Continuum random phase approximation self-consistent approaches to the theory of isobaric analog resonances

O. A. Rumyantsev and M. H. Urin
Moscow Engineering Physics Institute, 115409, Moscow, Russia
(Received 6 July 1992; revised manuscript received 25 June 1993)

Two approaches (exact and approximate) to the isobaric analog resonance (IAR) theory within the random phase approximation in the continuum are considered. Both of them are based on the partial self-consistency condition which is the result of the isospin symmetry of the nuclear Hamiltonian. The evaluations of the IAR partial proton widths for near magic nuclei over a wide atomic mass region are performed. The results obtained within the framework of these approaches are compared with each other and with the experimental data. The method of the calculation of the Coulomb correction to the IAR transition density is also given.

PACS number(s): 24.30.Gd, 24.10.Cn, 21.60.Jz

Within the framework of any isobaric analog resonance (IAR) theory, the charge independence of nuclear forces, proton Coulomb interaction, and single-particle continuum should be taken into account. All approaches to the IAR theory based on the use of the mean nuclear field and effective nucleon interaction can be divided into the approaches without explicit use and with the use of the approximate isospin conservation in medium and heavy nuclei.

The calculation of the Fermi strength function within the continuum random phase approximation (RPA) is the practically carried out element of the first group of the approaches. Within the framework of these approaches the mean field is calculated by means of the Hartree-Fock method with the Skyrme forces [1] or is parametrized directly [2]. In the last case, the isovector part of the mean nuclear field (the symmetry potential v) should be the coordinate with the isovector part of the nucleon interaction in the particle-hole channel. If one uses the Landau-Migdal forces $\frac{1}{2}F'\tau_1\tau_2\delta(\mathbf{r}_1-\mathbf{r}_2)$ the corresponding self-consistency condition has the form (the partial self-consistency [3])

$$v(r)=F'\rho(r)\ , \tag{1}$$

where $\rho(r)$ is the excess neutron density which depends on $v(r)$ through the single-particle wave functions. The Fermi strength function exhibits the maximum which corresponds to the IAR. The width of this maximum is the total proton escape width Γ_p.

The approach to the calculation of the IAR proton partial widths Γ_λ has been proposed in Ref. [4]. That approach falls in the first group of the approaches too. The shortcomings of Ref. [4] are the somewhat artificial procedure of taking into account the single-particle continuum and the absence of the systematic comparison of the results with experimental data. Moreover, within the framework of the first group of the approaches there are no methods for the evaluation of the parameters of the isospin forbidden processes (the IAR spreading width and the partial widths of the IAR direct neutron decay). The values of these parameters are essentially less as compared with the relevant values for other giant resonances.

Within the framework of the approaches which are based on the explicit use of the approximate isospin conservation in medium and heavy nuclei, any IAR relaxation parameters can be analyzed. These parameters are determined (except for the radiation widths) by the variable part of the nuclear mean Coulomb field [5–7]. The main shortcoming of such approaches is the nonconsistent consideration of the strong coupling of the IAR proton decay channels.

The objects of the present paper are the following: (i) to evaluate the IAR proton partial widths Γ_λ within the continuum RPA without the explicit use of the approximate isospin conservation taking into account the self-consistency condition (1); (ii) to formulate the consistent method for the consideration of the strong coupling of the IAR proton decay channels as well as the method for the calculation of the Coulomb correction to the IAR transition density and to calculate on that basis the widths Γ_λ; (iii) to compare the Γ_λ values obtained by these methods with each other and with the corresponding experimental data for near magic nuclei over wide atomic mass region.

If one takes into account the self-consistency condition (1), the IAR proton partial widths Γ_λ can be evaluated within the continuum RPA by the same method as in the case of other giant resonances. The method has been formulated recently [8] and consists in the calculation of the S matrix of the proton scattering via the IAR virtual excitation.

Let $\chi_\lambda(r)$ be the radial wave functions of neutron and proton single-particle states ($\lambda=n,l,j$ for bound states, $\lambda=\varepsilon,l,j$ for continuum states): $(h_{(\lambda)}-\varepsilon)\chi_\lambda=0$. Here $h_{(\lambda)}=h^0_{(\lambda)}+\frac{1}{2}\tau^{(3)}v(r)+\frac{1}{2}(1-\tau^{(3)})U_C(r)$ is the radial part of the single-particle Hamiltonian, $(\lambda)=l,j$, $h^0_{(\lambda)}$ is the isoscalar part of this Hamiltonian, and $U_C(r)$ is the mean Coulomb field. The single-particle wave functions $\chi_\lambda(r)$ are chosen to be real and in the case of continuum to be normalized to a δ function of the energy.

According to Ref. [8] the S matrix diagonal elements of the proton scattering via the IAR virtual excitation can be represented in the form

$$S_{\lambda\lambda}(\varepsilon^p)=\exp(2i\delta_\lambda)\left[1-2\pi i N_\lambda^n\int\chi_\lambda^p(r)v_\lambda(r,\omega)\chi_\lambda^n(r)dr\right], \tag{2}$$

$$v_\lambda(r,\omega)=\frac{F'}{4\pi r^2}\left[\chi_\lambda^p(r)\chi_\lambda^n(r)+\int A(rr',\omega)v_\lambda(r',\omega)dr'\right]. \tag{3}$$

Here λ is also the decay channel index determined by the proton quantum numbers, $\omega=\varepsilon^p-\varepsilon_\lambda^n$, $\delta_\lambda(\varepsilon^p)$ is the phase of proton potential scattering, and N_λ^n is the number of neutrons filling the subshell λ in the parent nucleus (the nuclei without nucleon pairing are only considered). The inhomogeneous term in Eq. (3) represents the charge-exchange external field acting upon the nucleus in the process of proton scattering. That field arose at the expense of the isovector part of the particle-hole interaction. The quantity $A(rr',\omega)$ in Eq. (3) is the radial part of the monopole particle-hole propagator for the relevant charge-exchange channel

$$A(rr',\omega)=\sum_\nu N_\nu^n\chi_\nu^n(r)\chi_\nu^n(r')g_{(\nu)}^p(rr',\varepsilon_\nu^n+\omega)+\sum_\nu N_\nu^p\chi_\nu^p(r)\chi_\nu^p(r')g_{(\nu)}^n(rr',\varepsilon_\nu^p-\omega). \tag{4}$$

Here $g_{(\nu)}(rr',\varepsilon)$ is the Green function of the relevant radial Schrödinger equation: $(\varepsilon-h_{(\nu)})g_{(\nu)}(rr',\varepsilon)=\delta(r-r')$. The integral equations of type (3) are widely used within the framework of the finite Fermi-system theory [9].

One of the poles in the ω dependence of the effective field $v_\lambda(\mathbf{r},\omega)$ corresponds to the IAR. To evaluate the proton widths Γ_λ and $\Gamma_p=\sum_\lambda\Gamma_\lambda$ employing the S matrix calculated according to Eqs. (1)–(4) it is necessary to parametrize this S matrix in the vicinity of the IAR energy by means of the Breit-Wigner formula

$$S_{\lambda\lambda}(\varepsilon^p)=\exp(2i\xi_\lambda)\left[1-\frac{i\Gamma_\lambda}{\varepsilon^p-\varepsilon^{res}+i\Gamma_p/2}\right]. \tag{5}$$

Thus, Eqs. (1)–(5) correspond to the exact calculation of the IAR partial proton widths within the continuum RPA.

Let us briefly consider the shell-model self-consistent IAR theory in which the approximate isospin conservation in medium and heavy nuclei is explicitly taken into account. Under the assumption that the Coulomb scattering amplitude (as well as the neutron and proton mass differences, isobaric noninvariance of the nuclear forces) is neglected and the Coulomb proton-nucleus interaction $U_C(\mathbf{r})$ is replaced by an average value Δ_C (Δ_C is the Coulomb displacement energy), the nuclear isospin is the exact quantum number. Within this approximation the isobaric analog state (IAS) having the wave function $|A\rangle=(2T)^{-1/2}\sum_a\tau_a^{(-)}|0\rangle$ (T is the isospin of the relevant parent nucleus state) is the exact eigenstate of the shell-model Hamiltonian. The homogeneous equation for the corresponding eigenfield within the continuum RPA has the form (see also [9])

$$\bar{v}(r)=\frac{F'}{4\pi r^2}\int\bar{A}(rr',\omega=\Delta_C)\bar{v}(r')dr'=F'\bar{\rho}(r). \tag{6}$$

The bar above any quantity in Eq. (6) denotes that this quantity is calculated with the use of the single-particle Hamiltonian $\bar{h}_{(\lambda)}=h_{(\lambda)}^0+\frac{1}{2}\tau^{(3)}\bar{v}+\frac{1}{2}(1-\tau^{(3)})\Delta_C$ without the account of the variable part of the mean Coulomb field. Within the considered approximation the IAS energy equals Δ_C and the IAS transition density $\bar{\rho}_{tr}(r)$ equals $(2T)^{-1/2}\bar{\rho}(r)$, where $\bar{\rho}(r)$ is the excess neutron density calculated by the use of the eigenfunctions of the Hamiltonian $\bar{h}$. The last equality in (6) [compare with Eq. (1)] is obtained by the use of the relation: $\bar{v}(r)=\bar{h}^n-\bar{h}^p+\Delta_C$.

Within the framework of this approach, the mixing of the nuclear states having the different isospin values, as well as the IAS decay, (except for the radiation decay) is caused by the variable part of the mean Coulomb field $V_C=U_C-\Delta_C$. To the lowest order in $\hat{V}_C=\sum_a\frac{1}{2}(1-\tau_a^{(3)})V_C(a)$ the Coulomb displacement energy can be found from the equation

$$\int\bar{\rho}_{tr}V_Cd\mathbf{r}=\langle A|\hat{V}_C^{(-)}|0\rangle=0, \tag{7}$$

where $\hat{V}_C^{(-)}=\sum_aV_C(a)\tau_a^{(-)}$. The matrix element which determines the amplitude of the mixing of the IAS and 0^+ state $|s\rangle$ having the "normal" isospin $(T-1)$ equals

$$\langle s|\hat{V}_C|A\rangle=(2T)^{-1/2}\langle s|\hat{V}_C^{(-)}|0\rangle. \tag{8}$$

Equations (7) and (8) make it possible to state that in linear approximation the correction to the IAS transition density at the expense of the field $\hat{V}_C$ coincides with the correction to the density matrix of the relevant parent nucleus state at the expense of the field $(2T)^{-1/2}\hat{V}_C^{(-)}$:

$$(2T)^{1/2}\delta\bar{\rho}_{tr}(r)=\frac{1}{4\pi r^2}\int\bar{A}(rr',\omega=\Delta_C)\tilde{V}_C(r')dr', \tag{9}$$

where the effective field $\tilde{V}_C$ satisfies the following equation:

$$\tilde{V}_C(r)=V_C(r)+\frac{F'}{4\pi r^2}\int\bar{A}(rr',\omega=\Delta_C)\tilde{V}_C(r')dr'. \tag{10}$$

In view of Eqs. (6) and (7) the solution of this equation is determined with an accuracy of the function which is proportional to $\bar{v}(r)$. To eliminate the admixture of the solution of the homogeneous Eq. (6) to $\tilde{V}_C(r)$ one should exclude the IAS contribution from the propagator $\bar{A}(rr',\Delta_C)$. For this purpose one should replace in Eqs. (9) and (10) the propagator $\bar{A}(rr',\Delta_C)$ by $\delta\bar{A}(rr')=\bar{A}(rr',\Delta_C)-\alpha\bar{\rho}(r)\bar{\rho}(r')/\int\bar{\rho}(r)\bar{v}(r)d\mathbf{r}$, where parameter α is found from the condition $\int\bar{v}(r)\delta\bar{A}(rr')\bar{v}(r')dr\,dr'=0$ and in accordance to Eq. (6) equals unity.

To take into account the real single-particle continuum we use instead of $\delta\bar{A}(rr')$ the quantity

$$\delta A(rr')=A(rr',\Delta_C)-\alpha\bar{\rho}(r)\bar{\rho}(r')/\int\bar{\rho}(r)\bar{v}(r)d\mathbf{r},$$

where

$$\alpha=\int\bar{v}(r)A(rr',\Delta_C)\bar{v}(r')dr\,dr'.$$

Here the propagator $A(rr',\Delta_C)$ is calculated by the use of the mean Coulomb field $U_C(r)$ and symmetry potential $\bar{v}(r)$. Because of the approximate isospin conservation in medium and heavy nuclei the α values are close to unity (0.95–1). Finally, the equation for the field $\tilde{V}_C(r)$ has the form

$$\tilde{V}_C(r)=V_C(r)+\frac{F'}{4\pi r^2}\int \delta A(rr')\tilde{V}_C(r')dr' . \quad (11)$$

Equations (9) and (11) correspond to the self-consistent method of the consideration of the strong coupling of the real and virtual IAR proton decay channels within the continuum RPA.

The accuracy of the method can be examined by means of comparison of the results of IAR proton width calculations with the corresponding values obtained according to Eqs. (2)–(5). The expression obtained in Refs. [5,7] for the IAR partial proton widths through the effective Coulomb field $\tilde{V}_C(r)$ is not changed:

$$\Gamma_\lambda=\pi N_\lambda^n T^{-1}\left|\int \chi_\lambda^p(r)\tilde{V}_C(r)\chi_\lambda^n(r)dr\right|^2 . \quad (12)$$

Here the single-particle wave functions are also calculated by use of the mean Coulomb field $U_C(r)$ and symmetry potential $\bar{v}(r)$. It should be noted that the neglect of the IAR proton channel coupling [i.e., the substitution $\tilde{V}_C \rightarrow V_C$ in Eq. (12)] corresponds to the Lane model [10] or to the approach given in Ref. [11] and results in a remarkable increase of the calculated Γ_λ values [7].

The approximate approach to the calculations of the widths Γ_λ according to Eqs. (11) and (12) is easier than the approach which is based on the S matrix calculation and is presented in the previous section. The values of the partial widths Γ_λ obtained by both methods are compared further on. Here we draw attention to the statement that one would expect the stability of the calculated widths Γ_λ to the variation of the parameters of the model due to the self-consistency conditions (1) and (6).

The parameters of the isoscalar part of the nuclear mean field, as well as of the mean Coulomb field, which are necessary for the evaluation of the widths Γ_λ within the framework of the shell-model approach are chosen according to Ref. [7]. The quasiparticle interaction strength F' is determined from the condition (1) where the symmetry potential is chosen according to Ref. [7] and the neutron excess density $\rho(r)$ is chosen to be proportional to the radial dependence of the nuclear mean field [5].

For a comparison with the experimental values of widths Γ_λ, the calculated values were reevaluated according to the potential barrier penetrability to the experimental proton channel energy and multiplied by the experimental value of the spectroscopic factor of the relevant single-particle state. The results of the calculations performed according to Eqs. (1)–(5), and (11) and (12) are given in Table I (all experimental data are taken from Ref. [7]). From the results given in the table it follows that (i) the approximate approach to the evaluation of the widths Γ_λ ensures good accuracy; (ii) the results obtained by means of both methods are in satisfactory agreement with the experimental data; and (iii) the largest uncertainty in the evaluation of widths Γ_λ is caused by the use of the experimental values of the spectroscopic factors of the single-particle states (the relevant values are given in Ref. [7]). The last circumstance allows one to return to the problem of the use of the experimental and calculated IAR proton partial widths for the determination of the spectroscopic factors of the low energy parent nucleus states.

TABLE I. The IAR proton partial widths.

Parent nucleus	λ^a	Γ_λ^b (keV)	Γ_λ^c (keV)	Γ_λ^{exp} (keV)
^{49}Ca	$p_{3/2}$	3.3	3.5	1.9±0.2
^{91}Zr	$d_{5/2}$	3.4	3.3	4.0±0.5
	$d_{3/2}$	17	17	15±3
	$g_{7/2}$	0.83	0.78	2.5±0.5
	$s_{1/2}$	26	23	17±3
^{139}Ba	$f_{7/2}$	22	21	16±2
	$p_{3/2}$	38	36	26±3
	$p_{1/2}$	35	34	22±2
	$h_{9/2}$	1.8	1.8	1.4
	$f_{5/2}$	13	12	10±1
^{208}Pb	$p_{1/2}$	62	61	51.6±1.7
	$f_{5/2}$	26	25	26[d]
	$p_{3/2}$	77	75	65[d]
	$f_{7/2}$	8.4	8.2	5.0±0.5
^{209}Pb	$g_{9/2}$	29	30	22.6±0.7
	$i_{11/2}$	1.4	1.3	1.6±0.4
	$j_{15/2}$	0.9	0.9	0.9±0.8
	$d_{5/2}$	78	78	50.2±1.0
	$s_{1/2}$	74	76	56.6±3.4
	$g_{7/2}$	51	51	42.9±3.6
	$d_{3/2}$	65	68	62.8±5.4

[a]If some resonances have the same quantum numbers the resonance with the largest spectroscopic factor is given.
[b]Partial width values calculated according to Eqs. (1)–(5).
[c]Partial width values calculated according to Eqs. (6), (7), and (12).
[d]P. Von Brentano (private communication).

The main results obtained in the present paper are as follows.

(i) Within the framework of the shell model the quantitative description of the IAR partial proton widths is given by the use of the continuum RPA. Due to the partial self-consistency these widths are mainly determined by the mean Coulomb field and, therefore, are expected to be stable to the variations of the parameters of the shell model.

(ii) The continuum RPA self-consistent description of the Coulomb correction to the IAR transition density is also given. (This quantity determines the partial widths for the isospin-forbidden IAR decays.) The accuracy of this description is established.

The next step in the analysis of the IAR partial proton widths consists in a generalization of the proposed approach to the case of nuclei with strong nucleon pairing. This generalization opens the door to the interpretation of a great body of experimental data on the IAR partial proton widths for these nuclei.

[1] N. Auerbach and N. Van Giai, Phys. Lett. **72B**, 289 (1978).
[2] N. I. Pyatov *et al.*, Yad. Fiz. **34**, 600 (1981) [Sov. J. Nucl. Phys. **34**, 335 (1981)].
[3] B. L. Birbrair and V. A. Sadovnikova, Yad. Fiz. **20**, 645 (1974) [Sov. J. Nucl. Phys. **20**, 347 (1974)].
[4] N. Van Giai *et al.*, Phys. Lett. B **233**, 1 (1989).
[5] M. H. Urin, Fiz. Elem. Chastits. At. Yadra **11**, 991 (1980) [Sov. J. Part. Nucl. **11**, 395 (1980)].
[6] N. Auerbach, Phys. Rep. C **98**, 273 (1983).
[7] V. G. Guba and M. H. Urin, Nucl. Phys. **A460**, 222 (1986).
[8] S. E. Muraviev and M. H. Urin, Phys. Lett. B **280**, 1 (1992).
[9] A. B. Migdal, *Theory of Finite Fermi Systems and Application to Atomic Nuclei* (Wiley, New York, 1967).
[10] A. M. Lane and J. M. Soper, Phys. Rev. Lett. **7**, 420 (1961).
[11] N. Auerbach *et al.*, Rev. Mod. Phys. **44**, 48 (1972).