Study of in-medium NN inelastic cross section from relativistic Boltzmann-Uehling-Uhlenbeck approach

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We have derived the explicit expressions for calculating in-medium $NN \rightarrow N\Delta$ cross sections, which are simultaneously given with the other integrands of the transport model such as the mean field, NN elastic cross section, as well as the transport equation itself based on the effective Lagrangian. Our results can reproduce the experimental data of free $\sigma_{pp\rightarrow pp\pi^0+pn\pi^+}$ nicely. The in-medium inelastic cross section $\sigma^*_{NN\rightarrow N\Delta}$ is calculated up to twice the nuclear matter density and is in agreement with the Dirac-Brueckner calculation.

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I. INTRODUCTION

There is increasing interest in developing a more strict and self-consistent dynamic theory for intermediate and high energy heavy ion collisions (HIC) for it has been found that the information for properties of hot and dense nuclear matter, i.e., the equation of state (EOS), comes out very indirectly. Botermans and Malfliet [1] proposed an approach in which the time evolution of the HIC process is described by the relativistic kinetic equation and the G matrix served as a dynamical input for the two-body interaction. However, it has great practical difficulty to acquire a complete numerical solution with their approach. More practically, in Refs. [2–4] a self-consistent relativistic Boltzmann-Uehling-Uhlenbeck (RBUU) equation is derived based on the effective Lagrangian, in which both mean field and in-medium NN cross sections are treated self-consistently and can be calculated simultaneously so that the medium effect can be taken into account automatically. However, in Refs. [2–4] only the nucleonic degree of freedom is taken into account. As is well known for high energy HIC the delta plays a very important role. The $NN \rightarrow N\Delta$ channel in the two-body collision term becomes more and more important as the colliding energy increases. And, on the other hand, as the energy increases the density of matter produced in the HIC becomes higher. The role of medium effects becomes more pronounced. One has to include the in-medium $NN \rightarrow N\Delta$ cross section in the two-body collision term.

The in-medium $NN \rightarrow N\Delta$ cross section has been studied by ter Haar and Malfliet with the Dirac-Brueckner approach [5]. They found that the $NN \rightarrow N\Delta$ cross sections were suppressed strongly at high density. Later Bertsch and Brown *et al.* [6] pointed out that the screen and antiscreen effects of the medium on the interaction enhanced the $NN \rightarrow N\Delta$ cross section at high density substantially. However, a nonrelativistic description is used in their work. It seems highly desirable to treat all the ingredients of the transport model such as mean field, elastic NN cross sections, inelastic NN cross sections, as well as the transport equation itself within the same framework in order to keep self-consistency. The aim of this paper is to study the in-medium $NN \rightarrow N\Delta$ cross section based on the same framework with Refs. [2–4], where a σ-ω type effective Lagrangian is used. In order to study $NN \rightarrow N\Delta$ cross sections the delta and pion degrees of freedom have to be included in the effective Lagrangian in addition to nucleon and σ, ω mesons. Serot and Walecka in Ref. [7] indicated that inclusion of the pion only changes the coupling constants g_σ and g_ω by less than 10% and has only a small effect on nuclear matter saturation property at the Hartree-Fock (HF) level, and we will not change the parameters of Ref. [4] for σ, ω mesons in this work.

This paper is organized as follows. In Sec. II we will derive the inelastic part of the collision term and a brief review of the model is also given in this section. In Sec. III we give the numerical results of both the free and effective inelastic cross section. Finally, a brief summary and outlook is given in Sec. IV.

II. FORMALISM

In this section we will give the derivation of the inelastic NN cross section through construction of the collision part of the kinetic equation. First of all, we write down the total effective Lagrangian used in the model. In order to consider the inelastic scattering cross section for $NN \rightarrow N\Delta$ reaction the Δ and π degrees of freedom have to be taken into account. The Lagrangian density for a system of nucleons and deltas interacting through σ, ω, π mesons can be written as

$$L = L_F + L_I \ . \tag{1}$$

Here L_F is the Lagrangian density for free nucleon, delta, and meson fields:

$$L_F = \bar{\psi}[i\gamma_\mu\partial^\mu - M_N]\psi + \bar{\psi}_{\Delta\nu}[i\gamma_\mu\partial^\mu - M_\Delta]\psi^\nu_\Delta + \tfrac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - U(\sigma) - \tfrac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \tfrac{1}{2}m^2_\omega\omega_\mu\omega^\mu + \tfrac{1}{2}(\partial_\mu\boldsymbol{\pi}\partial^\mu\boldsymbol{\pi} - m^2_\pi\boldsymbol{\pi}^2)\ , \tag{2}$$

$$U(\sigma) = \tfrac{1}{2}m^2_\sigma\sigma^2 + \tfrac{1}{3}b(g_\sigma\sigma)^3 + \tfrac{1}{4}c(g_\sigma\sigma)^4\ . \tag{3}$$

L_I is the interaction Lagrangian density

$$L_I = g_\sigma\bar{\psi}(x)\psi(x)\sigma(x) + g_\sigma\bar{\psi}_{\Delta\nu}(x)\psi^\nu_\Delta(x)\sigma(x) - g_\omega\bar{\psi}(x)\gamma_\mu\psi(x)\omega^\mu(x) - g_\omega\bar{\psi}_{\Delta\nu}(x)\gamma_\mu\psi^\nu_\Delta(x)\omega^\mu(x)$$
$$+\frac{f_\pi}{m_\pi}\bar{\psi}(x)\gamma_\mu\gamma_5\boldsymbol{\tau}\cdot\psi(x)\partial^\mu\boldsymbol{\pi}(x) - \frac{f^*}{m_\pi}\bar{\psi}_{\Delta\mu}(x)\partial^\mu\boldsymbol{\pi}(x)\cdot\mathbf{S}^+\psi(x) - \frac{f^*}{m_\pi}\bar{\psi}(x)\mathbf{S}\psi_{\Delta\mu}(x)\cdot\partial^\mu\boldsymbol{\pi}(x)\ , \tag{4}$$

where $\psi_{\Delta\mu}$ is the Rarita-Schwinger spinor of the Δ baryon, the symbols and notation are the same as in [7]. Here we have assumed that the coupling to scalar and vector mesons for both nucleons and deltas are the same following the same arguments of [8].

For a nonequilibrium problem it is convenient to make use of the closed time-path Green's function technique [9]. The nucleon Green's function in the interaction picture can be written as

$$iG_{12} = \left\langle T\left[\exp\left(-i\oint dx\, H_I(x)\right)\psi(1)\bar{\psi}(2)\right]\right\rangle\ . \tag{5}$$

Here $T[\ ]$ denotes the time-ordered product, $\oint dx \equiv \oint dt\, d\mathbf{x}$, $\oint$ stands for the integral along the contour which is given in Fig. 1.

According to a specified time of the field operator $\psi(1)$ and $\bar{\psi}(2)$ on the contour we have four different Green's function G^{--}_{12}, G^{++}_{12}, G^{-+}_{12}, G^{+-}_{12} consisting of a matrix

$$G_{12} = \begin{pmatrix} G^{--}_{12} & G^{-+}_{12} \\ G^{+-}_{12} & G^{++}_{12} \end{pmatrix}\ . \tag{6}$$

Other particle's Green functions can also be expressed in the same way. In Appendix A we give the zero-order Green's functions of nucleon, delta, and π mesons used in the present paper.

The corresponding Dyson equation for nucleon's Green's function has the form

$$iG_{12} = iG^0_{12} + \oint dx_3\oint dx_4 G^0_{14}\Sigma(4,3)iG_{32}\ . \tag{7}$$

The Dyson equation for the delta particle has the similar formulation [16]. The Dyson equations for delta's and nucleon's are coupled with each other through self-energy $\Sigma(4,3)$ which is also a matrix like G_{12}. Detailed discussions of Dyson equation for the delta particle go beyond the scope of this paper.

Under Born approximation the nucleon self-energy $\Sigma(4,3)$ becomes

$$\Sigma(4,3) = \Sigma_{\rm HF}(4,3) + \Sigma_{\rm Born}(4,3)\ . \tag{8}$$

$\Sigma_{\rm HF}(4,3)$ includes the Hartree term and the Fock term

$$\Sigma_{\rm HF}(4,3) = \Sigma_{\rm H}(4,3) + \Sigma_{\rm F}(4,3)\ . \tag{9}$$

The Born term $\Sigma_{\rm Born}(4,3)$ is illustrated by Fig. 2, where a doubled line denotes delta particle, solid and dashed lines represent nucleon and meson, respectively. Since in the intermediate energy regime the one delta production is dominant, only one doubled line appears in each corresponding diagram. Figures 2(a) and 2(b) are related to the following process (see also Fig. 3) which contribute to the elastic cross section for $N + N \to N + N$ reaction. We have studied this part in detail in Ref. [4]. Figures 2(c)–(f) are related to the process of Fig. 4, which contribute to the inelastic cross section for $N + N \to N + \Delta$ reaction. The detailed expressions of parts corresponding to Figs. 2(c)–(f) are given as

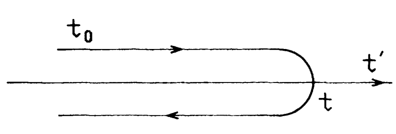

FIG. 1. Contour along the time axis for an evaluation of the operator expectation value.

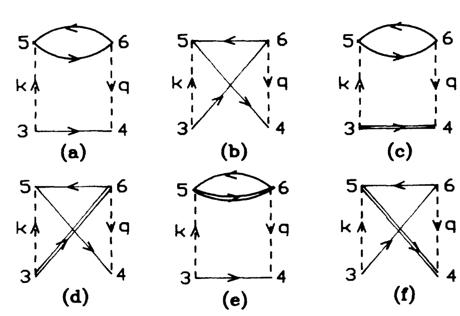

FIG. 2. Feynman diagrams for lowest-order contributions to the collision term.

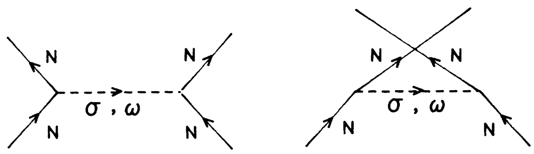

FIG. 3. Feynman diagrams for NN elastic scattering.

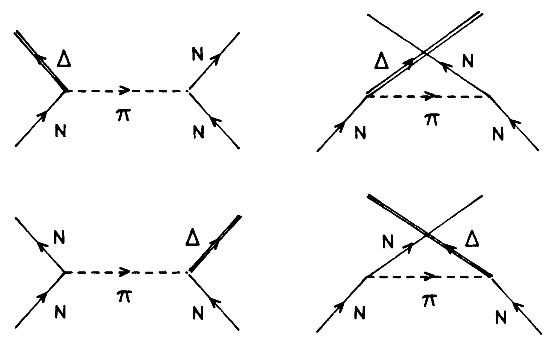

FIG. 4. Feynman diagrams for NN inelastic scattering.

$$\Sigma^{(c)}(4,3) = \sum_{T_4 t_5 t_6} \int dx_5 \int dx_6 \left\langle t \left| \frac{f^*}{m_\pi} S_j \right| T_4 \right\rangle q^\nu k^\mu G^0_{\nu\mu}(4,3) \left\langle T_4 \left| \frac{f^*}{m_\pi} S_i^+ \right| t \right\rangle \times \mathrm{tr}\left\{ \left\langle t_6 \left| \frac{f_\pi}{m_\pi} \not{k} \gamma_5 \tau_i \right| t_5 \right\rangle G^0(5,6) \left\langle t_5 \left| \frac{f_\pi}{m_\pi} \not{q} \gamma_5 \tau_j \right| t_6 \right\rangle G^0(6,5) \right\} \Delta^0_\pi(5,3)\Delta^0_\pi(4,6) \,, \tag{10}$$

$$\Sigma^{(d)}(4,3) = -\sum_{t_4 t_5 T_6} \int dx_5 \int dx_6 \left\langle t \left| \frac{f_\pi}{m_\pi} \not{q} \gamma_5 \tau_j \right| t_4 \right\rangle G^0(4,5) \left\langle t_4 \left| \frac{f_\pi}{m_\pi} \not{k} \gamma_5 \tau_i \right| t_5 \right\rangle \times G^0(5,6) \left\langle t_5 \left| \frac{f^*}{m_\pi} S_j \right| T_6 \right\rangle G^0_{\rho\mu}(6,3) q^\rho k^\mu \left\langle T_6 \left| \frac{f^*}{m_\pi} S_i^+ \right| t \right\rangle \Delta^0_\pi(5,3)\Delta^0_\pi(4,6) \,, \tag{11}$$

$$\Sigma^{(e)}(4,3) = \sum_{t_4 t_5 T_6} \int dx_5 \int dx_6 \left\langle t \left| \frac{f_\pi}{m_\pi} \not{q} \gamma_5 \tau_j \right| t_4 \right\rangle G^0(4,3) \left\langle t_4 \left| \frac{f_\pi}{m_\pi} \not{k} \gamma_5 \tau_i \right| t \right\rangle \times \mathrm{tr}\left\{ \left\langle T_6 \left| \frac{f^*}{m_\pi} S_i^+ \right| t_5 \right\rangle G^0(5,6) \left\langle t_5 \left| \frac{f^*}{m_\pi} S_j \right| T_6 \right\rangle G^0_{\rho\sigma}(6,5) q^\rho k^\sigma \right\} \Delta^0_\pi(5,3)\Delta^0_\pi(4,6) \,, \tag{12}$$

$$\Sigma^{(f)}(4,3) = -\sum_{T_4 t_5 t_6} \int dx_5 \int dx_6 \left\langle t \left| \frac{f^*}{m_\pi} S_j \right| T_4 \right\rangle G^0_{\nu\sigma}(4,5) q^\nu k^\sigma \left\langle T_4 \left| \frac{f^*}{m_\pi} S_i^+ \right| t_5 \right\rangle \times G^0(5,6) \left\langle t_5 \left| \frac{f_\pi}{m_\pi} \not{q} \gamma_5 \tau_j \right| t_6 \right\rangle G^0(6,3) \left\langle t_6 \left| \frac{f_\pi}{m_\pi} \not{k} \gamma_5 \tau_i \right| t \right\rangle \Delta^0_\pi(5,3)\Delta^0_\pi(4,6) \,. \tag{13}$$

Here t represents the isospin of the nucleon and capital T is that of delta. k, q is the four-momentum transfer of pion.

By making Wigner transformations on both sides of Eq. (7) and adopting the semiclassical approximation in which G and Σ are assumed to be peaked around $x' = x_1 - x_2$ and smoothly changing with $x = \frac{1}{2}(x_1 + x_2)$, the equation of motion for $G^{-+}(X,P)$, which we are interested in, has the form

$$[\gamma_\mu K^\mu(X,P) - M(X,P)] iG^{-+}(X,P) = F_C(X,P) \,. \tag{14}$$

Here

$$K^\mu(X,P) = P^\mu - \Sigma^\mu_{\mathrm{HF}}(X,P) + i[\partial^\mu_X + \partial^X_\nu \Sigma^\mu_{\mathrm{HF}}(X,P)\partial^\nu_P - \partial^\nu_P \Sigma^\mu_{\mathrm{HF}}(X,P)\partial^X_\nu] \,, \tag{15}$$

$$M(X,P) = M_N + \Sigma^S_{\mathrm{HF}}(X,P) - i[\partial^X_\nu \Sigma^S_{\mathrm{HF}}(X,P)\partial^\nu_P - \partial^\nu_P \Sigma^S_{\mathrm{HF}}(X,P)\partial^X_\nu] \,, \tag{16}$$

$$F_C(X,P) = \Sigma^{+-}_{\mathrm{Born}}(X,P) iG^{-+}(X,P) - \Sigma^{-+}_{\mathrm{Born}}(X,P) iG^{+-}(X,P) \,. \tag{17}$$

Then we make Clifford-algebra decomposition in spin- and isospin-saturated system and introduce the quasiparticle approximation and achieve the following kinetic equation,

$$\{[\partial_x^\mu - \Sigma_{\rm HF}^{\mu\nu}(x,p)\partial_\nu^p - \partial_p^\nu \Sigma_{\rm HF}^\mu(x,p)\partial_\nu^x]\frac{p_\mu(x,p)}{m^*(x,p)} + [\partial_\nu^x \Sigma_{\rm HF}^S(x,p)\partial_p^\nu - \partial_p^\nu \Sigma_{\rm HF}^S(x,p)\partial_\nu^x]\}{\rm tr}[iG^{-+}(x,p)]$$
$$= {\rm tr}[\Sigma_{\rm Born}^{+-}(x,p)G^{-+}(x,p) - \Sigma_{\rm Born}^{-+}(x,p)G^{+-}(x,p)] \ , \quad (18)$$

where

$$\Sigma_{\rm HF}^{\mu\nu}(x,p) = \partial_x^\mu \Sigma_{\rm HF}^\nu(x,p) - \partial_x^\nu \Sigma_{\rm HF}^\mu(x,p) \ , \quad (19)$$

$$p^\mu(x,p) = P^\mu - \Sigma_{\rm HF}^\mu(x,p) \ , \quad (20)$$

$$m^*(x,p) = M_N + \Sigma_{\rm HF}^S(x,p) \ . \quad (21)$$

We further define a distribution function $f(\mathbf{x},\mathbf{p},\tau)$ as

$$\tfrac{1}{4}{\rm tr}[iG^{-+}(x,p)] = -\frac{\pi m^*}{E^*}\delta(p_0 - E^*(p))f(\mathbf{x},\mathbf{p},\tau) \ . \quad (22)$$

Finally, we obtain the equation

$$\{[\partial_x^\mu - \Sigma_{\rm HF}^{\mu\nu}(x,p)\partial_\nu^p - \partial_p^\nu \Sigma_{\rm HF}^\mu(x,p)\partial_\nu^x]p_\mu + m^*[\partial_\nu^x \Sigma_{\rm HF}^S(x,p)\partial_p^\nu - \partial_p^\nu \Sigma_{\rm HF}^S(x,p)\partial_\nu^x]\}\frac{f(\mathbf{x},\mathbf{p},\tau)}{E^*} = C(x,p) \ . \quad (23)$$

The left-hand side of Eq. (23) is the mean field part and the right-hand side is the collision term which includes two parts, that is, the elastic part and inelastic part

$$C(x,p) = C_{\rm el}(x,p) + C_{\rm in}(x,p) \ . \quad (24)$$

For the detailed expressions of Hartree-Fock self-energy terms and the elastic part of collision terms can be found in [4]. The inelastic part of collision terms reads as

$$C_{\rm in}(x,p) = \frac{1}{2}\int\frac{d^3p_2}{(2\pi)^3}\int\frac{d^3p_3}{(2\pi)^3}\int\frac{d^3p_4}{(2\pi)^3}(2\pi)^4\delta^{(4)}(p+p_2-p_3-p_4)W_{\rm in}(p,p_2,p_3,p_4)(F_2-F_1) \ , \quad (25)$$

where

$$F_1 = f(\mathbf{x},\mathbf{p},\tau)f(\mathbf{x},\mathbf{p}_2,\tau)[1-f_\Delta(\mathbf{x},\mathbf{p}_3,\tau)][1-f(\mathbf{x},\mathbf{p}_4,\tau)] \ , \quad (26)$$

$$F_2 = [1-f(\mathbf{x},\mathbf{p},\tau)][1-f(\mathbf{x},\mathbf{p}_2,\tau)]f_\Delta(\mathbf{x},\mathbf{p}_3,\tau)f(\mathbf{x},\mathbf{p}_4,\tau) \ , \quad (27)$$

and $W_{\rm in}(p,p_2,p_3,p_4)$ is the transition probability,

$$W_{\rm in}(p,p_2,p_3,p_4) = G_1(p,p_2,p_3,p_4) + G_2(p,p_2,p_3,p_4) + p_3 \leftrightarrow p_4 \ , \quad (28)$$

$$G_1(p,p_2,p_3,p_4) = \frac{(f_\pi/m_\pi)^2(f^*/m_\pi)^2}{16E^*(p)E^*(p_2)E_\Delta^*(p_3)E^*(p_4)}(T_c\Phi_c - T_d\Phi_d) \ , \quad (29)$$

$$G_2(p,p_2,p_3,p_4) = \frac{(f_\pi/m_\pi)^2(f^*/m_\pi)^2}{16E^*(p)E^*(p_2)E^*(p_3)E_\Delta^*(p_4)}(T_e\Phi_e - T_f\Phi_f) \ . \quad (30)$$

Here T_c, T_d, T_e, T_f are the isospin matrix and Φ_c, Φ_d, Φ_e, Φ_f are the spin matrix. The subscripts c, d, e, f denote the terms contributed from Figs. 2(c)–(f), respectively. The concrete expressions for $T_{c\sim f}$ and $\Phi_{c\sim f}$ are

$$T_c = \sum_{t_2t_4T_3}\langle t|S_j|T_3\rangle\langle T_3|S_i^+|t\rangle\langle t_2|\tau_j|t_4\rangle\langle t_4|\tau_i|t_2\rangle \ , \quad (31)$$

$$T_d = \sum_{t_2t_4T_3}\langle t|\tau_j|t_4\rangle\langle t_4|\tau_i|t_2\rangle\langle t_2|S_j|T_3\rangle\langle T_3|S_i^+|t\rangle \ , \quad (32)$$

$$T_e = \sum_{t_2 t_3 T_4} \langle t|\tau_j|t_3\rangle\langle t_3|\tau_i|t\rangle\langle T_4|S_i^+|t_2\rangle\langle t_2|S_j|T_4\rangle \ , \tag{33}$$

$$T_f = \sum_{t_2 t_3 T_4} \langle t|S_j|T_4\rangle\langle T_4|S_i^+|t_2\rangle\langle t_2|\tau_j|t_3\rangle\langle t_3|\tau_i|t\rangle \ , \tag{34}$$

$$\Phi_c = \mathrm{tr}\{(\not p_3 + m_\Delta^*)(p-p_3)^\nu(p-p_3)^\mu D_{\nu\mu}(p_3)\mathrm{tr}[(\not p - \not p_3)\gamma_5(\not p_2 + m^*)(\not p - \not p_3)\gamma_5(\not p_4 + m^*)](\not p + m^*)\} \times \frac{1}{(p-p_3)^2 - m_\pi^2}\frac{1}{(p-p_3)^2 - m_\pi^2} \ , \tag{35}$$

$$\Phi_d = \mathrm{tr}\{(\not p - \not p_4)\gamma_5(\not p_4 + m^*)(\not p - \not p_3)\gamma_5(\not p_2 + m^*)(\not p_3 + m_\Delta^*)D_{\rho\mu}(p_3)(p-p_4)^\rho(p-p_3)^\mu(\not p + m^*)\} \times \frac{1}{(p-p_4)^2 - m_\pi^2}\frac{1}{(p-p_3)^2 - m_\pi^2} \ , \tag{36}$$

$$\Phi_e = \mathrm{tr}\{(\not p - \not p_3)\gamma_5(\not p + m^*)(\not p - \not p_3)\gamma_5\mathrm{tr}[(\not p_2 + m^*)(\not p_4 + m_\Delta^*)(p-p_3)^\rho(p-p_3)^\sigma D_{\rho\sigma}(p_4)](\not p + m^*)\} \times \frac{1}{(p-p_3)^2 - m_\pi^2}\frac{1}{(p-p_3)^2 - m_\pi^2} \ , \tag{37}$$

$$\Phi_f = \mathrm{tr}\{(\not p_4 + m_\Delta^*)(p-p_4)^\nu(p-p_3)^\sigma D_{\nu\sigma}(p_4)(\not p_2 + m^*)(\not p - \not p_4)\gamma_5(\not p_3 + m^*)(\not p - \not p_3)\gamma_5(\not p + m^*)\} \times \frac{1}{(p-p_3)^2 - m_\pi^2}\frac{1}{(p-p_4)^2 - m_\pi^2} \ . \tag{38}$$

The detailed expression of $D_{\mu\nu}$ is given in Appendix A. The relation between $W_{\rm in}(p,p_2,p_3,p_4)$ and differential cross section is [10]

$$\int \frac{d^3p_3}{(2\pi)^3}\int \frac{d^3p_4}{(2\pi)^3}(2\pi)^4\delta^{(4)}(p+p_2-p_3-p_4)W_{\rm in}(p,p_2,p_3,p_4) = \int v\sigma_{\rm in}(s,t)d\Omega \ . \tag{39}$$

Insert Eq. (39) into (25) and we find

$$C_{\rm in}(x,p) = \frac{1}{2}\int \frac{d^3p_2}{(2\pi)^3}v\sigma_{\rm in}(s,t)(F_2 - F_1)d\Omega. \tag{40}$$

Here v is the Møller velocity, $\sigma_{\rm in}(s,t)$ is the inelastic differential cross section. The detailed expressions of $\sigma_{\rm in}(s,t)$ are given in Appendix B.

III. NUMERICAL RESULTS

Before coming to the calculation of $NN \to N\Delta$ cross section we have to make some preparation. Firstly, for application to the calculation of HIC, the delta particle should be a decay particle, which in the formalism given in Sec. II is treated as an elementary particle. To take into account the decay width of the Δ isobar, following Ref. [11] we introduce a distribution function $f(M_\Delta)$ of Lorentz form,

$$f(M_\Delta) = \tfrac{1}{4}\Gamma^2(q)/[(M_\Delta - M_0)^2 + \tfrac{1}{4}\Gamma^2(q)], \tag{41}$$

where

$$q^2 = \frac{[M_\Delta^2 - (M_N + m_\pi)^2][M_\Delta^2 - (M_N - m_\pi)^2]}{4M_\Delta^2} \tag{42}$$

and width $\Gamma(q)$ is

$$\Gamma(q) = \{0.47/[1 + 0.6(q/m_\pi)^2]\}q^3/m_\pi^2. \tag{43}$$

The centroid mass of delta $\langle M_\Delta\rangle$ is given by

$$\langle M_\Delta\rangle = \frac{\int_{M_N+m_\pi}^{\sqrt{S}-M_N} f(M_\Delta)M_\Delta dM_\Delta}{\int_{M_N+m_\pi}^{\sqrt{S}-M_N} f(M_\Delta)dM_\Delta}. \tag{44}$$

Here S is the total energy of two colliding particles in the free space, M_N and m_π are the masses of nucleon and pion in free space, and M_0 and Γ_0 are the mass and width of Δ-isobar resonance, respectively. The commonly used values of above quantities are as follows [12]:

$$M_N = 939 \text{ MeV}, \quad m_\pi = 138 \text{ MeV}, \quad M_0 = 1232 \text{ MeV},$$

$$\Gamma_0 = 110 \ \text{MeV}, \quad \frac{f_\pi^2}{4\pi} = 0.080, \quad \frac{f^{*2}}{4\pi} = 0.37.$$

Secondly, the effects arising from the finite size of hadrons and a part of the short-range correlations have to be taken into account, therefore a phenomenological form factor is introduced at each vertex. For the $NN\pi$ vertex we take the commonly used form

$$F_{NN\pi}(t) = \frac{\Lambda^2}{\Lambda^2 - t}. \tag{45}$$

However, for the $N\Delta\pi$ vertex there exist various versions of the form factor [13,14]. Here we choose the mixed version of the cut off expression. Considering the mass distribution function of the delta particle we introduce the form factor for the $N\Delta\pi$ vertex as the following form:

$$F_{N\Delta\pi}(t,\langle M_\Delta\rangle) = \frac{\Lambda^2}{\Lambda^2 - t}\left(\frac{\frac{1}{4}\Gamma^2(\langle q\rangle)}{(\langle M_\Delta\rangle - M_0)^2 + \frac{1}{4}\Gamma_0^2}\right)^{1/4}. \tag{46}$$

Here the center-of-mass momentum $\langle q\rangle$ is obtained from Eq. (42) with the M_Δ replaced by $\langle M_\Delta\rangle$. If $\langle M_\Delta\rangle = M_0$, the difference between $F_{N\Delta\pi}(t,\langle M_\Delta\rangle)$ and $F_{NN\pi}(t)$ vanishes. The free parameter for calculating the inelastic cross sections left is only the cutoff mass Λ which will be fixed by fitting the experimental data of the free inelastic cross section.

A. The free inelastic cross section

After averaging over the initial state and eliminating the double counting at the final state the cross section for $N + N \to N + \Delta$ process reads as

$$\sigma_{NN\to N\Delta} = \frac{1}{32}\int \sigma_{\text{in}}(s,t)d\Omega, \tag{47}$$

$\sigma_{\text{in}}(s,t)$ is given in Appendix B. The relation between $\sigma_{NN\to N\Delta}$ and $\sigma_{pp\to pp\pi^0+pn\pi^+}$ is

$$\sigma_{pp\to pp\pi^0+pn\pi^+} = \tfrac{4}{3}\sigma_{NN\to N\Delta}. \tag{48}$$

The factor $\frac{4}{3}$ arises from the Clebsch-Gordan coefficient of the isospin. The results for the free cross section of $\sigma_{pp\to pp\pi^0+pn\pi^+}$ are displayed in Fig. 5 for different values of Λ. The dots are the experimental data taken from [15]. When $\Lambda = 510$ MeV the best fit to the experimental data of free $\sigma_{pp\to pp\pi^0+pn\pi^+}$ can be reached, the other two values of Λ are taken from Ref. [6]. The Λ value

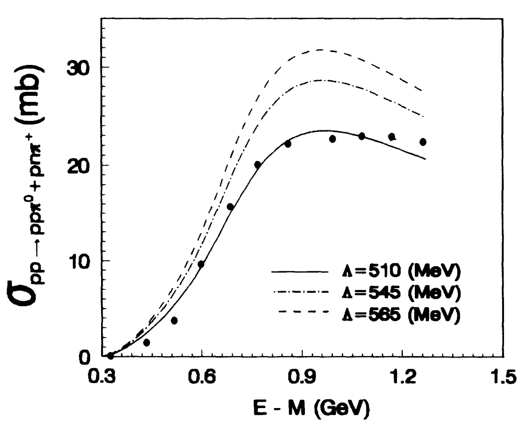

FIG. 5. Free scattering cross section for reactions $pp \to pp\pi^0$ and $pp \to pn\pi^+$. The dots are the experimental data from [15].

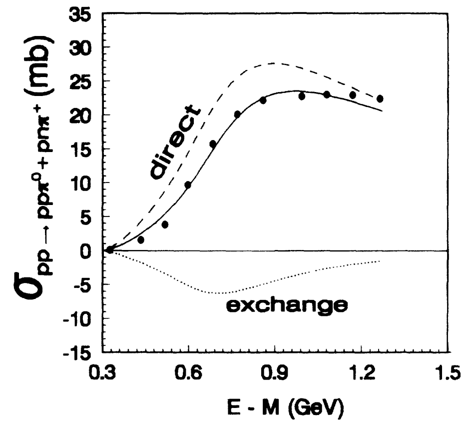

FIG. 6. The contributions of direct diagrams and exchange diagrams.

seems to be small, it is because we have not taken the ρ meson into account in the effective Lagrangian used. To simulate the effect of the ρ exchange a small cutoff mass for the pion has to be used [6]. Considering only one free parameter our results are rather good. In Fig. 6 we show the contributions of the direct term and exchange term, respectively, one can easily find that the cancellation effect of the exchange term could not be ignored. (The free scattering cross section for the $N + N \to N + \Delta$ reaction can be found in Fig. 8, see later.)

B. The in-medium inelastic cross section

In the medium, nucleons (deltas) interact with each other and the effective mass of nucleon (delta) decreases with increasing density. For simplicity let us eliminate the Fock term in the mean field and then the effective mass for nucleon and delta is

$$m^* = M_N - g_\sigma\langle\sigma\rangle, \tag{49}$$

$$m^*_\Delta = \langle M_\Delta\rangle - g_\sigma\langle\sigma\rangle. \tag{50}$$

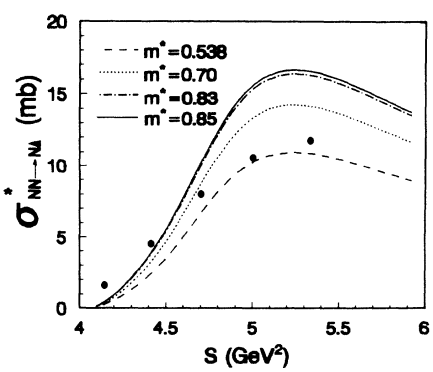

FIG. 7. The dependence of in-medium $NN \to N\Delta$ cross section on the effective mass, the dots are Dirac-Brueckner results taken from [5].

TABLE I. m_σ = 550 MeV and m_ω=783 MeV are used for all cases.

	g_σ	g_ω	bg_σ^3	cg_σ^4	$E_{\rm bin}$	m^*	K (MeV)	ρ_0
Set A	11.24	14.03			−15.45	0.538	561.7	0.145
Set B	9.40	10.95	−0.69	40.44	−15.57	0.70	380	0.145
Set C	6.90	7.54	−40.49	383.07	−15.76	0.83	380	0.145
Set D	7.937	6.696	42.35	157.55	−16.00	0.85	210	0.153

First let us investigate the influence of the different equations of state, which is characterized by saturation effective mass and compressibility, on the in-medium $NN \to N\Delta$ cross section. Parameters corresponding to different equations of state used in this calculation are given in Table I.

Figure 7 shows the effective $NN \to N\Delta$ cross section from different parameter sets at normal density $\rho = \rho_0$. The dots are the results of Dirac-Brueckner (DB) calculations [5]. One can find that $\sigma^*_{NN\to N\Delta}$ is sensitive to the effective mass and insensitive to compressibility. Our results are in agreement with the DB approach considering in their calculations with $m^* = 0.605$. Then we study the dependence of effective $NN \to N\Delta$ cross sections on the density and energy. Figure 8 shows the $\sigma^*_{NN\to N\Delta}$ at different densities and energies. The calculation is done for parameter set B. It can be found that the $\sigma^*_{NN\to N\Delta}$ substantial decreases with the increase of density especially at large momentum and high density. From this calculation we find a strong medium effect on the $NN \to N\Delta$ cross section. It would be very important to take this effect into account in the calculations of heavy ion collision.

Figure 9 displays the effective differential inelastic cross section as a function of c.m. scattering angle for different densities and energies. It is shown from Fig. 9 that the differential cross section becomes steeper with the increase of energy. There exists an evident density dependence at small angle but at large angle the density dependence is not very pronounced.

IV. SUMMARY AND OUTLOOK

In this work we have provided the explicit expressions for calculating the in-medium $NN \to N\Delta$ cross section in transport model calculation which is simultaneously given with other integrands such as the mean fields, NN elastic cross section, of relativistic kinetic equation as well as the RBUU type kinetic equation itself based on the effective Lagrangian by using the closed time path Green's function technique. With only one parameter of cutoff mass which is fixed by fitting the experimental data, the theoretical prediction can reproduce the experimental free $NN \to N\Delta$ cross section nicely. The in-medium $NN \to N\Delta$ cross section shows an obvious dependence on the saturation effective mass. The $NN \to N\Delta$ cross section and the differential $NN \to N\Delta$ cross section at forward angle are suppresed at high density. The screen and antiscreen effects of the medium of the interaction has not been included in the present investigation of the medium effect, which could be important for high density according to the studies of Bertsch *et al.* [6]. However, a nonrelativistic description was used in their study. For our case the relativistic description should be used for consistency with this work; work with this aspect is in progress.

After the presentation of our model and the determination of parameters of effective Lagrangian from this work as well as our previous work [2–4] the optical potential, the NN elastic and inelastic cross section can be calculated simultaneously. As a first time a comprehensive agreement with experimental data of optical potential, mean free path in nuclear matter as well

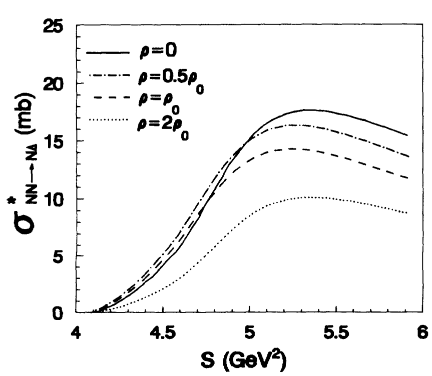

FIG. 8. The in-medium $NN \to N\Delta$ cross section with different densities and energies.

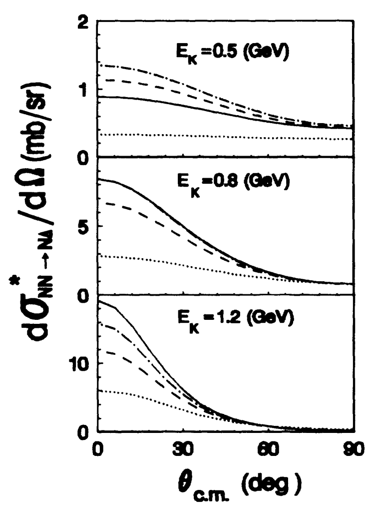

FIG. 9. The in-medium $NN \to N\Delta$ differential cross sections as a function of c.m. scattering angle $\theta_{\rm c.m.}$ for different nucleon kinetic energies and nuclear densities. The different curves correspond to different densities, which is the same as Fig. 8.

as free n-n scattering cross section can be reached till $E \sim 1$ GeV/nucleon. We believe that our model is a promising and suitable approach for the studies of HIC at BEVALAC and SIS energy range. The application for relativistic heavy ion collision is underway.

ACKNOWLEDGMENTS

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APPENDIX A

For the convenience of the reader in this appendix we give the expressions of the zero-order Green's function used in this paper.

(1) Nucleon

$$iG^0(1,2) = i\int \frac{d^4k}{(2\pi)^4} G^0(x,k) e^{-ik(x_1-x_2)}, \tag{A1}$$

$$G^{0\mp\mp}(x,k) = (\not{k} + M_N)\left(\frac{\pm 1}{k^2 - M_N^2 \pm i\epsilon} + \frac{\pi i}{E(k)}\delta(k_0 - E(k)) f(x,k)\right), \tag{A2}$$

$$G^{0+-}(x,k) = -\frac{\pi i}{E(k)}\delta(k_0 - E(k))[1 - f(x,k)](\not{k} + M_N), \tag{A3}$$

$$G^{0-+}(x,k) = \frac{\pi i}{E(k)}\delta(k_0 - E(k)) f(x,k)(\not{k} + M_N). \tag{A4}$$

(2) Delta

$$iG^0_{\mu\nu}(1,2) = i\int \frac{d^4k}{(2\pi)^4} G^0_{\mu\nu}(x,k) e^{-ik(x_1-x_2)}, \tag{A5}$$

$$G^{0\mp\mp}_{\mu\nu}(x,k) = (\not{k} + M_\Delta) D_{\mu\nu}\left(\frac{\pm 1}{k^2 - M_\Delta^2 \pm i\epsilon} + \frac{\pi i}{E(k)}\delta(k_0 - E(k)) f_\Delta(x,k)\right), \tag{A6}$$

$$G^{0+-}_{\mu\nu}(x,k) = -\frac{\pi i}{E(k)}\delta(k_0 - E(k))[1 - f_\Delta(x,k)](\not{k} + M_\Delta) D_{\mu\nu}, \tag{A7}$$

$$G^{0-+}_{\mu\nu}(x,k) = \frac{\pi i}{E(k)}\delta(k_0 - E(k)) f_\Delta(x,k)(\not{k} + M_\Delta) D_{\mu\nu}, \tag{A8}$$

$$D_{\mu\nu} = -g_{\mu\nu} + \frac{1}{3}\gamma_\mu\gamma_\nu + \frac{1}{3M_\Delta}(\gamma_\mu k_\nu - \gamma_\nu k_\mu) + \frac{2}{3M_\Delta^2} k_\mu k_\nu. \tag{A9}$$

(3) Pion

$$i\Delta^0_\pi(1,2) = i\int \frac{d^4k}{(2\pi)^4} \Delta^0_\pi(x,k) e^{-ik(x_1-x_2)}, \tag{A10}$$

$$\Delta^{0\mp\mp}_\pi(x,k) = \frac{\pm 1}{k^2 - m_\pi^2 \pm i\epsilon} - 2\pi i\delta(k^2 - m_\pi^2) f_\pi(x,k), \tag{A11}$$

$$\Delta^{0\pm\mp}_\pi(x,k) = -2\pi i\delta(k^2 - m_\pi^2)[\theta(\pm k_0) + f_\pi(x,k)]. \tag{A12}$$

In our theoretical framework the negative energy states are neglected. Here $f(x,k)$, $f_\Delta(x,k), f_\pi(x,k)$ are nucleon, delta, pion distribution function, respectively. The numbers 1, 2 denote x_1, x_2.

APPENDIX B

In this appendix we present the analytical expressions of inelastic differential cross section which are obtained by computing Eqs. (31)–(38) and finally transforming it into the center of mass of two particle system:

$$\sigma_{\rm in}(s,t) = \frac{16}{(2\pi)^2 s}\left(\frac{f_\pi}{m_\pi}\right)^2\left(\frac{f^*}{m_\pi}\right)^2\left(\frac{(s-m^{*2}-m_\Delta^{*2})^2-4m^{*2}m_\Delta^{*2}}{s(s-4m^{*2})}\right)^{1/2}[D(s,t)+E(s,t)+(s,t\leftrightarrow u)], \tag{B1}$$

where

$$D(s,t) = -\frac{m^{*2}t[(m_\Delta^*+m^*)^2-t]^2[(m_\Delta^*-m^*)^2-t]}{6m_\Delta^{*2}(t-m_\pi^2)^2}, \tag{B2}$$

$$E(s,t) = -\frac{m^{*2}}{12m_\Delta^{*2}(t-m_\pi^2)(u-m_\pi^2)}[E_1+E_2+E_3+E_4+E_5+E_6], \tag{B3}$$

$$E_1 = m_\Delta^{*2}[(8s-3t)m^{*2}t-2(s+3t)m^{*4}+3m^{*6}-2s^2t+2t^3], \tag{B4}$$

$$E_2 = m_\Delta^{*3}m^*[(2s+t)t-2(s+t)m^{*2}+6m^{*4}], \tag{B5}$$

$$E_3 = m_\Delta^* m^*[(2s-t)m^{*2}t+(s+3t)m^{*4}-(s+t)st-3m^{*6}], \tag{B6}$$

$$E_4 = m_\Delta^{*5}m^*[s-t-3m^{*2}]+m_\Delta^{*4}[(s-3t)m^{*2}+2st-t^2], \tag{B7}$$

$$E_5 = (s+9t)m^{*6}+(s+6t)(s+t)m^{*2}t-6(s+2t)m^{*4}t, \tag{B8}$$

$$E_6 = -m_\Delta^{*6}m^{*2}-2m^{*8}-t^2(s+t)^2, \tag{B9}$$

where D represents the contribution of direct diagrams and E is that of exchange diagrams and

$$s = (p+p_2)^2 = [E^*(p)+E^*(p_2)]^2-(\mathbf{p}+\mathbf{p}_2)^2, \tag{B10}$$

$$t = (p-p_3)^2 = \tfrac{1}{2}(3m^{*2}+m_\Delta^{*2}-s)+2|\mathbf{p}||\mathbf{p}_3|\cos\theta, \tag{B11}$$

$$u = (p-p_4)^2 = 3m^{*2}+m_\Delta^{*2}-s-t. \tag{B12}$$

θ is the scattering angle in c.m. system and

$$|\mathbf{p}| = \tfrac{1}{2}\sqrt{(s-4m^{*2})}, \tag{B13}$$

$$|\mathbf{p}_3| = \frac{1}{2}\frac{\sqrt{(s-m^{*2}-m_\Delta^{*2})^2-4m^{*2}m_\Delta^{*2}}}{\sqrt{s}}, \tag{B14}$$

$$E^*(p) = \sqrt{\mathbf{p}^2+m^{*2}}, \tag{B15}$$

$$E^*(p_2) = \sqrt{\mathbf{p}_2^2+m^{*2}}, \tag{B16}$$

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