Low-lying surface vibrations in the pair-hopping model

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The pair-hopping model of large amplitude motion is applied to the calculation of the frequency of low-lying quadrupole and octupole vibrations in atomic nuclei. Theory provides an overall account of the experimental findings.

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The lowest excited states of even-even nuclei display, with very few exceptions, quadrupole and octupole character (cf., e.g., Ref. [1]). Although these states carry a small fraction (5–10%) of the energy weighted sum rule, the associated transition probabilities are much larger than that of single-particle transitions. Furthermore, they are excited with large cross sections by projectiles which are absorbed at the nuclear surface. They are known as collective surface vibrations, and are intimately connected with the plastic behavior of the atomic nucleus, as manifested in static deformations and in fission and exotic decay processes. The superfluid tunneling model of large amplitude nuclear motion provides a quantitative description of these processes (cf. Refs. [2,3] and references therein). In the present paper we shall apply it to the calculation of the frequency of both quadrupole and octupole low-lying surface vibrations.

The Hamiltonian of the model is

$$\left(-\frac{\hbar^2}{2D}\frac{d^2}{d\xi^2}+V(\xi)\right)\psi_n(\xi)=E_n\psi_n(\xi), \tag{1}$$

where ξ is a deformation variable. In the present case this variable will be identified with the quadrupole and actupole parameters β_2 and β_3.

The energies of adiabatic deformation of the nucleus is schematically depicted in Fig. 1. As a function of the deformation, the degeneracy of single-particle levels is lifted except for the Kramers degeneracy connected to time reversal, leading to orbitals which are twofold degenerate. At each crossing, pairs of particles in time-reversal states jump from occupied to empty levels under the influence of the residual nuclear interaction. The system thus evolves from one Hartree-Fock configuration to the next. This is the essence of the pair-hopping model to describe plastic or adiabatic motion of the atomic nucleus.

To make the model a working tool one needs to be able to calculate the inertia and the potential energy function. A fair approximation to this function is the harmonic model

$$V(\beta_2)=\tfrac{1}{2}C_\lambda\beta_\lambda^2. \tag{2}$$

The index λ labels the multipolarity of the deformation. The restoring force can be calculated making use of the liquid drop model. This is because in a motion where the surface fluctuates with a frequency of the order of 10^{21} s^{-1}, the detailed motion of the nucleons associated with frequencies almost 2 orders of magnitude larger must be quite irrelevant. Consequently, the surface tension γ ($\gamma \approx 0.95$ MeV fm^{-2}) is sufficient to characterize the shape of the system, and the restoring force parameter can be written as [1]

$$C_\lambda=\gamma(\lambda-1)(\lambda+2)R_0^2-\frac{3}{2\pi}\frac{\lambda-1}{2\lambda+1}\frac{e^2Z^2}{R_c}. \tag{3}$$

The two radii entering in the expression are the nuclear and Coulomb radii, $R_0 = 1.2A^{1/3}$ and $R_c = 1.25A^{1/3}$, respectively, A being the mass number. The quantity Z indicates the proton number of the system.

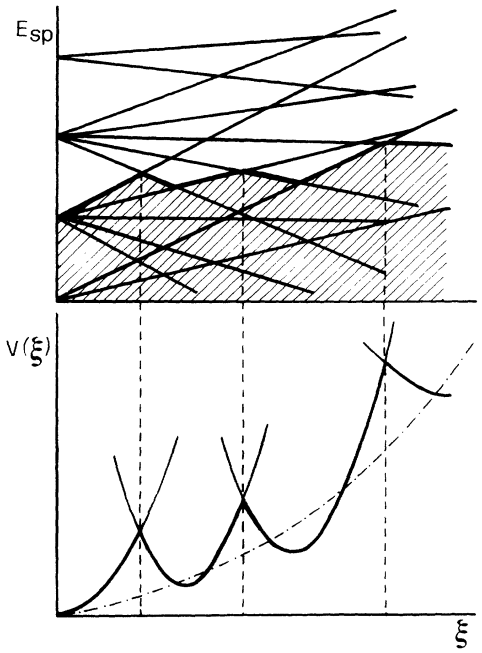

FIG. 1. Schematic picture of Hartree-Fock states as a function of the deformation, as well as the associated single-particle levels.

In what follows we shall use the approximate relation $Z \approx A/2.4$ and we shall neglect the difference between nuclear and Coulomb radii. In this way we obtain

$$C_\lambda = 1.4(\lambda-1)(\lambda+2)(1-0.002A)A^{2/3}\ \text{MeV}$$

$$(\lambda = 2,3). \quad (4)$$

The inertia associated with the motion is more difficult to obtain. It is customary to derive its expression from the comparison of a matrix representation of Eq. (1) and the associated matrix of a microscopic description in terms of Hartree-Fock configurations. Assuming that the two-body interaction v is state independent and acts only between nearest neighbors one obtains

$$\frac{D_\lambda}{\hbar_2} = -\frac{1}{2v}\left(\frac{dn}{d\beta_\lambda}\right)^2, \quad (5)$$

where $dn/d\beta_\lambda$ is the density of level crossings per unit deformation energy. The structure of this relation is quite plausible. The inertia is larger the larger the number of particles that have to be moved around in the motion. On the other hand, the larger the interaction, the smaller is the inertia, because it is easier to make a pair of particles jump at a crossing. To proceed further one needs an estimate of the matrix element v. A natural choice for the residual interaction which induces pairs of particles in time-reversal states to jump from an occupied to an empty level is the pairing interaction.

Most medium-heavy nuclei are superfluid in their ground state. Consequently, the jumping process can be viewed as a sequential "pickup" of a pair of nucleons from the occupied levels, and the "stripping" onto an empty level. Both processes are described by the matrix element

$$\langle \text{BCS}|P^+|\text{BCS}\rangle = \Delta/G, \quad (6)$$

where $|\text{BCS}\rangle$ is the wave function describing the condensate of protons and neutrons, $P^+ = \sum_{\nu>0} a_\nu^\dagger a_{\bar{\nu}}^\dagger$ is the two-particle transfer operator, Δ is the pairing gap, and G is the pairing coupling constant. The matrix element v can thus be written

$$v = -2\left[\frac{G}{4}\left(\frac{\Delta}{G}\right)^2\right] = -\frac{\Delta^2}{2G}. \quad (7)$$

The factor $\frac{1}{4}$ is associated with the fact that of all the four possible jumps of pairs of nucleons between the occupied and the empty levels, one chooses only those between down- and up-sloping levels (cf. Fig. 1). The factor 2 arises from the fact that one has to consider jumps of both protons and neutrons. Inserting the standard values of $\Delta = 12A^{-1/2}$ MeV and $G = 25A^{-1}$ MeV in Eq. (7) one obtains

$$v = -2.9\ \text{MeV}. \quad (8)$$

The quantity $dn/d\beta_\lambda$ can be estimated quite accurately realizing that the Fermi distribution in momentum space is spherical for each local minimum [3,4]. Between crossings, the Fermi surface gets distorted. In fact, it elongates in correspondence to a spatial reduction of the nuclear radius and it retracts when the nuclear radius becomes larger. Each time that the volume outside the original Fermi sphere contains two nucleons, it is possible to fill the depopulated momentum zones below the Fermi energy and restore spherical symmetry. This means that the system has moved from one local minimum to the next (cf. Fig. 1), and that a pair of nucleons have changed orbital. Making use of such a model one ob-

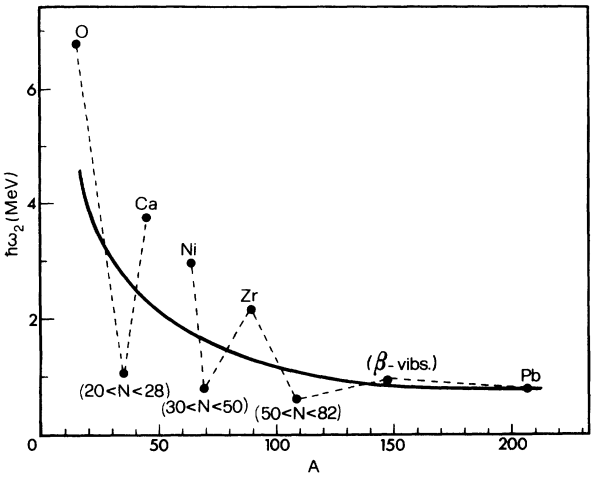

FIG. 2. Average energy of the lowest 2^+ state in nuclei as a function of the mass number. The data are taken from Ref. [1], Vol. I, Table 2-17, p. 196. The points labeled with the integral of neutron numbers include all nuclei in the corresponding sector, but leave out those with magic numbers of neutrons, protons, or both. These are included in the averaging labeled by the typical closed shell nuclei (Ca, Ni, Sn, Zr, and Pb). In this last case the anomalous case of ^{208}Pb was left out. The dashed line is to guide the eye. The continuous curve was calculated making use of Eq. (11) with $\lambda = 2$, carrying out similar averagings as done with the data.

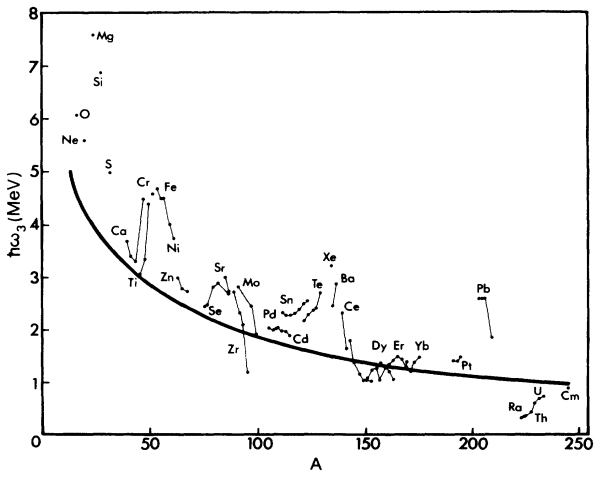

FIG. 3. Systematics of the energies of the lowest 3^- states, as taken from Ref. [1], Vol. II, Fig. 6.40, p. 560. The continuous curve was calculated making use of Eq. (11) with $\lambda = 3$.

tains for $\lambda = 2$ and 3 the approximate expression

$$\frac{dn}{d\beta_\lambda} \approx \frac{1}{4}\sqrt{\frac{2\lambda+1}{3\pi}}A. \tag{9}$$

We are now in position to calculate the inertia of the modes, which can be written as

$$\frac{D_\lambda}{\hbar^2} \approx (2\lambda+1)\times 10^{-3}A^2 \text{ MeV}^{-1}. \tag{10}$$

The basic frequencies associated with the harmonic motions for $\lambda = 2, 3$ are

$$\hbar\omega_\lambda = \sqrt{\frac{\hbar^2 C_\lambda}{D_\lambda}} \approx \sqrt{\frac{(\lambda-1)(\lambda+2)}{(2\lambda+1)}}(1-0.001A)\frac{38}{A^{2/3}} \text{ MeV}. \tag{11}$$

Note that the zero-point motion associated with these modes is given by

$$\beta_\lambda = \sqrt{\frac{\hbar\omega_\lambda}{2C_\lambda}} \approx \frac{3.7A^{-2/3}}{[(\lambda-1)(\lambda+2)(2\lambda+1)]^{1/4}}. \tag{12}$$

From the above equation and from Eq. (9) one can estimate that the number of crossings associated with the modes (11) is

$$n \approx \left(\frac{(2\lambda+1)}{(\lambda-1)(\lambda+2)}\right)^{1/4} 0.3A^{1/3} \approx 0.3A^{1/3}. \tag{13}$$

This number is approximately unity already for $A \approx 40$, and makes plausible the use of a large amplitude theory to describe the low-lying surface vibration of atomic nuclei.

In Figs. 2 and 3 we show the function (11) for $\lambda = 2$ and 3 in comparison with the data. We see that there are large fluctuations about the curves. These are associated with shell effects, which greatly affect the amount of pairing. However, on average, the smooth curves provide an overall account of the experimental findings.

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