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Self-energy of the pion

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We reexamine a recent calculation of the effect of dressing on the pion propagator in the one-pion-exchange potential. Our results confirm the qualitative features of the earlier work, namely that the correction can be represented as the exchange of an effective π' meson. However, at a quantitative level this approximation does not work well over a wide range of momentum transfer unless the mass of the π' is made too large to be of significance in nucleon nucleon scattering.

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There is now considerable evidence that the form factor for the emission of an off-mass-shell pion by a free nucleon is relatively soft [1–3]. In a dipole parametrization a mass less than 1 GeV is typical. On the other hand, conventional one-boson-exchange potentials (OBEP) typically require a much harder πNN form factor in order to reproduce the experimental phase shifts and deuteron properties [4,5]. Recently it has been proven possible to obtain equally good fits with a soft form factor provided an additional, heavy pion (π') exchange is included [6]. (An interesting alternative has been proposed by the Bochum group [7].)

Clearly it is of considerable interest to establish the physical mechanism behind the additional short-distance pseudoscalar exchange. It need not be a real π' meson, but could be a convenient representation of a more complicated short-distance physics involving quark-gluon or quark-meson exchange [8–10]. Saito's novel suggestion was that the radiative corrections associated with the internal structure of the pion itself might lead to a pion propagator that could be simulated by the exchange of an elementary pion *and* a heavier π' [11]. His suggestion echoed earlier work by Goldman *et al.* on the off-shell variation of the ρ-ω mixing amplitude [12]; see also Ref. [13].

In Saito's work the pion proagator was modeled as the propagator of an elementary π meson coupled to a q-$\bar{q}$ pair. As in Refs. [12,13], the propagators of the q-$\bar{q}$ pair were taken to be free Dirac propagators (with quark mass m). While this introduces an unphysical threshold at $2m$, it is not necessarily a fatal flaw in the spacelike region, where we need the propagator for NN scattering. Indeed there is a physical cut which begins at $(m_\pi + m_\rho)$ and by choosing m to be a typical constituent quark mass ($\sim$ 400 MeV) one might expect to simulate the effect of this cut.

In the model of Saito the renormalized pion propagator is written in the form

$$G(q^2) = \frac{i}{q^2 - \Sigma(q^2) - m_\pi^{(0)2}}, \tag{1}$$

where $\Sigma(q^2)$ is

$$\Sigma(q^2) = i6g^2D\int \frac{d^Dk}{(2\pi)^D}\frac{-k^2+\frac{1}{4}q^2+m^2}{[(k+\frac{q}{2})^2-m^2][(k-\frac{q}{2})^2-m^2k]}, \tag{2}$$

and the factor of 6 arises from color and isospin. For $D = 4$ the integral in Eq. (2) is highly singular. In Ref. [11] it was rewritten as a sum of three terms:

$$\Sigma = \Sigma_a + \Sigma_b + \Sigma_c\ , \tag{3}$$

where

$$i\Sigma_a = g^2\frac{3}{4\pi^4}\int d^4k\frac{1}{(k+\frac{q}{2})^2-m^2+i\epsilon}, \tag{4}$$

$$i\Sigma_b = g^2\frac{3}{4\pi^4}\int d^4k\frac{1}{(k-\frac{q}{2})^2-m^2+i\epsilon}, \tag{5}$$

$$i\Sigma_c = -g^2q^2\frac{3}{4\pi^4}\int d^4k\frac{1}{[(k+\frac{q}{2})^2-m^2+i\epsilon][(k-\frac{q}{2})^2-m^2+i\epsilon]}. \tag{6}$$

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Under an appropriate change of variables it appears that Σ_a and Σ_b are both independent of q^2 and can be incorporated into the bare mass term. This argument is not correct for a subtle reason. Both of the integrals in Eqs. (4) and (5) are quadratically divergent and it is known from the study of the axial anomaly that linear shifts in the integration variable are not permitted in this case [14]. There is in fact a surface term proportional to q^2. Fortunately for the analysis of Saito, this would affect only Σ_1 and Σ_2 (the second and third terms in a Taylor expansion about $q^2 = 0$), and it is Σ_2 alone which determines $\Sigma_R(q^2)$; see Eq. (12) below.

We have chosen to evaluate Eq. (2) directly using dimensional regularization [15], rather than relying on the expansion (3). Our result for $\Sigma(q^2)$ is:

$$\Sigma(q^2) = \frac{6g^2}{8\pi^2}\int_0^1 dx\, 2p(q^2)\Gamma(\frac{\epsilon}{2}-1) + 7q^2x(1-x) - m^2 + 2p(q^2)\ln[q^2x(1-x) - m^2] \tag{7}$$

where $p(q^2) = 3q^2x(1-x) - m^2$, m being the fermion mass. We can remove the divergences in this expression by adding counter terms to the Lagrangian, and bearing in mind the conditions we wish to impose on the renormalized self-energy, $\Sigma^R(q^2)$, in order that the pion propagator reproduces the physical properties of the pion in free space, namely

$$\Sigma^R(m_\pi^2) = 0,\ \frac{\partial}{\partial q^2}\Sigma^R(m_\pi^2) = 0, \tag{8}$$

To ensure this we add the following counterterms to the Lagrangian,

$$\mathcal{L}_{\rm CT} = -\frac{1}{2}\alpha\boldsymbol{\pi}\cdot(\Box + m_\pi^2)\boldsymbol{\pi} + \frac{1}{2!}\beta\boldsymbol{\pi}^2, \tag{9}$$

where

$$\alpha = \frac{\partial}{\partial q^2}\Sigma(m_\pi^2),\ \beta = \Sigma(m_\pi^2). \tag{10}$$

This gives us

$$\Sigma^R(q^2) = \Sigma(q^2) - \beta - (q^2 - m_\pi^2)\alpha\ , \tag{11}$$

and

$$G^R(q^2) = \frac{i}{q^2 - m_\pi^2 - \Sigma^R(q^2)}, \tag{12}$$

where $\Sigma^R(q^2)$ vanishes as $(q^2 - m_\pi^2)^2$ at the physical pion mass.

After some algebra we find that $\Sigma^R(q^2)$ takes the form

$$\Sigma^R(q^2) = \frac{6g^2}{4\pi^2}\int_0^1 dx\left[p(q^2)\ln\left(\frac{q^2x(1-x)-m^2}{m_\pi^2x(1-x)-m^2}\right) - (q^2-m_\pi^2)\left(\frac{1}{2} + \frac{2m^2x(1-x)}{m_\pi^2x(1-x)-m^2}\right)\right], \tag{13}$$

which becomes

$$\begin{aligned}\Sigma^R(q^2) = \frac{6g^2}{4\pi^2}\Bigg\{&-2q^2\left(\frac{m^2}{q^2} - \frac{m^2}{m_\pi^2}\right) - \frac{1}{2}(q^2 - m_\pi^2)\\ &+4q^2\left[\left(\frac{m^2}{q^2} - \frac{1}{4}\right)^{\frac{3}{2}}\arctan\left(\frac{1}{\sqrt{\frac{4m^2}{q^2}-1}}\right) - \left(\frac{m^2}{m_\pi^2} - \frac{1}{4}\right)^{\frac{3}{2}}\arctan\left(\frac{1}{\sqrt{\frac{4m^2}{m_\pi^2}-1}}\right)\right]\\ &+(3q^2 - 4m^2)\left[\sqrt{\frac{m^2}{q^2} - \frac{1}{4}}\arctan\left(\frac{1}{\sqrt{\frac{4m^2}{q^2}-1}}\right) - \sqrt{\frac{m^2}{m_\pi^2} - \frac{1}{4}}\arctan\left(\frac{1}{\sqrt{\frac{4m^2}{m_\pi^2}-1}}\right)\right]\\ &-2(q^2 - m_\pi^2)\frac{m^2}{m_\pi^2}\left[1 - 4\left(\frac{m^2}{m_\pi^2}\right)\frac{1}{\sqrt{\frac{4m^2}{m_\pi^2}-1}}\arctan\left(\frac{1}{\sqrt{\frac{4m^2}{m_\pi^2}-1}}\right)\right]\Bigg\},\end{aligned} \tag{14}$$

for $0 < q^2 < 4m^2$. For $q^2 < 0$ we have

$$\begin{aligned}\Sigma^R(q^2) = \frac{6g^2}{4\pi^2}\Bigg\{&-2q^2\left(\frac{m^2}{q^2} - \frac{m^2}{m_\pi^2}\right) - \frac{1}{2}(q^2 - m_\pi^2)\\ &+4q^2\left[\frac{1}{2}\left(\frac{m^2}{|q^2|} + \frac{1}{4}\right)^{\frac{3}{2}}\ln\left(\frac{\sqrt{\frac{4m^2}{|q^2|}+1}-1}{\sqrt{\frac{4m^{+2}}{|q^2|}+1}+1}\right) - \left(\frac{m^2}{m_\pi^2} - \frac{1}{4}\right)^{\frac{3}{2}}\arctan\left(\frac{1}{\sqrt{\frac{4m^2}{m_\pi^2}-1}}\right)\right]\\ &+(3q^2 - 4m^2)\left[\frac{1}{2}\sqrt{\frac{m^2}{|q^2|} - \frac{1}{4}}\ln\left(\frac{\sqrt{\frac{4m^2}{|q^2|}+1}-1}{\sqrt{\frac{4m^2}{|q^2|}+1}+1}\right) + \sqrt{\frac{m^2}{m_\pi^2} - \frac{1}{4}}\arctan\left(\frac{1}{\sqrt{\frac{4m^2}{m_\pi^2}-1}}\right)\right]\\ &-2(q^2 - m_\pi^2)\frac{m^2}{m_\pi^2}\left[1 - 4\left(\frac{m^2}{m_\pi^2}\right)\frac{1}{\sqrt{\frac{4m^2}{m_\pi^2}-1}}\arctan\left(\frac{1}{\sqrt{\frac{4m^2}{m_\pi^2}-1}}\right)\right]\Bigg\},\end{aligned} \tag{15}$$

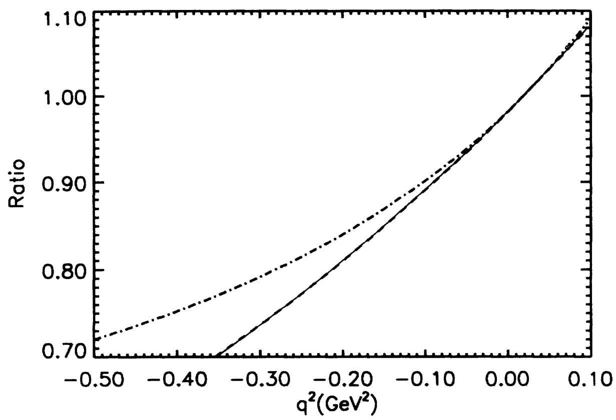

FIG. 1. The ratio of the free to renormalized pion propagators as a function of q^2. The solid curve is our exact result, the dashed curve is a simple parametrization [with mass parameter $\Lambda = 1.295$ GeV; see Eq. (18)], and the dash-dotted curve is the result assuming π plus π' exchange [Eq. (19) with $M = \Lambda$].

An appropriate value for $g_{\pi q}$ can be determined from the pion-nucleon coupling constant $\frac{g_{\pi N}^2}{4\pi} = 14.6$ [16]. An analysis within the consituent quark model yields the following relation

$$g_{\pi q} = \frac{3}{5}\frac{m_q}{m_N} g_{\pi N}. \tag{16}$$

In Fig. 1 we show the ratio (represented by the solid line) of the free to the renormalized pion propagator as a function of q^2. (The quark mass is set at 400 MeV for the reasons explained earlier.) In order to clarify its similarity to the phenomenological introduction of a π' meson we recall that $\Sigma^R(q^2)$ is proportional to $(q^2 - m_\pi^2)^2$. One might then approximate $\Sigma^R(q^2)$ as

$$\Sigma^R(q^2) \sim \frac{c(q^2 - m_\pi^2)^2}{(q^2 - \Lambda^2)}, \tag{17}$$

with c a dimensionless constant and Λ a mass parameter. In this approximation the ratio $R = G(q^2)/G^R(q^2)$ is

$$R = 1 - \frac{c(q^2 - m_\pi^2)}{(q^2 - \Lambda^2)}. \tag{18}$$

The dashed line in Fig. 1 which is almost identical to the solid curve shows the fit obtained for $\Lambda = 1.29$ GeV (with $c = 1.63$).

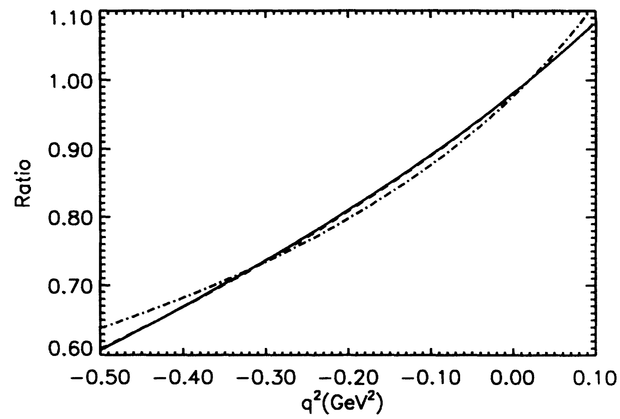

FIG. 2. Same as Fig. 1 but with the parameters in the π' case adjusted to give a best fit [$M = 2.16$ GeV and $C = 5.73$ in Eq. (19)].

If the renormalized pion propagator were to be approximated by the sum of elementary π and π' exchanges (with π' mass M) we should instead find

$$R = \left(1 + C\frac{q^2 - m_\pi^2}{q^2 - M^2}\right)^{-1}, \tag{19}$$

where $C = g_{\pi' N}^2 / g_{\pi N}^2$. To first order we would identify $\Lambda = M$ and $c = C$ and the result of this choice is shown as the dot-dashed line in Fig. 1. It clearly is not a good representation of the renormalized propagator. In fact, in order to fit even moderately well over the range of q^2 shown the π' mass must be made considerably larger. Our best fit using Eq. (19) is shown in Fig. 2 where we used a π' mass $M = 2.0$ GeV and $C = 5.73$. (The other two curves are as in Fig. 1.) While the corresponding $\pi' N$ coupling constant is in the range quoted in Ref. [6] the mass is far too large for this π' to play any role in NN scattering.

In conclusion, while the very interesting suggestion of Saito has been confirmed qualitatively, we are forced to conclude that this is not the source of the π' meson needed in NN scattering.

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