Few-nucleon forces from chiral Lagrangians

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(Received 10 January 1994)

Few-nucleon forces are considered from the point of view of effective chiral Lagrangians. It is argued that such forces naturally arise at the same order in chiral perturbation theory as some important features of the two-nucleon force. In particular, the leading few-nucleon forces cancel against the leading recoil correction in the iteration of the two-body potential. The remaining three-body potential is presented in momentum and coordinate spaces. It is dominated by contributions of the delta isobar of (i) two-pion range, which are not new, and (ii) shorter range, which involve an undetermined parameter.

PACS number(s): 21.30.+y, 12.39.Fe

I. INTRODUCTION

Getting the correct binding energy of nuclei from the underlying dynamics has been a long-standing problem in nuclear physics. Such remarkable progress has been achieved on few-nucleon calculations (for a clear introduction, see [1]) that nowadays they undoubtedly provide important information about their input, two- and sometimes three-nucleon (NN and $3N$) potentials. *What* information, however, is still debatable.

The traditional view [1] is that there is already some circumstantial evidence for the existence of $3N$ forces. The strongest indication comes from the fact that most realistic NN potentials underbind the triton by 0.5–1.1 MeV and the α particle by 4–5 MeV. There are also discrepancies between data and calculations with NN potentials only for the ^{3}H/^{3}He rms charge radii, the asymptotic normalization ratio C_2/C_0, and the nd spin doublet scattering length, which can all be improved by including $3N$ forces adjusted to reproduce ^{3}H binding. Other indirect evidence comes from improved nuclear saturation properties when $3N$ forces are considered [2].

This interpretation has been challenged recently [3]. It has been argued that certain $3N$ observables display a much larger sensitivity to some NN potential parameters than NN data do. As a consequence, a fine tuning of the NN potential is possible that is of little effect in the NN system but improves $3N$ fittings considerably. An example is the large uncertainties in the NN ε_1 mixing parameter that existed prior to the recent Nijmegen partial-wave analysis [4]: they allowed a static, one-boson exchange version of the Bonn potential with a particularly low deuteron D-state probability P_D to yield almost the correct triton binding without resorting to $3N$ forces.

Here I want to discuss what our theoretical prejudices are from the viewpoint of chiral symmetry. Chiral symmetry is a spontaneously broken (approximate) symmetry of quantum chromodynamics (QCD) that governs much of low-energy hadronic physics, but has not yet been incorporated systematically in the description of nuclear systems. Recently, following a program put forward by Weinberg [5], a two-body potential derived to third order in chiral perturbation theory was shown to reproduce qualitatively some features of the NN interaction [6] and quantitatively NN bound state data and phase shifts up to laboratory energies around 100 MeV [7,8]. In the following I will argue that a general chiral Lagrangian naturally explains many of the features of nuclear systems, so that it could be used as a guide for what ingredients we should expect to need in a specific problem. In particular, it will be found that few-nucleon forces arise at the same level as some important features of the NN force (such as the short-range tensor force and the spin-isospin independent central attraction) and are therefore expected to play a non-negligible role in few-nucleon systems. Clearly this line of reasoning is no substitute for the above debate concerning what data are actually telling us, but it does suggest that its best framework is one in which *both* NN and $3N$ forces are included simultaneously and consistently from the start. (From this standpoint, getting the correct binding from NN forces alone can only be considered a success after $3N$ forces calculated with the same assumptions—e.g., same mesons exchanged and same couplings and cutoffs—are shown to be irrelevant.)

After first reviewing in Sec. II some of the consequences of chiral symmetry to nuclear forces, I derive few-body potentials in momentum space, the leading terms (which are shown to cancel against part of the recoil correction of the iterated two-body potential) in Sec. III, and the dominant remaining three-body force in Sec. IV. (Some of these points have already been mentioned in [6] and [9].) The corresponding coordinate space potentials are presented in the Appendix, and Sec. V concludes the paper.

II. GENERALITIES

Most of traditional nuclear physics concerns processes involving momenta up to a few hundred MeV. (Approx-

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imate) chiral symmetry is the single most important ingredient in an effective theory of hadronic processes (at such low energies) compatible with the theory of strong interactions, QCD. The pion is the (pseudo-)Goldstone boson of the spontaneous breaking of $SU(2)_L \times SU(2)_R$ to $SU(2)_V$, a fact that has (literally, too) far-reaching consequences. One is the lightness of the pion. It is essential to include it in the effective Lagrangian with the nucleon and possibly the delta isobar, while heavier mesons can be integrated out of the theory. Not surprisingly, one finds a dominance of pion exchange in few-nucleon systems [10]. The other consequence is that the symmetry restricts the form of the interaction terms in the effective Lagrangian, while the details of QCD dynamics are buried in coupling constants (not fixed by symmetry).

With the assumption of naturalness—the notion that when expressed in the appropriate scale, these coupling constants are of $O(1)$—one can show that chiral symmetry turns a simple power counting argument into a classification scheme for the strength of interactions [5]. An arbitrary diagram contributing to a given process can be obtained by sewing together irreducible diagrams, which are those that cannot be separated in two by cutting the lines of initial or final particles in an intermediate state. For processes where all momenta Q are of $O(m_\pi)$, an irreducible diagram with $2A$ nucleon (and any number of pion) external lines, L loops, C separately connected pieces, and V_i vertices of type i which contain d_i derivatives (or powers of pion mass) and n_i baryon fields is of $O(Q^\nu)$ where

$$\nu = 4 - A + 2(L - C) + \sum_i V_i \Delta_i \tag{1}$$

with

$$\Delta_i = d_i + \frac{n_i}{2} - 2 \geq 0 \tag{2}$$

referred to as the index of the interaction i.

Therefore, at energies small compared to some characteristic QCD scale M (of the order of a typical hadronic energy scale, say the rho mass), the most important interactions are those with smaller indices. Say $\boldsymbol{\pi}$ denotes the pion field of mass m_π and decay constant F_π ($\simeq$190 MeV), N (Δ) is a two-spinor (four-component spinor) in both spin and isospin spaces that represents the nucleon (delta) of mass m_N (m_Δ), $\frac{1}{2}\vec{\sigma}(\boldsymbol{t})$ is the generator of spin (isospin) transformations in the $\frac{1}{2}$ representation, $\vec{S}(\boldsymbol{T})$ is the transition operator that satisfies $S_i S_j^+ = \frac{1}{3}(2\delta_{ij} - i\varepsilon_{ijk}\sigma_k)$ $[T_a T_b^+ = \frac{1}{6}(\delta_{ab} - i\varepsilon_{abc}t_c)]$, $\boldsymbol{t}^{(3/2)}$ is the isospin generator in the $\frac{3}{2}$ representation, and D is shorthand for

$$D = 1 + \frac{\boldsymbol{\pi}^2}{F_\pi^2},$$

then the Lagrangian to lowest order ($\Delta_i = 0$) is

$$\begin{aligned}\mathcal{L}_{(0)} = & -\tfrac{1}{2}D^{-2}\partial_\mu \boldsymbol{\pi}\cdot\partial^\mu\boldsymbol{\pi} - \tfrac{1}{2}m_\pi^2 D^{-1}\boldsymbol{\pi}^2 + \bar{N}\left(i\partial_0 - \frac{2D^{-1}}{F_\pi^2}\boldsymbol{t}\cdot(\boldsymbol{\pi}\times\dot{\boldsymbol{\pi}}) - m_N - \frac{2g_A D^{-1}}{F_\pi}\boldsymbol{t}\cdot\vec{\sigma}\cdot\vec{\nabla}\boldsymbol{\pi}\right)N \\ & +\bar{\Delta}\left(i\partial_0 - \frac{2D^{-1}}{F_\pi^2}\boldsymbol{t}^{(3/2)}\cdot(\boldsymbol{\pi}\times\dot{\boldsymbol{\pi}}) - m_\Delta\right)\Delta - \frac{2h_A}{F_\pi}D^{-1}\left[\bar{N}(\boldsymbol{T}\cdot\vec{S}\cdot\vec{\nabla}\boldsymbol{\pi})\Delta + \text{H.c.}\right] \\ & -\tfrac{1}{2}C_S\bar{N}N\bar{N}N - \tfrac{1}{2}C_T\bar{N}\vec{\sigma}N\cdot\bar{N}\vec{\sigma}N - D_T\bar{N}\boldsymbol{t}\vec{\sigma}N\cdot(\bar{N}\boldsymbol{T}\vec{S}\Delta + \text{H.c.}) + \cdots\end{aligned} \tag{3}$$

and the first order ($\Delta_i = 1$) terms are

$$\begin{aligned}\mathcal{L}_{(1)} = & \frac{-B_1}{F_\pi^2}D^{-2}\bar{N}N[(\vec{\nabla}\boldsymbol{\pi})^2 - \dot{\boldsymbol{\pi}}^2] - \frac{B_2}{F_\pi^2}D^{-2}\bar{N}\boldsymbol{t}\vec{\sigma}N\cdot(\vec{\nabla}\boldsymbol{\pi}\times\vec{\nabla}\boldsymbol{\pi}) - \frac{B_3 m_\pi^2}{F_\pi^2}D^{-1}\bar{N}N\boldsymbol{\pi}^2 \\ & -\frac{D_1}{F_\pi}D^{-1}\bar{N}N\bar{N}(\boldsymbol{t}\cdot\vec{\sigma}\cdot\vec{\nabla}\boldsymbol{\pi})N - \frac{D_2}{F_\pi}D^{-1}(\bar{N}\boldsymbol{t}\vec{\sigma}N\times\bar{N}\boldsymbol{t}\vec{\sigma}N)\cdot\vec{\nabla}\boldsymbol{\pi} \\ & -\tfrac{1}{2}E_1\bar{N}N\bar{N}\boldsymbol{t}N\cdot\bar{N}\boldsymbol{t}N - \tfrac{1}{2}E_2\bar{N}N\bar{N}\boldsymbol{t}\vec{\sigma}N\cdot\bar{N}\boldsymbol{t}\vec{\sigma}N - \frac{E_3}{2}(\bar{N}\boldsymbol{t}\vec{\sigma}N\times\bar{N}\boldsymbol{t}\vec{\sigma}N)\cdot\bar{N}\boldsymbol{t}\vec{\sigma}N + \cdots .\end{aligned} \tag{4}$$

Here $g_A, h_A, C_S, C_T, D_T, B_{1,2,3}, D_{1,2}$, and $E_{1,2,3}$ are undetermined constants to be obtained either by solving QCD or by fitting data. Note that I (i) only show those terms relevant for what follows and omit others that have more isobars and (ii) have applied Fierz reordering to terms with four and six nucleons in order to rewrite six other possible combinations of $\vec{\sigma}, \boldsymbol{t}$ in terms of those shown above.

Equations (1) and (2) provide the basis for a perturbative expansion in powers of Q/M. For systems with at most one nucleon ($A \leq 1$), they tell us that dominant contributions are due to tree graphs generated by (3) and (4), which just reproduce the time-honored current algebra results. Consideration of higher-order Lagrangians and loops allows a systematic accounting of corrections (for a review see [11]).

For systems with several nucleons ($A \geq 2$), the same power counting yields the main features of traditional nuclear physics.

The first feature is that a nucleus is basically made out of nucleons. At the level of nuclear dynamics itself that is because the nucleon mass is large compared to M so that reducible diagrams have small energy denominators. This leads to a picture where nucleons interact nonperturbatively through a nuclear potential; it is in the potential that the perturbative contributions of pions and deltas (and, indirectly, everything else) are. When external probes (pions and photons) are brought into play, the leading contributions come from diagrams with the maximum number of connected pieces: the probe interacts with each nucleon separately—that is the impulse approximation. First corrections to such an approximation, which are of pion-exchange type, have one less connected piece and so are expected to be of order $(m_\pi/M)^2 \sim 5\%$. Although meson exchange currents can be larger in some situations, a systematic chiral Lagrangian analysis carried out by Park, Min, and Rho [12] confirms this typical estimate. A similar remark applies to threshold pion-deuteron scattering [9] and pion photoproduction and electroproduction on light nuclei [13].

The second feature is that nucleons are nonrelativistic. This is again because the massive nucleon is not disturbed much by the little kicks it receives from other particles. At the level of the potential, this is why one-pion exchange is essentially static, corrections having $\Delta_i = 2$ and so being again a few percent. I will return shortly to their energy dependence and their effect on the iteration of the potential.

The third aspect is that nucleons interact mainly via pairwise forces. This is again a consequence of decreasing the number of connected pieces: few-body forces are smaller than two-body forces by a factor $(m_\pi/M)^2$. In triton, for example, we can estimate the two-body contribution per pair $\langle V_{2b}\rangle$ to the potential-energy expectation value $\langle V\rangle \approx -50$ MeV [1], as $\langle V_{2b}\rangle \sim \frac{1}{3}\langle V\rangle \sim -15$ MeV; the three-body contribution is thus expected to be $\langle V_{3b}\rangle \sim \pm 0.5$ MeV. Notice that this is a nontrivial result, since models without chiral symmetry exist (e.g., that in [14]) where two-pion exchange is large and therefore that yield large $3N$ forces. That pseudovector coupling of the pion implies smaller three-body forces was noted already in the 1950s [15] and the role of chiral symmetry was emphasized in the late 1960s [16]. I now turn to the systematic derivation of the implications of chiral symmetry to the multinucleon dynamics.

III. LEADING FORCES AND ENERGY DEPENDENCE

For a nucleus with A nucleons, we see from (1) and (2) that the smallest possible power of the small momentum Q is

$$\nu_{\rm min} = 6 - 3A. \tag{5}$$

It corresponds to tree ($L = 0$) diagrams constructed out of the lowest-order Lagrangian ($\Delta_i = 0$) with the maximum number of separately connected pieces ($C = A-1$). To this order, then, the nuclear potential V is simply a sum over all pairs (ij),

$$V^{(0)}(\vec{r}_{1,\ldots,}\vec{r}_A) = \sum_{(ij)} V_2^{(0)}(\vec{r}_i - \vec{r}_j) \tag{6}$$

of two-body potentials $V_2^{(0)}$ consisting of static one-pion exchange (OPE) plus two contact terms [5].

This is obviously a very crude approximation to the NN potential. It does allow us to see how energies of the order of 10 MeV arise from QCD: very roughly $\langle V_{2b}\rangle \sim \langle V_2^{(0)}\rangle \sim (1/2\pi^2)(g_A/F_\pi)^2 m_\pi^3 \sim -10$ MeV, not too far off the above triton estimate. However, a quantitative analysis requires corrections, which have already been calculated [6] up to

$$\nu = 9 - 3A = \nu_{\rm min} + 3 \tag{7}$$

($L = 0$, $\sum_i V_i\Delta_i = 1,2,3$; $L = 1$, $\sum_i V_i\Delta_i = 0,1$) and used to fit NN scattering and bound state data [7]. I refer the reader to Ref. [8] for the complete expressions. I just mention some of the results. Of particular importance here is that at $\nu_{\rm min}+2$, recoil has to be accounted for in OPE, which leads to a dependence on the energy $2m_N + E$ of the incoming nucleons. Denoting by $\vec{p}_i$ ($\vec{p}\,'_i$) the initial (final) momentum of nucleon i, $\vec{P}_{ij} \equiv \vec{p}_i + \vec{p}_j$, $\vec{k}_{ij} \equiv \frac{1}{2}(\vec{p}\,'_i - \vec{p}_j)$, $\vec{q}_{ij} \equiv \vec{p}_i - \vec{p}\,'_i = \vec{p}\,'_j - \vec{p}_j$, and $w_{ij} \equiv \sqrt{\vec{q}_{ij}^{\,2} + m_\pi^2}$, we get to this order (7), in momentum space,

$$\sum_{n=0}^{3} V_2^{(n)}(\vec{q}_{ij}, \vec{k}_{ij}; E) = C_S + C_T\vec{\sigma}_i\cdot\vec{\sigma}_j - \left(\frac{2g_A}{F_\pi}\right)^2 \boldsymbol{t}_i\cdot\boldsymbol{t}_j \frac{\vec{q}_{ij}\cdot\vec{\sigma}_i\vec{q}_{ij}\cdot\vec{\sigma}_j}{w_{ij}^2}\left[1 + \frac{1}{w_{ij}}\left(E - \frac{1}{m_N}(\vec{k}_{ij}^2 + \tfrac{1}{4}\vec{q}_{ij}^{\,2} + \tfrac{1}{4}\vec{P}_{ij}^{\,2})\right)\right] + \cdots. \tag{8}$$

Here I displayed the leading piece plus OPE recoil but hid in the dots the most interesting parts, namely, two-pion exchange (TPE) and contact terms. They provide the ingredients (such as short-range tensor and spin-orbit forces and intermediate range attraction) that are necessary for a reasonable fitting of phase shifts, which we indeed achieve (with a cutoff at the rho mass), up to around 100 MeV laboratory energies. (To go further in energy presumably requires even higher-order corrections. They can be included, but involve the more complicated calculation of two-loop diagrams and the introduction of many more undetermined parameters due to a host of new contact terms.)

Since these corrections contain such important information in the case of the two-body system, it is just natural to consider their effects in $A \geq 3$ nuclei. No calculation of the triton has been carried out with our two-body potential, so we cannot be quantitative as to how much room it leaves for three-body forces. If the D-state admixture can serve as a guide, one might expect not much underbinding since our $P_D \approx 5\%$ is lower than most realistic NN potentials. In any case, and this is the important point, consistency of the approach requires that we evaluate *all* corrections to a certain order. To the order given by (7), we also encounter forces involving irreducibly three nucleons V_3 and two pairs of nucleons $V_{2,2}$ for both of which $C = A - 2$. We then write

$$\sum_{n=0}^{3} V^{(n)}(\vec{r}_1,...,\vec{r}_A)$$
$$= \sum_{(ij)}\sum_{n=0}^{3} V_2^{(n)}(\vec{r}_i,\vec{r}_j) + \sum_{(ijk)}\sum_{n=2}^{3} V_3^{(n)}(\vec{r}_i,\vec{r}_j,\vec{r}_k) + \sum_{(ij;kl)}\sum_{n=2}^{3} V_{2,2}^{(n)}(\vec{r}_i - \vec{r}_j;\vec{r}_k - \vec{r}_l), \quad (9)$$

where the second sum extends over all triplets (ijk) and the third over all pairs of pairs $(ij;kl)$.

I now move to touch on these few-body parts.

The largest contribution is expected to come at $\nu = \nu_{\min} + 2$, being due to tree ($L = 0$) diagrams given by $\mathcal{L}_{(0)}$ in (3) ($\sum_i V_i\Delta_i = 0$). If we ignore the isobar for a while, the corresponding diagrams for the three-body potential are given in Fig. 1. One finds [5] that the various orderings of Fig. 1(c) add to zero, while Figs. 1(a) and 1(b) yield

$$V_3^{(2)no\Delta}(\vec{q}_{ij},\vec{q}_{jk}) = 2\left(\frac{g_A}{F_\pi}\right)^2 \frac{1}{w_{jk}^3}\vec{\sigma}_k\cdot\vec{q}_{jk}[t_i\cdot t_k(C_S\vec{\sigma}_i + C_T\vec{\sigma}_j)\cdot\vec{q}_{jk} + t_j\cdot t_k(C_S\vec{\sigma}_j + C_T\vec{\sigma}_i)\cdot\vec{q}_{jk}]$$
$$-4\left(\frac{g_A}{F_\pi}\right)^4 \frac{w_{ij}+w_{jk}}{w_{ij}^3 w_{jk}^3}\vec{\sigma}_i\cdot\vec{q}_{ij}\vec{\sigma}_k\cdot\vec{q}_{jk}[t_i\cdot t_k\vec{q}_{ij}\cdot\vec{q}_{jk} - 2t_j\cdot(t_i\times t_k)\vec{\sigma}_j\cdot(\vec{q}_{ij}\times\vec{q}_{jk})]$$
$$+\text{two cyclic permutations of } (ijk). \quad (10)$$

The second term is just TPE [Fig. 1(b)] and has been calculated long ago by Brueckner, Levinson, and Mahmoud [15]. The first term comes in part from the contact terms in (3), first considered by Weinberg [5]. (Note, however, that I correct here the corresponding result in [5].) The double-pair potential, in turn, comes from the diagrams in Fig. 2 and is

$$V_{2,2}^{(2)}(\vec{q}_{ij},\vec{q}_{kl}) = \left(\frac{2g_A}{F_\pi}\right)^2 \left\{C_S + C_T\vec{\sigma}_i\cdot\vec{\sigma}_j + \left(\frac{2g_A}{F_\pi}\right)^2 t_i\cdot t_j\frac{\vec{\sigma}_i\cdot\vec{q}_{ij}\vec{\sigma}_j\cdot\vec{q}_{ij}}{w_{ij}^2}\right\} t_k\cdot t_l\frac{\vec{\sigma}_k\cdot\vec{q}_{kl}\vec{\sigma}_l\cdot\vec{q}_{kl}}{w_{kl}^3}$$
$$+\text{interchange of } (ij) \text{ and } (kl). \quad (11)$$

(The coordinate space versions of these potentials are given in the Appendix.)

What is the effect of these leading few-body forces? The remarkable fact is that they are canceled by the energy dependence of the two-body potential (8) when the latter is iterated in the Lippmann-Schwinger equation.

To see this, consider the diagrams of Fig. 3, which are all orderings where nucleon j emits or absorbs a pion (that flies to or from nucleon k), before getting in touch with nucleon i. According to our power counting, Figs. 3(a) and 3(b) are the most important, because they are iterations of the NN potential, which is given by (8). Their sum will be proportional to

$$\frac{1}{E(\vec{p}_j-\vec{q}_{jk})+E(\vec{p}_k+\vec{q}_{jk})-E(\vec{p}_j)-E(\vec{p}_k)}\left[\frac{1}{E(\vec{p}_j-\vec{q}_{jk})+w_{jk}-E(\vec{p}_j)} + \frac{1}{E(\vec{p}_k+\vec{q}_{jk})+w_{jk}-E(\vec{p}_k)}\right]$$
$$= \frac{2}{w_{jk}[E(\vec{p}_j-\vec{q}_{jk})+E(\vec{p}_k+\vec{q}_{jk})-E(\vec{p}_j)-E(\vec{p}_k)]} - \frac{1}{w_{jk}^2}\left[1+O\left(\frac{E}{w}\right)\right]. \quad (12)$$

The first term is just the iteration of the leading order potential $V_2^{(0)}$; it is large because the difference in nucleon energies is small, $O(m_\pi^2/m_N)$, while w is $O(m_\pi)$. The second term is the iteration of the recoil correction to OPE shown in (8); the small energy denominator is canceled by the small recoil energy. The leading $3N$ force is expected to be of the same order; it is given by Fig. 3(c) [which is just Fig. 1(a)], which is proportional to

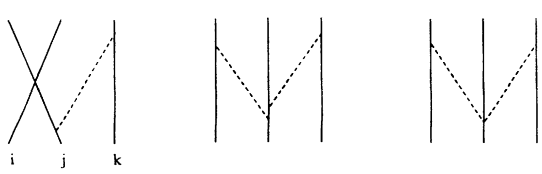

FIG. 1. Tree graphs contributing to the $3N$ potential. All other time orderings and permutations are to be considered as long as there is at least one pion in intermediate states. Solid lines are nucleons and dashed lines pions.

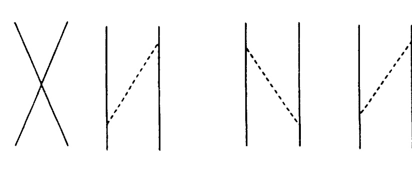

FIG. 2. Tree graphs contributing to the double-pair potential. All other time orderings and permutations are to be included as long as there is at least one pion in intermediate states.

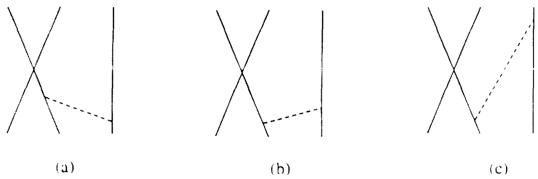

FIG. 3. Diagrams representing (a) and (b) part of the iteration of the NN potential whose energy dependence approximately cancels (c) the contribution from part of the $3N$ force. The same cancellation applies to other time orderings.

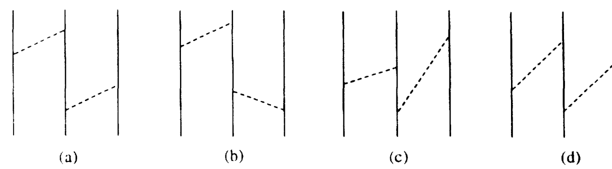

FIG. 4. Example of the cancellation analogous to the one in Fig. 3 for the TPE sector. In (a) and (b) recoil is considered in the pion line to the right. The same cancellation occurs in three other sets of four TPE diagrams corresponding to different orderings.

$$\frac{1}{w_{jk}+E(\vec{p}_j-\vec{q}_{jk})-E(\vec{p}_j)} \times \frac{1}{w_{jk}+E(\vec{p}^{\,\prime}_i)+E(\vec{p}^{\,\prime}_j)-E(\vec{p}_i)-E(\vec{p}_j)} = \frac{1}{w_{jk}^2}\left[1+O\left(\frac{E}{w}\right)\right]. \quad (13)$$

Here the first term is already the leading $3N$ force (10) for which nucleons are static. The important point now is that, because the diagrams in Fig. 3 differ *only* in their energy denominators, the $1/w^2$ terms cancel when we compute the T matrix or, equivalently, when we solve the Faddeev equations.

It is not difficult to show in the same way that a similar cancellation happens also in the $3N$ TPE piece (Fig. 4) and the double-pair force (Fig. 5). This cancellation has been noted before (in the case of the TPE $3N$ force) [17], but its model independence and generality are particularly clear in our context. More recently, Friar and Coon [18] have emphasized that choosing an energy-independent potential then leads to no $3N$ TPE forces (of this type) at all. In other words, in a few-body calculation both V_3 of (10) and $V_{2,2}$ of (11) can be omitted as long as we do the same to the recoil term in (8). (This is nicely exemplified [19] by comparing the triton binding energies, one from the full Bonn potential plus the above TPE $3N$ forces and the other from its energy-independent version obtained using the folded diagram technique.) Conversely, it is clear that it makes no sense to use an energy-dependent NN potential in a few-body calculation without at the same time including $3N$ and double-$2N$ forces calculated in the respective framework.

IV. REMAINING FEW-BODY FORCES

I now go to next order, $\nu=\nu_{\min}+3$, which still comes from tree ($L=0$) diagrams, but now have one vertex from $\mathcal{L}_{(1)}$ in (4) ($\sum_i V_i\Delta_i=1$). In the case of the double-pair force, the diagrams are still the same as in Fig. 2, but there are no corresponding $\bar{N}N\pi$ or $\bar{N}N\bar{N}N$ vertices in (4), so that

$$V_{2,2}^{(3)}(\vec{q}_{ij},\vec{q}_{kl})=0. \quad (14)$$

As for the $3N$ force, the same remark applies to Figs. 1(a) and 1(b); there are contributions though, from Figs. 1(c) and 6, that are readily calculated,

$$\begin{aligned} V_3^{(3)}(\vec{q}_{ij},\vec{q}_{jk}) ={}& E_1\boldsymbol{t}_i\cdot\boldsymbol{t}_k+E_2\vec{\sigma}_i\cdot\vec{\sigma}_k\boldsymbol{t}_i\cdot\boldsymbol{t}_k+E_3\vec{\sigma}_j\cdot(\vec{\sigma}_i\times\vec{\sigma}_k)\boldsymbol{t}_j\cdot(\boldsymbol{t}_i\times\boldsymbol{t}_k) \\ &-\frac{2g_A}{F_\pi^2}\frac{1}{w_{jk}^2}\vec{\sigma}_k\cdot\vec{q}_{jk}[D_1(\boldsymbol{t}_i\cdot\boldsymbol{t}_k\vec{\sigma}_i+\boldsymbol{t}_j\cdot\boldsymbol{t}_k\vec{\sigma}_j)-2D_2\boldsymbol{t}_j\cdot(\boldsymbol{t}_i\times\boldsymbol{t}_k)\vec{\sigma}_i\times\vec{\sigma}_j]\cdot\vec{q}_{jk} \\ &+2\left(\frac{2g_A}{F_\pi^2}\right)^2\frac{1}{w_{ij}^2w_{jk}^2}\vec{\sigma}_i\cdot\vec{q}_{ij}\vec{\sigma}_k\cdot\vec{q}_{jk}[\boldsymbol{t}_i\cdot\boldsymbol{t}_k(B_1\vec{q}_{ij}\cdot\vec{q}_{jk}+B_3m_\pi^2)-B_2\boldsymbol{t}_j\cdot(\boldsymbol{t}_i\times\boldsymbol{t}_k)\vec{\sigma}_j\cdot(\vec{q}_{ij}\times\vec{q}_{jk})] \\ &+\text{two cyclic permutations of }(ijk) \end{aligned} \quad (15)$$

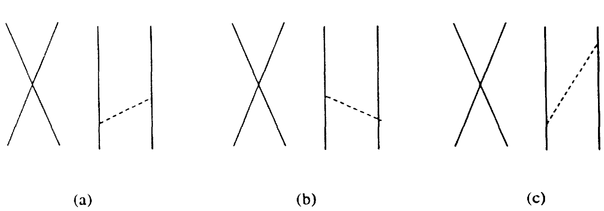

FIG. 5. Same cancellation as in Figs. 3 and 4, but for the double-pair potential. An analogous result holds for other orderings and for TPE diagrams.

(see the Appendix for the coordinate space version). Hence this $3N$ force has eight undetermined parameters. Of course, three of them (the B_i's) can be fixed once a systematic chiral Lagrangian analysis of πN scattering is carried out. Two others (the D_i's) can in principle be determined by processes such as π-deuteron scattering, or π production and/or absorption on NN systems, but it is unlikely that this could be done without much more accurate data than currently available. More important, the three remaining parameters (the E_i's) can only be determined from data involving $3N$ systems, so we do

FIG. 6. Other tree diagrams contributing to the $3N$ potential. Both orderings of (a) must be considered, as well as permutations.

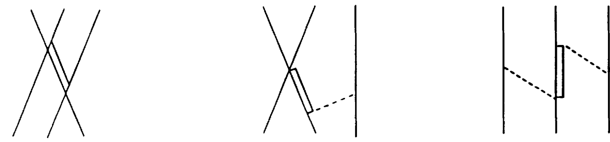

FIG. 7. Tree graphs with isobar contributing to the $3N$ potential. All other time orderings and permutations are to be considered. A double line represents the Δ isobar.

not have great predictive power. There are, of course, more than three measured quantities in these systems, and again in principle the above force is testable. The problem is, it seems that all current data can be fitted by appending to realistic NN potentials a "reasonable" $3N$ force with just *one* parameter that is fixed by the triton binding energy [1].

The situation is not completely hopeless, though, because I have been ignoring the Δ isobar. If the Δ is integrated out of the theory, its contributions appear only indirectly, in the coefficients of the general chiral Lagrangian. The mass difference to the nucleon sets both the suppression factor of these coefficients and the limit of applicability of the effective theory. If this difference were of order M or larger, no changes in the power counting arguments given above would be necessary. As it happens, though, $m_\Delta - m_N$ is only $\sim 2m_\pi$, which is closer to m_π than to M: coefficients receiving a contribution from the delta are anomalously large and the theory is valid only at very low energies. It is more convenient, then, to keep the Δ explicitly as a degree of freedom in the Lagrangian, as we did above, and treat it as the nucleon field, as far as power counting goes. That is what was done in our study of the NN potential [7,8]; here it implies an additional $3N$ force of order $\nu = \nu_{\min} + 2$, obtained from the graphs of Fig. 7 in the static limit, where all vertices are from (3). Not surprisingly, it has the form (15), but it is suppressed only by $m_\Delta - m_N$:

$$V_3^{(2)\mathrm{one}\Delta}(\vec{q}_{ij}, \vec{q}_{jk}) = V_3^{(3)}(\vec{q}_{ij}, \vec{q}_{jk}) \tag{16}$$

with

$$\begin{aligned}
E_1 &\to 0, \\
E_2 &\to \frac{1}{9}\frac{D_T^2}{m_\Delta - m_N}, \\
E_3 &\to -\frac{1}{18}\frac{D_T^2}{m_\Delta - m_N}, \\
D_1 &\to -\frac{4}{9}\frac{D_T h_A}{m_\Delta - m_N}, \\
D_2 &\to \frac{2}{9}\frac{D_T h_A}{m_\Delta - m_N}, \\
B_1 &\to -\frac{4}{9}\frac{h_A^2}{m_\Delta - m_N}, \\
B_2 &\to -\frac{2}{9}\frac{h_A^2}{m_\Delta - m_N}, \\
B_3 &\to 0.
\end{aligned}$$

Again, these forces (15) and (16) have some known elements, corresponding to the TPE pieces, because they are obviously related to the πN scattering amplitude. The importance of the Δ was recognized early and the TPE piece in (16) [h_A^2 terms, Fig. 7(c)] is simply the old Fujita-Miyazawa force [20]. Here it appears as the leading three-body force remaining after the cancellation exhibited in Sec. III and it is accompanied by shorter-range contributions depending on only one parameter (D_T). Similarly, the relevance of current algebra was noted in the 1960s [16]. Chiral symmetry has been implemented in this context by Coelho, Das, and Robilotta [21] using a chiral Lagrangian involving the ρ and the Δ in conjunction with a parametrization of the isoscalar amplitude. The TPE in (15) is the same as theirs, but hopefully it is clear that its derivation here is as model independent as possible (it does not involve any explicit assumptions about QCD dynamics in the form of the ρ) and comes from a perturbative expansion. A similar force was obtained by Coon and co-workers [22] by extrapolating amplitudes off mass shell using dispersion relations. The connection between these two approaches was examined in [23], the main difference arising from a term present in the isoscalar amplitude of the latter that generates a contact term similar to those in Fig. 6(a). It was also pointed out that this and other short-range terms that arise from the TPE in (15) are responsible for an extreme sensitivity of triton quantities on the cutoff parameter, which is introduced to regularize the coordinate space potential, but should otherwise not affect the potential much.

This problem can now be reinterpreted. First, I note that these troublesome short-range terms (except for the one from the parametrization of the isoscalar amplitude) have exactly the same structure as our new contact terms of Fig. 6 and so cannot be distinguished from them in a calculation using the complete $3N$ potential (15) and (16). The mentioned sensitivity is, therefore, only a dependence on terms that contain *bona fide* parameters (the D_i's and E_i's) of the general chiral Lagrangian. It is no more surprising than the sensitivity to, say, the $\pi N\Delta$ coupling h_A. Such short-range terms should not be omitted as suggested in [23], if a complete description is the goal; rather, it is the more general potentials (15) and (16) that should be used. Second, in the approach presented in this paper, the cutoff is not an independent parameter anyway: it is the same parameter that was used in the NN potential, the fitting of which yielded the values of some of the other parameters appearing in

(15) and (16) (g_A, h_A, and the B_i's). Changes of the cutoff parameter are compensated to a certain extent by changes in all the other parameters of the complete potential.

Calculations involving the above TPE pieces have been performed [24,25]: they tend to increase the binding of light nuclei by approximately the correct amount, although details of course depend on the NN potential and parameters used. A short-range $3N$ force was suggested on purely phenomenological grounds [24] in order to provide the necessary repulsion needed for nuclear matter saturation. As mentioned above, the dominant short-range terms (which have, however, a different structure than that assumed in [24]) are given by (16).

I stop here. I have discussed all there is to the order given by (7). As mentioned before, the inclusion of higher orders in the NN potential involves two-loop diagrams and lots of new contact terms. At the level of the $3N$ potential, one is also required to consider one-loop graphs. And then there are $4N$ forces. All such contributions are smaller by powers of m_π/M.

V. CONCLUSION

In conclusion, I have argued that naturalness of QCD implies that few-nucleon forces are generically smaller than the dominant NN forces, but appear at the same level as some other important features of the NN potential. In particular, the leading (static) $3N$ and double-pair forces are canceled by the leading energy dependence of the iterated NN force. The remaining $3N$ force has not only terms related to πN scattering, but also shorter-range components. It is expected to be dominated by the Fujita-Miyazawa force plus a shorter-range term that depends on only one undetermined parameter (D_T); they should be $O(m_\pi^2/M^2)$ and so some 5% of the NN contribution. Finally, $4N$ forces are expected to be $O(m_\pi^4/M^4)$, less than 1%, so that $4N$ systems should be underbound by pure NN forces by roughly four times the triton underbinding.

A complete chiral Lagrangian approach to third order would consist of calculating nuclear properties using the NN potential of Refs. [7,8] plus the few-nucleon forces in (10), (11), (15), and (16) [or simply, the NN chiral potential without the energy-dependent term plus (15) and (16)]. The proliferation of undetermined parameters suggests an alternative, more phenomenological approach: to take any realistic, energy-independent NN potential (which presumably describes the essential phenomenology of the NN system even in a multinucleon environment, as attested by the approximately equal binding such potentials produce), to add to it only the $3N$ potential (16), to fit the one free parameter to triton binding, and then to predict other $A = 3$ and $A \geq 4$ properties.

ACKNOWLEDGMENTS

I am grateful to S. Weinberg, C. Ordóñez, L. Ray, and especially J. Friar for discussions, encouragement, and comments on the manuscript. This work was supported in part by the U.S. Department of Energy.

APPENDIX

Here I present the potentials (10), (11), (15), and (16) in coordinate space, where I define

$$\vec{r}_{ij} = \vec{r}_i - \vec{r}_j = r_{ij}\hat{\mathbf{r}}_{ij},$$

$$\cos\theta_j = \hat{\mathbf{r}}_{ij} \cdot \hat{\mathbf{r}}_{jk}\,,$$

$$S_{nm}(\hat{\mathbf{r}}_1, \hat{\mathbf{r}}_2) = 3\vec{\sigma}_n \cdot \hat{\mathbf{r}}_1 \vec{\sigma}_m \cdot \hat{\mathbf{r}}_2 - \hat{\mathbf{r}}_1 \cdot \hat{\mathbf{r}}_2 \vec{\sigma}_n \cdot \vec{\sigma}_m.$$

The spin and isospin dependence of the three-body potentials (10), (15), and (16) can be written as

$$\begin{aligned} V_3^{(n)}(\vec{r}_{ij}, \vec{r}_{jk}) = \boldsymbol{t}_i \cdot \boldsymbol{t}_k \{ & V_s^{(n)}(\vec{r}_{ij}, \vec{r}_{jk}) + \vec{\sigma}_i \cdot \vec{\sigma}_k V_{\sigma 1}^{(n)}(\vec{r}_{ij}, \vec{r}_{jk}) + \vec{\sigma}_i \cdot \vec{\sigma}_j V_{\sigma 2}^{(n)}(\vec{r}_{ij}, \vec{r}_{jk}) + \vec{\sigma}_j \cdot \vec{\sigma}_k V_{\sigma 2}^{(n)}(\vec{r}_{jk}, \vec{r}_{ij}) \\ & + S_{ik}(\hat{\mathbf{r}}_{ij}, \hat{\mathbf{r}}_{ij}) V_{S1}^{(n)}(\vec{r}_{ij}, \vec{r}_{jk}) + S_{ik}(\hat{\mathbf{r}}_{jk}, \hat{\mathbf{r}}_{jk}) V_{S1}^{(n)}(\vec{r}_{jk}, \vec{r}_{ij}) \\ & + S_{ij}(\hat{\mathbf{r}}_{ij}, \hat{\mathbf{r}}_{ij}) V_{S2}^{(n)}(\vec{r}_{ij}, \vec{r}_{jk}) + S_{jk}(\hat{\mathbf{r}}_{jk}, \hat{\mathbf{r}}_{jk}) V_{S2}^{(n)}(\vec{r}_{jk}, \vec{r}_{ij}) + S_{ik}(\hat{\mathbf{r}}_{ij}, \hat{\mathbf{r}}_{jk}) V_{Sx}^{(n)}(\vec{r}_{ij}, \vec{r}_{jk}) \} \\ & + i\boldsymbol{t}_j \cdot (\boldsymbol{t}_i \times \boldsymbol{t}_k) \{ i\vec{\sigma}_j \cdot (\vec{\sigma}_i \times \vec{\sigma}_k) V_{\sigma\sigma}^{(n)}(\vec{r}_{ij}, \vec{r}_{jk}) + [S_{ij}(\hat{\mathbf{r}}_{ij}, \hat{\mathbf{r}}_{ij}), \vec{\sigma}_j \cdot \vec{\sigma}_k] V_{S\sigma}^{(n)}(\vec{r}_{ij}, \vec{r}_{jk}) \\ & - [S_{jk}((\hat{\mathbf{r}}_{jk}, \hat{\mathbf{r}}_{jk}), \vec{\sigma}_i \cdot \vec{\sigma}_j] V_{S\sigma}^{(n)}(\vec{r}_{jk}, \vec{r}_{ij}) + [S_{ij}(\hat{\mathbf{r}}_{ij}, \hat{\mathbf{r}}_{ij}), S_{jk}(\hat{\mathbf{r}}_{jk}, \hat{\mathbf{r}}_{jk})] V_{SS}^{(n)}(\vec{r}_{ij}, \vec{r}_{jk}) \} \\ & + \text{two cyclic permutations of } (ijk). \end{aligned}$$

For the potential (10) the radial functions $V^{(n)}$ are given by

$$V_s^{(2)}(\vec{r}_{ij}, \vec{r}_{jk}) = 0,$$

$$\begin{aligned} V_{\sigma 1}^{(2)}(\vec{r}_{ij}, \vec{r}_{jk}) = -\frac{2}{3}\left(\frac{g_A}{F_\pi}\right)^2 \Bigg\{ & \frac{2}{3}\left(\frac{g_A}{F_\pi}\right)^2 [m_\pi^4 I_{3,0}(r_{ij}) I_{2,0}(r_{jk}) + (3\cos^2\theta_j - 1) I_{3,2}(r_{ij}) I_{2,2}(r_{jk})] \\ & + \left[C_S - \frac{2}{3}\left(\frac{g_A}{F_\pi}\right)^2\right] [m_\pi^2 I_{3,0}(r_{ij}) - I_{1,0}(r_{ij})] I_{0,0}(r_{jk}) \Bigg\} + (r_{ij} \leftrightarrow r_{jk}), \end{aligned}$$

$$V_{\sigma 2}^{(2)}(\vec{r}_{ij},\vec{r}_{jk}) = -\frac{2}{3}\left(\frac{g_A}{F_\pi}\right)^2 C_T[m_\pi^2 I_{3,0}(r_{ij}) - I_{1,0}(r_{ij})]I_{0,0}(r_{jk}),$$

$$V_{S1}^{(2)}(\vec{r}_{ij},\vec{r}_{jk}) = -\frac{2}{3}\left(\frac{g_A}{F_\pi}\right)^2 \left\{\frac{2}{3}\left(\frac{g_A}{F_\pi}\right)^2 \{I_{3,2}(r_{ij})[m_\pi^2 I_{2,0}(r_{jk}) - I_{2,2}(r_{jk})] + I_{2,2}(r_{ij})[m_\pi^2 I_{3,0}(r_{jk}) - I_{3,2}(r_{jk}) - I_{1,0}(r_{jk})]\} + \left[C_S - \frac{2}{3}\left(\frac{g_A}{F_\pi}\right)^2\right] I_{3,2}(r_{ij})I_{0,0}(r_{jk})\right\},$$

$$V_{S2}^{(2)}(\vec{r}_{ij},\vec{r}_{jk}) = -\frac{2}{3}\left(\frac{g_A}{F_\pi}\right)^2 C_T I_{3,2}(r_{ij})I_{0,0}(r_{jk}),$$

$$V_{Sx}^{(2)}(\vec{r}_{ij},\vec{r}_{jk}) = -\frac{4}{3}\left(\frac{g_A}{F_\pi}\right)^4 \cos\theta_j I_{3,2}(r_{ij})I_{2,2}(r_{jk}) + (r_{ij} \leftrightarrow r_{jk}),$$

$$V_{\sigma\sigma}^{(2)}(\vec{r}_{ij},\vec{r}_{jk}) = -\frac{8}{9}\left(\frac{g_A}{F_\pi}\right)^4 [m_\pi^2 I_{3,0}(r_{ij}) - I_{1,0}(r_{ij})][m_\pi^2 I_{2,0}(r_{jk}) - I_{0,0}(r_{jk})] + (r_{ij} \leftrightarrow r_{jk}),$$

$$V_{S\sigma}^{(2)}(\vec{r}_{ij},\vec{r}_{jk}) = -\frac{4}{9}\left(\frac{g_A}{F_\pi}\right)^4 \{I_{2,2}(r_{ij})[m_\pi^2 I_{3,0}(r_{jk}) - I_{1,0}(r_{jk})] + I_{3,2}(r_{ij})[m_\pi^2 I_{2,0}(r_{jk}) - I_{0,0}(r_{jk})]\},$$

$$V_{SS}^{(2)}(\vec{r}_{ij},\vec{r}_{jk}) = -\frac{4}{9}\left(\frac{g_A}{F_\pi}\right)^4 I_{3,2}(r_{ij})I_{2,2}(r_{jk}) + (r_{ij} \leftrightarrow r_{jk}).$$

For the potential (15), they are instead

$$V_s^{(3)}(\vec{r}_{ij},\vec{r}_{jk}) = E_1 I_{0,0}(r_{ij})I_{0,0}(r_{jk}),$$

$$V_{\sigma 1}^{(3)}(\vec{r}_{ij},\vec{r}_{jk}) = \frac{2}{9}\left(\frac{2g_A}{F_\pi^2}\right)^2 \{B_1[m_\pi^4 I_{2,0}(r_{ij})I_{2,0}(r_{jk}) + (3\cos^2\theta_j - 1)I_{2,2}(r_{ij})I_{2,2}(r_{jk})] - 3B_3 m_\pi^2 \cos\theta_j I_{2,1}(r_{ij})I_{2,1}(r_{jk})\} + \frac{2g_A}{3F_\pi^2}D'_1 m_\pi^2[I_{2,0}(r_{ij})I_{0,0}(r_{jk}) + I_{0,0}(r_{ij})I_{2,0}(r_{jk})] + E'_2 I_{0,0}(r_{ij})I_{0,0}(r_{jk}),$$

$$V_{\sigma 2}^{(3)}(\vec{r}_{ij},\vec{r}_{jk}) = 0,$$

$$V_{S1}^{(3)}(\vec{r}_{ij},\vec{r}_{jk}) = \frac{2}{9}\left(\frac{2g_A}{F_\pi^2}\right)^2 B_1 I_{2,2}(r_{ij})[m_\pi^2 I_{2,0}(r_{jk}) - I_{2,2}(r_{jk})] + \frac{2}{3}\frac{g_A}{F_\pi^2}D'_1 I_{2,2}(r_{ij})I_{0,0}(r_{jk}),$$

$$V_{S2}^{(3)}(\vec{r}_{ij},\vec{r}_{jk}) = 0,$$

$$V_{Sx}^{(3)}(\vec{r}_{ij},\vec{r}_{jk}) = \frac{2}{3}\left(\frac{2g_A}{F_\pi^2}\right)^2 \{B_1\cos\theta_j I_{2,2}(r_{ij})I_{2,2}(r_{jk}) - B_3 m_\pi^2 I_{2,1}(r_{ij})I_{2,1}(r_{jk})\},$$

$$V_{\sigma\sigma}^{(3)}(\vec{r}_{ij},\vec{r}_{jk}) = \frac{2}{9}\left(\frac{2g_A}{F_\pi^2}\right)^2 B_2 m_\pi^4 I_{2,0}(r_{ij})I_{2,0}(r_{jk}) - \frac{2}{3}\frac{g_A}{F_\pi^2}D'_2 m_\pi^2[I_{2,0}(r_{ij})I_{0,0}(r_{jk}) + I_{0,0}(r_{ij})I_{2,0}(r_{jk})] - E'_3 I_{0,0}(r_{ij})I_{0,0}(r_{jk}),$$

$$V_{S\sigma}^{(3)}(\vec{r}_{ij},\vec{r}_{jk}) = \frac{1}{9}\left(\frac{2g_A}{F_\pi^2}\right)^2 B_2 m_\pi^2 I_{2,2}(r_{ij})I_{2,0}(r_{jk}) - \frac{2g_A}{3F_\pi^2}D'_2 I_{2,2}(r_{ij})I_{0,0}(r_{jk}),$$

$$V_{SS}^{(3)}(\vec{r}_{ij},\vec{r}_{jk}) = \frac{1}{9}\left(\frac{2g_A}{F_\pi^2}\right)^2 B_2 I_{2,2}(r_{ij})I_{2,2}(r_{jk}),$$

where

$$D'_1 \equiv D_1 - \frac{4g_A B_1}{3F_\pi^2},$$

$$D'_2 \equiv D_2 + \frac{4g_A B_2}{3F_\pi^2},$$

$$E'_2 \equiv E_2 - \frac{4}{3}\frac{g_A}{F_\pi^2}\left(D_1 - \frac{2}{3}\frac{g_A B_1}{F_\pi^2}\right),$$

$$E'_3 \equiv E_3 - \frac{4}{3}\frac{g_A}{F_\pi^2}\left(D_2 + \frac{2}{3}\frac{g_A B_2}{F_\pi^2}\right).$$

The Δ potential follows from the above $V^{(3)}$ and (16).

On the other hand, the double-pair potential (11) is

$$V_{2,2}^{(2)}(\vec{r}_{ij},\vec{r}_{kl}) = -\frac{4}{3}\left(\frac{g_A}{F_\pi}\right)^2 \boldsymbol{t}_i\cdot\boldsymbol{t}_j$$
$$\times\{\vec{\sigma}_i\cdot\vec{\sigma}_j[m_\pi^2 I_{3,0}(r_{ij}) - I_{1,0}(r_{ij})] + S_{ij}(\hat{\mathbf{r}}_{ij},\hat{\mathbf{r}}_{ij})I_{3,2}(r_{ij})\}(C_S+\vec{\sigma}_k\cdot\vec{\sigma}_l C_T)I_{0,0}(r_{kl})$$
$$+\frac{16}{9}\left(\frac{g_A}{F_\pi}\right)^4 \boldsymbol{t}_i\cdot\boldsymbol{t}_j\boldsymbol{t}_k\cdot\boldsymbol{t}_l\{\vec{\sigma}_i\cdot\vec{\sigma}_j[m_\pi^2 I_{2,0}(r_{ij}) - I_{0,0}(r_{ij})] + S_{ij}(\hat{\mathbf{r}}_{ij},\hat{\mathbf{r}}_{ij})I_{2,2}(r_{ij})\}$$
$$\times\{\vec{\sigma}_k\cdot\vec{\sigma}_l[m_\pi^2 I_{3,0}(r_{kl}) - I_{1,0}(r_{kl})] + S_{kl}(\hat{\mathbf{r}}_{kl},\hat{\mathbf{r}}_{kl})I_{3,2}(r_{kl})\} + (ij)\leftrightarrow(kl).$$

In the above expressions the I's are Fourier transforms with a cutoff function $F(\mathbf{q}^2;\Lambda)$ of parameter $\Lambda \le M$:

$$I_{k,0}(r;m_\pi,\Lambda) = \int \frac{d^3q}{(2\pi)^3}\frac{\exp(i\vec{q}\cdot\vec{r})F(\vec{q}^{\,2};\Lambda)}{(\vec{q}^{\,2}+m_\pi^2)^{k/2}},$$
$$I_{k,0}(r) = I_{k,0}(r;m_\pi,\Lambda),$$
$$I_{k,1}(r) = I'_{k,0}(r),$$
$$I_{k,2}(r) = I''_{k,0}(r) - \frac{1}{r}I'_{k,0}(r).$$

When the cutoff is removed

$$I_{0,0}(r) \overset{\Lambda\to\infty}{\longrightarrow} \delta(r),$$
$$I_{1,0}(r) \overset{\Lambda\to\infty}{\longrightarrow} \frac{m_\pi}{2\pi^2 r}K_1(m_\pi r),$$
$$I_{2,n}(r) \overset{\Lambda\to\infty}{\longrightarrow} \frac{(-m_\pi)^n}{4\pi r}\exp(-m_\pi r)a_n(r),$$
$$I_{3,n}(r) \overset{\Lambda\to\infty}{\longrightarrow} \frac{(-m_\pi)^n}{2\pi^2}K_n(m_\pi r),$$

where K_n are the modified Bessel functions and

$$a_0(r) = 1,$$
$$a_1(r) = 1 + \frac{1}{m_\pi r},$$
$$a_2(r) = 1 + \frac{3}{m_\pi r} + \frac{3}{(m_\pi r)^2}.$$

In the NN potential [7,8] the cutoff function was taken to be Gaussian,

$$F(\vec{q}^{\,2};\Lambda) = \exp(-\vec{q}^{\,2}/\Lambda^2),$$

in which case [26]

$$I_{0,0}(r) = \left(\frac{\Lambda}{4\pi}\right)^2 \frac{\sqrt{\pi}}{2}g(r),$$
$$I_{1,0}(r) = \frac{2}{\pi}\int_0^\infty d\lambda\,\exp\left(-\frac{\lambda^2}{\Lambda^2}\right)I_{2,0}(r;\sqrt{m_\pi^2+\lambda^2},\Lambda),$$
$$I_{2,n}(r) = \frac{m_\pi^n}{2}[a_n(r)f(r) - (-1)^n a_n(-r)f(-r) - b_n(r)g(r)],$$
$$I_{3,n}(r) = \frac{2}{\pi}\int_0^\infty \frac{d\lambda}{\lambda^2}\left[I_{2,n}(r;m_\pi,\Lambda) - \exp\left(-\frac{\lambda^2}{\Lambda^2}\right)I_{2,n}(r;\sqrt{m_\pi^2+\lambda^2},\Lambda)\right],$$

where

$$g(r) = \frac{\Lambda}{4\pi}\frac{2}{\sqrt{\pi}}\exp\left(-\frac{\Lambda^2 r^2}{4}\right),$$
$$f(r) = \frac{1}{4\pi r}\exp\left(-m_\pi r + \frac{m_\pi^2}{\Lambda^2}\right)\mathrm{erfc}\left(-\frac{\Lambda r}{2}+\frac{m_\pi}{\Lambda}\right),$$

$\mathrm{erfc}(x) = 2/\sqrt{\pi}\int_x^\infty dt\,\exp(-t^2)$ is the error function, and

$$b_0(r) = 0,$$
$$b_1(r) = \frac{1}{m_\pi r},$$
$$b_2(r) = \frac{3}{(m_\pi r)^2}(1+\tfrac{1}{6}\Lambda^2 r^2).$$

[1] J. L. Friar, Czech. J. Phys. **43**, 259 (1993); B. F. Gibson, Nucl. Phys. **A543**, 1c (1992).
[2] B. D. Day, Comments Nucl. Part. Phys. **11**, 115 (1983).
[3] I. Slaus, R. Machleidt, W. Tornow, W. Glöckle, and W. Witala, Comments Nucl. Part. Phys. **20**, 85 (1991).
[4] V. G. J. Stoks, R. A. M. Klomp, M. C. M. Rentmeester, and J. J. de Swart, Phys. Rev. C **48**, 792 (1993).
[5] S. Weinberg, Phys. Lett. B **251**, 288 (1990); Nucl. Phys. **B363**, 3 (1991).
[6] C. Ordóñez and U. van Kolck, Phys. Lett. B **291**, 459 (1992).
[7] C. Ordóñez, L. Ray, and U. van Kolck, Phys. Rev. Lett. **72**, 1982 (1994).
[8] C. Ordóñez, L. Ray, and U. van Kolck, in preparation.
[9] S. Weinberg, Phys. Lett. B **295**, 114 (1992).
[10] J. L. Friar, B. F. Gibson, and G. L. Payne, Phys. Rev. C **30**, 1084 (1984).
[11] U.-G. Meissner, Rep. Prog. Phys. **56**, 903 (1993).
[12] T.-S. Park, D.-P. Min, and M. Rho, Phys. Rep. **233**, 341 (1993).
[13] S. Beane, C.-Y. Lee, and U. van Kolck (unpublished).
[14] A. Klein, Phys. Rev. **90**, 1101 (1953).
[15] K. A. Brueckner, C. A. Levinson, and H. M. Mahmoud, Phys. Rev. **95**, 217 (1954).
[16] G. E. Brown, A. M. Green, and W. J. Gerace, Nucl. Phys. **A115**, 435 (1968); G. E. Brown, Comments Nucl. Part. Phys. **5**, 6 (1972).
[17] S. N. Yang and W. Glöckle, Phys. Rev. C **33**, 1774 (1986);

S. A. Coon and J. L. Friar, *ibid.* **34**, 1060 (1986).
[18] J. L. Friar and S. A. Coon (unpublished).
[19] K. Holinde, Nucl. Phys. **A543**, 143c (1992).
[20] J. Fujita and H. Miyazawa, Prog. Theor. Phys. **17**, 360 (1957).
[21] H. T. Coelho, T. K. Das, and M. R. Robilotta, Phys. Rev. C **28**, 1812 (1983).
[22] S. A. Coon, M. D. Scadron, P. C. McNamee, B. R. Barrett, D. W. E. Blatt, and B. H. J. McKellar, Nucl. Phys. **A317**, 242 (1979); S. A. Coon and W. Glöckle, Phys. Rev. C **23**, 1790 (1981).
[23] M. R. Robilotta and H. T. Coelho, Nucl. Phys. **A460**, 645 (1986).
[24] J. Carlson, V. R. Pandharipande, and R. B. Wiringa, Nucl. Phys. **A401**, 59 (1983).
[25] C. R. Chen, G. L. Payne, J. L. Friar, and B. F. Gibson, Phys. Rev. C **33**, 1740 (1986).
[26] Th. A. Rijken, Ann. Phys. (N.Y.) **208**, 253 (1991).