Intermittency and correlations in ^{16}O + Ag/Br interactions at 2.1 GeV/nucleon

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An increase of the scaled factorial moments with decreasing bin size of phase space—the phenomenon popularly known as intermittency—is a common feature in multiparticle production in relativistic high energy interactions. The data of ^{16}O + Ag/Br interactions at 2.1 GeV/nucleon also revealed the same feature which, usually, is identified as the revelation of singularity structure of correlation functions. In this article, we extend our analysis of ^{16}O + Ag/Br interactions at 2.1 GeV/nucleon data in terms of other tools (giving correlations) such as the scaled factorial correlators, the factorial cumulant moments, and the split-bin correlation functions. The study reveals that the information on correlations of our particle production data are contained in two-particle dynamical correlations only and that the two-particle dynamical correlations are due to a resonancelike production mechanism.

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The study of correlations in multiparticle production in high energy relativistic interactions entered into a new era with the appearance of the power-law dependence [1],

$$\ln F_q(\delta) = -a_q \ln \delta + b_q \quad \text{as } \delta \to 0 \ , \tag{1}$$

of the scaled factorial moments (giving correlations), F_q of order q, of the distribution of produced particles in small phase-space bins of size δ, which are smaller than the usual resonance correlation length. Termed "intermittency," the study of the super-short-range correlations started with the analysis of the ultrahigh energy cosmic ray event [2] of heavy-ion interaction. Subsequently, a wide range [3–22] of experimental and theoretical attempts have been made for understanding the origin of the power-law behavior. Initially, only the random cascade model or the α model [1] and no other model could provide a possible explanation to the "scale-invariant" intermittency phenomenon. However, the scenario has improved with different schools of thought [23–25] appearing in interpreting the power law and thus ideally providing a situation for better understanding of the phenomenon through analysis of experimental data in complementary approaches. There has been a considerable amount of criticism against the α model as well as against the use of the scaled factorial moments (SFMs) as an experimental tool [26,27]. In fact, on the experimental front—though almost all the available data of high-energy interactions have already been analyzed in terms of SFMs—a universally acceptable interpretation remains to be identified: other approaches or tools for a similar type of correlation study have not yet been fully utilized.

We have ^{16}O + Ag/Br interaction data at 2.1 GeV/nucleon, thoroughly analyzed in terms of SFMs with the "positive indication" of the power-law phenomenon in the distribution of produced particles and thus extending the incident energy range of the validity of the phenomenon [20]. This article presents analyses of the same data sample in terms of some aspects of other similar type of tools (for the study of correlations), namely the scaled factorial correlators (SFCs) [1,26,28], the factorial cumulant moments (FCMs) [23], and the split-bin correlation functions (SBCFs) [24], all of these being closely related to SFMs. Each set of correlation functions has merit and will be helpful, if available for all experiments, in the search for the origin of the power-law phenomenon. Our data were obtained by irradiating, horizontally, an Ilford G5 emulsion stack of pellicles of dimensions $10\times5\times0.06$ cm^3 with the ^{16}O beam of Bevalac, Berkeley at 2.1 GeV/nucleon incident energy. A sample of 731 events of central ^{16}O + Ag/Br interactions were selected through scanning of the emulsion plates under Leitz Ortholux microscope. Following the usual emulsion methodology, tracks of produced secondaries were identified and emission angles θ of these secondaries were measured, using a semiautomatic measuring system with 1 μm resolution along X and Y axes and 0.5 μm along the Z axis. The detail of the experiment is available in Ref. [20].

The definition of SFMs, which are largely used for analyzing the data sample with a range of multiplicities (and, in [20], for analyzing the data in consideration), are given by

$$F_q = M^{q-1} \sum_{j=1}^{M} n_j(n_j-1)\cdots(n_j-q+1)/\langle n\rangle^q \ , \tag{2}$$

where M is the number of bins (of size δ) into which the initial available phase space is divided, n_j is the number of particles in the jth bin, and $\langle n\rangle$ is the average multiplicity of the particles within the considered phase space (generally rapidity or pseudorapidity). These are horizontal SFMs and, since these moments are sensitive to the shape of the density distribution, corrections are made by dividing the SFMs by the correction factor [22],

$$R_q = (1/M) \sum_{j=1}^{M} M^q \langle n_j\rangle^q / \langle n\rangle^q \ , \tag{3}$$

where

$$\langle n_j \rangle = \frac{1}{N_{\text{event}}} \sum_{i}^{N_{\text{event}}} n_{j,i} \ .$$

Now the random cascade model or the α model, which was developed to explain the power-law behavior of the SFMs, predicts another power law in terms of scaled factorial correlators (SFCs) [1], F_{pq}, given by

$$\langle F_{pq} \rangle = (\Delta/D)^{a_{pq}} \ , \tag{4}$$

where the SFC of order pq,

$$\langle F_{pq} \rangle = \frac{[\langle n_i(n_i-1)\cdots(n_i-p+1)n_j(n_j-1)\cdots(n_j-q+1)\rangle]}{[\langle n_i(n_i-1)\cdots(n_i-p+1)\rangle\langle n_j(n_j-1)\cdots(n_j-q+1)\rangle]} \ , \tag{5}$$

characterizes correlations between multiplicity fluctuations in two bins i and j, with multiplicities n_i and n_j, respectively, separated by a distance D. Δ is the considered phase space such that $\delta = \Delta/D$ is the bin size. The α model predicts the δ independence of F_{pq}. While the SFMs calculate nonstatistical fluctuations in small phase-space bins, the SFCs correlate the fluctuations in different bins and thus SFCs are likely to reveal additional information. The theory of noise suppression in SFMs is valid for SFCs also.

So far, hadron-hadron [28] interaction data at 250 GeV/c, hadron-nucleus [29] interaction data at 200 GeV/c, and nucleus-nucleus [26] interaction data at 200 GeV/nucleon have been analyzed in terms of SFCs. We study the behavior of SFCs in the pseudorapidity ($\eta = -\ln\tan\theta 2$) space of width $\Delta\eta$=4.0 around the peak of the distribution [20]. Precisely, this is the range of the distribution where the SFM analysis has already been carried out. We calculate SFCs for four different values of the bin size by dividing the selected width ($\Delta\eta$=4.0) by M = 10, 20, 30, and 40. For a particular $\delta\eta = \Delta\eta/M$, the SFCs are calculated for all possible combinations of bins i and j and then to study the variation with D, the SFCs for different D values (bin-bin separation) are averaged separately for a given D. The variation of $\ln\langle F_{pq}\rangle$ as a function of $-\ln D$ is shown in Fig. 1. The positive slopes a_{pq}, given in Table I, clearly show that the correlated moments increase with a decreasing correlation length, D. The decrease in slopes, for the same order of pq, with decreasing bin size is also evident from Table I. Of course, the scaling [Eq. (4)] of the SFCs is valid for a certain range of D values. Because of the bending, appearing in Fig. 1, the slopes a_{pq} have been obtained from the linear fits restricted to a certain range of D values, which are mentioned in Table I. As the slopes of the plots depend on the selection of the D range, for a reasonable comparison, we keep the maximum of the D range the same for all four values of $\delta\eta$. The minimum of the range in each case is the $\delta\eta$ value. We find that scaling holds for our hadronization data at 2.1 GeV/nucleon within

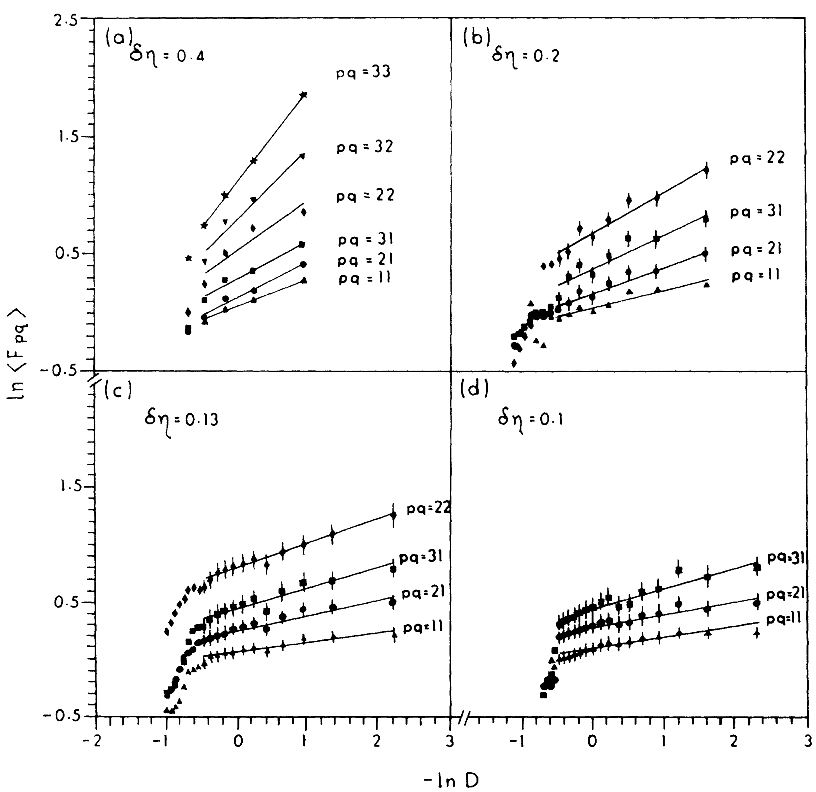

FIG. 1. Plots of $\ln\langle F_{pq}\rangle$ vs $-\ln D$ for ^{16}O + Ag/Br data at 2.1 GeV/nucleon for different values of $\delta\eta$.

TABLE I. Values of slope parameters for scaled factorial correlators studied in ^{16}O + Ag/Br interactions at 2.1 GeV/nucleon.

M	Order of correlators	Slope of the plots
	11	0.193±0.008
	21	0.306±0.018
	31	0.317±0.018
10	22	0.507±0.024
(0.40≤ D ≤1.60)	32	0.606±0.027
	33	0.824±0.031
	11	0.157±0.007
20	21	0.224±0.009
(0.20≤ D ≤1.60)	31	0.245±0.012
	22	0.345±0.020
	11	0.094±0.007
30	21	0.133±0.008
(0.13≤ D ≤1.60)	31	0.182±0.011
	22	0.212±0.021
	11	0.079±0.008
40	21	0.107±0.008
(0.10≤ D ≤1.60)	31	0.161±0.010

the range of the correlation length $D \leq 1.60$ in the pseudorapidity distribution. The $\delta\eta$ independence has also been observed (not shown). However, as already shown in Ref. [28], the $\delta\eta$ independence seems to be common to any model with standard short-range correlations. Further, the drop in the plots in Fig. 1 for higher values of D ($D \geq 1.60$) may be explained in terms of exponential short-range correlation function [28].

The results from the analysis of our 2.1 GeV/nucleon data in terms of SFCs are qualitatively identical with those already available from the hadron-hadron [28], the hadron-nucleus [29], and the nucleus-nucleus [26] data at the higher side of the available accelerator energy range. Thus, like the scaling behavior of the SFMs, the scaling behavior of SFCs also may appear to be a general phenomenon in multiparticle production. In theory, attempts are made in predicting physics in terms of the slope parameters of the power laws. Experimentally, however, the slope parameters from SFM analyses do not reveal any consistent dependence on incident energy and/or type of interactions (discussed in [20]). The results available from a few analyses in terms of SFCs also are not sufficient for a conclusive study of dependence of slope parameters. But the similar type of observations in the study of the power-law behavior of the SFCs in all of the analyses carried out so far, including different types of interactions and at widely varied energy range (extending to a few GeV/nucleon with this present), reveal that experimentally the α model has not yet provided any confirmable, discriminating information for interpreting the "intermittency phenomenon" for different types of high energy interactions. This situation demands the need for trying alternative approaches in the search for the origin of the power law. There have been a number of alternative approaches, and discussion on all these is beyond the scope of this article. To start with, we include here the analysis of data in terms of some aspects of factorial cumulant moments (FCMs) and split-bin correlation functions (SBCFs), which are related to SFMs but are distinct correlation functions having a number of experimental advantages over SFMs and SFCs.

Carruthers and collaborators advocated [23] the use of the FCMs in the analysis of multiparticle production data, suggesting that the observed increase in SFMs is due to the short-range correlations and therefore the introduction of the power law [Eq. (1)] may not be essential.

In the case of M equal bins of size $\delta\eta = \Delta\eta/M$, where n_j is the number of particles in the jth bin, ρ_q is the q-particle density correlation function, and the bin-averaged (vertical) factorial cumulant moments are defined as

$$K_q(\delta\eta) = \frac{1}{M(\delta\eta)^q} \sum_j \int_{\Omega_j} \prod_i d\eta_i C_q(\eta_1 , \dots , \eta_q)/(\bar{\rho}_j)^q , \tag{6}$$

where C_q are the cumulant correlation functions, giving, e.g., the two- and three-particle correlation functions,

$$\begin{aligned} C_2(\eta_1,\eta_2) &= \rho_2(\eta_1,\eta_2) - \rho_1(\eta_1)\rho_1(\eta_2) , \\ C_3(\eta_1,\eta_2,\eta_3) &= \rho_3(\eta_1,\eta_2,\eta_3) - \rho_1(\eta_1)\rho_2(\eta_2,\eta_3) \\ &\quad -\rho_1(\eta_2)\rho_2(\eta_3,\eta_1) - \rho_1(\eta_3)\rho_2(\eta_1,\eta_2) \\ &\quad +2\rho_1(\eta_1)\rho_1(\eta_2)\rho_1(\eta_3) , \end{aligned} \tag{7}$$

for $q = 2$ and 3, respectively.

The FCMs are connected to the bin-averaged (vertical) factorial moments [not exactly to the horizontal factorial moments, given by Eq. (2)],

$$\begin{aligned} F_q(\delta\eta) &= \frac{1}{M} \sum_{j=1}^{M} \langle n_j(n_j-1)\cdots(n_j-q+1)\rangle/\langle n_j\rangle^q \\ &= \frac{1}{M(\delta\eta)^q} \sum_{j=1}^{M} \int_{\Omega_j} \prod_i d\eta_j \rho_2(\eta_1 , \dots , \eta_q)/(\bar{\rho}_j)^q , \end{aligned} \tag{8}$$

by the relations

$$\begin{aligned} &F_2 = 1 + K_2 , \\ &F_3 = 1 + 3K_2 + K_3 , \\ &F_4 + 1 + 6K_2 + 3(K_2)^2 + 4K_3 + K_4 , \text{ etc.} \end{aligned} \tag{9}$$

These decomposed forms of F_q moments allow the removal of lower-order background contributions to obtain the true contributions of correlations of order q. Using measured data of F_q, one can obtain K_q from Eq. (9).

In Ref. [20], we have calculated horizontal factorial moments [given by Eq. (2)], corrected for the shape of the distribution. Equation (2) is not precisely the same as Eq. (8). So, the K_q moments calculated for the ^{16}O + Ag/Br interaction at 2.1 GeV/nucleon, from the corrected SFMs, using Eq. (9), give approximate values only. Even these approximate K_q moments, when plotted against the decreasing bin size (as shown in Fig. 2)

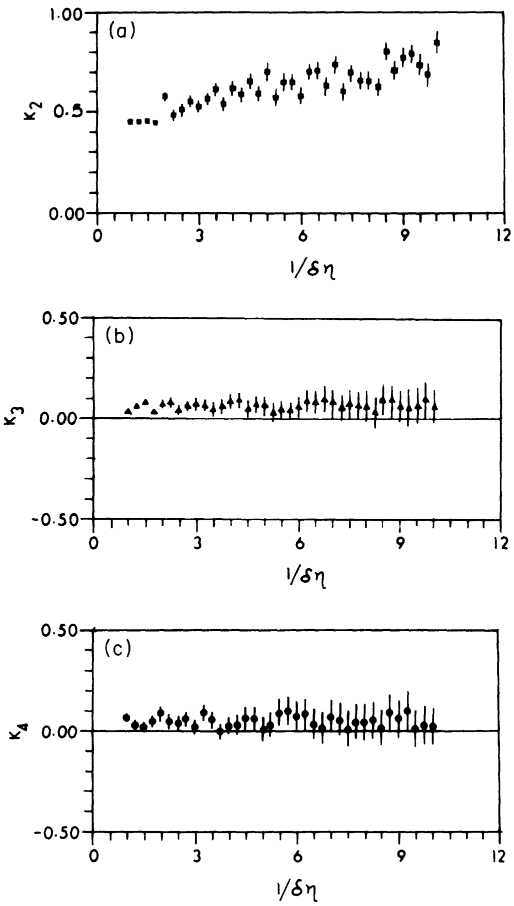

FIG. 2. Plots of cumulant moments K_q for (a) $q = 2$, (b) $q = 3$, and (c) $q = 4$ against $1/\delta\eta$ for ^{16}O + Ag/Br data at 2.1 GeV/nucleon.

are found to give significantly nonzero values only for $q = 2$, while those for $q = 3$ and $q = 4$ are compatible with zero.

Thus our analysis in terms of factorial cumulant moments or the K moments reveal that the observed increase in F_q for the ^{16}O + Ag/Br data at 2.1 GeV/nucleon, as has been shown in Ref. [20], is entirely due to two-particle dynamical correlations and that the higher-order correlations are absent in the data. Interestingly, this observation of the existence of only two-particle correlations at this lower end of relativistic energies is identical with those in similar analyses of heavy-ion interaction data at a higher range of relativistic accelerator energies [23]. However, a reasonable comparison between K_2 moments of our data and those obtained from corrected, horizontal SFMs of KLM data of ^{16}O + emulsion interaction at 200 GeV/nucleon [23] reveals that the K_2 moments increase with decreasing c.m. energy of the interaction.

The indication of the presence of only two-particle dynamical correlations in both the data sets leads to the query of whether the cause for the two-particle dynamical correlations for these widely varied range of energies is the same or else they have different production mechanisms. At this stage, to identify a possible source of dynamical correlations in our 2.1 GeV/nucleon data, we use a simple tool, namely the split-bin correlation functions (SBCFs) [24], prescribed for the test of the different production mechanism. As an experimental tool for studying correlations, also, the SBCFs have advantages over SFMs and SFCs. Since almost all the information on correlations are contained in the second moments, and this is true for virtually all correlation functions when correlations are small [24], we analyze SBCFs of second order only.

If each of the M equal bins (as discussed earlier) of the pseudorapidity space is divided in two sub-bins denoted by L (left) and R (right) and if n_j^L (n_j^R) is the number of particles in the left (right) sub-bin of the jth bin, we have the second-order SBCF,

$$S_2(M) = M \sum_{j=1}^{M} \langle n_j^L n_j^R \rangle / \langle N^L N^R \rangle \ , \tag{10}$$

where N^L (N^R) is the number of particles in the left (right) half of the pseudorapidity window. Here also, as in the case of SFM analysis, these moments need to be corrected for the nonflat single-particle distribution by subtracting the component,

$$S_2^c(M) = M \sum_{j=1}^{M} \langle n_j^L \rangle \langle n_j^R \rangle / \langle N^L \rangle \langle N^R \rangle \ . \tag{11}$$

SBCFs are less susceptible to any systematic errors involving detector efficiency and, therefore, can be measured more accurately than the SFMs and SFCs. As experimental tools, SBCFs have many other advantages over other similar types of correlation functions [24]. However, for the present analysis, we use the SBCFs mainly for identifying the process or mechanism likely to be responsible for the two-particle dynamical correlations of our hadronization data. It comes from comparison of SBCFs S_2 and $S_2^{(\phi)}$, where $S_2^{(\phi)}$ is obtained by dividing each of the pseudorapidity bins into two equal sub-bins in azimuthal angle ϕ. While S_2 is likely to be larger than $S_2^{(\phi)}$ for "resonancelike" mechanism, $S_2 < S_2^{(\phi)}$ indicates that the correlations are most likely due to "jetlike" production mechanism. $S_2^{(\phi)}$ is given by

$$S_2^{(\phi)}(M) = M \sum_{j=1}^{M} \langle n_j^+ n_j^- \rangle / \langle N^+ N^- \rangle \ , \tag{12}$$

where n_j^+ (n_j^-) is the number of particles in the jth bin with $\cos\phi > 0$ $(\cos\phi < 0)$ and N^+ (N^-) is the number of particles in the pseudorapidity window with $\cos\phi > 0$ $(\cos\phi < 0)$. We calculate S_2 and $S_2^{(\phi)}$, both corrected for nonflat single-particle density distribution, and plot $\ln S_2$ and $\ln S_2^{(\phi)}$ against $-\ln\delta\eta$ (for $1.0 \geq \delta\eta \geq 0.1$) in Fig. 3, along with the plot of the second-order SFMs (from Ref. [20]) for comparison. Like SFMs, the SBCFs also increase (as can be seen from the plots in Fig. 3) with decreasing bin size. Further, from Fig. 3, where S_2

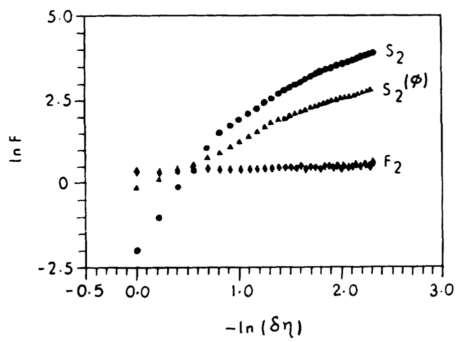

FIG. 3. Plots of logarithms of split-bin correlation functions, $\ln S_2$ and $\ln S_2^{(\phi)}$ as well as $\ln F_2$ against $-\ln \delta\eta$ for ^{16}O + Ag/Br interactions at 2.1 GeV/nucleon.

is larger than $S_2^{(\phi)}$, we find, within the framework of the simple test carried out, that the two-particle dynamical correlations even below the range of the so-called "usual resonance correlation length" are due to resonancelike production mechanism.

Thus, by extending the study of our nucleus-nucleus data at 2.1 GeV/nucleon in the α model as well as in alternative approaches, we have a broader range of information on the study of correlations in multiparticle production. The idea of removal of lower-order background contributions from F_q moments to obtain the true contributions of correlations of order q [23] is definitely an improvement in the correlation study. The effects of resonancelike correlations at Bevalac energies are not new. The SBCF analysis reveals the effect even below the range of the usual resonance correlation length.

Lastly, a comparison with data from other experiments would have made this study more meaningful. But, unfortunately, we do not yet have any analysis data available in terms of SFCs, FCMs, or SBCFs at the considered energy range. It will be worthwhile having information on other experimental data at this energy range analyzed in terms of these tools of correlation study in multiparticle production.

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