Superdeformed and hyperdeformed states in ^{56}Ni

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Resonances in ^{12}C+^{12}C and ^{24}Mg+^{24}Mg scattering have been associated with hyperdeformed configurations in the harmonic oscillator shell model and the alpha-cluster model. Resonances in ^{28}Si+^{28}Si scattering do not seem to fit into this picture so well. We present alpha-cluster model calculations for superdeformed and hyperdeformed states in ^{56}Ni. One configuration identified in our calculations corresponds to a 2:1 harmonic oscillator configuration. The others are more complex; however, none gives a totally satisfactory explanation of the ^{28}Si+^{28}Si resonances.

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Intermediate structure resonances have been found in the ^{28}Si(^{28}Si,^{28}Si*)^{28}Si* reaction [1]. These resonances are narrow peaks ($\Gamma \simeq 200$ keV) superimposed on broad structures at excitation energies of 60 to 70 MeV and are correlated in the excitation functions for a number of final channels [2]. Correlation analysis and level density comparisons exclude the possibility that these peaks arise from statistical fluctuations or that they are normal isolated compound-nucleus states. The angular dependence of the large-angle cross sections are well fitted by the squares of single Legendre polynomials of different orders ranging from $L = 36\hbar$ up to $L = 42\hbar$ [3]. In a later experiment, Dichter *et al.* measured the excitation function of the ^{40}Ca(^{16}O,^{28}Si*)^{28}Si* reaction [4] over the same range of excitation energies as in Ref. [1]. Again, intermediate width resonances ($\Gamma \simeq 200$ keV) were observed, and they were found to be strongly correlated with the peaks found in the excitation function of the ^{28}Si(^{28}Si,^{28}Si*)^{28}Si* reaction. As pointed out by Dichter *et al.* [4], there should be a mechanism which accounts for the aforementioned observations.

In spite of decades of effort, which includes both extensive experimental and theoretical endeavours, the resonance phenomena exhibited in heavy-ion reactions are only just beginning to be understood [5]. The theoretical interpretations of such heavy-ion resonances, so far, can be generally catalogued into two distinct classes: (1) dynamical models [6–8] in which these resonances are viewed as being a consequence of both weak absorption and the existence of pockets in the interaction potential of the two constituent ions, and (2) compound nucleus models [9–12] in which the shell stabilization of highly deformed compound nuclei may result in shape isomeric states having a large overlap with heavy-ion decay channels.

Several attempts have been made to explain the resonances observed in the ^{28}Si+^{28}Si and ^{40}Ca+^{16}O systems. Dynamical models, which successfully explain many resonances, fail for the ^{40}Ca+^{16}O system in that they can neither account for the experimental widths, nor can they reproduce the quasibound resonances for $30\hbar \leq L \leq 40\hbar$ in this system as shown, for example, in the calculations by Langanke *et al.* [13]. This is no surprise if one bears in mind that the intermediate width structures found in both the ^{28}Si+^{28}Si and ^{40}Ca+^{16}O systems are strongly correlated. The strong correlation suggests that the resonances proceed through the ^{56}Ni compound nucleus, and they might be the manifestation of shape isomeric superdeformed or hyperdeformed states in secondary minima in the potential energy surfaces. A few theoretical calculations [13–16] within the framework of the deformed shell model do indeed explain some features of these resonances. In particular, in the cranked deformed harmonic-oscillator model calculations of Bengtsson *et al.* [16], shell corrections for $L = 40\hbar$ show the existence of a potential-energy minimum which corresponds to a hyperdeformed, prolate shape with a 3:1 major to minor axis ratio. This result strongly suggests that intermediate width resonance structures found in the ^{28}Si+^{28}Si and ^{40}Ca+^{16}O systems arise from such a 3:1 hyperdeformed state. However, the problem as a whole is still far from being fully solved. For example, the experimentally measured resonant energies of the ^{28}Si+^{28}Si system lie some 15 MeV above the bottom of the calculated hyperdeformed potential-energy minimum, and besides the *barrier* is just 5 MeV. Here we present cranked Bloch-Brink (BB) alpha-cluster model [17–19] calculations for three configurations: one with a 2:1 major to minor axis ratio, a triaxially deformed configuration, and one with approximately a 3:1 major to minor axis ratio. We attempt to understand the experimentally observed resonances in terms of these configurations.

The cranked BB calculations have been performed as described in detail in Ref. [20]. However, unlike the work on ^{24}Mg, we did not make the exhaustive grid search which is usually employed to identify all the possible stable cluster configurations. Rather, we have concentrated on searching for those configurations which correspond to large deformations. Since we are interested in configurations corresponding to specific heavy-ion resonances, we start the alpha-cluster minimization calculations with the position coordinates corresponding to two lighter nuclei separated by some distance, so that the two densities just overlap. For example, for the triaxial alpha-cluster configuration in ^{56}Ni, we put two oblate ^{28}Si configu-

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rations together, the two symmetry axes being parallel to each other and perpendicular to the separation position vector. The alpha-cluster position parameters are thus merged into one set for ^{56}Ni, defined by the distance between the two centers of mass of the ^{28}Si nuclei. The search then proceeded through two steps: firstly, the binding energy was minimized by variation of the three parameters μ_x, μ_y, and μ_z, which are defined elsewhere [20], along with the oscillator frequency denoted as ω_0. Essentially, this means that the internal geometry of the clusters was fixed, while the x, y, and z length scales were varied. Secondly, all the parameters were freed to allow a completely unconstrained variational minimization of the binding energy. Using this method we found three configurations which could be cranked above $L = 34\hbar$ in ^{56}Ni. The first configuration was found starting with the alpha-cluster positions for ^{40}Ca+^{16}O [21], the second, as described above, from two oblate ^{28}Si nuclei, and the third from two prolate ^{28}Si nuclei end to end. Figure 1 shows the density profile contours for these configurations. We also obtained a few other configurations, however, all of these became unstable above $L = 25\hbar$ in the cranking calculations.

To obtain the moments of inertia, we tracked these local minima as a function of the rotational frequency in the cranking model. All the calculations were performed using the Brink-Boeker B1 nucleon-nucleon force [22] because of its simplicity and ability to reproduce rms radii [23]. As already noted previously [24], the B1 force does not predict the absolute values of binding energies in agreement with experiment. Here, for such a heavy nucleus where the applicability of the alpha-cluster model is questionable, we abandoned any attempt to predict absolute energies. However, for the record, the binding energies obtained were 450.44 MeV, 410.87 MeV, and

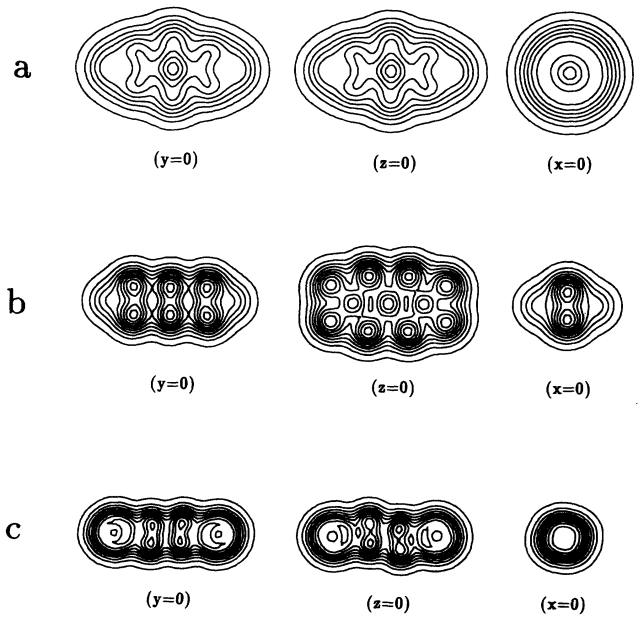

FIG. 1. Density profile contours for (a) 2:1, (b) triaxial, and (c) 3:1 alpha-cluster configurations in ^{56}Ni, from the Bloch-Brink alpha-cluster model calculations.

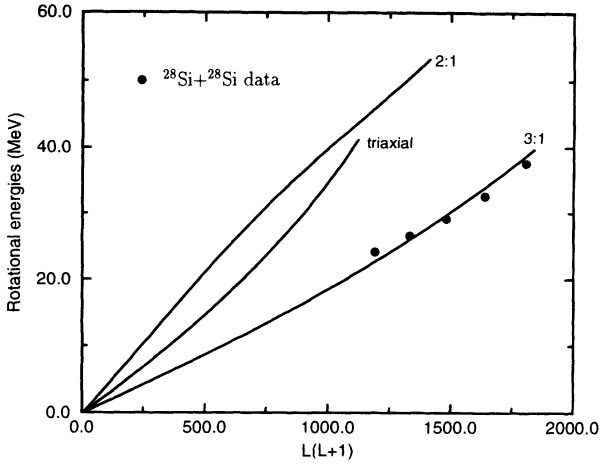

FIG. 2. Rotational energies for the 2:1, triaxial, and 3:1 alpha-cluster configurations in ^{56}Ni, as a function of $L(L+1)$ from the cranked cluster model. We only show the rotational energies obtained by cranking about the z axis for which all the configurations have the largest moments of inertia. For the purposes of display, all the bandheads are shifted to be at the origin. Also shown in the figure are the energies of experimental resonances offset to align them with the 3:1 configuration.

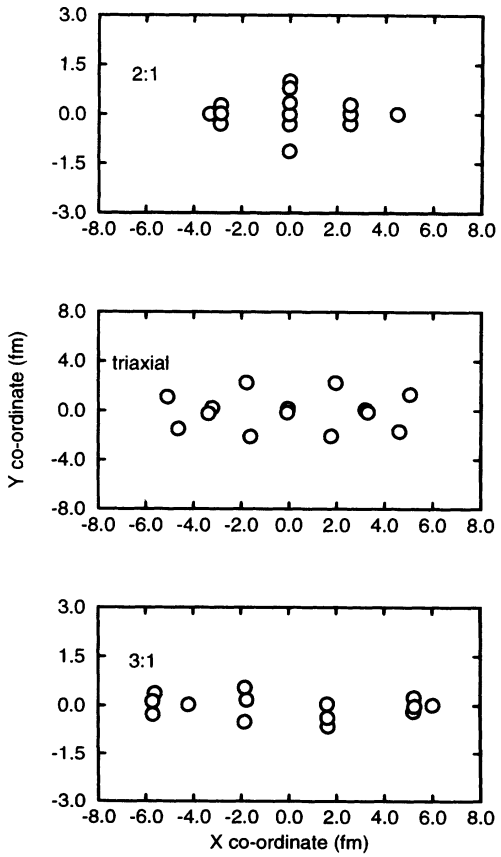

FIG. 3. Geometric arrangement of alpha clusters for the 2:1, triaxial, and 3:1 configurations in ^{56}Ni, obtained using the Bloch-Brink alpha-cluster model.

TABLE I. Shell model configurations for the 2:1, triaxial, and 3:1 alpha-cluster configurations in ^{56}Ni.

Bandhead	Shell model configurations
2:1	$([000][100][010][001][200][020][002][110][101][011][300][210][201][400])^4$
Triaxial	$([000][100][010][001][200][020][110][101][300][210][201][120][400][310])^4$
3:1	$([000][100][010][001][200][110][101][300][210][201][400][310][301][500])^4$

400.84 MeV, respectively, for the 2:1, triaxial, and 3:1 configurations. Figure 2 shows the calculated rotational energies relative to the bandheads for the three configurations. We have also plotted the energies of the resonances observed in the ^{28}Si+^{28}Si and ^{40}Ca+^{16}O systems for comparison. The cranked calculations for the three bands terminate at different spins.

It is the configuration constructed from two prolate ^{28}Si nuclei that appears to have the correct moment of inertia to explain the experimental results, and this is the only configuration which is stable up to the spins observed experimentally. Indeed, the structure obtained from two oblate ^{28}Si nuclei is the least stable at high spin. The ^{40}Ca-^{16}O structure, which, by the way, can also be formed from an oblate ^{28}Si and a prolate ^{28}Si, is more stable, but the moment of inertia does not agree with the experimental data. Hence the alpha-cluster model calculations which have been successful for the ^{12}C+^{12}C and ^{24}Mg+^{24}Mg resonances, do not seem to provide a consistent interpretation of the ^{28}Si+^{28}Si resonances, unless one invokes mixing of the oblate and prolate states in ^{28}Si or some other mixing of harmonic oscillator orbitals. However, we might then expect the prolate states in ^{28}Si to be strongly excited in the reaction, whereas there is no clear evidence that they are.

The Harvey prescription has previously been used successfully to interpret these and other heavy-ion resonances [12,25]. This prescription is related to the shell model limit of the alpha-cluster model [26]. The shell model configurations obtained from our cluster calculations are shown in Table I. These were obtained by taking the shell model limit as described elsewhere [26]. Applying the Harvey prescription to the shell model configurations in Table I it can be seen that the 2:1 state can break up into ^{40}Ca+^{16}O, while the 3:1 state can break up into two prolate ^{28}Si nuclei end to end, thus, for these two configurations, the shell model structure has been preserved during the minimization. For the triaxial configuration, this is not the case and the shell model configuration in Table I cannot be obtained from two oblate ^{28}Si nuclei using the Harvey prescription. Only the ^{40}Ca+^{16}O alpha-cluster configuration corresponds to an axially symmetric superdeformed oscillator configuration (2:1, in fact). The third alpha-cluster configuration is related to the 3:1 ^{16}O-^{16}O-^{16}O structure in ^{48}Cr [24], viz. it can be obtained by adding two alpha clusters to the ^{16}O-^{16}O-^{16}O structure of Ref. [24]. The alpha-cluster positions are illustrated schematically in Fig. 3 for all three configurations. It can be seen that the 2:1 and 3:1 configurations show mass asymmetry, which is known to be important in this region for both 2:1 and 3:1 configurations [27].

In conclusion, the alpha-cluster model does not seem to account for ^{28}Si+^{28}Si resonances in a totally satisfactory way. The alpha-cluster configuration with the correct moment of inertia cannot be formed diabatically from two oblate ^{28}Si nuclei. It may be that the neglect of spin-orbit effects is a problem for such a heavy system, or that the reaction mechanism may be more complex than we have assumed. However, we have found a configuration which predicts the moment of inertia correctly, and this is related to the 3:1 configuration in ^{48}Cr, which has been proposed to explain the resonances in ^{24}Mg+^{24}Mg scattering.

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