units of meV.

		$\Gamma_\gamma(c)$	$\Gamma_\gamma(d)$
	Expt.	32[a]	8.2[a]
^{37}Ar	$\alpha = 0$	38.4	10.2
	$\alpha = 0.408$	48.1	0.3
	Expt.	>69[b]	> 0.2[a]
^{37}K	$\alpha = 0$	48.4	13.5
	$\alpha = 0.408$	61.2	0.3

[a]These values are taken from the paper of Endt [22].
[b]This value is taken from the study of Iliadis *et al.* [23].

here are too detailed for the nature of fit characterizing the USD interaction. The possibility of contribution from the forbidden $M1$ body operator [17] was investigated, but it does not explain the discrepancies.

We have seen that mixing the USD wave functions for $|c\rangle$ and $|d\rangle$ can explain the suppression of proton emission seen in the actual $|d\rangle$ state. Possible explanations for why this mixing was necessary are all related to inadequacies of the USD interaction in one way or another. For example, it is easy to use first-order perturbation theory to estimate that, for the mixing corresponding to $\alpha = 0.41$, a mixing matrix element of $\langle c|V'|d\rangle \approx -150$ to -200 keV is necessary. If we now use the USD wave functions with a simple one-body Coulomb potential [18] for V', we estimate $\langle c|V'|d\rangle \approx -20$ to -180 keV. Thus we see that a possible contributing factor to the mixing of $|c\rangle$ and $|d\rangle$ is an isospin-violating Coulomb interaction. Also, the USD interaction was limited to 0-$\hbar\omega$ excitations and it is possible that configuration mixing induced by multi-$\hbar\omega$ excitations out of the s-d shell could contribute to the mixing of these states. This is consistent with what was suggested above with regard to the γ widths: perhaps all of these effects require more detailed accuracy in wave functions than provided by the USD fits. Neither of these possibilities is easily testable.

IV. NEUTRINO CROSS SECTIONS FOR ^{37}CL

As was mentioned above, one reason that the GT strength in this system is of interest is because of the ^{37}Cl solar neutrino detector [19]. Because of our better understanding of the GT strength in the $A = 37$ system, we are now able to make a more accurate estimate of the solar neutrino cross section. Using the techniques of Bahcall [20] as outlined in our previous paper, we have computed the ^{8}B solar neutrino cross section of ^{37}Cl as a function of Γ_p/Γ_γ in Fig. 4(a).

It can be seen that as Γ_p/Γ_γ increases, the cross section decreases. This decrease is a result of the factor of 100 decrease in the strength postulated in the 3.171-MeV state. Although the strength in the 1.410-MeV state increases as Γ_p/Γ_γ increases, it does not make up for the decrease in the strength in the former state. Figure 2 shows that the changes in $B_{\rm GT}(3239)$ are much greater than those of $B_{\rm GT}(1371)$, as a result of the ^{37}K decay phase space. It can also be seen that the case of no mixing of the Fermi strength into its neighboring state has a stronger cross section than the case of full mixing. This difference can be understood when one remembers that the amount of mixing sets the overall normalization of the β^+ decay experiment. The case of no mixing implies that all of the Fermi strength is in the IAS and hence the strength in all other states is as large as possible, since all other states were measured relative to the IAS.

If we use the limits on Γ_p/Γ_γ which were obtained in Sec. II, and the fact that the cross section does not change much over this range, it is possible to set limits on the neutrino cross section. The vertical lines in Fig. 4(a) show these limits on Γ_p/Γ_γ. The actual neutrino cross section is bounded by these lines and between the two extremes of Coulomb mixing. Thus the neutrino cross section is given by

$$\sigma_{^8\rm B} = 1.11 \pm 0.08 \; (1.10 \pm 0.06) \times 10^{-42}\ \rm cm^2 \ , \qquad (13)$$

where the uncertainties are 3σ and the quantity in parentheses is the cross section obtained using the tighter lower limit from peak d. Also shown in Fig. 4(a) is the Bahcall and Holstein estimate [21] of $1.06 \pm 0.1 \times 10^{-42}$ cm^2 for the ^{8}B neutrino cross section. This value is less than, but well within the uncertainties of, our estimates using the newest data. The main difference is that the new experiments have allowed us to reduce the uncertainty in the cross section. Bahcall's 3σ uncertainty was approximately 10%, whereas our uncertainty is 6% to 8%. Our estimates for the cross section make the solar neutrino problem slightly worse.

In Fig. 4(b) we have computed the ^{37}Cl cross section for supernova neutrinos, assuming a Fermi-Dirac distribution with zero chemical potential and a temperature of

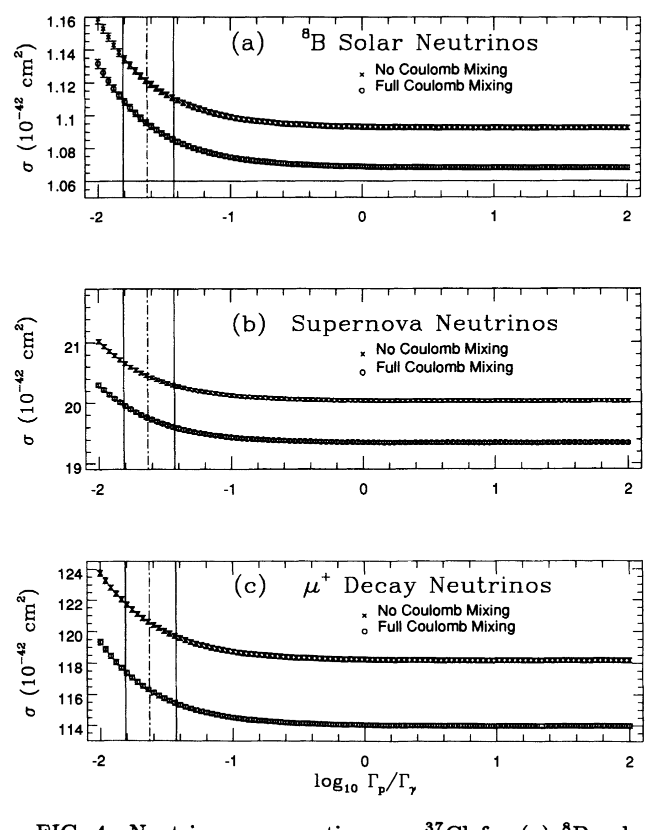

FIG. 4. Neutrino cross sections on ^{37}Cl for (a) ^{8}B solar neutrinos, (b) supernova neutrinos, and (c) neutrinos from μ^+ decay. The two extreme cases for Coulomb mixing of the Fermi strength into a state near the IAS are shown. The actual cross section will lie in the region bounded by these curves. The limits obtained for Γ_p/Γ_γ in Sec. II are the vertical lines in each plot. The dash-dotted vertical line shows the tighter lower limit for Γ_p/Γ_γ, obtained from peak d in Fig. 1. The value of $\Gamma_p/\Gamma_\gamma = 1/40$, obtained by Iliadis *et al.* [9], lies just to the right of this line. The horizontal line in (a) is the best estimate given by Bahcall and Holstein [21].

4.5 MeV, as was used in our previous paper. The same trends discussed above for ^{8}B solar neutrinos are evident here. The cross section is roughly 20 times larger than in the solar case because the supernova neutrino spectrum is more energetic. The limits which we obtain the supernova cross section are

$$\sigma_{\rm SN} = 20.14 \pm 1.69\ (20.03 \pm 1.36) \times 10^{-42}\ {\rm cm}^2\ , \quad (14)$$

where the uncertainties are again 3σ.

In Fig. 4(c) we have computed the ^{37}Cl cross section for μ^+ decay. The same trends with regard to Γ_p/Γ_γ and Coulomb mixing of the Fermi strength are evident. The cross section is roughly 100 times larger than the solar neutrino cross section because the neutrino spectrum from μ^+ decay has an even higher-energy distribution than the supernova spectrum. The limits we obtain for the cross section are

$$\sigma_{\mu^+} = 118.6 \pm 10.0\ (118.0 \pm 8.1) \times 10^{-42}\ {\rm cm}^2\ , \quad (15)$$

where the uncertainties are again 3σ. García, in his thesis, estimates a value of $106 \pm 3 \times 10^{-42}$ cm^2. His value is slightly weaker than our estimate because of our amplification of the strength in the 3.171-MeV state.

The uncertainties in our neutrino cross sections were partially the result of uncertainties in the value of Γ_p/Γ_γ. Using the value determined by Iliadis *et al.* [9], our cross sections become

$$\sigma_{^8{\rm B}} = 1.11 \pm 0.04 \times 10^{-42}\ {\rm cm}^2\ , \quad (16)$$

$$\sigma_{\rm SN} = 20.08 \pm 1.13 \times 10^{-42}\ {\rm cm}^2\ , \quad (17)$$

$$\sigma_{\mu^+} = 118.3 \pm 6.9 \times 10^{-42}\ {\rm cm}^2\ . \quad (18)$$

The uncertainty in the ^{8}B cross section is now a factor of 2 smaller than Bahcall's conservative estimate of the uncertainty. The uncertainties are now just the result of experimental uncertainties and the uncertainty over the amount of Coulomb mixing of Fermi strength near the IAS. Even if the latter uncertainty could be resolved, questions on isospin invariance would prevent us from making more precise estimates of cross sections.

Our estimates for the cross section to neutrinos from muon decay are approximately 20% larger than Bahcall's estimate [20] of 72×10^{-42} cm^2, which used the nuclear physics discussed in Bahcall and Holstein [21]. The reason for this difference is the large amount of strength near 7 MeV which was seen in both (p,n) experiments [1,2] and by García *et al.* [3]. No previous ^{37}Ca β^+ decay experiment was able to measure strength reliably at such high energies because of the unfavorable phase space. García *et al.*'s experiment has confirmed the existence of this strength, and it accounts for the enhancement of the cross sections for μ^+ neutrinos and the ^{8}B solar neutrinos, relative to the Bahcall estimates. We have not included the effects of forbidden transitions here. Such transitions, estimated in Bahcall and Holstein [21] and Bahcall [20], may enhance neutrino cross sections by 4% for ^{8}B neutrinos and 6% for μ^+ neutrinos.

TABLE V. Capture rates for a ^{37}Cl detector using the standard solar model. 1 SNU is 10^{-36} neutrino captures per second per atom. The neutrino sources are as defined in Bahcall [20]; the neutrino fluxes and their uncertainties used here are as given in Table 6.5 of Bahcall [20].

Neutrino source	Capture rate (SNU)
pp	0.0
pep	2.18(−1)±2.3(−2)
hep	3.49(−2)±1.9(−3)
^{7}Be	1.13±2.0(−1)
^{8}B	6.42±2.39
^{13}N	9.21(−2)±4.61(−2)
^{15}O	3.16(−1)±1.83(−1)
^{17}F	3.29(−3)±1.52(−3)
Total	8.21±2.40

For all three neutrino cross sections computed here, it can be seen that the largest remaining contributor to uncertainties is the amount of Coulomb mixing of Fermi strength in the IAS. In order to resolve this uncertainty, it would be necessary to obtain an absolute measurement of the total amount of strength in the IAS. The absolute strength in the IAS varies by only approximately 5% between the extremes of no mixing and total mixing, so that an extremely accurate measurement would be required. It appears unlikely that such a measurement will be possible in the near future. Even if such a measurement were possible, the uncertainties become limited by the contributions of forbidden transitions, and how accurately isospin invariance is satisfied. It thus appears that the present experiments have constrained the ^{37}Cl neutrino cross sections as accurately as will be possible for the near future.

Moreover, there is little to be gained from further precision in the cross section. Table V lists the total neutrino capture rate for the ^{37}Cl detector using the strength distribution obtained here and the neutrino fluxes as given in Table 6.5 of Bahcall [20]. This table is to be compared with Table 10.2 of Bahcall [20]. Although we have reduced the uncertainty of the most important cross section by a factor of 2, the uncertainty in the capture rate has only dropped to 29% from Bahcall's value of 33%. The reason is the large uncertainty (37%) in the ^{8}B solar neutrino flux.

V. CONCLUSIONS

We have seen that all of the differences in the low-lying Gamow-Teller strength between the García *et al.* [3] ^{37}Ca$(\beta^+)^{37}$K experiment and the ^{37}Cl$(p,n)^{37}$Ar experiments of Rapaport *et al.* [1] and Wells *et al.* [2] can be understood by taking into account a suppressed proton emission width for the 3.239-MeV state of ^{37}K. Requiring that the experiments be consistent, and allowing the proton emission width of this state to vary, we have been able to set limits on the ratio of the proton width to the γ emission width. The tightest lower limit came from assuming isospin invariance for GT strength in the 3.239-MeV state in ^{37}K and the 3.171 MeV in ^{37}Ar. It was found that the recent measurement of Iliadis *et al.* [9] was within the tightest limits.

Using the newly calibrated GT strength distribution, we have been able to compute neutrino cross sections on ^{37}Cl from solar neutrinos, supernova neutrinos, and μ^+ decay neutrinos, and they are given in Eqs. (16)–(18), respectively. The 3σ uncertainties for all of these cross sections are smaller than the Bahcall results because of improved knowledge of the GT strength distribution from the experiments. Our cross sections are larger than the Bahcall estimates because of GT strength in the vicinity of 7.5 MeV which was not in previous decay experiments and thus not known to Bahcall. The difference becomes larger as the neutrino spectrum becomes harder. For ^{8}B neutrinos, the difference is only 5%, while for μ^+ decay neutrinos, the difference is 20%. The largest source of uncertainty in the ^{8}B cross section is now the ^{8}B solar neutrino flux.

At any rate, the determination of Γ_p/Γ_γ by Iliadis *et al.* confirms that the amount of low-lying GT strength measured in (p,n) experiments and the β^+ decay experiment are consistent. Future analysis of the newest (p,n) experiment will determine how well the measurements agree for higher daughter excitation energies. At present, the ability of (p,n) experiments to measure GT strength appears to have been vindicated.

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