Properties of intruder bands in the Os-Pt-Hg-Pb region

G. D. Dracoulis
Department of Nuclear Physics, Research School of Physical Sciences and Engineering, The Australian National University, Canberra ACT 0200, Australia
(Received 5 October 1993)

Band mixing is used to reproduce the collective bands associated with shape coexistence in even-even osmium, platinum, mercury, and lead nuclei. The properties of the unperturbed bands are examined. The dependence of the unperturbed $2^+ \rightarrow 0^+$ energies of the prolate intruder bands on $N_\pi N_\nu$ is similar to that of the unperturbed normal bands if the intruder configurations are assumed to involve 4-proton, 4-hole excitations. The variation is reproducible using a restricted 2-component model of interacting proton and neutron shells.

PACS number(s): 21.10.Re, 21.60.−n, 27.70.+q

Intruder bands arising from shape coexistence are a well established feature in the light mercury isotopes, as reviewed recently by Wood *et al.* [1]. Analogous effects have been suggested in the light platinum nuclei and osmium nuclei, and most recently in the very neutron-deficient lead nuclei [2]. They attract the label of "intruder" not from the traditional association of the parity intruders in the Nilsson model, but from the point of view of excitations of particles from below a shell, into orbitals above the shell, which because of their slope with deformation, intrude into the low-energy part of the spectrum. A pertinent feature is that the moments of inertia of the (prolate) bands are very similar, implying a common correlated configuration. The objective of the present note is to examine the possible classification of the bands in terms of the number of valence particles involved in the configurations.

The similarity alluded to above is apparent in the plot of excitation energy against $I(I+1)$, shown in the lower panel of Fig. 1, for some selected cases. At medium spins, the bands are parallel, the main differences between isotopes being the difference in excitation energy of the intruder band relative to the "normal" band. A reconstruction of the yrast and nonyrast states in ^{184}Hg using two mixing bands is shown in the upper panel of the same figure.

Included in the figure is the recent result [2,3] for ^{186}Pb, which, with ^{188}Pb, represent the first candidates for the prolate-deformed bands in lead. These nuclei are predicted [4–6] to show *three* minima in the potential well; a deep minimum at sphericity and secondary minima at oblate and prolate deformation. (Evidence for the *oblate* minimum in the lead nuclei has been presented previously, e.g., Ref. [7].) Although this situation differs from that in mercury and platinum because of the presence of the favored spherical configuration, the similarity between the moments of inertia of the collective bands in lead and mercury was one of the arguments [2] for associating the bands in lead with the prolate deformation.

A more sensitive view of the band structure is given by the alignments, which are shown in Fig. 2. The reference parameters have been chosen to suit the prolate deformed band in the mercury isotopes. Since the alignment is given by

$$\begin{aligned} i &= I_x(\omega) - (\mathfrak{I}_0\omega + \mathfrak{I}_1\omega^3)_{\mathrm{ref}} \\ &= [\mathfrak{I}_0(g) - \mathfrak{I}_0(d)]\omega + [\mathfrak{I}_1(g) - \mathfrak{I}_1(d)]\omega^3 \ , \end{aligned} \tag{1}$$

where g and d are used to label the normal and intruder bands, the coefficient of the term linear in ω will be negative when there is a change from a small to a large deformation. Superimposed on that effect, as in the cases shown in Fig. 2, a negative slope occurs because the first transition is also largely a transition between bands, hence it reflects the difference in excitation energy be-

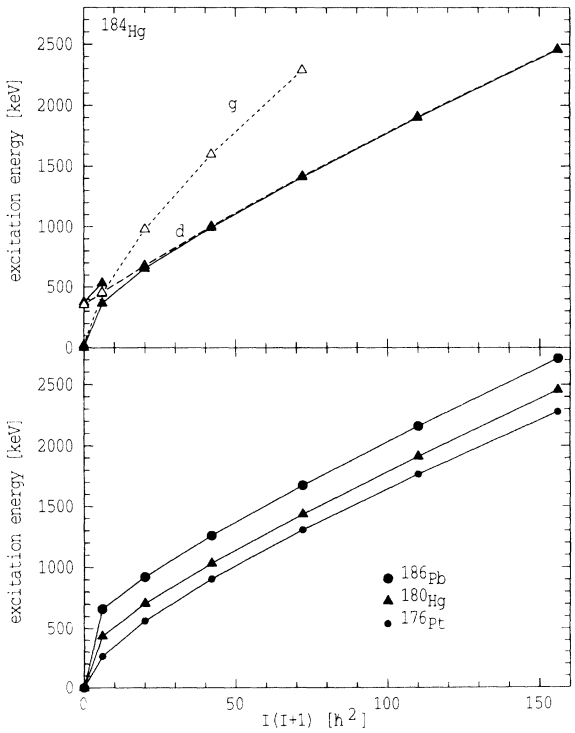

FIG. 1. Excitation energies of yrast bands in selected nuclei (lower panel) and for yrast and nonyrast states in ^{184}Hg (upper panel). The dashed lines and open symbols in the upper panel show the unperturbed g and d bands, which when mixed reproduce experiment.

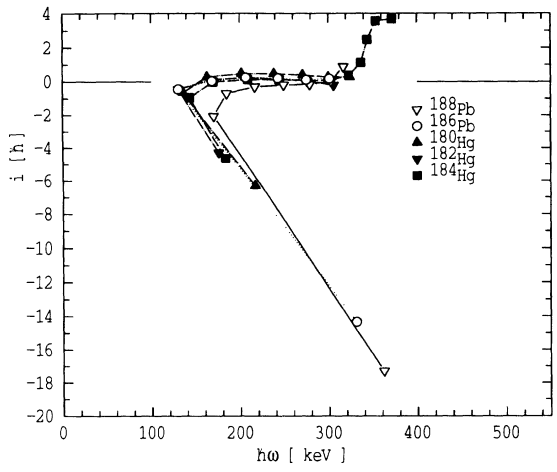

FIG. 2. Comparison of alignments for very neutron-deficient mercury, and lead nuclei. The same reference parameters, $\mathfrak{I}_0 = 27$ MeV^{-1} $\hbar^2$ and $\mathfrak{I}_1 = 190$ MeV^{-3} $\hbar^4$, have been used for all bands.

tween them. It is also clear from the figure that the bands at large deformation, in the spin and frequency range where the apparent alignment is close to zero, are not identical, as evidenced by the ^{188}Pb case. It has a slightly negative alignment in this region, implying a slightly lower collectivity for its intruder band.

If subtle differences between the bands in different nuclei are to be interpreted, it is prudent to remove the perturbations caused by the band crossings. The approach we have used previously (as discussed, for example, in Ref. [8]) is to use band mixing to reproduce experimental energies, and then to examine the properties of the *unperturbed* bands so deduced.

The properties of the mixing bands that can be used are well constrained when the perturbations are localized to the crossing region, as occurs when the shape differences are large. The parameters are constrained further if nonyrast states (such as excited 0^+ states) are also known. That is the case in some of the light platinum and mercury isotopes, and we have identified corresponding states in the light osmium isotopes [9,10]. Reproduction of the nonyrast states is included in the present band-mixing calculations.

The results of these and other calculations are listed in Table I. We focus here on the energy of the $2^+ \rightarrow 0^+$ transition, a simple band property, often used in global examinations of collective properties and their classification [11]. (The behavior of an effective quadrupole moment obtained from the same transition energies was shown in Ref. [12]. A more complicated measure of the unperturbed moments of inertia was used in the cases listed in Ref. [13].) The upper panel of Fig. 3 shows the observed, yrast $2^+ \rightarrow 0^+$ energies for a range of nuclei, plotted against the product of valence proton bosons, N_π, and neutron bosons, N_ν, counted from the nearest closed shells. Some of the irregularities presumably arise from the perturbations caused by band crossings. The lower panel shows the *unperturbed* $2^+ \rightarrow 0^+$ energies extracted from the band-mixing calculations for both the normal

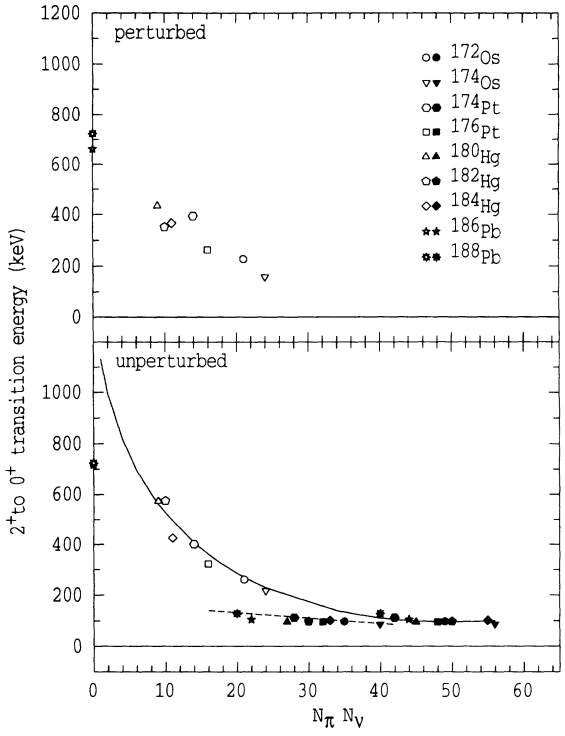

FIG. 3. Observed energies of $2^+ \rightarrow 0^+$ transitions (upper panel) and unperturbed energies extracted from the band-mixing calculations (lower panel). The filled symbols represent the intruder bands. The dashed line in the lower panel connects intruder bands if counted as proton 2-particle, 2-hole excitations. The solid line is the calculated dependence from a modified form of the 2-component model discussed in the text. (The points for the unperturbed $2^+ \rightarrow 0^+$ transitions for the normal states in Pb are to be seen as lower limits.)

and intruder bands with the intruder values plotted at two alternative values of $N_\pi N_\nu$, as will be discussed below. (Note that the points for the normal bands in the two lead cases should be seen as limits, since the band crossing apparently occurs near or below the 2^+ state of the spherical configuration, so that its energy cannot be determined without more experimental data than is presently available.)

Counting of valence particles for the intruder band follows the procedure of giving equal weight to holes and particles. In lead, for example, the normal band has $N_\pi = 0$, while if the intruder involves a 2-proton excitation, a 2-hole, 2-particle state with $N_\pi = 2$ is formed. In ^{186}Pb, therefore, the number of valence neutron pairs is $N_\nu = (126 - 104)/2 = 11$, so that the product $N_\pi N_\nu = 0$ and 22 for the normal and intruder states, respectively. If the excitation forming the intruder involves excitation of two proton pairs, it is 4-hole, 4-particle in character giving N_π (intruder)= 4 and $N_\pi N_\nu = 44$, etc.

Various schemes have recently been suggested for classifying such states as members of multiplets (somewhat analogous to isobaric analog multiplets) expected from concepts such as I spin, which treats the protons and neutrons separately, and F spin, which combines them.

(See, for example, Barrett *et al.* [13], Nazarewicz [6], and Heyde *et al.* [14].) Depending on the validity of these concepts, bands in different nuclei identifiable as multiplet members could have similar properties.

Bengtsson and Nazarewicz [5] and Nazarewicz [6] discussed shape coexistence in lead nuclei using the configuration constrained shell-correction approach and suggested that, while the excitation of a pair of $h_{9/2}$ protons on the oblate side into the $\Omega^\pi = \frac{9}{2}^-[505]$ orbital was the main component in the oblate-deformed structure, more complicated configurations were involved on the prolate side. The preference for prolate deformation was driven by occupation of the $\frac{1}{2}^-[541]$, $h_{9/2}$ proton orbital, but the wave function contained $[N = 4]^{-4}(1h_{9/2} \otimes 2f_{7/2})^4$ (i.e., 4-hole, 4-particle) and even higher-order excitations. (Separation of those configurations from mean field and pairing is nontrivial, as stressed recently by Tajima *et al.* [15].)

Although boson counting in such complex cases might be problematic, there is a correlation evident in the lower panel of Fig. 3. The scatter in the upper panel is partly removed by the use of *unperturbed* normal states (open symbols), while the *unperturbed* values for the intruder bands (filled symbols) continue the monotonic dependence on $N_\pi N_\nu$ if the proton configurations are "counted" as 4-hole, 4-particle excitations. The alternative values obtained assuming 2-hole, 2-particle excitations are connected by the dashed line.

While a monotonic dependence on $N_\pi N_\nu$ for both normal and intruder states has appeal, and could be taken as supporting the more complicated configurations suggested, it is not necessarily expected. For example, Heyde and Sau [16] use a schematic 2-component shell model to take into account proton and neutron degrees of freedom explicitly, albeit by assuming notional single-j proton and neutron shells,which interact via a quadrupole-quadrupole force. The 2^+ state energy as a function of the shell filling is given in their model by

$$E_{2^+\to 0^+} = \epsilon_2^{+(0)} - \frac{\kappa F}{5}\left[N_\pi N_\nu\left(1 - \frac{N_\pi}{\Omega_\pi}\right)\left(1 - \frac{N_\nu}{\Omega_\nu}\right)\right]^{1/2}, \quad (2)$$

TABLE I. Properties of the unperturbed bands used in the band-mixing calculations.

Nucleus	band	E_0 (keV)[a]	V (keV)[b]	$E_{2^+\to 0^+}$	Calculation (Ref.)	Experiment (Ref.)
^{172}Os	g	90	245	260	[10][c]	[10]
	d	669		97		
^{174}Os	g	199	264	216	[9][c]	[9]
	d	349		87		
^{174}Pt	g	10	82	401	This work	[18]
	d	702		112		
^{176}Pt	g	82	210	322	[13,19][d]	[20,21]
	d	361		96		
^{178}Pt	g	271	211	265	[13,16][d]	[20,22]
	d	164		95		
^{180}Hg	g	35	121	573	[13]	[23]
	d	422		95		
^{182}Hg	g	64	144	574	[13]	[24]
	d	328		97		
^{184}Hg	g	20	84	426	This work[c]	[25,26]
	d	354		101		
^{186}Pb	g	2	44	(717)	This work	[2,3]
	d	590		105		
^{188}Pb	g	2	29	(728)	This work	[2]
	d	678		128		

[a]Unperturbed band-head energies. See Ref. [8] for formulation of the mixing bands in the variable moment-of-inertia model.
[b]Spin-independent interactions.
[c]Band-mixing extended to include reproduction of nonyrast states.
[d]Band-mixing includes reproduction of nonyrast 0^+ state only.

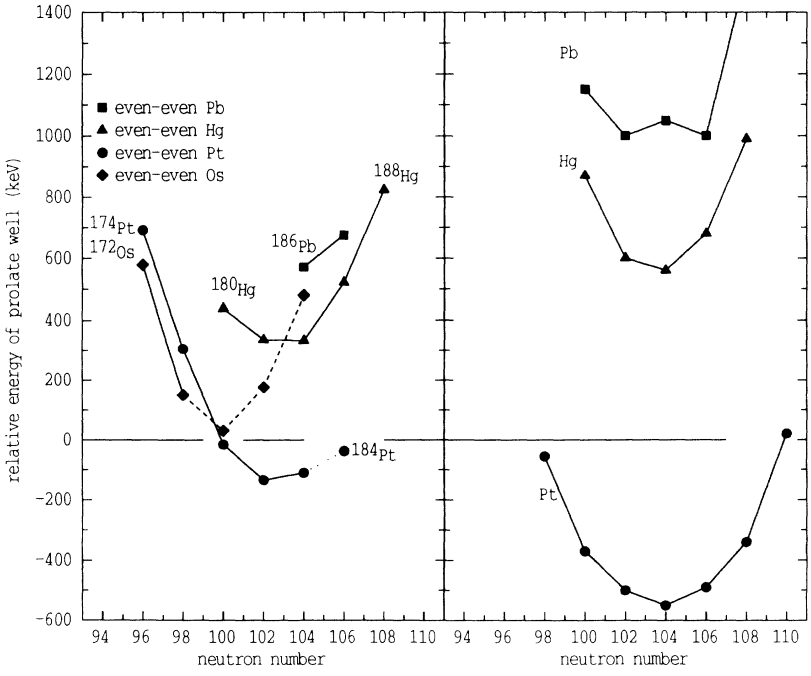

FIG. 4. Relative energies of the prolate coexisting wells from experiment (left) and theory (right). Experimental values correspond to the unperturbed bandhead energy differences. (The heavier osmium cases are dashed since the independence of the coexisting wells in those cases is open to doubt (Ref. [9]).) Theoretical values correspond to the calculated prolate well minima relative to the oblate minima in the Hg cases, and to the spherical minimum in the Pb cases.

where F depends on the radial matrix elements and would be constant for a given set of orbitals and κ defines the interaction strength between protons and neutrons.

Equation (2) does not give a single curve as a function of the product $N_\pi N_\nu$, since it depends separately on the values of the degeneracies Ω and number of pairs N for the proton and neutron shells. Significant differences would be expected for the same value of $N_\pi N_\nu$ produced by different values of $(N_\pi + N_\nu)$. The single curve shown in the lower panel of Fig. 3 is instead forced by expanding Eq. (2) and replacing $(N_\pi + N_\nu)$ by the values of $2\sqrt{N_\pi N_\nu}$. (The other parameter values, $\Omega_\pi = \Omega_\nu = 14$, $\kappa F = 1990$ keV, and $\epsilon_2^{+(0)} = 1500$ keV, are similar to the values used [16] for the Ba region of $\Omega_\pi = \Omega_\nu = 16$, $\kappa F = 1825$ keV, and $\epsilon_2^{+(0)} = 1344$ keV.) With this arbitrary change the small differences in moments of inertia between the intruder bands themselves can be described. The intruder band in ^{188}Pb, for example, has the smallest value of $N_\pi N_\nu$ (40), and correspondingly the highest *unperturbed* $2^+ \rightarrow 0^+$ energy, of the set.

A comment on the other properties of the interacting bands used in the band-mixing analyses, specifically the unperturbed bandhead energies and spin-independent interaction matrix elements listed in Table I, is also appropriate. The energy differences, which could be associated with the energy difference between the minima in the potential wells, are shown in Fig. 4 on the left. Included in the experimental plot are the new osmium results [9,10]. The theoretical values, distilled from a number of sources (including Refs. [17] and [6]), are shown on the right. There is apparently a general discrepancy between theory and experiment for lead, mercury, and platinum. This discrepancy is due at least partly to the difficulty in treating pairing accurately. The matrix elements between the normal and intruder bands are not precisely determined but they vary from small, ~ 30 keV in the lead cases, to large, ~ 300 keV in the osmium cases, consistent with the minima in the potential well being more distinct in the higher-Z cases. The interaction strengths could provide a useful test of theories, since they imply demands on both the detailed configurations, and inclusion of the effects of dynamics.

I am grateful to Kris Heyde for pointing out the predictions of the 2-component model, and to my colleagues in the Department of Nuclear Physics for continuing discussions.

[1] J. L. Wood, K. Heyde, W. Nazarewicz, M. Huyse, and P. Van Duppen, Phys. Rep. **215**, 101 (1992).
[2] J. Heese, K. H. Maier, H. Grawe, J. Grebosz, H. Kluge, W. Meczynski, M. Schramm, R. Schubert, K. Spohr, and J. Styczen, Phys. Lett. B **302**, 390 (1993).
[3] A. M. Baxter *et al.*, Phys. Rev. C **48**, R2140 (1993).
[4] F. R. May, V. V. Pashkevich, and S. Frauendorf, Phys. Lett. B **68**, 113 (1977).
[5] R. Bengtsson and W. Nazarewicz, Z. Phys. **334**, 269 (1989).
[6] W. Nazarewicz, Phys. Lett. B **305**, 195 (1993).
[7] J. Penninga, W. H. A. Hesselink, A. Balanda, A. Stock, H. Verheul, J. van Klinken, H. J. Riezebos, and M. J. A. deVoigt, Nucl. Phys. **A471**, 535 (1987).
[8] G. D. Dracoulis, R. A. Bark, A. E. Stuchbery, A. P. Byrne, A. M. Baxter, and F. Reiss, Nucl. Phys. **A486**, 414 (1988).
[9] T. Kibédi, G. D. Dracoulis, A. P. Byrne, P. M. Davidson, and S. Kuyucak, Nucl. Phys. **A567**, 183 (1994).
[10] P. M. Davidson, G. D. Dracoulis, T. Kibédi, A. P. Byrne, S. S. Anderssen, A. M. Baxter, B. Fabricius, G. J. Lane, and A. E. Stuchbery, Nucl. Phys. **A568**, 90 (1994).
[11] R. F. Casten, Nucl. Phys. **A443**, 1 (1985).
[12] G. D. Dracoulis, in *Proceedings of the International Conference on the Future of Nuclear Spectroscopy, Crete, 1993*, edited by a C. A. Kalfas, W. Gelletly, S. Harrisopoulos, D. Loukas, and R. Vlastou (National Technical University Press, Athens, in press).
[13] B. R. Barrett, G. D. Dracoulis, and R. A. Bark, Phys. Rev. C **43**, R926 (1991).
[14] K. Heyde, P. van Isacker, and J. L. Wood, Phys. Rev. C **49**, 559 (1994).
[15] N. Tajima, H. Flocard, P. Bonche, J. Dobaczewski, and P.-H. Heenen, Nucl. Phys. **A551**, 409 (1993).
[16] K. Heyde and J. Sau, Phys. Rev. C **33**, 1050 (1986).
[17] R. Bengtsson, T. Bengtsson, J. Dudek, G. Leander, W. Nazarewicz, and J.-Y. Zhang, Phys. Lett. B **183**, 1 (1987).
[18] G. D. Dracoulis, B. Fabricius, A. E. Stuchbery, A. O. Macchiavelli, W. Korten, F. Azaiez, E. Ruble, M.-A. Deleplanque, R. M. Diamond, and F. S. Stephens, Phys. Rev. C **44**, R1246 (1991).
[19] P. M. Davidson, G. D. Dracoulis, T. Kibédi, A. P. Byrne, S. S. Anderssen, A. M. Baxter, B. Fabricius, G. J. Lane, and A. E. Stuchbery (unpublished).
[20] G. D. Dracoulis, A. E. Stuchbery, A. P. Byrne, A. R. Poletti, S. J. Poletti, J. Gerl, and R. A. Bark, J. Phys. G **12**, L97 (1986).
[21] J. Wauters, P. Dendooven, M. Huyse, G. Reusen, P. Van Duppen, R. Kirchner, O. Klepper, and E. Roeckl, Z. Phys. A **345**, 21 (1993).
[22] E. Hagberg, P. G. Hansen, P. Hornshoj, S. Mattsson, and P. Tidemand-Petersson, Nucl. Phys. A **318**, 29 (1979).
[23] G. D. Dracoulis, A. E. Stuchbery, A. O. Macchiavelli, C. W. Beausang, J. Burde, M.-A. Deleplanque, R. M. Diamond, and F. S. Stephens, Phys. Lett. B **108**, 365 (1988).
[24] C. W. Ma, A. V. Ramayya, J. H. Hamilton, S. J. Robinson, M. E. Barclay, K. Zhao, J. D. Cole, E. F. Zganjar, and E. H. Spejewski, Phys. Lett. B **139**, 276 (1984).
[25] J. D. Cole *et al.*, Phys. Rev. Lett. **37**, 1185 (1976).
[26] C. W. Ma, A. V. Ramayya, J. H. Hamilton, S. J. Robinson, J. D. Cole, E. F. Zganjar, E. H. Spejewski, R. Bengtsson, W. Nazarewicz, and J.-Y. Zhang, Phys. Lett. B **167**, 277 (1986).