Coulomb instability of hot nuclei with derivative scalar coupling

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The relativistic nuclear mean-field model with derivative scalar coupling suggested by Zimanyi and Moszkowski is extended to asymmetric nuclear matter by including the ρ meson degree of freedom in the Lagrangian. The extended model is then used to study the Coulomb instability of asymmetric nuclear matter at finite temperature. The critical temperature for the liquid-gas phase transition in nuclear matter and its dependence on the asymmetry parameter are calculated. The limiting temperature $T_{\rm lim}$, which reflects Coulomb instability of hot nuclei is studied. The calculated results are compared with those given by quantum hadrodynamics (QHD) models.

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I. INTRODUCTION

Although the mean-field theory (MFT) of the quantum hadrodynamics (QHD) model is very successful in describing the properties of both nuclear matter and finite nuclei [1], it has two major shortcomings. First, the effective nucleon mass becomes very small for nuclear matter at moderately high density and temperature [2]. This strong change in the effective mass has serious effects in the calculation of the production of new particles in heavy-ion collisions. Second, the QHD model gives a compression modulus $K = 540$ MeV, which is much larger than its empirical value [3,4]. Recently, Zimanyi and Moszkowski suggested a new model with derivative scalar coupling [5] (referred to as the ZM-I model in the rest of this paper). In the mean-field approximation, Zimanyi and Moszkowski obtained a nuclear equation of state for infinite symmetric nuclear matter at zero temperature, which is much softer than that in the Walecka model. It yields much more reasonable values for two key properties of nuclear matter, namely, the compression modulus (225 MeV) and the effective nucleon mass (0.85) at the saturation point.

A successful model should describe well not only the properties of infinite symmetric nuclear matter at zero temperature, but also those of nuclear systems of finite size, asymmetric quantities, and at finite temperature. In a recent paper [6] we derived an equation of state for infinite nuclear matter at finite temperature by means of a real time Green's function method, starting from the ZM-I model. It is found that the equation of state obtained at finite temperature is much softer than that given by the Walecka model and is quite similar to that given by the nonrelativistic theory with the Skyrme effective nucleon-nucleon interaction (SkM*). As a result, the ZM-I model gives us a more reasonable critical temperature for the liquid-gas phase transition in infinite symmetric nuclear matter than the Walecka model. The next step is to investigate if the ZM-I model is still good for describing the properties of asymmetric and/or finite nuclear systems. This is the main purpose of this paper.

We would like to extend the formalism for symmetric nuclear matter [6] to the asymmetric case. The equation of state obtained is then used to study the liquid-gas phase transition in infinite asymmetric nuclear matter, where the critical temperature T_C for the liquid-gas phase transition in nuclear matter and its dependence on asymmetry are calculated.

Most of the existing calculations of the critical temperature for the liquid-gas phase transition in nuclear matter are based on nonrelativistic theories starting from effective nucleon-nucleon interactions, such as the Skyrme interaction [7–9], the Gogny interaction [10,11] etc. [12,13]. In a recent paper [14] we have discussed the asymmetry dependence of the critical temperature for the liquid-gas phase transition, starting from a quantum hadrodynamics model. It was found that in order to describe well the dependence of the properties of nuclear matter on asymmetry, we have to use the QHD-II model, where the ρ meson degree of freedom is taken into account. As a result, the main feature obtained in nonrelativistic theories with effective nucleon-nucleon interactions remains in the QHD-II model. It is, therefore, of interest to adopt the ZM model to study such a phase transition to see what role the ρ meson plays in the ZM model and compare the obtained results with those in the QHD-II model.

Furthermore, we shall also study the finite nuclear system at finite temperature. As pointed out by Levit and Bonche [15], another temperature, namely, the limiting temperature $T_{\rm lim}$, is important for a finite nuclear system with a Coulomb interaction. Below the limiting temperature $T_{\rm lim}$, the nucleus can exist in equilibrium with the surrounding vapor. But above $T_{\rm lim}$ the nucleus is unstable and shall fragment. This is the so-called Coulomb instability of hot nuclei. Recently, much effort [16,17] has been devoted to studying the Coulomb instability of asymmetric nuclear matter. But most of such studies are also based on a nonrelativistic treatment with effective nucleon-nucleon interactions. It is therefore of interest to investigate such an instability of hot nuclei in a relativistic approach. In Ref. [14] we also discussed the Coulomb instability of hot nuclei, starting from QHD models. It is found again that the QHD-I model (without the ρ meson) cannot describe well hot nuclei with two phases in equilibrium. For a sufficiently large asymmetry effect, it is necessary to include the ρ meson degree of freedom in the Lagrangian. How about the situation in the ZM model? This is also a problem we would like to investigate in this paper.

In Sec. II we will describe briefly the MFT of the ZM model for bulk nuclear matter. A two-phase model and the coexistence equations are described in Sec. III. The numerical results and some discussions are presented in Sec. IV. We then give some concluding remarks in Sec. V.

II. BULK MATTER IN THE ZM MODEL

The degrees of freedom in the ZM model are baryons, scalar mesons, and vector mesons. The outstanding feature of this model is that the coupling between baryon and scalar meson is of derivative form. As mentioned in the last section we would like to extend the ZM-I model to the asymmetric nuclear system by including the ρ meson degree of freedom in the Lagrangian of the nuclear system. Following the prescription for the QHD-II model [1], we take into account the coupling term between the nucleon and ρ meson fields besides the terms for the ρ meson field. The Lagrangian density for the extended ZM model (referred to as the ZM-II model) [5] is then

$$
\begin{aligned}
L = &-\bar{\psi}M\psi \\
&+\left[1+\frac{g_\sigma\sigma}{M}\right]\bar{\psi}[i\gamma_\mu\partial^\mu - g_\omega\gamma_\mu\omega^\mu - g_\rho\tfrac{1}{2}\gamma_\mu\tau\cdot\mathbf{b}^\mu]\psi \\
&+\tfrac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2) - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \tfrac{1}{2}m_\omega^2\omega_\mu\omega^\mu \\
&-\tfrac{1}{4}\mathbf{L}_{\mu\nu}\cdot\mathbf{L}^{\mu\nu} + \tfrac{1}{2}m_\rho^2\mathbf{b}_\mu\cdot\mathbf{b}^\mu \ , \qquad (1)
\end{aligned}
$$

where

$$F_{\mu\nu} \equiv \partial_\mu\omega_\nu - \partial_\nu\omega_\mu \qquad (2)$$

and

$$\mathbf{L}_{\mu\nu} \equiv \partial_\mu\mathbf{b}_\nu - \partial_\nu\mathbf{b}_\mu \ . \qquad (3)$$

In Eq. (1), M is the rest mass of nucleon and τ is the Pauli operator. σ, ω_μ, and $\mathbf{b}_\mu$ are the neutral scalar meson, neutral vector meson, and isovector vector meson fields, with parameters m_σ, g_σ, m_ω, g_ω, m_ρ, and g_ρ, respectively. By rescaling the fermion wave function

$$\psi \to \left[1+\frac{g_\sigma\sigma}{M}\right]^{-1/2}\psi \ , \qquad (4)$$

the Lagrangian given by Eq. (1) can be written in the following form which is more familiar to us:

$$
\begin{aligned}
L = &-\bar{\psi}[M^* - i\gamma_\mu\partial^\mu + g_\omega\gamma_\mu\omega^\mu + g_\rho\tfrac{1}{2}\gamma_\mu\tau\cdot b^\mu]\psi \\
&+\tfrac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2) - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \tfrac{1}{2}m_\omega^2\omega_\mu\omega^\mu \qquad (5)\\
&-\tfrac{1}{4}\mathbf{L}_{\mu\nu}\cdot\mathbf{L}^{\mu\nu} + \tfrac{1}{2}m_\rho^2\mathbf{b}_\mu\cdot\mathbf{b}^\mu, \qquad (6)
\end{aligned}
$$

where the effective mass M^* is defined as

$$M^* = m^*M, \quad m^* = \left[1+\frac{g_\sigma\sigma}{M}\right]^{-1} \qquad (7)$$

This gradient or derivative coupled Lagrangian is Lorentz invariant. The Lagrangian also contains couplings between scalar meson and vector mesons. Zimanyi and Moszkowski [5] derived the equation of state for symmetric nuclear matter at zero temperature in a mean-field approximation. In Ref. [4] we have derived the equation of state for nuclear matter at finite temperature by using a real time Green's function method with a first-order pair cutoff approximation. It is not difficult to extend the formalism to the case with a ρ meson field. Since the details can be found in Ref. [6] we will just give the main results here.

The single nucleon spectrum $E^*(k)$ in nuclear matter is given by

$$E^*(k) = \sqrt{\mathbf{k}^2 + M^{*2}} \ , \qquad (8)$$

where $\mathbf{k}$ and M^* are, respectively, the momentum and the effective mass of the nucleon. The effective mass M^* of the nucleon in nuclear matter is given by the relation

$$M^* = \left[1+\frac{g_\sigma\sigma_0}{M}\right]^{-1}M \ , \qquad (9)$$

where the mean field σ_0 of the scalar meson σ is expressed as

$$
\begin{aligned}
\sigma_0 = \frac{g_\sigma}{m_\sigma^2}m^{*2}\Bigg\{&\frac{\gamma_n}{(2\pi)^3}\int d^3k\frac{M^*}{E^*(k)}[n_n(k)+\bar{n}_n(k)] \\
&+\frac{\gamma_p}{(2\pi)^3}\int d^3k\frac{M^*}{E^*(k)}[n_p(k)+\bar{n}_p(k)]\Bigg\} \qquad (10)
\end{aligned}
$$

In Eq. (10), $\gamma_p = \gamma_n = 2$ is the spin degeneracy. $n_q(k)$ and $\bar{n}_q(k)$ are nucleon and antinucleon distributions, respectively, expressed as

$$n_q(k) = \{\exp[(E^*(k) - \nu_q)/k_BT] + 1\}^{-1} \qquad (11)$$

and

$$\bar{n}_q(k) = \{\exp[(E^*(k) + \nu_q)/k_BT] + 1\}^{-1} \quad (q = n, p) \ , \qquad (12)$$

where T is the temperature of the system studied, k_B the Boltzmann constant, and the quantity ν_q is related to the usual chemical potential μ_q by the equations

$$\nu_n = \mu_n - g_\omega\omega_0 + \tfrac{1}{2}g_\rho b_0 \qquad (13)$$

and

$$\nu_p = \mu_p - g_\omega \omega_0 - \tfrac{1}{2} g_\rho b_0 \ . \quad (14)$$

In above equations, ω_0 and b_0 are the time components of the mean fields for the ω and ρ mesons, with the expressions

$$\omega_0 = \frac{g_\omega}{m_\omega^2}\rho \quad (15)$$

and

$$b_0 = \frac{g_\rho}{2m_\rho^2}\rho_3 \ , \quad (16)$$

where $\rho_3 = \rho_p - \rho_n$. The neutron density ρ_n and proton density ρ_p determine the chemical potentials by the subsidiary conditions

$$\rho_q = \frac{\gamma_q}{(2\pi)^3}\int d^3k[n_q(k) - \bar{n}_q(k)] \ \ (q = n, p) \ . \quad (17)$$

The neutron density ρ_n and proton density ρ_p are also related to the total density ρ and asymmetry parameter α by the relations

$$\rho_n = (1+\alpha)\rho/2, \ \ \rho_p = (1-\alpha)\rho/2 \ . \quad (18)$$

Equations (8)–(18) form a closed set of equations for calculating the single nucleon spectrum $E^*(k)$, effective mass M^*, and chemical potentials μ_q $(q = n,p)$ self-consistently.

Having obtained the single nucleon spectrum $E^*(k)$ and the chemical potentials μ_q, one can easily calculate the thermodynamical potential Ω and then calculate all other thermodynamical quantities of the system. For example, the pressure of nuclear matter at a given temperature can be found by the ensemble average of the energy-momentum tensor $\hat{T}^{\mu\nu}$:

$$\begin{aligned} p &= \tfrac{1}{3}\langle \hat{T}_\omega^{ii}\rangle + \tfrac{1}{3}\langle \hat{T}_\sigma^{ii}\rangle + \tfrac{1}{3}\langle \hat{T}_\rho^{ii}\rangle + \tfrac{1}{3}\langle \hat{T}_N^{ii}\rangle \\ &= \tfrac{1}{2}m_\omega^2\omega_0^2 - \tfrac{1}{2}m_\sigma^2\sigma_0^2 + \tfrac{1}{2}m_\rho^2 b_0^2 \\ &\quad + \frac{1}{3}\frac{\gamma_n}{(2\pi)^3}\int d^3k \frac{\mathbf{k}^2}{\sqrt{\mathbf{k}^2 + M^{*2}}}[n_n(k) + \bar{n}_n(k)] \\ &\quad + \frac{1}{3}\frac{\gamma_p}{(2\pi)^3}\int d^3k \frac{\mathbf{k}^2}{\sqrt{\mathbf{k}^2 + M^{*2}}}[n_p(k) + \bar{n}_p(k)] \ , \end{aligned} \quad (19)$$

where the subscript N stands for the nucleon field.

III. TWO-PHASE EQUILIBRIUM MODEL

Since the main purpose of this paper is to investigate the change in the Coulomb instability of hot nuclei when the ZM model is used in describing the bulk nuclear matter instead of the QHD model, we will adopt the same model as used in Refs. [14–17]. We consider the hot nucleus as a uniformly charged drop of nuclear liquid at a given temperature T, with a sharp edge, and in both thermal mechanical and chemical equilibrium with the surrounding vapor. A set of two-phase coexistence equations is, therefore, obtained by requiring the equality of temperature T, pressure p, neutron chemical potential μ_n, and proton chemical potential μ_p of the liquid and vapor phases:

$$p(T,\rho_L,\alpha_L) + p_{\mathrm{Coul}}(\rho_L) + p_{\mathrm{surf}}(T,\rho_L) = p(T,\rho_V,\alpha_V) \ , \quad (20)$$

$$\mu_n(T,\rho_L,\alpha_L) = \mu_n(T,\rho_V,\alpha_V) \ , \quad (21)$$

$$\mu_p(T,\rho_L,\alpha_L) + \mu_{\mathrm{Coul}}(\rho_L) = \mu_p(T,\rho_V,\alpha_V) \ , \quad (22)$$

where the subscripts L and V stand for liquid and vapor, respectively. In the liquid phase, Coulomb and surface effects have been included.

In the MFT of the ZM model for infinite nuclear matter, the Coulomb interaction is switched off and the surface effect is not considered. When the Coulomb interaction is added, the single nucleon spectrum given by Eq. (8) should be added by an additional Coulomb potential energy. For simplicity, we will use an average Coulomb potential per proton in a uniformly charged sphere:

$$V_{\mathrm{Coul}}(\rho) = \frac{6}{5}\frac{Ze^2}{R} \ , \quad (23)$$

where Z and R are the charge number and radius of the liquid droplet. The exchange term of the Coulomb interaction has been neglected. When the Coulomb interaction is switched on, the chemical potential for protons has also an additional term $\mu_{\mathrm{Coul}} = V_{\mathrm{Coul}}$. The contribution of the Coulomb interaction to the presssure is expressed as

$$p_{\mathrm{Coul}}(\rho) = \frac{Z^2e^2}{5AR}\rho \ , \quad (24)$$

where $A = N + Z$ is the number of nucleons in the liquid droplet.

For the liquid droplet with a surface we should also consider the surface effect on pressure. Following Refs. [15] and [16], the formula for the temperature dependence of the pressure tension $\gamma(T)$ suggested by Goodman, Kapusta, and Mekjian [18] is used:

$$\gamma(T) = (1.14 \ \mathrm{MeV\,fm^{-2}})\left[1 + \frac{3T}{2T_C}\right]\left[1 - \frac{T}{T_C}\right]^{3/2} \ , \quad (25)$$

where T_C is the critical temperature for infinite symmetric nuclear matter. The additional pressure given by the surface tension of the liquid droplet is then

$$p_{\mathrm{surf}}(T,\rho) = -2\gamma(T)/R \ , \quad (26)$$

where the nuclear density ρ is related to the nuclear radius R by the relation $A = \frac{4}{3}\pi R^3\rho$ for a given nucleon number A [19].

IV. RESULTS AND DISCUSSIONS

By using the formalism given in Secs. II and III, we can discuss the properties of nuclear matter at finite temperature. In the numerical calculation, we choose the coupling parameters $C_\sigma^2 \equiv (M^2/m_\sigma^2)g_\sigma^2 = 169.2$,

$C_\omega^2 \equiv (M^2/m_\omega^2)g_\omega^2 = 59.1$, which have been taken in the ZM-I model to reproduce the equilibrium properties of nuclear matter at zero temperature [5]. The coupling constant g_ρ between the ρ meson and nucleon fields in the present ZM-II model is different from that used in the QHD-II model. In the QHD-II model, the coupling term is given between the ρ meson field and the original nucleon field. But in the ZM-II model, the coupling term is given between the ρ meson field and rescaling nucleon field. So the coupling constant g_ρ in the ZM-II model is related to the coupling constant g_ρ^0 in the QHD-II model by equation $g_\rho m^* = g_\rho^0$. Then we have the parameter

$$C_\rho^2 m^{*2} \equiv (M^{*2}/m_\rho^2)g_\rho^2 = (M^2/m_\rho^2)(g_\rho^0)^2 = 54.71 ,$$

which is determined from $\rho \to 2\pi$ decay [1].

A. Infinite nuclear matter

When discussing the liquid-gas phase transition in infinite symmetric nuclear matter, the equation of state (EOS) given by pressure-density (p-ρ) isotherms is equivalent to the one given by chemical-density (μ-ρ) isotherms, as pointed out by Jaqaman, Mekjian, and Zamick [7]. Therefore either of them can be used to calculate the critical temperature T_C for the liquid-gas phase transition. The critical temperature T_C can be determined either by the condition of the inflection point of p-ρ isotherms,

$$\left(\frac{\partial p}{\partial \rho}\right)_T = 0 \text{ and } \left(\frac{\partial^2 p}{\partial \rho^2}\right)_T = 0 , \qquad (27)$$

or by the condition of the inflection point of μ-ρ isotherms,

$$\left(\frac{\partial \mu}{\partial \rho}\right)_T = 0 \text{ and } \left(\frac{\partial^2 \mu}{\partial \rho^2}\right)_T = 0 . \qquad (28)$$

In the case of asymmetric nuclear matter, the situation changes. As mentioned by Jaqaman, Mekjian, and Zamick [7] and Song and Su [17], protons and neutrons are not in chemical equilibrium, although they may be in thermal equilibrium. So their chemical potentials are not related to each other. Since the proton and neutron have different chemical potentials, they shall also appear to have different critical temperatures T_C^p and T_C^n, respectively. But we cannot imagine that the kind of nucleons with high critical temperature can stick together after all other nucleons with lower critical temperature have boiled off. We therefore must choose the lesser of T_C^p and T_C^n as the correct critical temperature. Because of the definition of the single-particle spectrum $E^*(k)$ in relativistic theories such as the QHD and ZM models, the resultant chemical potential μ has also included the nucleon rest mass M. For convenience in the comparison between the present results and those given by the nonrelativistic theories, we define a reduced chemical potential $\tilde{\mu}$ as $\tilde{\mu} \equiv \mu - M$. We shall show $\tilde{\mu}$-ρ isotherms instead of μ-ρ isotherms. Since the difference between μ and $\tilde{\mu}$ is only a constant $M = 938$ MeV, the $\tilde{\mu}$-ρ isotherm has the same behavior as the corresponding μ-ρ isotherm. Instead of μ, we will discuss the reduced chemical potential $\tilde{\mu}$ in the following. For convenience, we will denote the chemical potential by $\tilde{\mu}$. In Fig. 1 we present the $\tilde{\mu}$-ρ isotherms for infinite asymmetric nuclear matter with asymmetry parameter $\alpha = 0.4$ and at various temperatures. In the case of asymmetric nuclear matter, the proton chemical potential $\tilde{\mu}_p$ and the neutron chemical potential $\tilde{\mu}_n$ separate, with $\tilde{\mu}_n$ moving up and $\tilde{\mu}_p$ moving down compared to the chemical potential for symmetric nuclear matter (also see Fig. 2). At lower temperatures, both $\tilde{\mu}_n$-ρ and $\tilde{\mu}_p$-ρ isotherms exhibit the form of two-phase coexistence, with an unphysical region for each. When the temperature T increases, the unphysical regions in the two kinds of isotherms get smaller. We can then find a critical temperature T_C^n for neutrons and a critical temperature T_C^p for protons. The result for the asymmetry parameter $\alpha = 0.4$ is $T_C^n = 14.45$ MeV and $T_C^p = 18.55$ MeV. As mentioned in the beginning of this subsection, we should choose the lower of the two critical temperatures T_C^n as the correct critical temperature for asymmetric nuclear matter.

In Fig. 2 we present the $\tilde{\mu}_q$-ρ isotherms for infinite nuclear matter with various asymmetry parameters α and at a temperature $T = 12$ MeV. For symmetric nuclear matter ($\alpha = 0$), the chemical potential for protons is equal to the one for neutrons, as expected. For asymmetric nuclear matter ($\alpha \neq 0$), the proton chemical potential

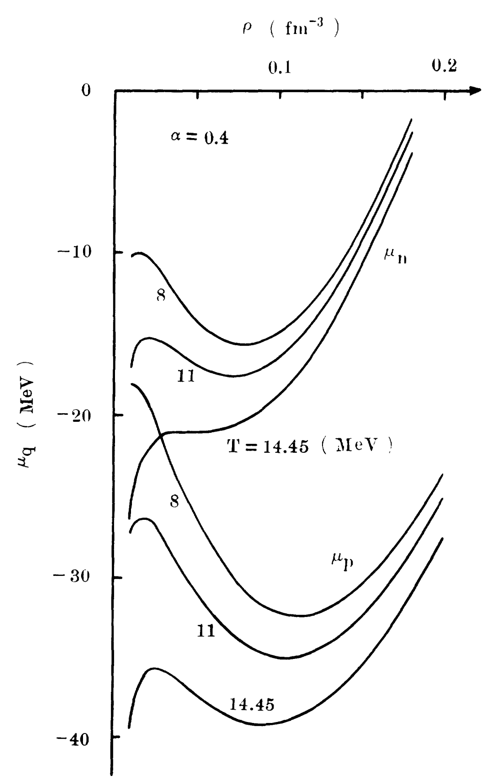

FIG. 1. $\tilde{\mu}_n$-ρ and $\tilde{\mu}_p$-ρ isotherms of infinite asymmetric nuclear matter with asymmetry parameter $\alpha = 0.4$ and at various temperature.

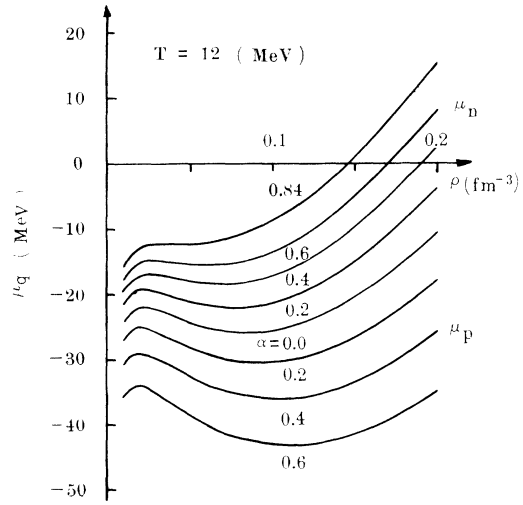

FIG. 2. $\tilde{\mu}_n$-ρ and $\tilde{\mu}_p$-ρ isotherms of infinite asymmetric nuclear matter with various asymmetry parameters and at a fixed temperature $T = 12$ MeV.

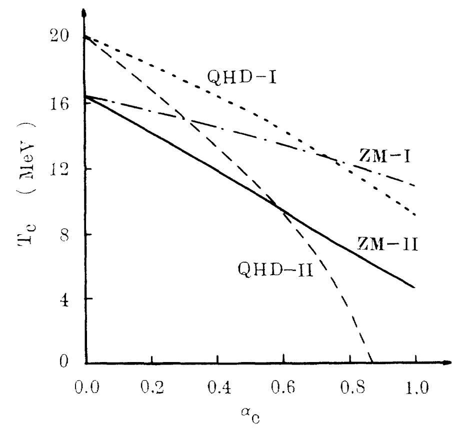

FIG. 3. Phase diagram of the critical temperature T_C versus the critical asymmetry parameter α_C for infinite nuclear matter. The solid curve is calculated with the ZM-II model, the dot-dashed curve with the ZM-I model, the dashed curve with the QHD-II model, and the dotted curve with the QHD-I model.

$\tilde{\mu}_p$ and neutron chemical potential $\tilde{\mu}_n$ separate, forming a gap between two curves. When the asymmetry parameter α increases, the gap between these two curves also increases. It can also be seen that for not too large asymmetry parameters α (< 0.84), each isotherm exhibits a typical two-phase coexistence form, with an unphysical region. When the asymmetry parameter α increases, the unphysical region becomes smaller for $\tilde{\mu}_n$-ρ isotherms and becomes larger for $\tilde{\mu}_p$-ρ isotherms. It is implicated that the critical temperature of neutron T_C^n decreases and the critical temperature of proton T_C^p increases as the asymmetry parameter α increases. At $\alpha = 0.84$, the unphysical region in the $\tilde{\mu}_n$-ρ isotherm disappears and there appears an inflection point. We may call this asymmetry parameter the critical asymmetry α_C for the liquid-gas phase transition in infinite nuclear matter at the fixed temperature T [9]. We can obtain a critical asymmetry parameter α_C for each given temperature T. The resulting T_C-α_C diagram is shown in Fig. 3, with a solid curve (denoted by ZM-II). The phase diagram separates the T-α space into two regions. In the exterior region, nuclear matter can exist in the gaseous phase only, while in the interior region both liquid and gaseous phases are allowed. For example, the critical asymmetry at $T = 12$ MeV is $\alpha = 0.84$, above which only the gaseous phase can exist in nuclear matter. For comparison, we have also shown in Fig. 3 the T_C-α_C phase diagrams calculated, respectively, from the ZM-I model with a dot-dashed curve, from the QHD-II model with a dashed curve, and from the QHD-I model with a dotted curve.

It is seen that the critical temperature T_C decreases monotonically as the asymmetry parameter α increases. This general trend is consistent with the results predicted by nonrelativistic theories [11,17] and the QHD-I models [14]. Compared to the curve given by the QHD-I model, the T_C-α_C curve calculated with the ZM-I model has small decreasing rate, although it has a lower T_C value than the QHD-I model for nuclear matter in the region of a small and moderate asymmetry parameter. Consequently, the two curves have a crossing point around $\alpha = 0.7$ and the critical temperature given by the ZM-I model becomes higher than that calculated with the QHD-I model in the region of the large asymmetry parameter. This result comes directly from the fact that the ZM-I model gives a larger effective mass than the QHD-I model, especially for higher density. As a result, the gap between μ_n and μ_p calculated with the ZM-I model is smaller than that calculated with the QHD-I model in the same conditions. It is this gap that causes the decrease of the critical temperature in asymmetric nuclear matter, as pointed out in the text where the Fig. 2 is discussed. The small gap given by the ZM-I model results in a small decreasing rate, compared to the situations in the QHD-I model.

Now let us discuss the effect of including the ρ meson degree of freedom. It is seen from Eqs. (13), (14), and (19) that the inclusion of the ρ meson degree of freedom gives an additional asymmetry effect on the chemical potentials μ_q and pressure p. As a result, the chemical potential of neutrons shifts up further and the chemical potential of protons shifts down further from the ZM-I results. As discussed in the last paragraph, the increase in the gap between μ_n and μ_p shall decrease the critical temperature in asymmetric nuclear matter. This result is explicitly shown in Fig. 3. The curve for the ZM-II model drops more quickly than that for the ZM-I model. There is a similar situation for the QHD models.

B. Finite nuclear matter

Now let us discuss the Coulomb instability of hot nuclei by calculating the limiting temperature $T_{\lim}$, above which the set of coexistence equations (20)–(22) has no solution.

We show in Fig. 4 by a solid curve the mass number de-

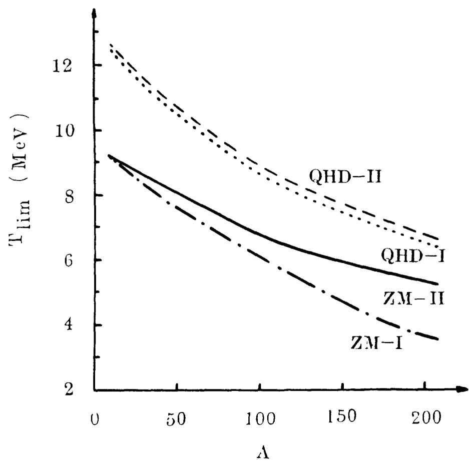

FIG. 4. Mass number dependence of limiting temperature $T_{\lim}$ calculated with the ZM-II model (solid curve) compared to those given, respectively, by the ZM-I model (dot-dashed curve), by the QHD-II model (dashed curve), and by the QHD-I model (dotted curve).

pendence of the limiting temperature $T_{\lim}$ for the nuclei along the β-stability line:

$$Z = 0.5A - 0.3 \times 10^{-2} A^{5/3} . \tag{29}$$

For a comparison we have also drawn the curves calculated, respectively, from the ZM-I model with a dot-dashed curve, from the QHD-II model with a dashed curve, and from the QHD-I model with a dotted curve. It is seen that the four curves have a similar trend: The limiting temperature decreases monotonically as the mass number A increases, but the rate of the decrease is smaller for larger A. It is also seen that the two curves with the ZM models are always lower than those given by the QHD models for the nuclei along the β stability. This result indicates that the hot nuclei described by QHD models are more stable than that described by the ZM model. This conclusion is consistent with the calculated critical temperatures of asymmetric nuclear matter (see Fig. 3), where the critical temperatures calculated with QHD models are higher than those given by the ZM models for asymmetry parameters of nuclei along the β-stability line ($\alpha < 0.3$). These results come from the same fact that the nuclear matter described by the ZM models is much softer than that described by the QHD models. Compared to those with the ZM-I model, the limiting temperature calculated with the ZM-II model has higher values and exhibits explicitly an additional asymmetry effect. As a result, the heavier hot nucleus becomes more stable when the ρ meson degree of freedom is included in the Lagrangian of the system. We present the solution of the coexistence equations (20)–(22) and the equilibrium values of $\tilde{\mu}_n$, $\tilde{\mu}_p$, and p at the limiting temperature in Table I. It is seen that the same feature as in the QHD-I model remains; i.e., the neutron chemical potential $\tilde{\mu}_n$ is always lower than the proton chemical potential $\tilde{\mu}_p$, which results in a negative asymmetry parameter α_V for the vapor phase. This feature is very different from that of the results in nonrelativistic theories, where the asymmetry parameter of vapor is always positive. The result that $\tilde{\mu}_n < \tilde{\mu}_p$ and $\alpha_V < 0$ comes from the fact that the gap between $\tilde{\mu}_n$ and $\tilde{\mu}_p$ calculated with these relativistic models (say, about 30 MeV for $\alpha = 0.4$, $\rho = 0.17$, and $T = 10$ MeV in the QHD-I model and about 25 MeV under the same conditions as above in the ZM-I model) is much smaller than that calculated with nonrelativistic theories from the effective nucleon-nucleon interaction (about 50 MeV in the same condition as above). After adding the contribution from Coulomb interaction, we then have the opposite results: $\tilde{\mu}_n < \tilde{\mu}_p$ in these relativistic models and $\tilde{\mu}_n > \tilde{\mu}_p$ in the nonrelativistic theories. Tracing back to the starting point of the ZM-I or QHD-I model, the unusual result comes from the fact the nucleon-nucleon interaction in the ZM-I or QHD-I model is independent of isospin. A model with isospin dependence shall give a quite different result. As mentioned above, the inclusion of the ρ meson degree of freedom gives an additional asymmetry effect. We present in Table II the solution of the coexistence equations (20)–(22) and the equilibrium values of $\tilde{\mu}_n$, $\tilde{\mu}_p$, and p at the limiting temperature in the ZM-II model. It is seen that after the ρ meson degree of freedom is included in the Lagrangian, the results are improved somewhat compared to those with the ZM-I model. The asymmetry parameters for the vapor phase become positive in the $A = 150, 208$ cases, while they are still negative in the $A = 10, 50, 109$ cases. The results are not desired, becuase the negative asymmetry parameter α_V in the vapor phase is not reasonable. Let us explain why we have these undesirable results in the ZM-II model. In order to make the points more clear, we show in Fig. 5 the $\tilde{\mu}_q$-ρ ($q = n, p$) isotherms for nuclear matter with an asymmetry parameter $\alpha = 0.2$ and at temperature $T = 6$ MeV calculated with the ZM-I model (dot-dashed curves) and the ZM-II model (solid curves). As a comparison, the results in the same conditions as above but with the QHD-I (dot-dashed curve) and QHD-

TABLE I. Equilibrium values of densities (in fm^{-3}), pressure (in MeV fm^{-3}), reduced chemical potentials (in MeV fm^{-3}), and asymmetry parameter for the nuclei along the β-stability line at the limiting temperature, with the ZM-I model.

A	$T_{\lim}$	ρ_L	ρ_V	α_V	$\tilde{\mu}_n$	$\tilde{\mu}_p$	p
10	9.2	0.159	0.0151	−0.080	−16.8	−15.1	0.063
50	7.7	0.155	0.0108	−0.258	−15.4	−10.7	0.043
109	5.8	0.157	0.0079	−0.481	−13.2	−6.0	0.025
150	4.8	0.157	0.0069	−0.616	−12.1	−3.8	0.018
208	3.6	0.157	0.0053	−0.784	−10.9	−1.7	0.011

TABLE II. Same quantities as in Table I, but with the ZM-II model.

A	$T_{\rm lim}$	ρ_L	ρ_V	α_V	$\tilde{\mu}_n$	$\tilde{\mu}_p$	p
10	9.2	0.159	0.0151	−0.013	−16.1	−15.8	0.062
50	8.1	0.152	0.0120	−0.032	−14.0	−13.3	0.047
109	6.7	0.152	0.0095	−0.022	−10.9	−10.5	0.032
150	6.0	0.151	0.0077	0.004	−9.2	−9.3	0.025
208	5.3	0.149	0.0063	0.076	−7.4	−8.5	0.019

II (solid curve) models are presented in Fig. 6. It can be seen from these two figures that the gap between $\tilde{\mu}_n$ and $\tilde{\mu}_p$ in the ZM-I model is smaller than that in the QHD-I model, especially in the high density region. For example, for $\rho = 0.17$ fm^{-3}, the difference $\tilde{\mu}_n - \tilde{\mu}_p$ is 11.7 MeV in the ZM-I model, compared to 15.5 MeV in the QHD-I model. This result comes from the fact that the effective nucleon mass in the ZM-I model is larger than that in the QHD-I model, especially in the higher density region. It is this point that makes the ZM-I model more reasonable in some aspects than the Walecka model. After including the ρ meson degree of freedom, the ZM-II model gives a larger asymmetry effect than the QHD-II model because of the different coupling constants used in the two models. As a result, the gap between $\tilde{\mu}_n$ and $\tilde{\mu}_p$ in the ZM-II model is comparable with that in the QHD-II model. For example, the gap in the above conditions is 23.1 MeV in the ZM-II model and 23.6 MeV in the QHD-II model. Now we should note the second point that the equilibrium densities of the liquid phase in the two models are quite different. Taking ^{208}Pb as example, $\rho_L = 0.181$ fm^{-3} in the QHD-II model, while $\rho_L = 0.149$ fm^{-3} in the ZM-II model. It is this difference that makes the chemical potential gap $\tilde{\mu}_n - \tilde{\mu}_p$ at the equilibrium point in the ZM-II model (20.5 MeV) much smaller than that in the QHD-II (25.0 MeV) model. But the Coulomb potentials in these two cases are quite close to each other ($V_{\rm Coul} = 20.4$ MeV at $\rho = 0.149$ fm^{-3} in the ZM-II model and $V_{\rm Coul} = 21.9$ MeV at $\rho = 0.181$ fm^{-3} in the QHD-II model). After adding the Coulomb potential, the neutron chemical potential is only slightly higher than the proton chemical potential in the ZM-II model, while the former is about 3 MeV higher than the latter in the QHD-II model. As a result, the equilibrium asymmetry parameter of the vapor phase in the ZM-II model is rather small (0.076), although it is still positive, compared to that in the QHD-II model (0.255). It is the same reason that cuases some lighter hot nuclei to have negative equilibrium asymmetry parameters for the vapor phase.

V. CONCLUDING REMARKS

We have studied the liquid-gas phase transition in asymmetric nuclear matter and the Coulomb instability of hot nuclei by means of the mean-field theories of the ZM-I and ZM-II models. We have also compared the calculated results with those given by the QHD models. From the results and discussions in the preceding sections we can arrive at the following conclusions.

(1) The critical temperature for the liquid-gas phase transition in nuclear matter decreases as the asymmetry parameter of nuclear matter increases. Such an asymmetry effect in the ZM-II model is much larger than in the ZM-I model, due to the inclusion of ρ mesons.

(2) Both critical and limiting temperatures in the ZM models are lower than those in the QHD models, which reflect the fact that nuclear matter described by the ZM model is softer than that by the QHD models.

(3) It is necessary to include the ρ meson degree of freedom in the Lagrangian of the nuclear system to account

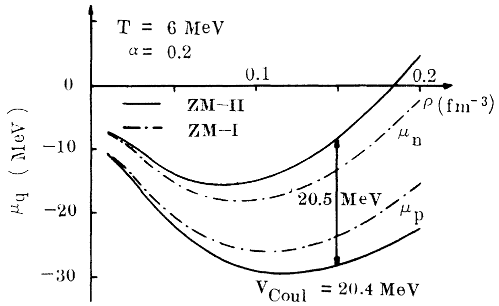

FIG. 5. $\tilde{\mu}$-ρ and $\tilde{\mu}_p$-ρ isotherms of infinite asymmetric nuclear matter with asymmetry parameter $\alpha = 0.2$ and at temperature $T = 6$ MeV calculated with the ZM-I model (dot-dashed curves) and the ZM-II model (solid curves).

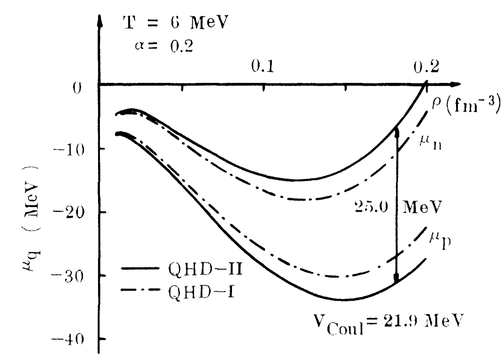

FIG. 6. Same as Fig. 5 but calculated with the QHD-I model (dot-dashed curve) and the QHD-II model (solid curve).

for the asymmetry effect. But it seems that the ρ meson freedom alone cannot give enough of an asymmetry effect in such a simple mean-field approximation. Some more elegant model is desired.

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