Reply to "Comment on 'Orbital $M1$ versus $E2$ strength in deformed nuclei: A new energy-weighted sum rule' "

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A recent article by us [Phys. Rev. C **47**, 2604 (1993)] has received criticism by Hamamoto and Nazarewicz (HN). In what follows we reply to points (a) to (f) raised in their work.

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(a),(b) The authors of Ref. [2] say that from their model in Ref. [3] they get

$$S_{\rm HE}(M1^{\rm orb}) > S_{\rm LE}(M1^{\rm orb}) , \tag{1}$$

in contrast to the conclusion that follows from our model in Ref. [1]

$$S_{\rm HE}(M1^{\rm orb}) = S_{\rm LE}(M1^{\rm orb}) . \tag{2}$$

It is not so surprising that both conclusions are different since they result from different models. In Ref. [1] a self-consistent deformed mean field is considered, while in Refs. [2,3] HN study the role of residual (particularly isovector quadrupole–quadrupole) interactions. As long as decay strengths into the ground state of high energy $M1$ excitations remain an open question, Eqs. (1) and (2) are nothing more (nor less) than outcomes from different models that call for experimental verification. However, it should be pointed out that (i) recent microscopic (QRPA) calculations [4] that take into account residual interactions and that implement the self-consistency of the mean field support our result in Eq. (2), (ii) the HN model gives much larger values for $S_{\rm LE}$ [see Eqs. (3) and (4) in Ref. [2]] than the experimental ones, even for small values of deformation [see Table I [1] of Ref. [1]].

Residual interactions are important to get the details of individual $M1$ excitations in agreement with experiment, but the choice of the residual interactions and of the valence space is not free from ambiguities and the magnitude of the LEWSR for magnetic excitations depends much on this choice. For the evaluation of the LEWSR this dependence can in principle be avoided on the basis of Thouless theorem [5], according to which only knowledge of the Hartree-Fock ground state is needed to compute LEWSR's for RPA excitations. This is the essential assumption underlying the approach taken in Ref. [1], and particularly in Secs. III and IV, where we exploited the properties following from the self-consistency of the mean field and the well-known phenomenology of rotational ground-state bands to derive a semiempirical equation [Eq. (37)] relating low-lying $M1$ and $E2$ excitations.

(c) The reason why g_R should appear in Eq. (14) of Ref. [1] comes from the separation between intrinsic and collective degrees of freedom (see, for instance, Ref. [6]). In the model considered in Secs. III and IV there is no separation into valence and core particles; in principle, all particles are allowed to contribute to orbital $M1$ excitations and to the collective rotation, but there is separation between collective and intrinsic wave functions (D matrix and Slater determinant) and the corresponding separation for operators. This is why we have to use the intrinsic orbital $M1$ operator given in Eq. (14) to remove the spurious rotation. This was already explained in Ref. [1] where we discussed the analogy with the effective charges for electric dipole excitations.

That Eq. (14) is the proper operator to be used in our model can be shown in any microscopic theory of the collective modes such as projected Hartree-Fock or variation after projection [6]. The simplest intuitive argument to derive Eq. (14) is that the orbital $M1$ operator

$$\boldsymbol{\mu}^{\rm orb} = \sum_i g_l^i \boldsymbol{l}_i \tag{3}$$

separates into a collective and an intrinsic operator. The collective operator must be

$$\boldsymbol{\mu}_{\rm col}^{\rm orb} = g_R \boldsymbol{L} \tag{4}$$

with $\boldsymbol{L} = \sum_i \boldsymbol{l}_i$, the total orbital angular momentum operator that contributes only to static magnetic moments of the band, not to excitations. Therefore the intrinsic operator must be

$$\begin{aligned}\boldsymbol{\mu}_{\rm int}^{\rm orb} &= \boldsymbol{\mu}^{\rm orb} - \boldsymbol{\mu}_{\rm col}^{\rm orb} \\ &= \sum_i (g_l^i - g_R)\boldsymbol{l}_i = (1-g_R)\boldsymbol{L}_\pi - g_R \boldsymbol{L}_\nu ,\end{aligned} \tag{5}$$

which gives back Eq. (14) of Ref. [1] (bare g_l^i values are used). Within the projected Hartree-Fock approximation one can show that using this operator is equivalent to orthogonalization of the $M1$ excitations to the spu-

[1] We note that there is a misprint in the list of experimental references under Table I in Ref. [1]: In "d," Ref. [3] should read Ref. [23].

rious rotational mode, with the collective gyromagnetic ratio given by a microscopic expression ($g_R = \mathcal{J}^\pi/\mathcal{J} \simeq \langle L_\pi^2\rangle/\langle L^2\rangle$) [6,7]. This equation has the right limit when g_R=0, and only becomes purely isovector when $g_R = \frac{1}{2}$, as it should. The limit $g_R = 0$ corresponds to no collective rotational mode (no ground-state rotational band) and therefore Eq. (5) goes into Eq. (3) as it should. The reason why $\boldsymbol{\mu}^{\rm orb}$ is often replaced by $\frac{1}{2}(\boldsymbol{L}_\pi - \boldsymbol{L}_\nu)$ is that in the scheme of residual interactions, as that discussed in Sec. II of Ref. [1] (see also Ref. [11]), the residual interaction commutes with the isoscalar part of $\boldsymbol{\mu}$, and therefore only the isovector part of $\boldsymbol{\mu}^{\rm orb}$ contributes to the LEWSR.

(*d*) The sum $B(M1^{\rm orb})$ value of the low frequency modes in the deformed H.O. model is linearly proportional to the deformation when pairing is neglected [8,9]. When pairing is taken into account, a δ^2 dependence of the $M1$ strengths at low energy is obtained [9]. However, this does not enter in contradiction with what we say in Ref. [1], nor does it invalidate our results there. It only means that the non-energy-weighted sum rule depends strongly on pairing, while, as already stated in Ref. [1], the LEWSR is not modified by explicitly including the pairing interaction between like particles because this interaction commutes with the orbital $M1$ operator. There is no contradiction between our model and experimental facts, and Table I of Ref. [1] shows, indeed, that the phenomenological expression we derive for $S_{\rm LE}(M1^{\rm orb})$ [Eq. (37) of Ref. [1], which goes beyond the H.O. model] describes quite nicely the experimental facts for all deformed isotopes for which the relevant experimental information is available.

(*e*) Within the scheme of Secs. III and IV of Ref. [1], removal of spurious contributions is taken care of by the use of the intrinsic operator. See Ref. [1] and our discussion in point (*c*) above.

(*f*) We agree with the two first sentences of this paragraph in Ref. [2]. This is why, as clearly stated in Ref. [1], we used the deformed H.O. model as a guideline to derive new relations between different observables that allow us to arrive at our final expression in Eq. (37) (see Sec. IV of Ref. [1]), instead of stopping at Eq. (16) and using the model in a conventional way. For instance, we avoided using $g_R = Z/A$, $\delta_o/\delta = 1$, and other similar approximations that are frequently used, which are in most cases quite unrealistic and do not strictly follow from our model. As clearly stated in Ref. [1], our basic approach in this respect is to consider the quantities Σ_i^ρ, ω_i as effective quantum numbers and H.O. frequencies corresponding to the expectation values of the observables in the true intrinsic ground state. This allows us to replace said quantities (or combinations of them) by the magnitudes corresponding to known properties of the ground-state band. Most of the remarks made in paragraph (*f*) of Ref. [2] are explicitly or implicitly taken into account in Ref. [1]. It should be understood that we are not discussing spin excitations in this paper and, indeed, the experimentalists take pains to isolate the orbital from the spin strength by exciting the $M1$ modes with both electrons and protons. We have discussed spin contributions in other works [10–12]. As already stated in point (*d*) above and in Ref. [1], the pairing interaction between like nucleons commutes with $\boldsymbol{\mu}^{\rm orb}$; it does not affect the LEWSR, nor is it expected to change drastically the relationship in Eq. (2). Likewise, high j components are expected to affect the detailed $M1$ strength distribution (i.e., the individual excitations) more than the LEWSR. In any case pairing, high j components, etc., affect both the orbital $M1$ excitations and the magnitudes appearing in the right-hand side of Eq. (37) of [1]. Hence, to the extent that the self-consistency of the mean field is properly implemented, such effects can be expected to be approximately taken care of by Eq. (37) in [1]. From the above arguments, one may also extract a rationale for the good agreement of this equation with the experiment seen in Table I of [1].

In summary, most of the issues raised by HN are answered by looking at Ref. [1] and at previous publications by us (see, in particular, Refs. [7] and [9]–[11]). The main physics we are trying to convey in Ref. [1] is that there exist collective orbital modes in nuclei, whose identity is not destroyed by the various residual interactions and single particle energy splittings. We believe this is the central point, that also emerges from microscopic calculations [9–11], concerning the existence of scissors modes. We did not intend to imply that the H.O. model is by itself sufficient to study the richness of $M1$ strength functions (which would be quite unrealistic). We close by noting that the fundamental difference between our approach and that of Hamamoto and Nazarewicz [2,3] can be summarized as follows. They emphasize the role of residual interactions and use schematic interactions to do detailed calculations of magnetic dipole excitations. We, on the other hand, emphasize the role of the self-consistency of the mean field and use a schematic approach to unearth the nature of collectivity. By showing that the double commutator of the orbital magnetic dipole operator with the quadrupole operator gives us back the quadrupole operator, we are able to establish a clear-cut connection between $B(M1)$ and $B(E2)$.

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