Poincaré covariant particle dynamics. I. Intranuclear cascade model

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To analyze the high energy heavy ion reactions performed at Lawrence Berkeley BEVALAC, Brookhaven AGS, and CERN SPS we present a dynamical model that is entirely Poincaré covariant. The description of strong interactions by Lorentz-scalar quasipotentials makes possible a relativistic extension of the cascade concept without losing its inherent simplicity. No field degrees of freedom appear explicitly, neither in elastic nucleon scattering nor in particle creation processes. Although various formalisms describe directly interacting relativistic particles with identical solutions for the two-particle case, they all suffer from different problems in many-particle systems. The basis of our approach is a Hamiltonian formulation for N pointlike nucleons, moving unconstrained in an $8N$-dimensional phase space. One additional Lorentz scalar is introduced for an appropriate parametrization of all trajectories, defining also the connection to the proper times of the individual interacting nucleons. The creation of particles, for which Hamiltonian dynamics does not provide a generic mechanism, is incorporated phenomenologically as a "perturbative" process. We describe heavy ion collisions as a sequence of two-particle reactions without any additional fit to experimental data. Pion yield and mass spectra of the heavy fragments agree reasonably well with BEVALAC results. We do not compare to experimental high energy data because the employed mechanism of particle production is certainly improper in that regime. However, the numerical results give an impression on how the reaction evolves.

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I. INTRODUCTION

Although the well-known field equations of QCD probably form the basis of the theory of strong interactions, no convincing description of any dynamical process associated with confinement has been given on these grounds up to this day. Hence we have to rely on soluble phenomenological models for most of the reactions in nuclear physics. In particular, this is true in the case of heavy ion reactions that are complicated by the large number of nucleons and the finite size of the nucleus.

Because the attempt to solve the many-particle problem by means of quantum field theory seems to be hopeless in nearly all cases of interest, at least for the purpose of an approximation to high energy heavy ion collisions it should be reasonable to recover the old idea of relativistic action at a distance where the field degrees of freedom are not explicitly taken into account.

An early, simple, and successful model for describing nucleus-nucleus collisions has been the cascade approach [1] where the reaction is simulated by a sequence of individual scatterings between nucleons. As long as those scatterings are fitted to the free nucleon-nucleon cross sections, the nucleons always appear as free elementary particles on their mass shell. Nevertheless, a lot of the experimental data up to bombarding energies of about 1 GeV per nucleon were well explained by such models [2], even though the underlying assumptions get worse with rising energy of the incident projectile.

One basic fault is the absence of manifest Poincaré covariance of the dynamics. If only relativistic kinematics is respected, which means the relation between energy, momentum, and mass of a particle as well as four-momentum conservation, the sequence of scatterings in space-time depends on the reference frame in a simulated heavy ion reaction [3]. A second point of criticism is the lack of attraction in the nucleon-nucleon interaction. Because the nucleons merely react according to elementary cross sections, nothing else but a repulsion comparable to hard-sphere scattering is simulated. Furthermore, it is obvious that no complex stable fragments in the final state are formed without any attractive part in the interaction. To enable a comparison to the experimental data some supplementary model [4] is needed. Finally, there is the defect of omitted quantum interference effects (we expect, however, an improvement of this approximation with increasing energies).

To solve all these problems we would like to have a theory of relativistic (quantum) mechanics. It should describe a system of directly interacting particles by a Hamiltonian phase space formalism using one time variable as parametrization. Such a theory unfortunately does not exist because of the no-interaction theorem [5]. Furthermore, most of the existing relativistic theories for directly interacting particles moving on ordinary world lines are useful in the case of two particles only [6–8]. A generalization to an arbitrary but fixed number of degrees of freedom often leads to mysterious equations. Some seem to be more appropriate to the many-particle problem, others are designed to allow for the desired quan-

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tization. But to get a practicable Poincaré-covariant description for a system of many particles in direct interaction (no fields), we cannot respect all conventional requirements that emerge from classical mechanics.

In this paper we choose the easiest way to avoid analytical difficulties by employing a many-time formalism: the phase space of an N-particle system is extended to $8N$ dimensions without respecting conventional constraints that restrict the movement to a $(6N + 1)$-dimensional subspace. Hence, to each nucleon i we assign a four-vector of independent canonical coordinates $x^{\mu}{}_i$ and a conjugate four-momentum $p^{\mu}{}_i$. Note that all components are dynamically independent quantities, and consequently the nucleons are off their mass shell in the case of interaction. Last but not least, we introduce an additional Lorentz scalar τ for an appropriate parametrization of the particle trajectories. This quantity acquires in part the role played by time in ordinary nonrelativistic Hamiltonian mechanics.

From the ladder approximation of the Bethe-Salpeter equation and a semiclassical approximation it is possible to deduce a Hamiltonian for the (inelastic) scattering of two free nucleons by means of a Wigner transformation [9]. Because this cannot be generalized to the case of many off-shell nucleons, we introduce an appropriate Hamiltonian in Sec. II A to describe the individual nucleon-nucleon elastic interactions based on phenomenological arguments [10,11].

An important consequence of our model is the distinction between canonical and physical coordinates calculable by a projection. This is not as strange as it seems to be. In Secs. II B–II D we discuss briefly some other formalisms and extract the equal-time description in the center-of-mass frame of two interacting particles. We will end up with the same equations of motion in all cases. Differences, however, emerge for a sequence of two-particle reactions, used to model a heavy ion reaction. This is one of the origins for differences to other models like relativistic quantum molecular dynamics (RQMD) [12].

In Hamiltonian dynamics the particle number is constant throughout, so particle creating processes have to be incorporated phenomenologically in a Poincaré-covariant manner. In this paper we restrict ourselves to incoherent pion production, and may consequently not expect agreement between calculation and experiment beyond the Lawrence Berkeley BEVALAC range. This is sufficent for principal considerations, but more refined mechanisms are discussed in [13].

The elastic scattering for off-shell nucleons is presented in Sec. III A, followed by the details on pion production in III B. Section III C contains a discussion on the parameters in the many-particle system and an investigation of the initial state for a heavy ion reaction. The analysis of the numerical results is given in Sec. IV, followed by the conclusions in Sec. V.

II. TWO-PARTICLE DYNAMICS

The early attempts to find a theory for particles interacting at a distance were rather disappointing [6]. In Hamiltonian theory there is a no-interaction theorem [5]. It rules out any interaction between N particles moving in $6N$-dimensional phase space $(\mathbf{x}_i, \mathbf{p}_i)$ if just Galilei invariance is replaced by Lorentz invariance. To be specific, we get a unique solution for the two sets of Poisson bracket relations given in (2.1) and (2.2).

Invariance of the equations of motion with respect to Lorentz transformations, space and time translations requires the existence of ten corresponding phase space functions

$$\begin{aligned} H &= P^0, & J^r &= \tfrac{1}{2}\varepsilon^{rst}M_{st}, \\ \mathbf{P} &= P^r, & K^r &= M^{r0}, \end{aligned} \qquad (r = 1,2,3)$$

representing the generators which have to obey the Lie algebra $[\, g = \mathrm{diag}\,(+\,-\,-\,-)\,]$:

$$\begin{aligned} \{P^{\mu}, P^{\nu}\} &= 0, \\ \{M^{\mu\nu}, P^{\lambda}\} &= -g^{\mu\lambda}P^{\nu} + g^{\nu\lambda}P^{\mu}, \\ \{M^{\mu\nu}, M^{\lambda\sigma}\} &= -g^{\mu\lambda}M^{\nu\sigma} - g^{\nu\sigma}M^{\mu\lambda} \\ &\quad + g^{\mu\sigma}M^{\nu\lambda} + g^{\nu\lambda}M^{\mu\sigma}. \end{aligned} \tag{2.1}$$

Identification of canonically conjugate variables $\mathbf{x}_i, \mathbf{p}_i$ with particle observables in three-dimensional configuration space leads to

$$\begin{aligned} \{x^r{}_i, P^s\} &= \delta^{rs}, \\ \{x^r{}_i, J^s\} &= \varepsilon^{rsv} g_{vt} x^t{}_i, \\ \{x^r{}_i, K^s\} &= x^s{}_i\{x^r{}_i, H\}. \end{aligned} \tag{2.2}$$

From (2.1) and (2.2) we get a unique solution for the generators:

$$H = \sum_i \sqrt{m_i^2 + \mathbf{p}_i{}^2}, \tag{2.3a}$$

$$\mathbf{P} = \sum_i \mathbf{p}_i, \tag{2.3b}$$

$$\mathbf{J} = \sum_i (\mathbf{x}_i \times \mathbf{p}_i)\,, \tag{2.3c}$$

$$\mathbf{K} = \sum_i \mathbf{x}_i\sqrt{m_i^2 + \mathbf{p}_i{}^2}. \tag{2.3d}$$

They describe a system of free particles, which can be seen from $\{\{\mathbf{x}_i, H\}, H\} = 0$, meaning $\frac{d\mathbf{x}_i}{dt} = \text{const}$.

To avoid this kind of result, at least some of the above requirements have to be omitted [6,7], and the extension of the phase space to $8N$ dimensions (x_i, p_i) seems to be the natural procedure. Then (2.1) is easily solved by $P^{\mu} = \sum_i p^{\mu}{}_i$ and $M^{\mu\nu} = \sum_i (x^{\mu}{}_i p^{\nu}{}_i - x^{\nu}{}_i p^{\mu}{}_i)$. As a consequence, physical positions and momenta as measured by any observer in the three-dimensional configuration space as a function of time have to be calculated by some projection to a subspace, which of course is not unique. Several of such formalisms have been suggested [14]. We will show, however, that in the two-particle case they all lead to equivalent sets of equations of motion.

For clarity we now proceed with a particular two-particle case, which is the basis of our cascade model.

The interaction is described by a quasipotential V that depends on the square of just one four-vector r representing some distance ($r^2 < 0$) of the particles. For details we refer to the discussion of the single Hamiltonian formalism in Sec. II A 2. We will show that in all formalisms we end up with the following equations of motion:

$$\frac{dp}{d\tau} = 0, \tag{2.4a}$$

$$\frac{dz}{d\tau} = \frac{p}{m}, \tag{2.4b}$$

$$\frac{dk}{d\tau} = 2\frac{\partial V}{\partial r^2}r, \tag{2.4c}$$

$$\frac{dr}{d\tau} = \frac{k}{\mu}, \tag{2.4d}$$

where p, z, k, r are four-vectors and τ is a Lorentz scalar. While $p = p_1 + p_2$ always stands for the conserved total momentum, the center-of-mass coordinate z, the relative momentum k, and the relative coordinate r vary slightly in their interpretation for different formalisms. In all cases, however, we have

$$(rp) = (kp) = 0, \tag{2.5}$$

which states that the zeroth components of r and k are zero in the center-of-mass system ($\mathbf{p} = 0$). Finally the scalar τ may be removed from (2.4) in favor of the time component z^0 because p^0 is conserved and positive. The details of (2.4), including its derivation, will now be given for different Hamiltonian and Lagrangian formalisms, but it should already be mentioned here that in general it is not possible to look at p, z, k, r as canonically conjugate variables.

A. Single Hamiltonian formalism

1. Equation of motion

The easiest way to realize a Poincaré-covariant formalism is to replace the nonrelativistic 12-dimensional Hamiltonian theory with Euclidean metric, using the time for parametrization, by a 16-dimensional Hamiltonian theory with Minkowski metric, using some scalar τ for parametrization. We then have to worry about the four additional variables, need an interpretation of τ, and have to establish a connection between the variables used and all observables as well as some observer time.

To do so we start with a scalar Hamiltonian of the usual form

$$H = \frac{m^2 - p^2}{2m} - \frac{q^2}{2\mu} + V, \tag{2.6}$$

which is a function of the two independent pairs of canonical variables: $p = p_1 + p_2$, $y = (m_1x_1 + m_2x_2)/m$ and $x = x_1 - x_2$, $q = (m_2p_1 - m_1p_2)/m$. The constants $m = m_1 + m_2$ and $\mu = m_1m_2/m$ give the total and the reduced mass of the free particles $\left(m_i = \lim_{V\to 0}\sqrt{p_i^2}\right)$. Thus we have given an interpretation for two of the redundant variables.

For the quasipotential we select $V = V(-\hat{x}^2)$, where $\hat{x} := x - \frac{(xp)}{p^2}p$ gives the component of x perpendicular to the total momentum ($\hat{x}p = 0$), which is nothing else but the equal-time spatial distance in the center-of-mass system. The further reasons for the restriction of V to just this dependence are discussed in Sec. II A 2.

By evaluation of $\frac{d}{d\tau} = \{H, \quad\}$ we get the equations of motion for the canonical variables

$$\frac{dp}{d\tau} = 0, \tag{2.7a}$$

$$\frac{dy}{d\tau} = \frac{p}{m} + 2\left(\frac{\partial V}{\partial \hat{x}^2}\right)\frac{xp}{p^2}\hat{x}, \tag{2.7b}$$

$$\frac{dq}{d\tau} = 2\left(\frac{\partial V}{\partial \hat{x}^2}\right)\hat{x}, \tag{2.7c}$$

$$\frac{dx}{d\tau} = \frac{q}{\mu}, \tag{2.7d}$$

which do not yet have the form of (2.4). Note that qp is a constant but not equal to zero for unequal-mass particles ($m_1 \neq m_2$). Hence, xp increases linearly with τ, and $\mathbf{y}$ is not at rest in the center-of-mass system. In contrast, the variable

$$z = \left(y + \frac{qp}{p^2}x\right) - \frac{xp}{p^2}q \tag{2.8}$$

(instead of y) is the physically appropriate center-of-mass coordinate of the system ($dz/d\tau = p/m$), and the physical coordinates of the single particles are

$$z_i = z \pm \frac{\mu}{m_i}x = x_i + (z - y). \tag{2.9}$$

Indeed we arrive at (2.4) and (2.5) by setting $k = \hat{q} := q - \frac{(qp)}{p^2}p$, and $r = \hat{x}$. The connection between the proper times of the individual particles and the parameter τ reads

$$d\tau_i = d\tau\,\frac{\sqrt{p_i^2}}{m_i}. \tag{2.10}$$

It should be noticed that z is not a canonical variable. However, the expression in brackets in (2.8), together with p, x, and $\hat{q}$ determines a canonical transformation when taking the conserved quantity qp/p^2 as an independent constant. So for the case $xp = 0$ we regain a canonical interpretation.

2. The variables of the quasipotential

When we look at the equations of motion for an arbitrary (scalar) quasipotential in (2.6), it becomes clear why V should depend on $\hat{x}^2$ only:

1. The total momentum p is conserved if V does not depend on y:
$dp/d\tau = \{H,p\} = \partial V/\partial y = 0;$
2. The effective masses will assume their constant free particle values ($\sqrt{p_i^2} = m_i$) for $V = 0$ if $\partial V/\partial x$ is perpendicular to p:

$$\frac{d}{d\tau}(p_i^2 - (m_i^2 + 2\mu V)) = \pm 2\,\frac{m_i}{m}\left(\frac{\partial V}{\partial x}p\right) = 0.$$

This is the case if V only depends on q, p, and the component $\hat{x}$ of x;

3. The variable z as defined in (2.8) satisfies the equation of motion for the center-of-mass ($dz/d\tau = p/m$) if, in addition, $\partial V/\partial p$ is perpendicular to p. This leaves only $\hat{x}$, $\hat{q}$, and $q_{\|} = q - \hat{q}$ as possible variables;

4. A (possibly too restrictive, but sufficient) condition to guarantee particle velocity $v_i = \left|d\mathbf{z}_i/dz_i^0\right|$ smaller than the speed of light is $\partial V/\partial q = 0$.

B. Multi-Hamiltonian

The more refined Hamiltonian theories make use of separate Hamiltonians H_i and parametrizations τ_i ($i = 1,2$) for each particle [15]

$$H_i = \frac{m_i^2 - p_i^2}{2m_i} + V_i(\hat{x}^2) \tag{2.11}$$

(notations as in the previous section). The equations of motion now read $\frac{\partial x_i}{\partial \tau_j} = \{H_j, x_i\}$ and $\frac{\partial p_i}{\partial \tau_j} = \{H_j, p_i\}$. Note that we have $\{H_i, x_j\} \neq 0$ ($i \neq j$). With these two Hamiltonians the motion of the particles no longer is along a one-dimensional line but on a two-dimensional sheet in the 16-dimensional phase space. In order to have a unique set of world lines, one gauge-fixing constraint is needed [16], although it is not our intention to discuss all possible definitions in this paper. Instead we will return to the single τ formulation by a suitable choice of $\tau_i(\tau)$.

The compatibility ("predictivity" [15]) condition

$$\begin{aligned} 0 &= \{H_1, H_2\} \\ &= \frac{2(\hat{x}\hat{q})}{m_1 m_2}\left(\frac{\partial}{\partial \hat{x}^2}(m_2V_2 - m_1V_1)\right) \end{aligned} \tag{2.12}$$

is satisfied most easily for equal quasipotentials $m_1V_1 = m_2V_2$. Applying a symmetry argument for particle interchange and an appropriate normalization we get $\tau = \tau_1 = \tau_2$. Now, for the canonically conjugate variables the same equations of motion as in (2.7) result from $\frac{d}{d\tau} = \{H, \quad\}$, where $H = H_1 + H_2$ and $V = V_1 + V_2$. Finally, define z as in (2.8) and also $r = \hat{x}$, $k = \hat{q}$ to reproduce the equal-time description in the center-of-mass frame as given by (2.4) and (2.5).

C. Multi-Hamiltonian with constraints

Considering (2.7) and (2.8), it seems to be quite natural to retain the canonical interpretation for the center-of-mass variable z by imposing the explicit constraint $xp = 0$ (leading to $\hat{x} = x$) for the relative coordinate. In other words, equal times ($x^0 = 0$) are required for the canonical variable x in the center-of-mass system. In the multi-Hamiltonian approach [17]

$$H_i = \frac{m_i^2 - p_i^2}{2m_i} + V_i(x^2), \tag{2.13}$$

the quasipotentials then should be independent of p. The compatibility condition ($0 = \{H_1, H_2\}$) now requires

$$\begin{aligned} 0 = (xp)\mu &\left(\frac{\partial}{\partial x^2}(V_1 + V_2)\right) \\ &+ (xq)\left(\frac{\partial}{\partial x^2}(m_2V_2 - m_1V_1)\right). \end{aligned} \tag{2.14}$$

With $m_1V_1 = m_2V_2$ and $xp = 0$ we obviously get that result. The possibility to identify canonical and physical coordinates is based on $\{H_1, x_2\} = \{H_2, x_1\} = 0$. However, well-defined world lines demand an appropriate connection between the parameters τ_1 and τ_2. To rewrite the equations of motion in terms of τ it is sufficient to know the derivatives $\tau_i' = \frac{d}{d\tau}\tau_i(\tau)$. The first of two necessary requirements is the conservation of the explicit constraint $xp = 0$ for the motion, and the second one is a normalization that respects the symmetry for particle interchange:

$$0 = \frac{d}{d\tau}(xp) = \tau_1'\,\frac{pp_1}{m_1} - \tau_2'\,\frac{pp_2}{m_2}, \tag{2.15a}$$

$$\frac{1}{2}(\tau_1' + \tau_2') = 1 - \frac{m_1 - m_2}{2\mu}\left(\frac{qp}{p^2}\right). \tag{2.15b}$$

These equations indeed define τ_i' properly because both $p^2 =$ const. and $qp =$ const. hold. Solving for τ_i', we get the equations for the canonical variables from $\frac{d}{d\tau} = \tau_1'\{H_1, \quad\} + \tau_2'\{H_2, \quad\}$:

$$\frac{dp}{d\tau} = 0, \tag{2.16a}$$

$$\frac{dy}{d\tau} = \frac{p}{m} - \frac{1}{\mu}\left(\frac{qp}{p^2}\right)\left[q + \frac{m_1 - m_2}{m_1 + m_2}p\right], \tag{2.16b}$$

$$\frac{dx}{d\tau} = \frac{1}{\mu}\left(q - \frac{qp}{p^2}p\right), \tag{2.16c}$$

$$\frac{dq}{d\tau} = 2\left(\frac{\partial V}{\partial x^2}\right)x, \tag{2.16d}$$

where $V = \tau_1'V_1 + \tau_2'V_2 = m_1V_1/\mu = m_2V_2/\mu$ in fact is independent of the momenta.

Since H_1 and H_2 are conserved and the quasipotentials are equal ($m_1V_1 = m_2V_2$), it is possible to force $p_1^2 - p_2^2 = m_1^2 - m_2^2$ by the initial conditions. Therefore (2.16) may equally well be deduced from the single Hamiltonian

$$\begin{aligned} H &= \tau_1'H_1 + \tau_2'H_2 \\ &= \frac{m^2 - p^2}{2m} - \frac{q^2}{2\mu} \\ &\quad + \frac{1}{\mu}\left(\frac{qp}{p^2}\right)\left(qp - \frac{m_1 - m_2}{2m}(m^2 - p^2)\right) + V(x^2). \end{aligned} \tag{2.17}$$

From this Hamiltonian, (2.4) and (2.5) are recovered by setting $r = \hat{x} = x$, $k = \hat{q} = q - \frac{qp}{p^2}p$ and

$$z = \frac{\left(y + \frac{qp}{p^2}x\right) - \frac{xp}{p^2}q}{1 + \frac{(m_1 - m_2)^2}{4m_1m_2}\left(1 - \left(\frac{m^2}{p^2}\right)^2\right)}, \tag{2.18}$$

where the denominator looks like a renormalization of the total mass.

Note that for equal-mass particles ($p_1^2 - p_2^2 = m_1^2 - m_2^2 = 0$) everything that may look a little strange does vanish, as it does in all other formalisms described here. Even though explicit constraints seem to be a powerful tool to remove the redundant variables [8], it is not possible to generalize $xp = 0$ to more than two particles.

D. Singular Lagrangian

Lagrangian and Hamiltonian formalisms usually are connected by a simple Legendre transformation. However, if it is impossible to eliminate the velocities in favor of the canonical momenta, a Lagrangian is called singular, and additional constraints exist. As an example of a singular Lagrangian [18] consider

$$L(x_i, \overset{\circ}{x}_i) = \sum_{i=1}^{2} \sqrt{\overset{\circ}{x}_i^2 \;(m_i^2 + 2m_i V_i(x^2))}, \qquad (2.19)$$

where $^\circ = \frac{d}{d\sigma}$ is the derivative with respect to the scalar σ used for parametrization (τ is reserved for later reparametrization). Because the Lagrangian is singular $\left[\det\left(\frac{\partial^2 L}{\partial \overset{\circ}{x}_i \partial \overset{\circ}{x}_j}\right) = 0\right]$, the canonical momenta $p_i = \frac{\partial L}{\partial \overset{\circ}{x}_i}$ cannot be used to obtain an explicit expression for the velocities $\overset{\circ}{x}_i$. Instead, by squaring p_i the following constraints emerge:

$$p_i^2 = m_i^2 + 2m_i V_i(x^2). \qquad (2.20)$$

This is the analog to the conservation of H_i in the multi-Hamiltonian approach.

The scalars $M_i(\sigma) = \sqrt{[m_i^2 + 2m_i V_i(x^2)]/\overset{\circ}{x}_i^2}$, $M(\sigma) = M_1 + M_2$, and $\mathcal{M}(\sigma) = M_1 M_2 / M$, depending on σ through x_i and $\overset{\circ}{x}_i$, will be used in what follows. In these terms the Euler-Lagrange equations read

$$\overset{\circ}{p}_i = \pm 2x \left\{ \frac{m_1}{M_1(\sigma)} \frac{\partial V_1}{\partial x^2} + \frac{m_2}{M_2(\sigma)} \frac{\partial V_2}{\partial x^2} \right\} \qquad (2.21a)$$

$$p_i = M_i(\sigma)\, \overset{\circ}{x}_i \,. \qquad (2.21b)$$

Because the mass constraints (2.20) have to hold for all σ, their derivatives lead to the same consequences as (2.14): $xp = 0$ and $m_1 V_1 = m_2 V_2 = \mu V$. A further differentiation of the constraint $xp = 0$ with respect to σ leads to a connection between the masslike functions $M_i(\sigma)$:

$$\frac{M_1(\sigma)}{M_2(\sigma)} = \frac{p^2 + (p_1^2 - p_2^2)}{p^2 - (p_1^2 - p_2^2)} = \frac{pp_1}{pp_2} = \text{const.} \qquad (2.22)$$

This can be regarded as the counterpart of (2.15). What is left is the freedom for some kind of normalization of the M_i by a world-line reparametrization. We define

$$\tau(\sigma) = \int_0^\sigma \frac{\mu}{\mathcal{M}(\sigma)}\, d\sigma, \qquad (2.23)$$

to rewrite (2.21) in terms of τ, and regain (2.16) as the equations of motion. Using (2.20) we have $p_1^2 - p_2^2 = m_1^2 - m_2^2$ for the considered case of equal quasipotentials ($m_1 V_1 = m_2 V_2$). Together with $r = x$, $k = \hat{q}$ and (2.18), nothing but (2.4) and (2.5) will emerge again.

III. POINCARÉ COVARIANT CASCADE

A. Relativistic nucleon-nucleon collisions

Generalizing the considerations of Sec. II A, the basis of our model are the solutions of Hamilton's equations of motion for two sets of four-vectors $\dot{x}_i = \{H, x_i\}$ and $\dot{p}_i = \{H, p_i\}$ resulting from the N-particle Hamiltonian:

$$H = \sum_{i=1}^{N} (m_i^2 - p_i^2)/2m_i + V(x, p). \qquad (3.1)$$

A Lorentz scalar τ is introduced for the parametrization of the solutions in phase space $(x_i(\tau), p_i(\tau))$. Hence, τ assumes in part the role played by time in ordinary nonrelativistic mechanics, but it is independent of all the zeroth components t_i in the four-vectors x_i. An inversion of the functions $t_i(\tau)$ and subsequent replacement of τ by some observation time (t) in the solutions of the equations of motion is possible for free motion only ($V = 0$). Even then $t_i(\tau) = t_j(\tau)$ is true only for particles of equal speed ($|\mathbf{p}_i|/E_i = |\mathbf{p}_j|/E_j$).

We suppose a system of particles with pair interactions mediated by Lorentz scalar quasipotentials depending on one Lorentz-scalar variable only. This is the square of $\hat{x} = x - p(xp)/p^2$, which is the component of the relative coordinate $x = x_1 - x_2$ that is orthogonal to the total momentum $p = p_1 + p_2$. Hence, $d = \sqrt{-\hat{x}^2}$ gives the spatial distance for two particles in their center-of-mass frame [$\hat{x}_{\text{c.m.}} = (0, \mathbf{r})$]. Another reason to choose this dependence is the mass shell condition $p_i^2 = m_i^2$ for free particles ($V = 0$), as discussed in Sec. II A 2. Any change in the potential $\Delta V = V - V'$ effects a change in the effective masses of the form

$$(p_i')^2 = p_i^2 - 2\mu(\Delta V). \qquad (3.2)$$

Describing a scattering of many particles by a superposition of interactions between pairs and, for example supposing N free particles ($V_{ij} = 0$) on their mass shells ($p_i^2 = m_i^2$) in the initial state, we then will find a final state, where all free particles ($V_{ij}' = 0$) are on their mass shells again [$(p_i')^2 = m_i^2$], irrespective of the interactions in between.

In the limit of small momenta we of course regain the classical nonrelativistic equations of motion. There V plays the role of an ordinary potential. Because we would like to describe interacting nucleons, a Yukawa form for the quasipotentials would be appropriate. But to get a cascadelike description in the many-particle problem of two colliding nuclei, we adopt the extreme simplification that V becomes a discontinuous step function:

$$V(d) = \begin{cases} 0 & \text{if } d \in [R_0, \infty[\\ -V_0 & \text{if } d \in [R_c, R_0[\\ \infty & \text{if } d \in [0, R_c\;[. \end{cases} \qquad (3.3)$$

Then all nucleons move on straight lines except at those points where V changes discontinuously. Using (3.3) the solution of the equations of motion reduces to a sudden change of 4-momenta and 4-coordinates:

$$p_i' = p_i \pm \Delta q, \qquad x_i' = x_i + \frac{xp}{p^2}\Delta q. \tag{3.4}$$

Note that the discontinuity of the canonical coordinates x_i is caused solely by the momentum dependence of the quasipotential V (precisely as in nonrelativistic mechanics) and thus is at most indirectly a consequence of phase space extension. The change in relative momentum Δq depends on the momentary distance $(-\hat{x}^2)$ and the direction of motion $(\hat{x}\hat{q})$ of the particles. Formulas for penetration into the quasipotential $(-)$ and the escape from it $(+)$ just differ in the signs:

$$\Delta q = -\left\{(\hat{x}\hat{q}) \mp \sqrt{(\hat{x}\hat{q})^2 \mp 2\mu V_0 \hat{x}^2}\right\}\frac{\hat{x}}{\hat{x}^2}. \tag{3.5}$$

For a reflection we have

$$\Delta q = -2(\hat{x}\hat{q})\frac{\hat{x}}{\hat{x}^2}. \tag{3.6}$$

B. Phenomenology of pion production

In this paper we restrict ourselves to incorporate a phenomenological model of pion production. All other reaction channels will be neglected. An investigation of particle creation at very high energies is given in a subsequent article [13]. Here we just examine the principles of energy loss in a relativistic system.

Because of energy and momentum conservation there is a minimum relative momentum $k_{\rm min}$ in the center-of-mass frame for two particles which allows for particle creation. If M_1 and M_2 are the masses in the initial and $(M_1 + M_2 + \Delta m)$ in the final state, we have

$$\begin{aligned} k_{\rm min}^2 &= 4(\Delta m/2)(M_1 + \Delta m/2)(M_2 + \Delta m/2) \\ &\quad \times (M_1 + M_2 + \Delta m/2)(M_1 + M_2 + \Delta m)^{-2}. \end{aligned} \tag{3.7}$$

So in the laboratory frame we find for the case of two colliding free nucleons ($M_1 = M_2 = m_N \approx 940$ MeV)

$$E_{\rm min}^{\rm lab} = 2\frac{k_{\rm min}^2}{m_N} = 2\Delta m\left\{1 + \frac{1}{4}\left(\frac{\Delta m}{m_N}\right)\right\}. \tag{3.8}$$

Omitting all relative motions and interactions in a nucleus, the lightest hadron (the pion) will be produced in a heavy ion reaction by a collision of nucleons from different nuclei at $E_{\rm lab}/A_P \geq 290$ MeV ($\Delta m = m_\pi \approx 140$ MeV). Next to the pion we may find the kaon ($m_K \approx 500$ MeV) at about $E_{\rm lab}/A_P \approx 1600$ MeV [$\Delta m \geq m_K + (m_\Lambda - m_N) \approx 675$ MeV] because there has to be a strange particle in the final state ($m_\Lambda \approx 1115$ MeV). By increasing the energy many species of mesons will be created, and the limit for antibaryon production ($\Delta m = 2m_N \approx 1880$ MeV) is reached at $E_{\rm lab}/A_P \geq 5600$ MeV. Since we restrict ourselves to pion production we may consequently not expect an agreement between calculation and experiment beyond 2 GeV per nucleon. Furthermore, we neglect the possibility of coherent multipion production, which tends to underestimate the pion yield with growing energy.

As usual we choose the $\Delta(1232)$ resonance to be the dominant intermediate state [1] in a nucleon-nucleon scattering. Hence, pion production is a two-step process, beginning with a collision-induced excitation of a nucleon to a delta that is followed by an independent delta decay:

$$N + N \longrightarrow N + \Delta, \qquad \Delta \longrightarrow N + \pi. \tag{3.9}$$

The reaction channel for pion production will be opened by every hard-core collision of two nucleons ($-\hat{x}^2 = R_c^2$) in a heavy ion reaction. Because the direct interaction between the nucleons by means of the quasipotential originates physically from virtual pion exchange, only real pions may be created. Then the approximation of neglected reabsorption should not be as bad as in other models. Apart from further restrictions, hence, we impose the subsidiary condition that the nucleons involved in a pion production have to be asymptotically free. That means they do not enter a bound state of their common quasipotential $V(\hat{x}^2)$, neither on account of a small relative momentum $k = \sqrt{-\hat{q}^2}$ nor because of a large relative angular momentum [$\hat{L}^2 = \hat{x}^2\hat{q}^2 - (\hat{x}\hat{q})^2$]:

$$(k^2 - 2\mu V_0)R_0^2 - \hat{L}^2 \geq 0. \tag{3.10}$$

We have to fix isospins, coordinates, and momenta of two particles in each case of (3.9). The Clebsch-Gordan coefficients of isospin coupling decide about the branching ratios for different charge divisions when delta resonances are produced from protons (p) and neutrons (n). For pp (nn) collision there is $\Delta^{++} : \Delta^+$ ($\Delta^- : \Delta^0$) creation with a ratio 3:1. A np-pair has probability $\frac{1}{2}$ to be in an isospin zero state, which rules out delta excitation. In all other cases we have $\Delta^+ : \Delta^0$ as 1:1. Decay of Δ^{++} and Δ^- is unique, while for Δ^+ (Δ^0) we have the branching ratio 1:2 for $\pi^+ : \pi^0$ ($\pi^- : \pi^0$).

Momentum and angular momentum conservation determine at most ten of 16 unknown components:

$$P = \sum_i p_i = \text{const}, \tag{3.11a}$$

$$J^{\mu\nu} = \sum_i (x_i^\mu p_i^\nu - x_i^\nu p_i^\mu) = \text{const}. \tag{3.11b}$$

Therefore, we fix the effective masses from the beginning. One nucleon may keep its effective mass:

$$M_1' = \sqrt{(p_1')^2} = M_1 = \sqrt{p_1^2}, \tag{3.12}$$

while the other gets excited by twice the pion mass, reflecting the experimental mass difference for nucleon and delta:

$$M_\Delta = \sqrt{p_\Delta^2} = M_2 + 2m_\pi = \sqrt{p_2^2} + 2m_\pi. \tag{3.13}$$

In the second step the nucleon recovers its original effective mass:

$$M_2' = \sqrt{(p_2')^2} = M_2 = \sqrt{p_2^2}, \tag{3.14}$$

and the pion is on its mass shell:

$$\sqrt{p_\pi^2} = m_\pi. \tag{3.15}$$

The further details of our mechanism for Δ production are best described in the center-of-mass frame of the colliding nucleon pair:

(i) we hold the time variable of the unexcited nucleon (t_1) fixed,

(ii) the effective time center $(M_i t_i + M_j t_j)/(M_i + M_j)$ of the nucleon pair ($i = 1, j = 2$) is transferred to the $(N + \Delta)$ state ($i=1, j=\Delta$),

(iii) in the reaction plane defined by $\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$ and $\mathbf{k} = (M_2\mathbf{p}_1 - M_1\mathbf{p}_2)/(M_1 + M_2)$ we suppose a reflection,

(iv) the direction of the new relative momentum $\mathbf{k}' = (M_\Delta \mathbf{p}_1' - M_1 \mathbf{p}_\Delta)/(M_1 + M_\Delta)$ is chosen at random within the reaction plane.

As a consequence of these settings we obtain an additional restriction in the pion production. Because the magnitude of the relative momentum $|\mathbf{k}'|$ is fixed by the effective masses and energy conservation,

$$(k')^2 = \frac{[p^2 - (M_1 + M_\Delta)^2][p^2 - (M_1 - M_\Delta)^2]}{4p^2}, \tag{3.16}$$

angular momentum conservation yields $|\mathbf{r}'| = R_c|\mathbf{k}|/|\mathbf{k}'|$. In the energetically possible limit $|\mathbf{k}'| \to 0$ we would have $|\mathbf{r}'| \to \infty$. To avoid this unphysical behavior we introduce the subsidiary condition $|\mathbf{r}'| \leq R_0$ that sets a cutoff to the relative momentum and thus suppresses the low energy reactions.

Delta decay is easily fixed in its rest frame. The direction of the pion momentum is chosen completely at random, and everything is located at one space-time position.

C. The nuclear many-particle system

Apart from usual assumptions concerning the nucleon distribution in the ground state of a nucleus, all free parameters of the model are connected to elementary particle reactions.

1. Model parameters in the many-particle system

The parameters of the quasipotential (R_0, R_c, V_0), introduced in Sec. III A, are fitted to experimental data. The total nucleon-nucleon cross section $\sigma_{\text{tot}} \approx 40$ mb determines $R_0 = \sqrt{\sigma_{\text{tot}}/\pi} \approx 1.13$ fm. We then get a good agreement to the elastic cross section if we set $R_c = 0.5$ fm and $V_0 = 60$ MeV$/c^2$.

If we strictly fix the radii R_0 and R_c in a simulated heavy ion reaction, there is no disintegration of the total collision into a sequence of well-separated nucleon-nucleon interactions despite the simple form (3.3) for the quasipotential. The reason is the momentum dependence of the invariant spacings between the particles ($d_{ij} = \sqrt{-\hat{x}_{ij}^2}$). Any sudden change of the quasipotential of nucleons 1 and 2 ($\Delta V_{12} \neq 0$) changes their individual momenta discontinuously. Hence, all spacings to the other nucleons (d_{1i}, d_{2i}, $i \neq 1, 2$) change discontinuously also, and for example a previous value $d_{13} > R_0$ may become $d_{13} < R_0$ because of the sudden change ΔV_{12} in the quasipotential of nucleons 1 and 2. Then it would be inconsistent to apply free particle motion to nucleon 3.

Instead of solving this many-particle problem analytically, we allow a variation in the parameters. We compose the heavy ion reaction from two-particle interactions and permit penetration into the quasipotential for $R_0 \leq 1.13$ fm, escape from the quasipotential or reflection from the inside for $R_0 \geq 1.13$ fm, and reflection at the hard core for $R_c \leq 0.5$ fm. This procedure of course becomes questionable if we get a considerable deviation from $R_0 = 1.13$ fm and $R_c = 0.5$ fm for a large number of interactions. Hence, our model—as any cascade model—is restricted to dilute systems.

A typical distribution for the parameters in a heavy ion reaction (cf. Fig. 1) clearly shows some interactions at $R_0 > 1.13$ fm resulting from reflections at the outer boundary of the quasipotential in a bound two-particle subsystem. All other deviations are rather small.

As is true for the radii R_0 and R_c, we also may not strictly fix the depth of the quasipotential V_0. In

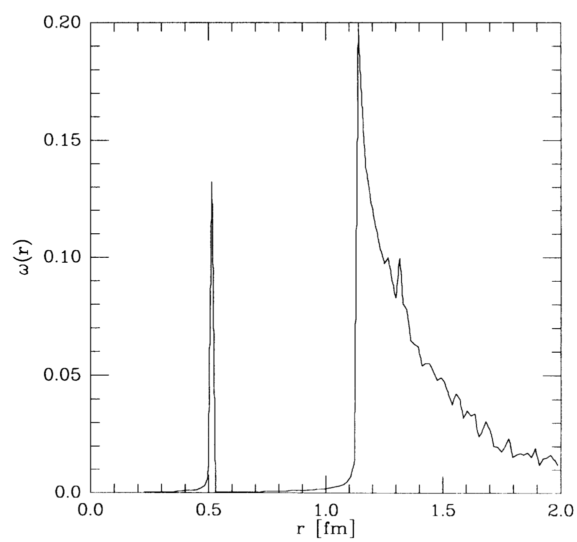

FIG. 1. Distribution for the parameters R_0 and R_c of the quasipotentials in the reaction Ar+Ar at 2 GeV per nucleon. Hard-core reflection occurs for 0 fm $\leq r \leq 0.5$ fm, penetrations into the quasipotentials for 0.5 fm $\leq r \leq 1.13$ fm. The region $r \geq 1.13$ fm is dominated by reflections from the inside, while escapes from the quasipotential nearly form the symmetrical counterpart to penetrations into the quasipotential. We normalized $\omega(r)$ to the value at $r = 1.13$ fm (not included in the figure).

a system of nucleons only ($m_i = m_N$) we find a decrease in the square of the effective mass by a constant value ($m_N V_0$) with every penetration into a quasipotential [cf. Eq. (3.2)]. If a certain number of interactions ($m_N/V_0 \leq 16$) is exceeded, p_i^2 would become negative, and we no longer could interpret it as the square of an effective mass. Hence, the actual depth for a penetration into the quasipotential has to decrease with an increasing number of simultaneous interactions, i.e., it has to be density-dependent.

Choosing from the many possible ways to set a value for V_0, we weaken the quasipotential by a direct dependence on the effective masses of the two interacting nucleons:

$$V_0 \longrightarrow \tilde{V}_0 = \left(\frac{p_1^2 - m_\pi^2}{m_1^2 - m_\pi^2}\right)\left(\frac{p_2^2 - m_\pi^2}{m_2^2 - m_\pi^2}\right) V_0 \leq V_0. \quad (3.17)$$

Thus the effective nucleon mass is greater than the pion mass even for an arbitrary number of interactions. A typical distribution for the effective nucleon masses in a heavy ion reaction at the instant with the greatest number of interactions is plotted in Fig. 2. The theoretical limit $\sqrt{p_i^2} = m_\pi = 0.15 m_N$ is far from being reached.

A further subsidiary condition follows from the absence of a self-consistent mean field. This many-particle effect of strong interactions is not reproduced in our purely mechanical model on the basis of two-nucleon interactions. Surely we can neglect the change in energy by an additional mean field for relativistic heavy ion collisions, but we have to account for the different origin of the off-shell behavior. In our model off-shell nucleons do not arise from collective modes of the whole system. Instead, the nucleons are divided into independent clusters at every stage of a simulated heavy ion reaction. All nucleons from one cluster interact directly or indirectly by means

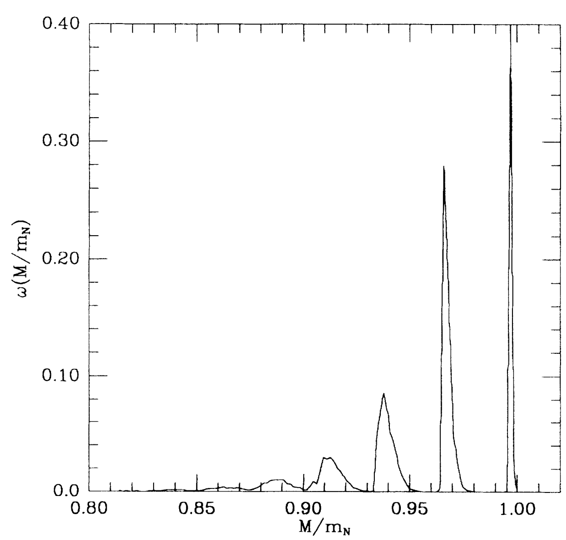

FIG. 2. Distribution for the effective masses ($M = \sqrt{p_i^2}$) in the reaction Ar+Ar at 2 GeV per nucleon at that instant where we find the greatest number of interactions. The originally discret values are contracted and smeared out due to the factor introduced in (3.17). We normalized $\omega(M/m_N)$ to the value at $M/m_N = 1$ (not included in the figure).

of a chain of two-nucleon quasipotentials, and there is no interaction between nucleons from different clusters. For example we have free nucleons ($p_i^2 = m_N^2$), two nucleons solely in interaction with each other [$p_i^2 = m_N(m_N - V_0)$], three nucleons linked in a row, and so on. Each of these subsystems may appear as a stable fragment in the final state, since there is no interaction between different clusters at that given stage. Consequently, we have to guarantee that clusters always may be interpreted as free nuclei. This is realized by a mass shell constraint for clusters during the whole reaction:

$$\sqrt{\left(\sum_{i=1}^{n} p_i\right)^2} \geq n \cdot m_N, \quad (3.18)$$

which holds regardless of the structure inside that cluster of n nucleons. So the nucleons may leave their mass shell ($\sqrt{p_i^2} < m_N \quad (i = 1, ..., n)$), but the cluster as a whole must remain "on-shell". If this inequality is violated by pion production or an escape from an inner interaction, we cancel that process and replace it by a reflection. Hence, all the nucleon-nucleon interactions in a cluster respect condition (3.18).

2. The nuclear ground state

For an arrangement of a nucleus in its ground state we start as usual from the distribution functions for noninteracting nucleons. A nucleus at rest is created according to Fermi distributions for coordinates and momenta whose radii take the values $r_0 A^{1/3}$ ($r_0 = 1.2$ fm) and $\sqrt{E_F(E_F + 2m_N)}$ ($E_F = 33.4$ MeV), respectively. The width of the surfaces in the distributions we arbitrarily set to 0 for the coordinates, and to $0.03 m_N = 28.2$ MeV for momenta. At first the time variables all are equal to zero, and the energies we determine from the mass shell condition $\sqrt{p_i^2} = m_N$.

Subsequently these distributions are changed on account of the interactions between the nucleons and the finite mass number of a nucleus. We select the coordinates successively and require all the invariant distances ($d_{ij} = \sqrt{-\hat{x}_{ij}^2}$) to be larger than R_c. Furthermore, we reduce the effective mass for every nucleon whose invariant distance to another one is in the interval $d_{ij} \in]R_c, R_0[$ according to (3.2). If a nucleus is prepared self-consistently in such a way it neither will be at rest ($\sum_{i=1}^{A} \mathbf{p}_i \neq 0$), nor will its center be located in the origin ($\sum_{i=1}^{A} m_i x_i \neq 0$). Hence, we have to perform suitable transformations in phase space.

As a consequence of the off-shell behavior of the nucleons, the distributions of momenta, energies, and coordinates broaden and the former sharp surface in the distribution of the coordinates is smeared out. The average number of interactions in the ground state is a little less than one. All the distributions of the nucleons in a nucleus as they emerge from the described procedure are shown in [11].

Finally, we state that we do not need equal-time variables in the canonical coordinates for a consistent description of interacting nucleons. Half of the width in the distribution of these time variables is caused by the interaction of the nucleons. The other part emerges from large spatial components contained in the Lorentz transformation to the nuclear rest frame due to the finite particle number. In all other distributions there are no essential finite size effects.

The initial state of a heavy ion reaction ($\tau = 0$) now consists of two nuclei of masses $M_P = \sqrt{\left(\sum_{i=1}^{A_P} p_i\right)^2}$ and $M_T = \sqrt{\left(\sum_{i=1}^{A_T} p_i\right)^2}$, where the projectile has the total momentum $P_P = M_P(1+\varepsilon, \sqrt{\varepsilon(\varepsilon+2)}\,\mathbf{e}_z)$ in the laboratory frame (target at rest) if $E_{\text{lab}} = A_P m_N \varepsilon$ is the bombarding energy. Furthermore, at $\tau = 0$ the two nuclei just touch each other, i.e., the first interaction for $\tau > 0$ is a penetration into the quasipotential of one nucleon in the projectile and one in the target. This fixes the centers $Y_P = (\sum_{i=1}^{A_P} m_i x_i)/M_P$ and $Y_T = (\sum_{i=1}^{A_T} m_i x_i)/M_T$ of the nuclei.

IV. NUMERICAL RESULTS

The reaction $^{40}_{18}\mathrm{Ar} + ^{40}_{18}\mathrm{Ar}$ was selected to illustrate a simulated heavy ion collision that is characterized by cluster formation and pion production. We examined this system in the energy range between 300 MeV and 100 GeV per nucleon. To get insight into a collision process we then focused on the reaction with 2 GeV per nucleon ($\varepsilon \approx 2.13$). Note that the system becomes purely academic beyond 2 GeV per nucleon, since all but incoherent pion production is neglected.

All presented results were obtained by averaging 1000 nearly central nucleus-nucleus collisions with a random impact parameter b smaller than one quarter of the sum of both nuclear radii. A single reaction is terminated when all clusters stay stable, i.e., the bound subsystems will not strongly decay to smaller clusters, radiate free nucleons or produce real pions.

For more detailed results and the investigation of a heavier system (Nb+Nb) we refer to [11].

A. Energy dependence

Although pions are created in a collision-induced process with an intermediate delta state as is done in the Cugnon cascade model [19], there is an essential difference that causes a clear reduction of the pion rate beyond 1 GeV per nucleon. For π^- this is shown in Fig. 3. By the fit of the pion production with the free nucleon-nucleon cross section an effective creation process is introduced that also contains contributions from multipion production. In our model pions are created in an elementary, dynamical process. Thus the cross section depends on more than the relative energy of the colliding nucleons (cf. Sec. III B). The excessive pion yield of the Cugnon cascade relative to the experimental data [20] then becomes increasingly reduced.

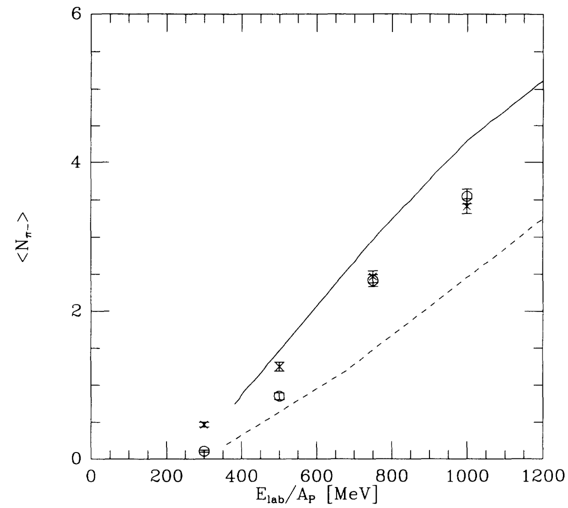

FIG. 3. Mean multiplicity of negatively charged pions in the reaction Ar+Ar (circles: $V_0 = 0$ MeV, crosses: $V_0 = 60$ MeV). Error bars mark the uncertainty in the average and not the deviation of single values. The dashed line represents experimental data from the reaction Ar+KCl [20] and the full line results from the Cugnon cascade [19].

In Fig. 4 we see the general behavior of this simplified system of nucleons and pions that underestimates the pion production increasingly with rising energy. However, it becomes clear that the average number of created pions is nearly independent of the strength of the attractive part in the nucleon-nucleon interaction (V_0) already for energies above 500 MeV.

The average number of stable fragments in the final state is shown in Fig. 5. We do not count free nucleons but otherwise include all fragments regardless of the number of nucleons contained in the cluster. These are

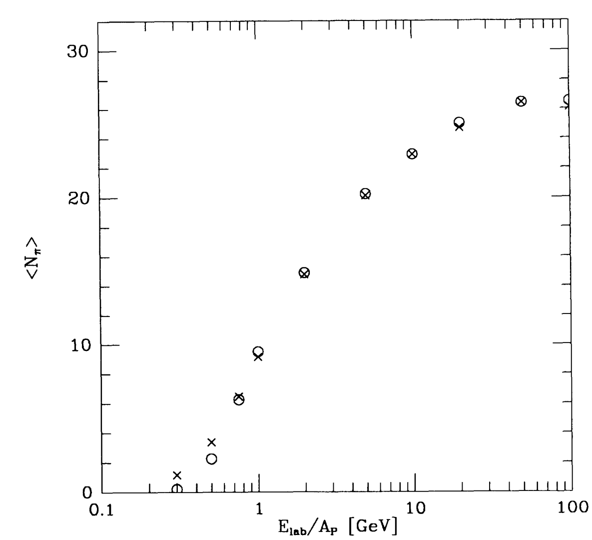

FIG. 4. Mean multiplicity of the total number of pions in the reaction Ar+Ar. Above 2 GeV per nucleon the production rate is increasingly underestimated because of the lack of appropriate reaction channels. However, we do not find an essential dependence on the depth in the quasipotential (circles: $V_0 = 0$ MeV, crosses: $V_0 = 60$ MeV).

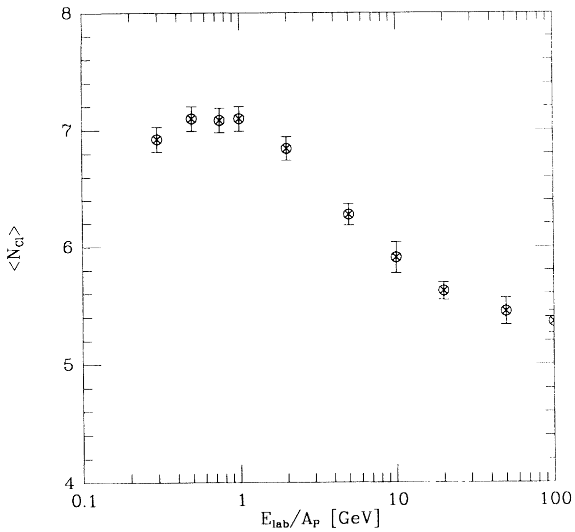

FIG. 5. Average number of stable fragments in the final state of the reaction Ar+Ar (regardless of the number of nucleons contained in one cluster). Error bars mark the uncertainty in the average and not the deviation of single values.

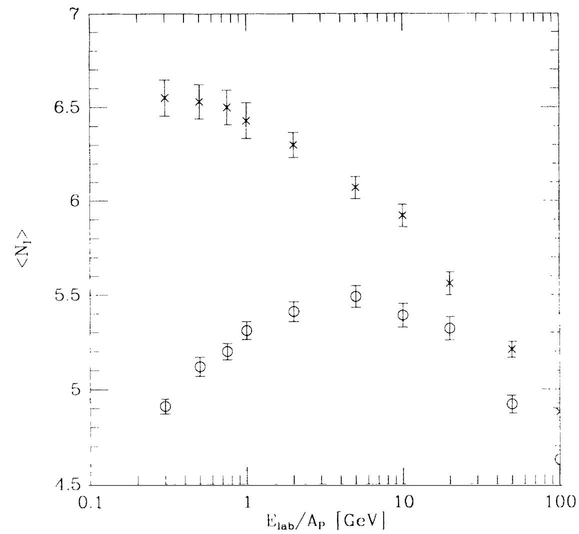

FIG. 6. Maximum number of simultaneous interactions that any nucleon had at any stage of the reaction Ar+Ar. This measure for the local nuclear density obviously increases by increasing the depth of the quasipotential (circles: $V_0 = 0$ MeV, crosses: $V_0 = 60$ MeV). Error bars mark the uncertainty in the average and not the deviation of single values.

mainly built by pairs of nucleons (cf. Fig. 7) and decrease only slightly in number with increasing energy. The latter effect will change, of course, for a reaction of heavier ions [11].

A further comparison of results with and without an attractive force between nucleons is presented in Fig. 6 ($V_0 = 0$ MeV and $V_0 = 60$ MeV, respectively). To get an idea on the highest local density obtained during a heavy ion collision we recorded the maximum number of simultaneous interactions that any of the individual nucleons had at any stage of the reaction. Obviously we find a clear increase of the nuclear density with increasing V_0 as expected, whereas the number of produced pions according to Fig. 4 is nearly uneffected. Hence, our results contradict the predictions made in [21]. By increasing the energy beyond 10 GeV per nucleon, the difference between the curves $V_0 = 0$ MeV and $V_0 = 60$ MeV becomes small, of course, since the average ratio of relative momentum and depth of the quasipotential becomes large.

Finally, we should add a remark on the mean value of τ at the end of the reaction. First of all τ_{end} does decrease with increasing energy of the projectile, but the ratio of τ_{end} to that value of τ where we find the maximum in the pion production (cf. Fig. 8) is more or less independent of energy. On the other hand, the physical time elapsed for an imagined free particle in the equal-speed system of both nuclei $\left[v = \sqrt{\frac{\varepsilon}{\varepsilon+2}} \text{ and } \varepsilon = E_{\mathrm{lab}}/(A_P m_N)\right]$ given in units of fm/c $\left[T_{\mathrm{end}} = \sqrt{\frac{\varepsilon+2}{2}}\ \tau_{\mathrm{end}}\right]$ is rather constant for reactions up to 10 GeV, but seems to increase at higher energies. This may be a consequence of the fact that in that regime particle production is not described adequately by our model.

B. Reaction statistics

A multiplicity distribution for the different fragments (mass number A) is given in Fig. 7. It is strongly decreasing with A even though in 1 event (of 1000) we found $A = 13$ in the final state. On the average the ratio of bound to free nucleons is about 1:4.

In Fig. 8 we show the mean multiplicity for pions, $\langle \frac{dN_\pi}{d\tau} \rangle$, as a function of the Lorentz scalar τ. The connection of the τ scale to a physical time scale may be deduced from an imagined free particle. For example, in the equal-speed system of both nuclei $\left[v = \sqrt{\frac{\varepsilon}{\varepsilon+2}} \approx 0.72\right]$ we find $T = \sqrt{\frac{\varepsilon+2}{2}}\ \tau$ [T in units fm/c] or $T = \sqrt{\frac{\varepsilon}{2}}\ \tau$

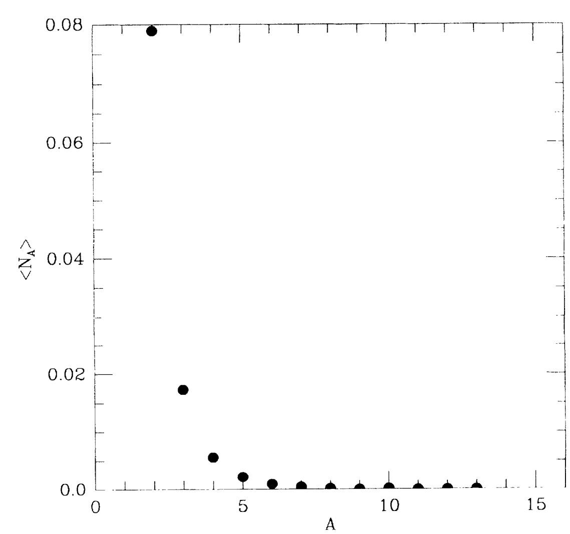

FIG. 7. Multiplicity distribution for the different fragments (mass number A) in the final state. As a normalization we have chosen one for the number of free nucleons (the point at $A = 1$, however, is omitted).

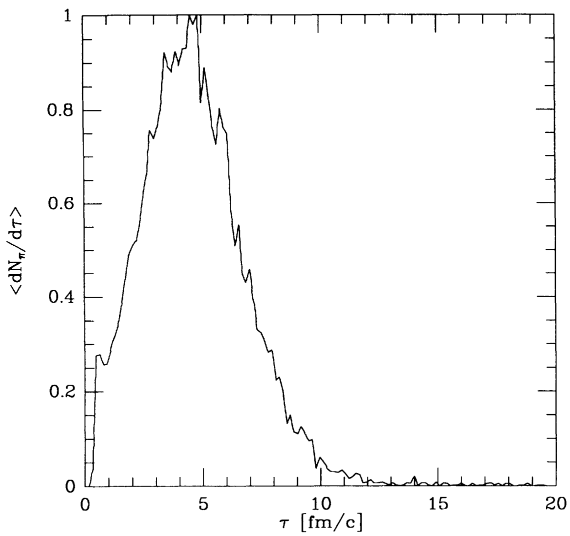

FIG. 8. Mean multiplicity for the pions as a function of τ. The maximum in the distribution is arbitrarily set to 1.

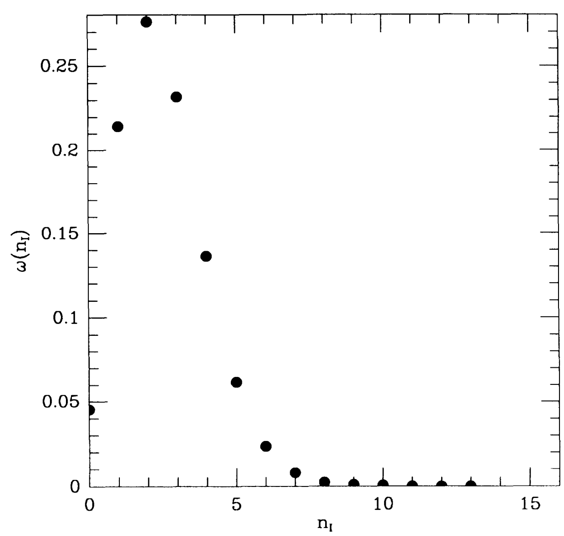

FIG. 9. Maximum number of simultaneous interactions n_I for a specified nucleon. Even though in one case $n_I = 13$ occurred we rarely find more than six simultaneous interactions. [Normalization, $\sum_{n_I} \omega(n_I) = 1$.]

[T in units fm/v]. Thus, the maximum in the production rate ($\tau \approx 5$ fm/c) occurs at $T \approx 7.2$ fm/$c \approx 5.2$ fm/v, and beyond $T \approx 28.7$ fm/$c \approx 20.6$ fm/v [$\tau = 20$ fm/c] pions very rarely emerge.

The number of pions produced by a specified nucleon is rather small. About 90% of the nucleons are involved in pion production only once during the reaction. Furthermore, none of the nucleons was responsible for more than six pions.

Figure 9 gives a first impression of the compression and the internal energy of the system. It shows the distribution of the maximum number of simultaneous interactions for a specified nucleon, i.e., the statistics of the highest local densities. We may conclude that the relative momentum available for particle creation becomes distributed quite rapidly among the nucleons. However, we always find a dilute nuclear system, since the maximum number of simultaneous interactions for a specified nucleon also remains small.

The rapid loss of relative momentum effected by nucleon-nucleon collisions is reflected in Fig. 10. We show the distributions for the relative momenta in the center of mass frame of the two interacting particles ($k = \sqrt{-\hat{q}^2}$). This is done separately just before (a) penetration into the quasipotential, (b) hard-core scattering, and (c) escape from the quasipotential. Although these reactions appear in short intervals on the average, the relative momentum of nucleons from different nuclei (right maximum) decreases considerably. For hard-core scattering and the escape from the quasipotential we already find a domination of the left maximum, containing mainly collisions between nucleons from the same nucleus in the initial state.

From Fig. 11 we get an impression of the kinematics in the final state. While the rapidity distribution $\left(y = \frac{1}{2}\ln\frac{E+p_\parallel}{E-p_\parallel}\right)$ of the pions has a single maximum at midrapidity, the nucleons and the fragments exhibit maxima at projectile and target rapidity as well as a good

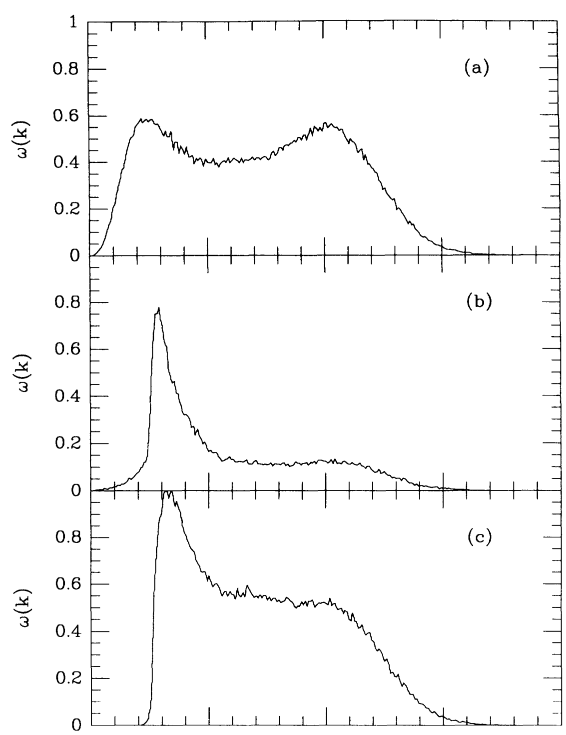

FIG. 10. Distribution for the relative momenta ($k = \sqrt{-\hat{q}^2}$) normalized to the free nucleon mass just before (a) penetration into the quasipotential, (b) hard-core scattering and (c) escape from the quasipotential in the reaction Ar+Ar at 2 GeV per nucleon. In (c) we arbitrarily set the maximum to one. The areas below the curves represent the frequency of the three different events.

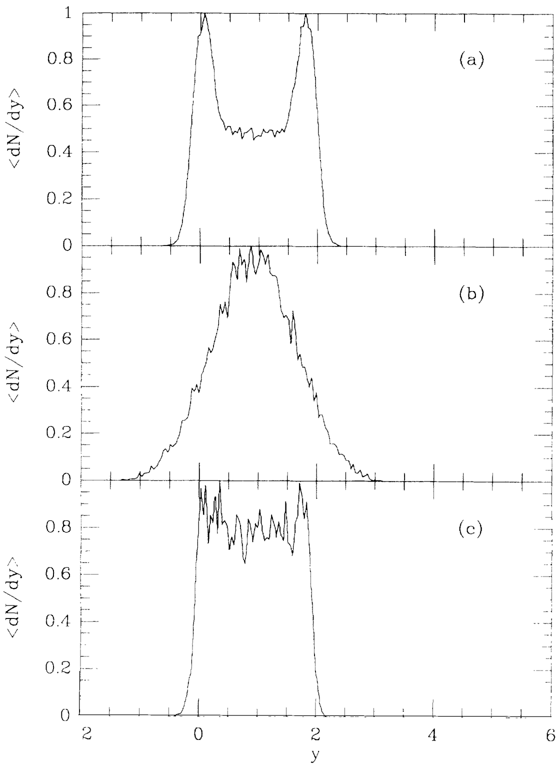

FIG. 11. Distributions of the rapidity $y = \frac{1}{2} \ln \frac{E+p_\parallel}{E-p_\parallel}$. For the reaction Ar+Ar at 2 GeV per nucleon we analyzed (a) the nucleons (free and bound), (b) the pions, and (c) the fragments in the final state. Normalization is arbitrary.

amount in between.

Often it is impossible to measure the rapidity (no particle identification) in an experiment. Then the pseudorapidity $[\eta = -\ln(\tan\Theta/2)]$ is presented instead. Both are comparable only in the limit $p_\perp \gg m$, which here does not apply. Hence, a statement about the rapidity is rather questionable when only the distribution of the pseudorapidity is known.

On the other hand, for a comparison of numerical results to the experimental data, it is essential to subject the numerical results of the model to the same "filter" of the final state that is effected by the detectors [22]. Because we do not aim at a detailed analysis of experimental data in this paper, we have not included such a presentation here.

V. CONCLUSIONS

By extension of phase space to $8N$ dimensions in order to guarantee a simple representation for the generators of the symmetry transformations (space-time displacements and rotations), a variety of ways become available to circumvent the no-interaction theorem. The main distinction between the different published models describing interacting relativistic particles regards the prescription for the elimination of redundant degrees of freedom. There the constraint Hamiltonian approach [8] seems to assume the major role, also incorporating the singular Lagrangian formalism and predictive mechanics, at least for the cases of interest.

It was not our intention to investigate these different formalisms with respect to their general degree of equivalence. Rather, we have shown here that, for the two-particle system all these different formalisms lead to an equivalent set of equations of motion and are thus physically indistinguishable, while for more than two particles none of these formalisms can be given a consistent physical interpretation. In our opinion, the single Hamiltonian formalism described in II A is the closest possible approach to what may be called physical intuition, mainly because the nonrelativistic limit is determined by applying Galilei instead of Lorentz transformations to the solutions in the center-of-mass frame.

We have therefore used this form to construct a relativistic intranuclear cascade model which allows two-particle quasipotentials when applied to sufficiently dilute systems. This restriction is a consequence of the necessary momentum dependence of the quasipotentials, enforced by the mass shell constraint for free nucleons, leading to the distinction between canonical and physical coordinates.

The results obtained for the reaction ${}^{40}_{18}\mathrm{Ar} + {}^{40}_{18}\mathrm{Ar}$ in the energy regime between 300 MeV and 2 GeV per nucleon are in reasonable agreement with the experimental data [20,22]. A particularly interesting result is that the pion yield is nearly unaffected by the attractive part of the nucleon-nucleon interaction, while the nucleon density increases as expected (cf. Figs. 3, 6). This is in obvious contradiction to an idea presented in [21].

Before more detailed comparisons to the many experimental data obtained by the BEVALAC groups can be meaningful, it is essential to subject the numerical results of the model to the same "filter" of the final state that is effected by the detectors [22].

The numerical results above a bombarding energy of 2 GeV per nucleon have to be assessed cautiously. While the direct nucleon-nucleon interaction becomes less important, the creation of particles will increasingly dominate the whole reaction. Hence, it is necessary to develop a concept of particle production compatible with the underlying Hamiltonian formalism even for a large number of created particles. Our present "perturbative" mechanism of pion production on the basis of the nucleon degrees of freedom is surely insufficient beyond BEVALAC energies.

A successful model for describing particle production has been devised by the Lund group [23], where new hadrons are created in recursively defined fragmentation processes, using a string picture for the $q\bar{q}$ interaction. The translation of that idea to our model consists in an application of our production mechanism to the degrees of freedom of quarks confined by a linear quasipotential inside the hadrons. In other words we are aiming at incorporating the quarks into a description of heavy ion reactions purely on the basis of particle dynamics, where all field degrees of freedom (including strings) are subsumed in quasipotentials. We call such a model "Poincaré-covariant collision dynamics" [13]. Whether this framework may be justified by other approaches like lattice QCD [24] or quark gluon transport theory [25] is yet an open question.

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