$\Lambda(1405)$ and meson-baryon interactions in a quark model

M. Arima,* S. Matsui, and K. Shimizu
Department of Physics, Sophia University, Tokyo 102, Japan
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We investigate the mass spectra of p-wave Λ^*'s by means of the constituent quark model with the effect of coupling to meson-baryon channels. The quark wave function of the lowest p-wave Λ^*, i.e., $\Lambda(1405)$, is found to be dominated by the antisymmetric flavor state which strongly couples to the $\bar{K}N$ and $\pi\Sigma$ channels. The significant downward shift of the $\Lambda(1405)$'s mass is attributed to its large attractive self-energy due to the $\bar{K}N\Lambda^*$ and $\pi\Sigma\Lambda^*$ coupling and its strong increase due to the presence of the cusp around the $\bar{K}N$ threshold.

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The unexpected failure of the constituent quark model in reproducing the mass of low lying Λ^* resonances has been a long-standing problem [1]. One of the resonances, $\Lambda(1405)$, is providing many interesting and difficult questions about its role in the $\bar{K}N$ interaction at low energies [2] and about its structure ($\bar{K}N$ bound state [3] or three-quark excited state [4]). Among them, the unusual light mass has been especially hard to understand in the quark model. In spite of several efforts, we do not know of any definite explanations for a mass difference between $\Lambda(1405)$ J=1/2 and $\Lambda(1520)$ J=3/2. Because they are described as a so-called LS partner in the quark model (both have orbital angular momentum 1 and intrinsic spin 1/2), we can naively expect that the LS force may bring out their mass difference. However, since the LS splitting is not observed in other baryon spectra [$N(1535)$ and $N(1520)$ are the typical cases of this degeneracy], the LS force is believed not to be effective for the mass spectra of the baryons for some unexplored reasons [5,6]. Therefore we must look for another mechanism which reproduces the mass spectra of the Λ^* resonances.

The effect of the meson-baryon coupling on the mass of the Λ^* has been studied by introducing the meson-quark interaction in the constituent quark model [7,8]. Although a consistent explanation of the available data (the $\bar{K}N$ scattering length and the masses of Λ^*) has not been obtained, the self-energy of the Λ^* due to the coupling with the $\bar{K}N$ and $\pi\Sigma$ channels seems to contribute to lower the mass of $\Lambda(1405)$ significantly.

In this paper, we concentrate on the mechanism which selectively moves $\Lambda(1405)$ down below the $\bar{K}N$ threshold and leaves other Λ^*'s at high energies which have been predicted by the constituent quark model without the meson-baryon couplings. We consider that it is worthwhile noting the reason why $\Lambda(1405)$ is solely affected by the $\bar{K}N$ and $\pi\Sigma$ interactions, although we should in the future give a consistent explanation of the available data including the masses of the J=3/2 Λ^*'s.

We refine the treatment of Ref. [7] for Λ^* so as to explore the behavior of $\Lambda(1405)$ in the Λ^* spectra by including several effects which have not been considered so far, e.g., the mixing of the quark wave functions within the $J^\pi = 1/2^-$ Λ^* which have different flavor symmetries and intrinsic spins due to the spin-spin and tensor interactions.

The Hamiltonian consists of four parts

$$H = H_0 + H_{SS} + H_T + H_I \ , \tag{1}$$

where the first three parts are essentially the same as those of Ref. [1]; H_0 consists of the phenomenological confining potential (quadratic type) and the kinetic energy of confined quarks, and H_{SS} (H_T) is the spin-spin (tensor) interaction due to the one-gluon-exchange potential (OGEP) between the constituent quarks. The effect of the flavor symmetry breaking due to the quark mass difference between m_u (u/d quark's mass) and m_s (s quark's mass) is considered in the OGEP, while we use the common value of the confining strength (i.e., the oscillator constant for quadratic confinement) for both the u/d quark and s quark. The wave functions of the baryons are constructed by assuming complete flavor SU(3) symmetry, and the details are given in Ref. [9].

We improve this quark model by including the meson-baryon interaction using the quark wave functions of the baryons. The meson-quark coupling H_I is given by

$$H_I = \frac{if_{Mqq}}{m_M} \sum_i^3 \left(\frac{\omega_M}{2m_i} [\boldsymbol{\sigma}_i \cdot \overleftarrow{\mathbf{p}}_i \mathbf{f}_i \cdot \boldsymbol{\phi}_i(\mathbf{k}) + \mathbf{f}_i \cdot \boldsymbol{\phi}_i(\mathbf{k}) \boldsymbol{\sigma}_i \cdot \overrightarrow{\mathbf{p}}_i] - \boldsymbol{\sigma}_i \cdot \mathbf{k} \mathbf{f}_i \cdot \boldsymbol{\phi}_i(\mathbf{k}) \right) + \text{H.c.} \ , \tag{2}$$

where $\mathbf{f}_i$ is the flavor SU(3) operator for the ith quark, m_i the ith quark mass, and m_M, $\mathbf{k}$, ω_M, and $\boldsymbol{\phi}_i$ are the mass, the momentum, the energy, and the field of the meson M, respectively. The effective meson-quark coupling constant is denoted by f_{Mqq} in Eq. (2). This interaction corresponds to the nonrelativistic form of the pseudovector coupling. The first and second terms which come from the operator $\gamma_0\partial_0$ in the pseudovector coupling play an important role for the s-wave meson-baryon interaction at low energies [9].

*Present address: Department of Physics, Osaka City University, Osaka 558, Japan.

The masses of Λ^* resonances are influenced by the coupling with the $\bar{K}N$ and $\pi\Sigma$ channels. This situation becomes clear by showing the effective Hamiltonian $H_{\rm eff}$ and the eigenvalue equation for Λ^*,

$$H_{\rm eff}|\nu\rangle\rangle = [H_0 + H_{SS} + H_T + \Sigma(E)]|\nu\rangle\rangle = E|\nu\rangle\rangle \ , \quad (3)$$

where $\Sigma(E)$ is the self-energy of the resonances due to the coupling with the meson-baryon channels and $|\nu\rangle\rangle$ ($\nu = 1,2,3$) corresponds to the physical states of the negative parity Λ^* resonances. The eigenstates of Eq. (3) are given by diagonalizing $H_{\rm eff}$ at energy E, and written as

$$|\nu\rangle\rangle = \sum_i^3 c_{\nu i}|\Lambda_i^*\rangle \ , \quad (4)$$

where $|\Lambda_i^*\rangle$ are the eigenstates of H_0. They differ from each other in their intrinsic spin and flavor symmetry. The $|\Lambda_1^*\rangle$ has **1** flavor symmetry with intrinsic spin 1/2, and $|\Lambda_2^*\rangle$ and $|\Lambda_3^*\rangle$ have **8** flavor symmetry with intrinsic spins 1/2 and 3/2, respectively.

The self-energy $\Sigma(E)$ consists of two terms: $\Sigma^{\bar{K}N}(E)$ due to the coupling with the $\bar{K}N$ channel and $\Sigma^{\pi\Sigma}(E)$ due to the coupling with the $\pi\Sigma$ channel. The matrix element of the $\Sigma^{\bar{K}N}(E)$ is written as

$$\langle\Lambda_i^*|\Sigma(E)|\Lambda_j^*\rangle = \int d^3k \langle\Lambda_i^*|H_I|\bar{K}N\rangle \times \frac{1}{E - E_{\bar{K}} - E_N + i\epsilon}\langle\bar{K}N|H_I|\Lambda_j^*\rangle \ , \quad (5)$$

where $\langle\Lambda_i^*|H_I|\bar{K}N\rangle$ is the vertex function of the $\bar{K}N\Lambda_i^*$ coupling for the kaon momentum $\mathbf{k}$. The $E_{\bar{K}}$ and E_N are the energies of the kaon and nucleon, respectively. $\Sigma^{\pi\Sigma}(E)$ can be similarly written by substituting $|\pi\Sigma\rangle$ for $|\bar{K}N\rangle$ in Eq. (5). The explicit forms of the vertex functions are as follows:

$$\langle\Lambda_1^*|H_I|\pi\Sigma\rangle = \frac{9}{2}\frac{f_{\pi qq}}{m_\pi\sqrt{\omega_\pi}}\left(\frac{\omega_\pi}{2m_u}[C(k) + D(k)] - 3C(k)\right)Y_{00}^*(\hat{k}) \ , \quad (6)$$

$$\langle\Lambda_2^*|H_I|\pi\Sigma\rangle = -\frac{1}{3}\langle\Lambda_1^*|H_I|\pi\Sigma\rangle \ , \quad (7)$$

$$\langle\Lambda_3^*|H_I|\pi\Sigma\rangle = -\frac{2}{3}\langle\Lambda_1^*|H_I|\pi\Sigma\rangle \ , \quad (8)$$

$$\langle\Lambda_1^*|H_I|\bar{K}N\rangle = -\frac{3}{2}\frac{\sqrt{6}f_{\bar{K}qq}}{m_{\bar{K}}\sqrt{\omega_{\bar{K}}}}\left(\frac{\omega_{\bar{K}}}{4\mu^+}[C(k) + D(k)] + \frac{3\omega_{\bar{K}}}{4\mu^-}C(k) - 3C(k)\right)Y_{00}^*(\hat{k}) \ , \quad (9)$$

$$\langle\Lambda_2^*|H_I|\bar{K}N\rangle = \langle\Lambda_1^*|H_I|\bar{K}N\rangle \ , \quad (10)$$

$$\langle\Lambda_3^*|H_I|\bar{K}N\rangle = 0 \ , \quad (11)$$

where $\mu^\pm = m_u m_s/(m_u \pm m_s)$. The functions $C(k)$ and $D(k)$ are given by

$$C(k) = \frac{2}{27}\sqrt{\frac{\pi}{\beta}}k^2 \exp\left(-\frac{k^2}{6\beta}\right) \ , \quad (12)$$

$$D(k) = \frac{4}{9}\sqrt{\pi\beta}\left(3 - \frac{k^2}{3\beta}\right)\exp\left(-\frac{k^2}{6\beta}\right) \ , \quad (13)$$

where β is the size parameter of the quark wave function for baryons. Note that $\Sigma^{\bar{K}N}(E)$ and $\Sigma^{\pi\Sigma}(E)$ have nondiagonal matrix elements between $|\Lambda_i^*\rangle$'s as the spin-spin and tensor interactions do. In this work, since we consider the resonance properties near the $\bar{K}N$ threshold, the scattering states for the calculation of Λ^*'s self-energies are restricted to the $\bar{K}N$ and $\pi\Sigma$ channels.

In order to determine the masses of the resonance state Λ^*, we need to look for the resonance energy of $\bar{K}N$ scattering. When we solve the $\bar{K}N$ and $\pi\Sigma$ scattering problems by means of the Λ^* isobar model, we can see that the peak of the $\bar{K}N$ scattering cross section appears approximately at the resonance energy E_R which satisfies

$$\mathrm{Re}\,\langle\langle\nu|E_R - H_0 - H_{SS} - H_T - \Sigma(E_R)|\nu\rangle\rangle = 0. \quad (14)$$

The inverse of the left-hand side of Eq. (14) is nothing but the Λ^*'s propagator, and the "zero" of its real part corresponds to the peak position of the cross section. These resonance energies also correspond to those which are determined by the partial wave analysis of experimental data.

Values of the parameters in our calculation are shown in Table I. They are almost the same as the conventional ones, i.e., the SU(6) values of the meson-quark coupling constants $f_{\pi qq}$ and $f_{\bar{K}qq}$ and the size parameter β of Ref. [9] for the quark wave function (Ref. [9] also treats the p-wave baryons). Other parameters related with the confinement of quarks and OGEP interactions are almost the same as those of Ref. [1].

Now we show our results. The mass spectra of the Λ^* are shown in Fig. 1. Here we show the experimental values, results of Ref. [1], our results without the coupling with the meson baryon channels, and our complete calculation including the coupling with the meson-baryon channels, respectively. At first sight, it might be mysterious why our results without the coupling with the meson baryon channels do not agree with those of Isgur and Karl's calculation. As clearly stated in Ref. [1], their calculations are refined based on the usual constituent quark model. So their results are different from our simple calculation. The important point is that the mass of the $\Lambda(1405)$ could not be explained even by their refined version of the usual constituent quark model.

TABLE I. The parameters of our calculation.

M_0 (MeV)	1800
m_u (MeV)	330
$\zeta \equiv m_u/m_s$	0.6
α_s	1.0
β (fm^{-2})	4.0
$f_{\pi qq}$	0.54
$f_{\bar{K}qq}$	1.89

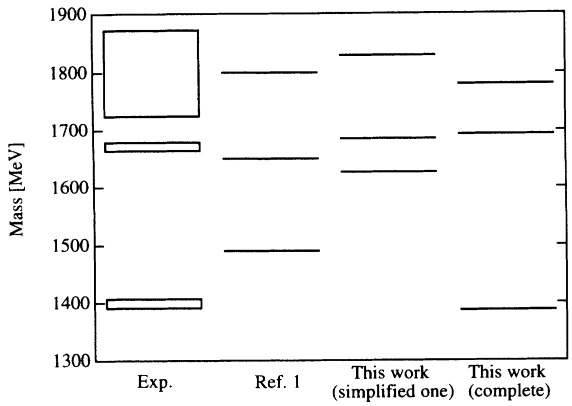

FIG. 1. The mass spectra of Λ^*. From the left column, there are the experimental values, the results of Ref. [1], our result of a simplified calculation with diagonalizing only the $H_0+H_{SS}+H_T$, and our complete calculation considering the coupling with meson-baryon channels, respectively.

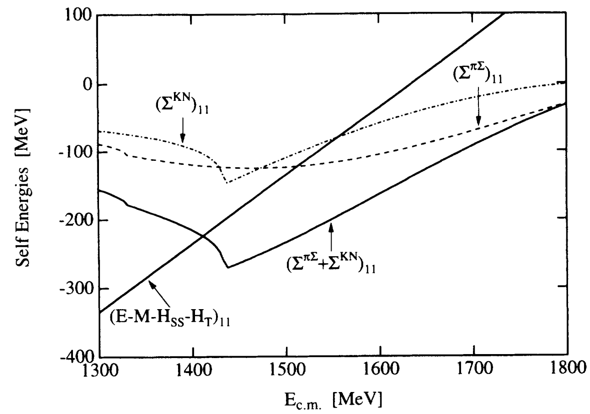

FIG. 2. The diagonal matrix element of self-energies $\Sigma^{\pi\Sigma}_{11}(E)$ (dashed curve) and $\Sigma^{\bar{K}N}_{11}(E)$ (dot-dashed curve). The solid curve corresponds to the sum of them, and the solid line to $\langle\Lambda^*_1|E-M-H_{SS}-H_T|\Lambda^*_1\rangle$.

Our complete calculation seems to agree well with the experimental values. Note that we did not perform a parameter search in this calculation. Although our calculation is a preliminary one, $\Lambda(1405)$ alone goes down below the $\bar{K}N$ threshold but the other Λ^* resonances stay at high energies. These three resonances stayed around 1600–1700 MeV when the $\bar{K}N$ and $\pi\Sigma$ interactions were not considered.

The mixing coefficients for $|\nu\rangle\rangle$, $c_{\nu i}$, are shown in Table II. These coefficients are complex numbers because the self-energy $\Sigma(E)$ has an imaginary part above the threshold. For $\Lambda(1405)$, say, $|1\rangle\rangle$, the coefficients c_{12} and c_{13} are small and c_{11} is almost real and equal to 1. This means that the contribution of the mixings of $|\Lambda^*_2\rangle$ and $|\Lambda^*_3\rangle$ to $\Lambda(1405)$ is small. Thus the wave function of $\Lambda(1405)$ mainly has **1** symmetry flavor state in the constituent quark model, and we can discuss its structure without taking into account other flavor symmetry states.

The large contribution of the self-energy for $|\Lambda^*_1\rangle$ is explained by looking at the vertex functions for the $\bar{K}N\Lambda^*$ and $\pi\Sigma\Lambda^*$ couplings calculated by the quark wave function. As Eqs. (6)-(11) show, $\langle\Lambda^*_1|H_I|\pi\Sigma\rangle$ is 3 or 3/2 times as large as other vertex functions, and $\langle\Lambda^*_1|H_I|\bar{K}N\rangle$ is the same as $\langle\Lambda^*_2|H_I|\bar{K}N\rangle$. The strong coupling of $|\Lambda^*_1\rangle$ with the $\pi\Sigma$ channel makes the diagonal element $\Sigma^{\pi\Sigma}_{11}(E) \equiv \langle\Lambda^*_1|\Sigma^{\pi\Sigma}(E)|\Lambda^*_1\rangle$ remarkably large as compared with the off-diagonal elements $\Sigma^{\pi\Sigma}_{12}(E)$ and $\Sigma^{\pi\Sigma}_{13}(E)$. Other terms in $H_{\rm eff}$ have also small off-diagonal elements, and so the coupling with the meson-baryon channels does not produce the mutual mixing among $|\Lambda^*_i\rangle$'s for $\Lambda(1405)$.

The mass of $\Lambda(1405)$ is almost determined by the diagonal element of the effective Hamiltonian, $\langle\Lambda^*_1|H_{\rm eff}|\Lambda^*_1\rangle$, because of the small mixing among $|\Lambda^*_i\rangle$'s. According to the energy dependence of the self-energy (see Fig. 2), we can clearly see the mechanism which pulls down the mass of $\Lambda(1405)$ in the following way. We show the self-energies $\Sigma^{\bar{K}N}(E)$, $\Sigma^{\pi\Sigma}(E)$, and the sum of them. The cross point between the self-energy and the straight line, which corresponds to $\langle\Lambda^*_1|E-M-H_{SS}-H_T|\Lambda^*_1\rangle$, approximately gives E_R. Note that there is a cusplike behavior in $\Sigma^{\bar{K}N}_{11}(E)$ at the $\bar{K}N$ threshold energy (about 1440 MeV) which is accompanied by the appearance of the imaginary part. The energy dependence of $\Sigma^{\pi\Sigma}_{11}(E)$ is, on the other hand, moderate in the energy region considered here. Because of this cusplike behavior in $\Sigma^{\bar{K}N}_{11}(E)$, the cross point appears at low energies below the $\bar{K}N$ threshold.

If the energy dependence of $\Sigma^{\bar{K}N}_{11}(E)$ is as smooth as that of $\Sigma^{\pi\Sigma}_{11}(E)$, the magnitude of the downward shift of the $\Lambda(1405)$'s mass is almost determined by the strength of $\Sigma^{\bar{K}N}_{11}(E)+\Sigma^{\pi\Sigma}_{11}(E)$ around 1630 MeV where $\Lambda(1405)$ stayed before H_I is switched on. However, there is a strong energy dependence due to this cusplike behavior of $\Sigma^{\bar{K}N}(E)$ in the self-energy of $\Lambda(1405)$. Therefore we cannot determine the mass of $\Lambda(1405)$ intuitively by neglecting the energy dependence of $\Sigma(E)$. Against our naive expectation, the cross point runs down to the low energy region beyond the $\bar{K}N$ threshold.

We next consider other Λ^* resonances with $J^\pi=1/2^-$. They remained to stay at high energies even if we take into account the coupling with the $\bar{K}N$ and $\pi\Sigma$ channels. Because the mixing between $|\Lambda^*_2\rangle$ and $|\Lambda^*_3\rangle$ is not negligible, it is not easy to see straightforwardly the positions of these resonances after H_I is switched on. Looking at the vertex functions in Eqs. (6)-(11), we can see that the couplings of these higher mass Λ^* with the $\pi\Sigma$ channel are small by a factor of 2 or 3 compared with those of $\Lambda(1405)$. We can also show that the off-diagonal elements of the self-energy $\Sigma^{\bar{K}N}(E)+\Sigma^{\pi\Sigma}(E)$ are about the half of $H_{SS}+H_T$ at the energy considered here (see

TABLE II. The mixing coefficients $c_{\mu i}$.

Energy (MeV)	c_{11}	c_{12}	c_{13}
1387	$0.92+0.015i$	$0.36-0.052i$	$-0.19-0.028i$
Energy (MeV)	c_{21}	c_{22}	c_{23}
1693	$0.41+0.071i$	$-0.59-0.24i$	$0.77-0.22i$
Energy (MeV)	c_{31}	c_{32}	c_{33}
1790	$0.011+0.26i$	$0.79-0.096i$	$-0.68-0.11i$

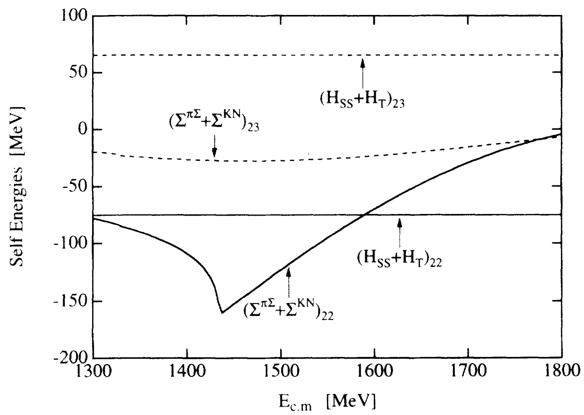

FIG. 3. The off-diagonal elements of self-energies for the second and third Λ^* resonances. The meanings of each line are shown in the figure.

Fig. 3). Then the mixing of $|\Lambda_2^*\rangle$ and $|\Lambda_3^*\rangle$ is not affected by including the self-energies, and their masses changed only by a few tenths MeV from the positions predicted by the usual constituent quark model. Therefore the effect of the meson-baryon interactions is rather small for these higher Λ^* compared with $\Lambda(1405)$.

We have shown the mechanism which pulls down $\Lambda(1405)$ below the $\bar{K}N$ threshold in the constituent quark model when we take into account the $\bar{K}N$ and $\pi\Sigma$ interactions. We find that the antisymmetric flavor state is dominant in the $\Lambda(1405)$'s wave function, and this flavor state strongly couples to the $\pi\Sigma$ channel. Because of this strong coupling, $\Lambda(1405)$ gains large attractive self-energies compared with other Λ^* resonances. The cusp-like behavior of $\Sigma^{\bar{K}N}(E)$ is also important to pull $\Lambda(1405)$ down to much lower energy below the $\bar{K}N$ threshold.

Because the energy dependence of the self-energies is smooth far from the threshold, and also because the coupling with the $\bar{K}N$ and $\pi\Sigma$ channels is weak, the masses of the other Λ^*'s are not affected very much even if we consider the effect of the meson-baryon interactions in the constituent quark model. They almost remain to stay at the positions predicted by the usual constituent quark model.

In order to derive more definite conclusions, we have to perform a systematic study including the low energy $\bar{K}N$ and $\pi\Sigma$ interactions. Then investigating the effects of the meson-baryon interaction to the overall baryon spectrum, we will be able to confirm our conclusion given in this Brief Report. We address these subjects in our future plan.

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