Pion electroproduction on proton and deuteron

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The cross sections for the $p(e,e'\pi^+)n$ and $d(e,e'\pi^+)2n$ reactions are calculated using a simple model with pion, nucleon and Δ-resonance degrees of freedom, for the kinematical conditions used in a recent Saclay experiment. The final-state interactions between the two outgoing neutrons are found to have a large effect on the $d(e,e'\pi^+)2n$ reaction. The calculated quenching of the π^+ production cross section for deuterium is in agreement with that observed experimentally. It is predicted that, due to the strong final-state interaction in the singlet S state of the two neutrons, the $e,e'\pi^+$ cross-section can have a large dependence on the orientation of deuteron spin. In principle this dependence can be observed with tensor polarized deuterium targets under suitable kinematical conditions.

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I. INTRODUCTION

The pion, as a mediator of the long-range nuclear force, is expected to play an important role in nuclear physics. All realistic models of the nucleon-nucleon interaction contain a tensor potential $v^{t\tau}(r_{ij})S_{ij}\tau_i\cdot\tau_j$, which is very similar to that in the one-pion-exchange potential (OPEP) between pointlike nucleons at $r_{ij}>1$ fm. In the many-body theories of nuclear ground states [1] this OPEP, suitably cutoff at $r_{ij}<1$ fm, provides most of the potential energy of the nucleus. However, there are many uncertainties in our understanding of the role of pion fields in nuclei. For example, recent experiments with medium [2] and high [3] energy proton beams do not seem to support the existence of OPEP between nucleons that are ~ 1 fm apart [4].

Pion electroproduction from nuclei can also be used to study the pion field in nuclei. Due to the weakness of the electromagnetic interaction the results of these experiments may be relatively simpler to analyze. The first such experiment, comparing the $p(e,e'\pi^+)n$ and $d(e,e'\pi^+)2n$ cross sections was recently carried out at Saclay [5]. Incident 645 MeV electrons were scattered at an angle of 36° and the cross sections were measured at two values of the photon-nucleon invariant mass $W=1160$ (1230) MeV. At these values the energy and momentum transferred by the electron are 290 (376) MeV and 414 (456) MeV/c. The Δ-resonance contributions are expected to be much larger at $W=1230$ MeV.

The π^+ were detected at an angle of 4° with the direction of the virtual photons. Under these kinematics the π^+ emitted from the interaction with protons have a fixed energy of 273 (369) MeV. In contrast the π^+ emitted in the $d(e,e'\pi^+)2n$ reaction have a continuous spectrum peaked at approximately these energies with a width of ~ 20 MeV. The radio R_{dp}, defined as

$$R_{dp}=\int\frac{d^4\sigma(\text{deuteron})}{dE_{e'}\,d\Omega_{e'}\,d\Omega_\pi dE_\pi}dE_\pi\Big/\frac{d^3\sigma(\text{proton})}{dE_{e'}\,d\Omega_{e'}d\Omega_\pi}\,, \tag{1.1}$$

was measured by integrating over the ~ 50 MeV wide peak region. The observed values of R_{dp} are 0.80 ± 0.05 (0.75 ± 0.07) at $W=1160$ (1230) MeV; their significant deviation from unity indicates nontrivial effects in this reaction.

The pion-nucleon scattering cross sections are of the order of 10 fm^2, whereas the mean separation $\bar{r}_{np}$ between the nucleons in the deuteron is 4 fm. Thus the effects of π-N final-state interactions are expected to be of the order

$$\bar{\sigma}_{\pi N}/4\pi\bar{r}_{np}^2\sim 0.05 \tag{1.2}$$

and are neglected in this initial study. In contrast, the energy in the relative motion between the two outgoing neutrons emitted in the $d(e,e'\pi^+)2n$ reaction is very small. Hence, the final-state interactions between them can have larger effects as in the case of threshold photoproduction [6].

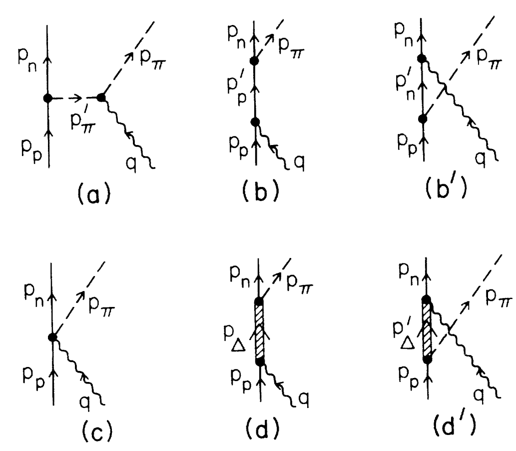

FIG. 1. Contributions to the $p(e,e'\pi^+)n$ cross section. The full, dashed, wavy, and thick lines, respectively, denote nucleon, pion, virtual photon, and Δ.

In the present work the cross sections measured by the Saclay experiments [5] are calculated by keeping the π, nucleon, and Δ-resonance degrees of freedom. The final-state interactions between the two outgoing neutrons are treated exactly, whereas those of the pion are neglected. The calculated values of R_{dp} are in agreement with experiment. Moreover, the theory predicts that, due to the strong final-state interactions in the singlet S state of the two neutrons, the $d(e,e'\pi^+)2n$ cross section has a large dependence on the orientation of the deuteron spin, particularly at $W=1160$ MeV. In principle this dependence can be observed with tensor polarized deuterium targets [7]. The $p(e,e'\pi^+)n$ and $d(e,e'\pi^+)2n$ reactions are discussed respectively in Secs. II and III, and a brief summary is given in Sec. IV.

II. PION ELECTROPRODUCTION FROM A NUCLEON

The amplitudes containing only the π, N, and Δ degrees of freedom, responsible for pion electroproduction from a nucleon, are shown in Fig. 1. In the literature [8,9] they are, respectively, called the pion pole [Fig. 1(a)], nucleon pole [Figs. 1(b) and 1(b')], seagull [Fig. 1(c)], and Δ pole [Figs. 1(d) and 1(d')]. They are used in the Born approximation with effective coupling constants and vertex form factors.

The cross section for the $p(e,e'\pi^+)n$ reaction of an unpolarized electron beam with stationary protons is given by [10,11]

$$\frac{d^3\sigma}{d\Omega_{e'}d\Omega_\pi dE_{e'}}=\sigma_M\frac{p_\pi^2}{(2\pi)^3}E_\pi m_N\left[\frac{1}{(E_\pi+E_n)p_\pi-E_\pi q\cos\theta_\pi}\right]$$
$$\times\{\nu_L|\rho^{fi}|^2+\nu_T(|J_x^{fi}|^2+|J_y^{fi}|^2)+\nu_{TT}(|J_x^{fi}|^2-|J_y^{fi}|^2)-2\nu_{LT}\mathrm{Re}(\rho^{fi}J_x^{fi*})\}\ , \tag{2.1}$$

where σ_M is the Mott cross section. The energy momentum of the incident electron, scattered electron, pion, and neutron are denoted by $E_e,\mathbf{k}$; $E_{e'},\mathbf{k}'$; $E_\pi,\mathbf{p}_\pi$; and $E_n,\mathbf{p}_n$, respectively. The electron scattering angle is denoted by θ_e, and the energy-momentum transfer by $\omega,\mathbf{q}$. Nonrelativistic expressions are used for the nucleon energies; the target proton has $E_p=m_N,\mathbf{p}_p=0$ so that

$$\omega=p_n^2/2m_N+E_\pi\ ,\quad \mathbf{q}=\mathbf{p}_\pi+\mathbf{p}_n\ . \tag{2.2}$$

The angle between $\mathbf{q}$ and $\mathbf{p}_\pi$ is θ_π, the z axis is chosen along $\mathbf{q}$ and $\mathbf{p}_\pi$ is in the xz plane. The virtual photon flux parameters ν_L, ν_T, ν_{TT}, and ν_{LT} are given by

$$Q^2=q^2-\omega^2\ , \tag{2.3}$$

$$\nu_L=Q^4/q^4\ , \tag{2.4}$$

$$\nu_T=\tan^2\left[\frac{\theta_e}{2}\right]+\frac{1}{2}\frac{Q^2}{q^2}\ , \tag{2.5}$$

$$\nu_{TT}=\frac{1}{2}\frac{Q^2}{q^2}\ , \tag{2.6}$$

$$\nu_{LT}=\frac{Q^2}{q^2}\left[\frac{Q^2}{q^2}+\tan^2\left[\frac{\theta_e}{2}\right]\right]^{1/2}\ . \tag{2.7}$$

The ρ^{fi}, J_x^{fi}, and J_y^{fi} denote transition amplitudes induced by the charge, and x and y components of the current, respectively. The effective charge and current operators for the $e,e'\pi^+$ reaction are defined such that their matrix elements between the initial and final nucleon states give the transition amplitudes. They have contributions from the processes shown in Fig. 1 and can be written as

$$\rho=\rho_\pi+\rho_N+\rho_{\mathrm{sg}}\ , \tag{2.8}$$

$$\mathbf{J}=\mathbf{J}_\pi+\mathbf{J}_\mathrm{N}+\mathbf{J}_{\mathrm{sg}}+\mathbf{J}_\Delta\ , \tag{2.9}$$

where the subscripts π, N, sg, and Δ, respectively denote pion-pole, nucleon-pole, seagull, and Δ-pole terms. The transition $N\to\Delta$ is mostly magnetic, and hence the ρ_Δ contribution is neglected.

The motion of the nucleons is treated nonrelativistically by keeping terms of order 1 in the charge and of order $1/m_N$ in the current. The pion- and nucleon-pole contributions are easily obtained from the Hamiltonians:

$$H_{\pi NN}=-\frac{f}{m_\pi}\tau_i\sigma\cdot\nabla\phi_i\ , \tag{2.10}$$

$$H_{\gamma NN}=e_NA_0-e_N\left[\frac{\mathbf{p}_\mathrm{N}+\mathbf{p}'_\mathrm{N}}{2m_N}\right]\cdot\mathbf{A}-\frac{e}{2m_N}\mu_N\sigma\cdot\nabla\times\mathbf{A}\ , \tag{2.11}$$

$$H_{\gamma\pi\pi}=ie[\phi_+\partial^\mu\phi_--\phi_-\partial^\mu\phi_+]A_\mu\ , \tag{2.12}$$

in which ϕ_i is the pion field with isospin i, σ is the nucleon spin, τ_i is the i component of the nucleon isospin, $\phi_\pm=(\phi_x\pm i\phi_y)/\sqrt{2}$, $A^\mu(A_0,\mathbf{A})$ is the electromagnetic field, and e_N and μ_N are the nucleon charge and magnetic moments. The value [12] $f^2/4\pi=0.075$ for the πNN coupling constants is used in this work. The nucleon- and pion-pole charge and current operators are given by

$$\rho_N=\frac{if}{m_\pi\sqrt{E_\pi}}\frac{\sigma\cdot\mathbf{p}_\pi}{\omega-q^2/2m_N}\tau_-\ , \tag{2.13}$$

$$\mathbf{J}_\mathrm{N}=\frac{f}{m_\pi\sqrt{E_\pi}}\sigma\cdot\mathbf{p}_\pi\tau_-\frac{1}{\omega-q^2/2m_N}\left[\frac{\mathbf{q}}{2m_N}+i\frac{\mu_p}{2m_N}\sigma\times\mathbf{q}\right]+\frac{if}{m_\pi\sqrt{E_\pi}}\tau_-\frac{1}{E_\pi+p_\pi^2/2m_N}\frac{\mu_n}{2m_N}\sigma\times\mathbf{q}\sigma\cdot\mathbf{p}_\pi\ , \tag{2.14}$$

$$\rho_\pi = \frac{if}{m_\pi \sqrt{E_\pi}} \frac{(2E_\pi - \omega)\boldsymbol{\sigma}\cdot(\mathbf{p}_\pi - \mathbf{q})}{(\omega - E_\pi)^2 - (\mathbf{q}-\mathbf{p}_\pi)^2 - m_\pi^2} \tau_- \ , \tag{2.15}$$

$$\mathbf{J}_\pi = \rho_\pi (2\mathbf{p}_\pi - \mathbf{q})/(2E_\pi - \omega) \ , \tag{2.16}$$

where τ_- changes a proton into neutron. Minimal substitution in the nonrelativistic $H_{\pi NN}$ [Eq. (2.11)] gives

$$\mathbf{J}_{\rm sg} = \frac{if}{m_\pi \sqrt{E_\pi}} \boldsymbol{\sigma} \tau_- \ , \tag{2.17}$$

and $\rho_{\rm sg}=0$ to order 1. The correction of order $1/m_N$ to $\rho_{\rm sg}$,

$$\rho_{\rm sg} = \frac{if}{m_\pi \sqrt{E_\pi}} \frac{\boldsymbol{\sigma}\cdot(\mathbf{q}-\mathbf{p}_\pi)}{2m_N} \tau_- \ , \tag{2.18}$$

is easily obtained by using the relativistic $\gamma^\mu \gamma_5 \partial_\mu$ πNN coupling; however, it gives negligible contributions in the present work. The Δ-pole terms are obtained from

$$H_{\gamma n \Delta} = -\mu^* \frac{e}{2m_N} \mathbf{S}\cdot\nabla\times \mathbf{A} T_z + \text{H.c.} \ , \tag{2.19}$$

$$H_{\pi N \Delta} = -\frac{f_\Delta}{m_\pi} \mathbf{S}\cdot\nabla\phi_i T_i + \text{H.c.} \ , \tag{2.20}$$

where μ^* is the $N\rightarrow\Delta$ transition magnetic moment, whose value is taken to be 3 [13], $f_\Delta^2/4\pi = 0.356$ is the $\pi N\Delta$ coupling constant obtained from the observed width $\Gamma = 115$ MeV of the Δ, and $\mathbf{S}$ and $\mathbf{T}$ are the transition spin and isospin operators. The $\mathbf{J}_\Delta$ is found to be

$$\mathbf{J}_\Delta = \frac{f_\Delta \mu^*}{3m_\pi \sqrt{E_\pi} 2m_N} \tau_- \left\{ \frac{\frac{2}{3}\mathbf{p}_\pi \times \mathbf{q} - (i/3)[\mathbf{p}_\pi \mathbf{q}\cdot\boldsymbol{\sigma} - \mathbf{p}_\pi \cdot \mathbf{q}\boldsymbol{\sigma}]}{\omega + m_N - m_\Delta - q^2/2m_\Delta + i\Gamma/2} - \frac{\frac{2}{3}\mathbf{p}_\pi \times \mathbf{q} + (i/3)[\mathbf{p}_\pi \mathbf{q}\cdot\boldsymbol{\sigma} - \mathbf{p}_\pi \cdot \mathbf{q}\boldsymbol{\sigma}]}{m_N - m_\Delta - p_\pi^2/2m_\Delta - E_\pi + i\Gamma/2} \right\} . \tag{2.21}$$

The $\mathbf{J}_\Delta$ is transverse, so that the continuity equation is

$$\omega(\rho_\pi + \rho_N) = \mathbf{q}\cdot(\mathbf{J}_{\rm sg} + \mathbf{J}_\pi + \mathbf{J}_N) \ , \tag{2.22}$$

and it is satisfied by the operators given above.

In parallel kinematics, $\theta_\pi = 0°$, and it can be easily verified that

$$\rho = \xi \sigma_z \tau_- \ , \tag{2.23}$$

$$J_{x,y} = \xi' \sigma_{x,y} \tau_- \ , \tag{2.24}$$

where ξ and ξ' are functions of the coupling constants and kinematical variables. Equations (2.23) and (2.24) impose conservation of the total angular momentum along the z axis. In the Saclay experiment $\theta_\pi = 4°$, therefore (2.23) and (2.24) are good approximations. The results for the $p(e,e'\pi^+)n$ cross section, using the Saclay [5] kinematics, are given in Table I, where the subscripts of σ denote initial and final nucleon spin, and $\sigma_{\downarrow\uparrow} = \sigma_{\uparrow\downarrow}$ and $\sigma_{\downarrow\downarrow} = \sigma_{\uparrow\uparrow}$. The first four rows give cross sections obtained from the N, π, sg, and Δ terms, respectively, while the last gives results including all terms. The current operators $\mathbf{J}_{\rm sg}$, $\mathbf{J}_\Delta$, and $\mathbf{J}_N$ give substantial contributions to the spin-flip cross section, while the charge operators ρ_π and ρ_N give most of the longitudinal cross section without spin flip. The total cross section is not dominated by the pion-pole term, and hence it cannot be used easily to measure the pion content of the proton.

The results given in Table I are for point hadrons, and the calculated cross section at $W = 1160$ MeV of 153 pb/MeV Sr^2, is much larger than the observed value [5] of 46±3 pb/MeV Sr^2, suggesting the importance of the vertex form factors omitted from the expressions of $\mathbf{J}$ and ρ. The primary interest of the present work is the ratio R_{dp} [Eq. (1.1)], which we hope to calculate with point hadron operators. Nevertheless it is interesting to estimate the magnitude of the form factors from the observed cross section.

In principle, each of the π, N, sg, and Δ contributions can have different form factors inserted in a way that preserves the validity of the continuity Eq. (2.24). In this crude estimate we approximate all of them by $G_E(q^2)F_{\pi NN}(p_\pi^2)$ appropriate for the nucleon-pole terms. Thus

$$|G_E(q^2)F_{\pi NN}(p_\pi^2)|^2 \sim \tfrac{46}{153} \tag{2.25}$$

for the $W = 1160$ MeV kinematics. Using the dipole fit

TABLE I. Cross sections for $p(e,e'\pi^+)n$ in pb/MeV Sr^2.

	1160 MeV			1230 MeV		
W	$\sigma_{\uparrow\uparrow}$	$\sigma_{\uparrow\downarrow}$	$\sigma_{\rm tot}$	$\sigma_{\uparrow\uparrow}$	$\sigma_{\uparrow\downarrow}$	$\sigma_{\rm tot}$
N	23.1	9.7	32.7	13.8	14.0	27.9
π	19.9	0.2	20.1	23.8	1.0	24.8
sg	0.2	51.3	51.5	0.0	50.1	50.1
Δ	0.1	10.6	10.7	0.4	55.0	55.5
All	97.2	55.3	153.0	78.5	140.0	219.0

[9] for the proton electric form factor $G_E(q^2)$ we obtain

$$F_{\pi NN}(p_\pi=232\ \mathrm{MeV}/c)\sim 0.69\ . \tag{2.26}$$

If the $F_{\pi NN}$ is parametrized as

$$F_{\pi NN}=\Lambda^2/(\Lambda^2+p_\pi^2)\ . \tag{2.27}$$

Eq. (2.27) implies a value of 347 MeV for Λ much smaller than that in models of N-N interaction. For example, the πNN Λ is 1.3 GeV in the Bonn [14] model.

III. PION ELECTROPRODUCTION FROM DEUTERON

The cross-section for the $d(e,e'\pi^+)2n$ reaction of an unpolarized electron beam with stationary deuterons is given by

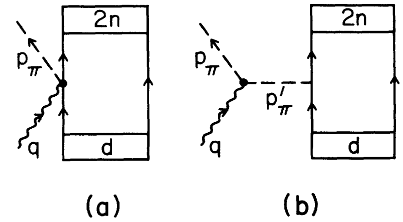

FIG. 2. Seagull and pion-pole contributions to the $d(e,e'\pi^+)2n$ cross section. The boxes marked d and $2n$ indicate the deuteron and final two-nucleon wave functions; see the caption of Fig. 1 for other notations.

$$\frac{d^4\sigma(M_d)}{d\Omega_e\, d\Omega_\pi dE_{e'} dE_\pi}=\sigma_M\frac{p_\pi E_\pi m_N p}{12(2\pi)^3}(v_L R_L+v_T R_T+v_{TT}R_{TT}-v_{LT}R_{LT})\ , \tag{3.1}$$

where M_d is the projection of the deuteron spin on the z $(=\hat{q})$ axis, and p is magnitude of the relative momentum between the outgoing neutrons. The p is kinematically determined; however, we have to integrate over the direction of p and sum over the spin states denoted by S, M_s of the $2n$ final state. The response functions R_L and R_T are given by

$$R_L=\sum_{S,M_s}\int\frac{d\Omega_p}{(2\pi)^3}|\rho(M_d,S,M_s,\Omega_p)|^2\ , \tag{3.2}$$

$$R_T=\sum_{S,M_s}\int\frac{d\Omega_p}{(2\pi)^3}[|J_x(M_d,S,M_s,\Omega_p)|^2 +|J_y(M_d,S,M_s,\Omega_p)|^2]\ , \tag{3.3}$$

where $\rho(M_d,S,M_s,\Omega_p)$ and $\mathbf{J}(M_d,S,M_s,\Omega_p)$ denote transition amplitudes between the initial deuteron in state M_d to the final $2n$ state $\mathbf{p}$, S, M_s via the charge and current operators. These amplitudes contain contributions from the π, N, Δ, and seagull terms:

$$\rho(M_d,S,M_s,\Omega_p)=\sum_\alpha\rho_\alpha(M_d,S,M_s,\Omega_p)\ , \tag{3.4}$$

$$\mathbf{J}(M_d,S,M_s,\Omega_p)=\sum_\alpha\mathbf{J}_\alpha(M_d,S,M_s,\Omega_p)\ , \tag{3.5}$$

$$\alpha=\pi,N,\Delta,\mathrm{sg}\ . \tag{3.6}$$

Expressions for R_{TT} and R_{LT} can be trivially obtained from Eq. (2.1) using the above notation.

In the Saclay experiments p is small, $p^2/m\sim 10$ MeV, and hence the final state interaction between the neutrons cannot be neglected. In the present work the initial deuteron as well as the final $2n$ wave functions are calculated exactly using the nonrelativistic Argonne model [15] of the N-N interaction. The final-state interactions of the outgoing pion with the other nucleon are neglected in this initial study.

There are no NN or $N\Delta$ intermediate states in the transitions illustrated in Fig. 2 via the π and sg terms. Therefore, the expressions for their contributions are simpler:

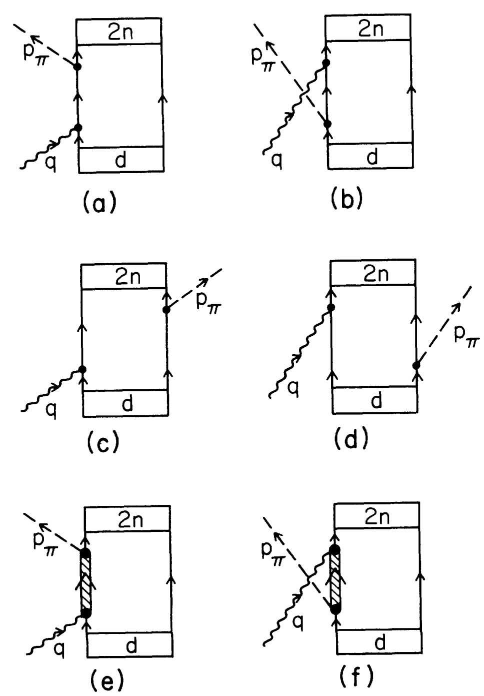

FIG. 3. The nucleon- and Δ-pole contributions to the $d(e,e'\pi^+)2n$ cross section. See captions of Figs. 1 and 2 for notation.

$$\mathbf{J}_{sg}(M_d,S,M_s,\Omega_p)=2\frac{if}{m_\pi\sqrt{E_\pi}}\langle S,M_s,\Omega_p|e^{i(\mathbf{q}-\mathbf{p}_\pi)\cdot(\mathbf{r}_1-\mathbf{r}_2)/2}\tau_-(1)\boldsymbol{\sigma}(1)|M_d\rangle\ , \tag{3.7}$$

$$\rho_\pi(M_d,S,M_s,\Omega_p)=2\frac{if}{m_\pi\sqrt{E_\pi}}\frac{2E_\pi-\omega}{[(\omega-E_\pi)^2-(\mathbf{q}-\mathbf{p}_p i)^2-m_\pi^2]}\langle S,M_s,\Omega_p|e^{i(\mathbf{q}-\mathbf{p}_\pi)\cdot(\mathbf{r}_1-\mathbf{r}_2)/2}\tau_-(1)\boldsymbol{\sigma}(1)\cdot(\mathbf{p}_\pi-\mathbf{q})|M_d\rangle\ , \tag{3.8}$$

$$\mathbf{J}_\pi(M_d,S,M_s,\Omega_p)=(2\mathbf{p}_\pi-\mathbf{q})/(2E_\pi-\omega)\rho_\pi(M_d,S,M_s,\Omega_p)\ . \tag{3.9}$$

In the above equations the contribution from nucleon 1 is doubled to take into account that from nucleon 2. In parallel kinematics, $2\mathbf{p}_\pi-\mathbf{q}$ is along the z axis, and $\mathbf{J}_\pi$ does not have a transverse component.

The nucleon- and Δ-pole contributions have NN and $N\Delta$ intermediate states as illustrated in Fig. 3. In this initial study we approximate them by plane waves $|\mathbf{k}_1\mathbf{k}_2\eta\rangle$, where $\mathbf{k}_1$ and $\mathbf{k}_2$ are momenta of particles 1 and 2, and η is a collective variable for the spin-isospin state. The contribution of diagram 3(a) to the transition amplitude via the charge operator is given by

$$T_\rho[3(\mathrm{a})]=2\int\frac{d^3k_1\,d^3k_2}{(2\pi)^6}\sum_\eta\langle S,M_s,\Omega_p|H_{\pi NN}|\mathbf{k}_1\mathbf{k}_2\eta\rangle\frac{1}{\omega+E_d-(\hbar^2/2m_N)(k_1^2+k_2^2)+i\varepsilon}\langle\mathbf{k}_1\mathbf{k}_2\eta|H_{\gamma NN}|M_d\rangle\ . \tag{3.10}$$

Let $\mathbf{r}$ and $\mathbf{r}'$ be the relative coordinates in the spatial integrals of the $H_{\pi NN}$ and $H_{\gamma NN}$ matrix elements. We obtain

$$T_\rho[3(\mathrm{a})]=-2\frac{ifm_N}{4\pi m_\pi\sqrt{E_\pi}}\int d^3r\,d^3r'\,\Psi_{2n}^\dagger(\mathbf{r},S,M_s,\Omega_p)\boldsymbol{\sigma}_1\cdot\mathbf{p}_\pi\tau_-(1)e^{-i\mathbf{p}_\pi\cdot\mathbf{r}/2}\frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}e^{i\mathbf{q}\cdot\mathbf{r}'/2}\Psi_d(\mathbf{r}',M_d)\ , \tag{3.11}$$

$$k^2/m_N=\omega+E_d-q^2/4m_N\ . \tag{3.12}$$

The other diagrams [3(b)–3(d)] cannot contribute to $\rho_N(M_d,S,M_s,\Omega_p)$; thus

$$\rho_N(M_d,S,M_s,\Omega_p)=T_\rho[3(\mathrm{a})]\ . \tag{3.13}$$

The expressions for $\mathbf{J}_N\ (M_d,S,M_s,\Omega_p)$ and $\mathbf{J}_\Delta\ (M_d,S,M_s,\Omega_p)$ are obtained in a similar way:

$$\begin{aligned}\mathbf{J}_\mathbf{N}(M_d,S,M_s,\Omega_p)=2\frac{f}{8\pi m_\pi\sqrt{E_\pi}}\int d^3r\,d^3r'\Psi_{2n}^\dagger(\mathbf{r},S,M_s,\Omega_p)\frac{1}{|\mathbf{r}-\mathbf{r}'|}\\
\times\{\boldsymbol{\sigma}_1\cdot\mathbf{p}_\pi\tau_-(1)e^{-i\mathbf{p}_\pi\cdot\mathbf{r}/2}e^{ik|\mathbf{r}-\mathbf{r}'|}e^{i\mathbf{q}\cdot\mathbf{r}'/2}\boldsymbol{\sigma}_1\times\mathbf{q}\mu_p\\
+\boldsymbol{\sigma}_1\times\mathbf{q}\mu_n e^{i\mathbf{q}\cdot\mathbf{r}/2}e^{ik'|\mathbf{r}-\mathbf{r}'|}e^{-i\mathbf{p}_\pi\cdot\mathbf{r}'/2}\boldsymbol{\sigma}_1\cdot\mathbf{p}_\pi\tau_-(1)\\
+\boldsymbol{\sigma}_2\cdot\mathbf{p}_\pi\tau_-(2)e^{i\mathbf{p}_\pi\cdot\mathbf{r}/2}e^{ik|\mathbf{r}-\mathbf{r}'|}e^{i\mathbf{q}\cdot\mathbf{r}'/2}\boldsymbol{\sigma}_1\times\mathbf{q}\mu_n\\
+\boldsymbol{\sigma}_1\times\mathbf{q}\mu_n e^{i\mathbf{q}\cdot\mathbf{r}/2}e^{ik'|\mathbf{r}-\mathbf{r}'|}e^{i\mathbf{p}_\pi\cdot\mathbf{r}'/2}\boldsymbol{\sigma}_2\cdot\mathbf{p}_\pi\tau_-(2)\\
-2\boldsymbol{\sigma}_1\cdot\mathbf{p}_\pi\tau_-(1)e^{-i\mathbf{p}_\pi\cdot\mathbf{r}/2}e^{ik|\mathbf{r}-\mathbf{r}'|}e^{i\mathbf{q}\cdot\mathbf{r}'/2}\nabla'\}\Psi_d(\mathbf{r}',Md)\ ,\end{aligned} \tag{3.14}$$

where

$$k'^2/m_N=E_d-p_\pi^2/4m_N-E_\pi\ . \tag{3.15}$$

The first four terms of $\mathbf{J}_\mathbf{N}\ (M_d,S,M_s,\Omega_p)$ come from diagrams 3(a)–3(e), respectively, and the last gives the contribution of the proton convection current to diagram 3(a). Finally,

$$\begin{aligned}\mathbf{J}_\Delta(M_d,S,M_s,\Omega_p)=-2\frac{f_\Delta\mu^*}{4\pi m_\pi\sqrt{E_\pi}}\frac{m_\Delta}{m_N+m_\Delta}\int d^3r\,d^3r'\,\Psi_{2n}^\dagger(\mathbf{r},S,M_s,\Omega_p)\frac{1}{|\mathbf{r}-\mathbf{r}'|}e^{-i\zeta(\mathbf{q}-\mathbf{p}_\pi)\cdot\mathbf{r}}\\
\times\left\{e^{-i\zeta'\mathbf{p}_\pi\cdot\mathbf{r}}\left[\frac{2}{3}\mathbf{p}_\pi\times\mathbf{q}-\frac{i}{3}(\mathbf{p}_\pi\boldsymbol{\sigma}_1\cdot\mathbf{q}-\mathbf{p}_\pi\cdot\mathbf{q}\boldsymbol{\sigma}_1)\right]e^{iK|\mathbf{r}-\mathbf{r}'|}e^{i\zeta'\mathbf{q}\cdot\mathbf{r}'}\right.\\
\left.-e^{i\zeta'\mathbf{q}\cdot\mathbf{r}}\left[\frac{2}{3}\mathbf{p}_\pi\times\mathbf{q}+\frac{i}{3}(\mathbf{p}_\pi\boldsymbol{\sigma}_1\cdot\mathbf{q}-\mathbf{p}_\pi\cdot\mathbf{q}\boldsymbol{\sigma}_1)\right]e^{iK'|\mathbf{r}-\mathbf{r}'|}e^{-i\zeta'\mathbf{p}_\pi\cdot\mathbf{r}'}\right\}\\
\times(\sqrt{2}/3)\tau_-(1)\Psi_d(\mathbf{r}',M_d)\ ,\end{aligned} \tag{3.16}$$

where

$$\zeta=\frac{1}{2}\left[\frac{m_N-m_\Delta}{m_N+m_\Delta}\right], \tag{3.17}$$

$$\zeta'=\frac{m_N}{m_N+m_\Delta}, \tag{3.18}$$

$$\frac{1}{2m_\Delta\zeta'}K^2=E_d+\omega-m_\Delta+m_N-\frac{q^2}{2(m_N+m_\Delta)}+\frac{i\Gamma}{2}. \tag{3.19}$$

$$\frac{1}{2m_\Delta\zeta'}K'^2=E_d-m_\Delta+m_N-E_\pi-\frac{p_\pi^2}{2(m_N+m_\Delta)}+\frac{i\Gamma}{2}. \tag{3.20}$$

The calculated $d(e,e'\pi^+)2n$ cross sections for polarized targets are shown in Figs. 4 and 5 for $W=1160$ and 1230 MeV, respectively. These cross sections have a narrow peak at the threshold due to the strong final-state interactions of the two neutrons in the $^1S_0$ quasibound state. They depend significantly on the value of M_d in the region of the $^1S_0$ peak, particularly at $W=1160$ MeV.

Let M_{2n} be the z component of the total angular momentum of the two neutrons in their center-of-mass frame. Conservation of angular momentum implies that in the near parallel kinematics ($\theta_\pi \sim 0°$) of the Saclay experiment:

$$M_{2n}=M_d \tag{3.21}$$

for longitudinal charge induced transitions and

$$M_{2n}=M_d\pm 1 \tag{3.22}$$

for transverse current induced transitions. The $^1S_0$ state has $M_{2n}=0$; hence the cross section in the peak region comes entirely from longitudinal (transverse) photons when $M_d=0$ (±1). In Table I, the $\sigma_{\uparrow\uparrow}$ for $p(e,e'\pi^+)n$ is essentially longitudinal, while the $\sigma_{\uparrow\downarrow}$ is transverse. At $W=1160$ MeV the longitudinal cross section for the proton is almost twice the transverse; hence the deuteron cross section in the peak region is much larger for $M_d=0$ than for $M_d=\pm1$ (Fig. 4). The spin-isospin structure of the effective charge and current operators is given by Eqs. (2.23) and (2.24). It can be easily verified that the ratio of their matrix elements is

$$\frac{\xi\langle S=0,nn|\sigma_z(1)\tau_-(1)+\sigma_z(2)\tau_-(2)|M_d=0\rangle}{\xi'\langle S=0,nn|\sigma_x(1)\tau_-(1)+\sigma_x(2)\tau_-(2)|M_d=1\rangle}=\sqrt{2}\frac{\xi}{\xi'}. \tag{3.23}$$

Thus we expect that, at the $^1S_0$ peak,

$$\frac{d\sigma(M_d=\pm1)}{d\sigma(M_d=0)}\approx\frac{1}{2}\frac{\sigma_{\uparrow\downarrow}}{\sigma_{\uparrow\uparrow}}. \tag{3.24}$$

The calculated values shown in Figs. 4 and 5 and Table I are consistent with this estimate.

The ratios R_{dp} [Eq. (1.1)] have been calculated by integrating the cross section for unpolarized deuterons over the missing mass region covered in the Saclay experiment. The calculated values of 0.75 (0.81) are in fair agreement with the observed values of 0.8 ± 0.05 (0.75 ± 0.07) at $W=1160$ (1232) MeV.

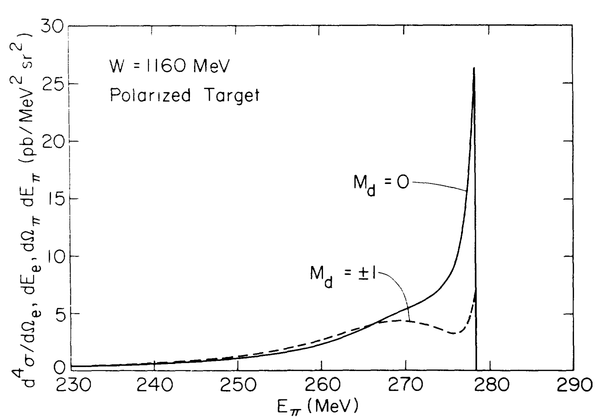

FIG. 4. The pion electroproduction cross section for polarized deuterons at $W=1160$ MeV. The full and dashed lines give results obtained for $M_d=0$ and $M_d=\pm1$ states, respectively. These results are for point hadrons without any vertex form factors.

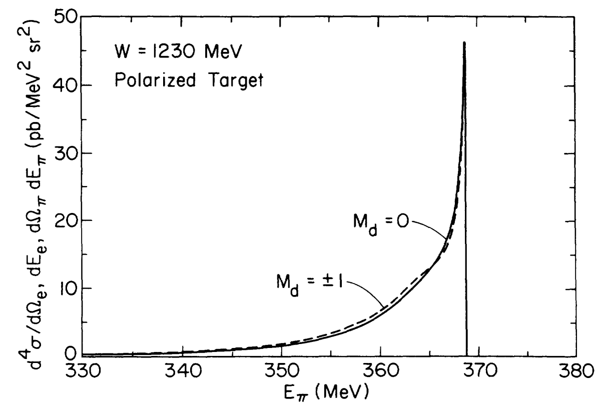

FIG. 5. The pion electroproduction cross section for polarized deuterons at $W=1230$ MeV. See caption of Fig. 4 for notation.

IV. CONCLUSIONS

The present results suggest that the quenching of pion production rate in the $d(e,e'\pi^+)2n$ reaction observed at Saclay is mostly due to the final state interactions between the two outgoing neutrons. The main approximation in this work is the neglect of the final-state interactions of the pion. Simple estimates using πN cross sections and the deuteron radius suggest that the pion final-state interactions should have less than 5% effect on the π^+ production rate. However, since the observed quenching is only $\sim$20% it may be necessary to include the final-state interactions of the pion to calculate it accurately.

The second, and perhaps more interesting, prediction of this work is the large effect of deuteron spin-direction on pion production rates. Hopefully this effect can be measured using tensor polarized deuterium targets. Its observation will provide a better understanding of pion electroproduction on deuteron, the simplest of all nuclei.

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