Ideal quarks and mesons in the relativistic quark model

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We propose a microscopic theory for interacting mesons and ideal quarks in the relativistic quark model using the time-dependent mean-field theory technique. For simplicity we examined the Nambu–Jona-Lasinio model. The dynamical chiral-symmetry breaking leads to a zero-frequency mode (pion) due to the restoration of chiral symmetry. The ideal quarks are represented as dressed particles independent of mean fields, and do not have the conventional properties of fermions. This is due to the constraints of eliminating the double counting of degrees of freedom between the mean fields and quarks. The small fluctuation around the static solution is then investigated. The pseudoscalar and scalar mesons are represented as the collective modes of the mean fields.

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The fundamental theory of hadrons is believed to be quantum chromodynamics (QCD), which is a non-Abelian gauge field theory. A technique for calculating QCD is the Monte Carlo estimation of the functional integral, which is called lattice QCD. On the other hand, there are various phenomenological models for the hadrons. One of them is a relativistic quark model [1] given by the Lagrangian density

$$L_{\text{eff}}(x) = \bar{\psi}(x) i \not{\partial} \psi(x) + \int d^4y\, \bar{\psi}(x)\lambda^a\gamma^\mu\psi(x)V(x-y) \times \bar{\psi}(y)\lambda^a\gamma_\mu\psi(y) \, , \tag{1}$$

where the λ^a are the Gell-Mann SU(3) matrices. This model might be derived from QCD [2]. Equation (1) is a nonlocal version of the Nambu–Jona-Lasinio (NJL) model [3]. The NJL model has recently received increasing attention in connection with the low-energy properties of the hadrons. The model spontaneously breaks the chiral symmetry, and the dynamics generate a mass for quarks. Nambu and Jona-Lasinio showed how to construct the various low-lying $(\psi\bar{\psi})$ bound states of the model by solving the Bethe-Salpeter equation. The chiral symmetry then requires the appearance of the Goldstone bosons (massless pions) as the collective $(\psi\bar{\psi})$ of massless quarks. As is well known, the NJL model does not include any confinement mechanism due to contact four-fermion interaction. In this paper, however, we will consider the NJL model instead of the relativistic quark model (1) for simplicity.

As is shown by Eguchi [4], the NJL Lagrangian density is approximately the same as the effective Lagrangian density obtained from the linear σ model [5] with radiative corrections. However, Eguchi's method includes the double counting of degrees of freedom between the quarks and mesons due to the introduction of the auxiliary fields (meson fields) σ and $\vec{\pi}$. Therefore, his argument is not a rigorous derivation.

Recently, Providência *et al.* [6] have investigated the dynamical chiral-symmetry breaking, the light-meson spectrum, and the properties of the pion in the NJL model using the time-dependent Hartree-Fock (TDHF) formalism. The light-meson spectrum corresponds to the collective excitations of the Dirac sea of massive particles. However, the TDHF method is limited only to systems having an even number of valence quarks. Therefore, to treat systems (nucleon or Δ) having an odd number of valence quarks, an extension of the conventional TDHF method is needed. The author [7] has recently proposed such an extension using the path-integral technique in the field of nuclear physics. The result obtained is identical to the boson-fermion expansion given by several other authors [8]. In this paper, we will apply our method to the NJL model, and propose a quantum-mechanical treatment for the description of the meson fields, the ideal quark fields, and the mutual interplay from the viewpoint of the mean-field theory. Constraints eliminating the double counting of degrees of freedom between the mean fields and quark fields are automatically derived. The ideal quarks are represented as dressed particles independent of the mean fields, and are considered to be constituent quarks. Thus, we can derive the mean-field Hamiltonian coupled with the ideal quarks. Furthermore, we will examine the small fluctuation around the static solution. Then we will find two kinds of collective modes. One is the pseudoscalar Goldstone boson (zero-frequency mode) identified with pion, and another is the scalar boson with twice the ideal-quark mass.

Let us start from the functional integral in the NJL model [3]

$$Z = \int D\psi D\psi^\dagger \exp\left[i\int L(x)d^4x\right] \, , \tag{2a}$$

where the Lagrangian density with the scalar and pseudoscalar interaction is given as

$$L(x) = \bar{\psi} i \not{\partial} \psi + g[(\bar{\psi}\psi)^2 + (\bar{\psi} i\gamma_5\psi)^2] \, . \tag{2b}$$

For the sake of simplifying the notations, we consider only one quark flavor $\mathrm{U}_R(1)\times\mathrm{U}_L(1)$. The generalization to the realistic case of two massless flavors is straightforward.

We first perform the plane wave expansion of the original quark fields:

$$\psi(x,t) = \sum_{\mathbf{p}} \begin{pmatrix} b_{\mathbf{p}1} \\ b_{\mathbf{p}2} \\ d^*_{-\mathbf{p}1} \\ d^*_{-\mathbf{p}2} \end{pmatrix} e^{i\mathbf{p}\cdot\mathbf{x}} , \qquad (3)$$

where $b_{\mathbf{p}s}$ and $d_{\mathbf{p}s}$ are the Grassmann variables of the quark and antiquark with momentum $\mathbf{p}$ and helicity $s = 1, 2$, respectively. Following the standard procedure we introduce auxiliary fields $(S_{\mathbf{pp}'}, P_{\mathbf{pp}'})$ in such a way that the Lagrangian becomes bilinear in the quark fields. This eliminates the quadratic interactions among the quark fields. Then the functional integral (2a) is rewritten as

$$Z = \int DbDb^*DdDd^*DSDP \exp\left(i\int L'\,dt\right) \qquad (4)$$

where the Lagrangian L' is redefined as

$$\begin{aligned} L' = {} & i\sum_{\mathbf{p}s}(b^*_{\mathbf{p}s}\dot{b}_{\mathbf{p}s} + d_{\mathbf{p}s}\dot{d}^*_{\mathbf{p}s}) - \sum_{\mathbf{p}s}\sigma\cdot\mathbf{p}(b^*_{\mathbf{p}s}d^*_{-\mathbf{p}s} + d_{-\mathbf{p}s}b_{\mathbf{p}s}) \\ & +2g\sum_{\mathbf{pq}s}\sum_{\mathbf{p}'\mathbf{q}'}[S_{\mathbf{pp}'}(b^*_{\mathbf{q}s}b_{\mathbf{q}'s} - d_{-\mathbf{q}s}d^*_{-\mathbf{q}'s}) + iP_{\mathbf{pp}'}(b^*_{\mathbf{q}s}d^*_{-\mathbf{q}'s} - d_{-\mathbf{q}s}b_{\mathbf{q}'s})]\delta_{\mathbf{p}-\mathbf{p}'+\mathbf{q}-\mathbf{q}'} \\ & -g\sum_{\mathbf{pq}}\sum_{\mathbf{p}'\mathbf{q}'}[S_{\mathbf{pp}'}S_{\mathbf{qq}'} + P_{\mathbf{pp}'}P_{\mathbf{qq}'}]\delta_{\mathbf{p}-\mathbf{p}'+\mathbf{q}-\mathbf{q}'} . \end{aligned} \qquad (5)$$

Here the dot denotes a time derivative and σ is the Pauli spin matrix. In the connection with the mean-field theory, it is now convenient to introduce a decomposition of the Hermitian matrix $(S_{\mathbf{pp}'}, P_{\mathbf{pp}'})$ in the following form

$$S_{\mathbf{pp}'} = 2\,\mathrm{Tr}(\gamma_0\rho_{\mathbf{pp}'}) , \quad P_{\mathbf{pp}'} = 2i\,\mathrm{Tr}(\gamma_0\gamma_5\rho_{\mathbf{pp}'}). \qquad (6)$$

Here $\rho_{\mathbf{pp}'}$ is the Hermitian matrix as follows:

$$\rho_{\mathbf{pp}'} = (U^T M U^*)_{\mathbf{pp}'} , \qquad (7)$$

where U and M are the unitary and Hermitian matrices, respectively:

$$U = \begin{pmatrix} u^\dagger & v \\ -v^\dagger & u \end{pmatrix} , \quad M = \begin{pmatrix} w & 0 \\ 0 & 1-\tilde{w} \end{pmatrix} \qquad (8)$$

The matrices u and v are given by the elements

$$u_{\mathbf{pp}'} = \left(\sqrt{1-\beta^\dagger\beta}\right)_{\mathbf{pp}'} , \quad v_{\mathbf{pp}'} = \beta_{\mathbf{pp}'} , \qquad (9)$$

where $\beta_{\mathbf{pp}'}$ is a complex variable. They automatically satisfy the normalization conditions: $(u^\dagger u + v^\dagger v)_{\mathbf{pp}'} = \delta_{\mathbf{pp}'}$, and w and $\tilde{w}$ are the Hermitian matrices.

Let us now go to the body-fixed frame of the quark fields by means of the unitary transformation (8). To do this, we introduce "ideal-quark" fields $(a_{\mathbf{p}s}, c_{\mathbf{p}s})$ by the unitary transformation:

$$\begin{pmatrix} a \\ c \end{pmatrix} = U \begin{pmatrix} b \\ d^* \end{pmatrix} = \begin{pmatrix} u^\dagger & v \\ -v^\dagger & u \end{pmatrix} \begin{pmatrix} b \\ d^* \end{pmatrix} \qquad (10)$$

The functional integral Z can then be written as

$$Z = \int DaDa^*DcDc^*D\beta D\beta^* DwD\tilde{w} \exp\left(i\int L'dt\right) \qquad (11)$$

From the stationary phase approximation about w and $\tilde{w}$ in the functional integral (11), for the arbitrary (u, v) the following conditions should hold:

$$w_{\mathbf{pp}'} = \frac{1}{2}\sum_s a^*_{\mathbf{p}s}a_{\mathbf{p}'s}, \quad \tilde{w}_{\mathbf{pp}'} = \frac{1}{2}\sum_s c^*_{\mathbf{p}s}c_{\mathbf{p}'s} , \qquad (12)$$

$$\phi^{(1)}_{\mathbf{pp}'} = \sum_s a^*_{\mathbf{p}s}c^*_{\mathbf{p}'s} = 0, \quad \phi^{(2)}_{\mathbf{pp}'} = \sum_s c_{\mathbf{p}s}a_{\mathbf{p}'s} = 0 . \qquad (13)$$

As mentioned before, there is the double counting of degrees of freedom due to the introduction of the auxiliary fields. The double counting is eliminated by the above constraints (12) and (13). Then the Lagrangian in the functional integral (11) is written by

$$L' = i\sum_{\mathbf{p}s}[v^\dagger\dot{v} + u\dot{u}^\dagger]_{\mathbf{pp}} + i\sum_{\mathbf{p}s}(a^*_{\mathbf{p}s}\dot{a}_{\mathbf{p}s} + c^*_{\mathbf{p}s}\dot{c}_{\mathbf{p}s}) - H' , \qquad (14)$$

where the Hamiltonian H' is given by

$$\begin{aligned} H' = {} & -i\sum_{\mathbf{p}s}[v^\dagger\dot{v} + u\dot{v}^\dagger]_{\mathbf{pp}} - \sum_{\mathbf{p}s}\sigma\cdot\mathbf{p}[uv + v^\dagger u^\dagger]_{\mathbf{pp}} \\ & -g\sum_{\mathbf{pp}'}\sum_{ss'}([v^\dagger v - uu^\dagger]_{\mathbf{pp}}[v^\dagger v - uu^\dagger]_{\mathbf{p}'\mathbf{p}'} - [v^\dagger u^\dagger - uv]_{\mathbf{pp}}[v^\dagger u^\dagger - uv]_{\mathbf{p}'\mathbf{p}'}) \\ & +\text{terms about } a,\ a^*,\ c, \text{ and } c^* . \end{aligned} \qquad (15)$$

From the variation $\delta L' = 0$ about β, β^*, a, a^*, c, and c^* under the constraints (13), the equations of motion are then given by

$$i\dot{\beta}_{\mathbf{pp}'} = [\beta_{\mathbf{pp}'}, H']_D, \quad i\dot{\beta}^*_{\mathbf{pp}'} = [\beta^*_{\mathbf{pp}'}, H']_D , \tag{16a}$$

$$i\dot{a}_{\mathbf{p}s} = [a_{\mathbf{p}s}, H']_D, \quad i\dot{a}^*_{\mathbf{p}s} = [a^*_{\mathbf{p}s}, H']_D , \tag{16b}$$

$$i\dot{c}_{\mathbf{p}s} = [c_{\mathbf{p}s}, H']_D, \quad i\dot{c}^*_{\mathbf{p}s} = [c^*_{\mathbf{p}s}, H']_D , \tag{16c}$$

where $[A,B]_D$ denotes the Dirac bracket [9]:

$$[A,B]_D = [A,B]_P + \sum_{\mathbf{pp}'\mathbf{qq}'} \{[A,\phi^{(1)}_{\mathbf{pp}'}]_P[\phi^{(1)}_{\mathbf{pp}'},\phi^{(2)}_{\mathbf{qq}'}]^{-1}_P[\phi^{(2)}_{\mathbf{qq}'},B]_P + [A,\phi^{(2)}_{\mathbf{pp}'}]_P[\phi^{(2)}_{\mathbf{pp}'},\phi^{(1)}_{\mathbf{qq}'}]^{-1}_P[\phi^{(1)}_{\mathbf{qq}'},B]_P\} . \tag{17}$$

The bracket $[A,B]_P$ is the Poisson bracket involving the Grassmann variables defined by Casalbuoni [10]. The canonical quantization can be performed by the replacements:

$$(\beta_{\mathbf{pp}'}, \beta^*_{\mathbf{pp}'}, a_{\mathbf{p}s}, a^*_{\mathbf{p}s}, c_{\mathbf{p}s}, c^*_{\mathbf{p}s}) \rightarrow (\hat{\beta}_{\mathbf{pp}'}, \hat{\beta}^*_{\mathbf{pp}'}, \hat{a}_{\mathbf{p}s}, \hat{a}^*_{\mathbf{p}s}, \hat{c}_{\mathbf{p}s}, \hat{c}^*_{\mathbf{p}s}) , \tag{18a}$$

$$[A,B]_D \rightarrow [\hat{A},\hat{B}]_\pm , \tag{18b}$$

where $[\hat{A},\hat{B}]_\pm$ means the commutation and anticommutation relations. Then, the commutation relations are expressed as

$$[\hat{a}_{\mathbf{p}s}, \hat{a}^*_{\mathbf{p}'s'}]_+ = \delta_{\mathbf{pp}'}\delta_{ss'} + \sum_{\mathbf{pp}'\mathbf{qq}'} \hat{c}^*_{\mathbf{p}s}[2\hat{S}]^{-1}_{\mathbf{pq},\mathbf{p}'\mathbf{q}'}\hat{c}_{\mathbf{p}'s'} , \tag{19a}$$

$$[\hat{c}_{\mathbf{p}s}, \hat{c}^*_{\mathbf{p}'s'}]_+ = \delta_{\mathbf{pp}'}\delta_{ss'} + \sum_{\mathbf{pp}'\mathbf{qq}'} \hat{a}^*_{\mathbf{p}s}[2\hat{S}]^{-1}_{\mathbf{pq},\mathbf{p}'\mathbf{q}'}\hat{a}_{\mathbf{p}'s'} , \tag{19b}$$

$$[\hat{a}_{\mathbf{p}s}, \hat{a}_{\mathbf{p}'s'}]_+ = [\hat{a}^*_{\mathbf{p}s}, \hat{a}^*_{\mathbf{p}'s'}]_+ = [\hat{c}_{\mathbf{p}s}, \hat{c}_{\mathbf{p}'s'}]_+ = [\hat{c}^*_{\mathbf{p}s}, \hat{c}^*_{\mathbf{p}'s'}]_+ = 0 , \tag{19c}$$

$$[\hat{\beta}_{\mathbf{pp}'}, \hat{\beta}^*_{\mathbf{qq}'}]_- = \delta_{\mathbf{pq}}\delta_{\mathbf{p}'\mathbf{q}'} , \quad [\hat{\beta}_{\mathbf{pp}'}, \hat{\beta}_{\mathbf{qq}'}]_- = [\hat{\beta}^*_{\mathbf{pp}'}, \hat{\beta}^*_{\mathbf{qq}'}]_- = 0 , \tag{19d}$$

where $\hat{S}$ is defined by

$$[2\hat{S}]_{\mathbf{pp}',\mathbf{qq}'} = [\hat{\phi}^{(1)}_{\mathbf{pq}}, \hat{\phi}^{(2)}_{\mathbf{p}'\mathbf{q}'}]_P . \tag{19e}$$

The above relations imply that the ideal quarks $(\hat{a}_{\mathbf{p}s}, \hat{c}_{\mathbf{p}s})$ do not satisfy the anticommutation relations of fermions. This is due to the constraints (13) eliminating the double counting of degrees of freedom between the mean fields and the quarks.

Let us present the description of the mechanism of chiral symmetry breaking by obtaining the static solution from the above Hamiltonian (15) without the ideal quark fields. We assume that the static solution is real value and $(u_{\mathbf{pp}'}, v_{\mathbf{pp}'})$ is diagonal, i.e., $u_{\mathbf{pp}'} = \delta_{\mathbf{pp}'}u_{\mathbf{p}}$ and $v_{\mathbf{pp}'} = \delta_{\mathbf{pp}'}u_{\mathbf{p}}$. Then the minimization of the static energy leads to the gap equation

$$m = 4g\sum_{\mathbf{p}}^{\Lambda} \frac{m}{E} , \tag{20}$$

where Λ is the cutoff parameter, and the ideal-quark mass m and the energy E are defined as

$$m = \frac{u_{\mathbf{p}}^2 - v_{\mathbf{p}}^2}{2u_{\mathbf{p}}v_{\mathbf{p}}}\sigma\cdot\mathbf{p}, \quad E = \frac{\sigma\cdot\mathbf{p}}{2u_{\mathbf{p}}v_{\mathbf{p}}} , \tag{21}$$

which satisfies the relationship

$$E^2 = m^2 + \mathbf{p}^2 . \tag{22}$$

From Eq. (21) and the normalization condition $u_{\mathbf{p}}^2 + v_{\mathbf{p}}^2 = 1$, the static solutions $u^0_{\mathbf{p}}$ and $v^0_{\mathbf{p}}$ are obtained as follows:

$$u^0_{\mathbf{p}} = \left[\frac{1}{2}\left(1+\frac{m}{E}\right)\right]^{1/2}, \quad v^0_{\mathbf{p}} = \left[\frac{1}{2}\left(1+\frac{m}{E}\right)\right]^{1/2}\frac{\sigma\cdot\mathbf{p}}{E+m} . \tag{23}$$

These static solutions spontaneously break the chiral symmetry.

We will next examine the small fluctuation around the static solution. To do this, we decompose the inverse of the unitary matrix into the static part and the fluctuation part:

$$U^{-1} = \begin{pmatrix} u & -v \\ v^\dagger & u^\dagger \end{pmatrix} = \begin{pmatrix} u^0 & -v^0 \\ v^0 & u^0 \end{pmatrix}\begin{pmatrix} \tilde{u} & -\tilde{v} \\ \tilde{v}^\dagger & \tilde{u}^\dagger \end{pmatrix} , \tag{24}$$

where $\tilde{u}_{\mathbf{pp}'}$ and $\tilde{v}_{\mathbf{pp}'}$ are the matrices given by the elements

$$\tilde{u}_{\mathbf{pp}'} = \left(\sqrt{1-\tilde{\beta}^\dagger\tilde{\beta}}\right)_{\mathbf{pp}'} , \quad \tilde{v}_{\mathbf{pp}'} = \tilde{\beta}_{\mathbf{pp}'} . \tag{25}$$

Substituting (24) into (16a), we obtain the linear equations by making the so-called random phase approximation (RPA):

$$i\dot{\tilde{\beta}}_{\mathbf{pp}} = 2E\tilde{\beta}_{\mathbf{pp}} - 16gu^0_{\mathbf{p}}v^0_{\mathbf{p}}\sum_{\mathbf{q}} u^0_{\mathbf{q}}v^0_{\mathbf{p}}(\tilde{\beta}_{\mathbf{qq}} + \tilde{\beta}^*_{\mathbf{qq}}) - 4g\sum_{\mathbf{q}}(\tilde{\beta}_{\mathbf{qq}} - \tilde{\beta}^*_{\mathbf{qq}}) , \tag{26}$$

Then the RPA Hamiltonian becomes

$$H_{\mathrm{RPA}} = 4\sum_{\mathbf{p}s}\sigma\cdot\mathbf{p}u^0_{\mathbf{p}}v^0_{\mathbf{p}}\tilde{\beta}^*_{\mathbf{pp}}\tilde{\beta}_{\mathbf{pp}} - 4g\sum_{\mathbf{pp}'}\sum_{ss'}[(v^0_{\mathbf{p}})^2 - (u^0_{\mathbf{p}})^2][(v^0_{\mathbf{p}'})^2 - (u^0_{\mathbf{p}'})^2]\tilde{\beta}^*_{\mathbf{p}'\mathbf{p}'}\tilde{\beta}_{\mathbf{p}'\mathbf{p}'}$$
$$-4g\sum_{\mathbf{pp}'}\sum_{ss'}u^0_{\mathbf{p}}v^0_{\mathbf{p}}u^0_{\mathbf{p}'}v^0_{\mathbf{p}'}(\tilde{\beta}_{\mathbf{pp}} + \tilde{\beta}^*_{\mathbf{pp}})(\tilde{\beta}_{\mathbf{p}'\mathbf{p}'} + \tilde{\beta}^*_{\mathbf{p}'\mathbf{p}'}) + g\sum_{\mathbf{pp}'}\sum_{ss'}(\tilde{\beta}_{\mathbf{pp}} - \tilde{\beta}^*_{\mathbf{pp}})(\tilde{\beta}_{\mathbf{p}'\mathbf{p}'} - \tilde{\beta}^*_{\mathbf{p}'\mathbf{p}'}) . \tag{27}$$

Let us next examine the eigenmodes and the eigenfunctions of the linear equations (26), which may be decoupled into equations for the pseudoscalar function and the scalar function.

(1) Pseudoscalar mode. Using the equilibrium condition or the gap equation (20), we obtain

$$\sum_{\mathbf{p}}^{\Lambda} \frac{1}{E}(\dot{\tilde{\beta}}_{\mathbf{pp}} + \dot{\tilde{\beta}}^{*}_{\mathbf{pp}}) = 0 \ . \tag{28}$$

This solution $\sum_{\mathbf{p}}^{\Lambda}(\tilde{\beta}_{\mathbf{pp}} + \tilde{\beta}^{*}_{\mathbf{pp}})/E$ is the zero-frequency meson mode (the Goldstone mode). The zero-frequency mode is a consequence of the violation of the chiral symmetry (γ_5 invariant). In other words, there is the degeneracy of the deformed vacuum state under the γ_5 gauge transformation. Since the axial charge is, in RPA, given by

$$\begin{aligned} Q_5 &= \int d^3x\, \bar{\psi}\gamma^0\gamma_5\psi \\ &= -2m \sum_{\mathbf{p}}^{\Lambda} \frac{1}{E}(\tilde{\beta}_{\mathbf{pp}} + \tilde{\beta}^{*}_{\mathbf{pp}}) = \pi \ , \end{aligned} \tag{29}$$

this charge conserves, in agreement with the Goldstone theorem.

(2) Scalar mode. The RPA equations (26) reduce to

$$\dot{P} = -4m^2 Q, \quad \dot{Q} = P \ , \tag{30}$$

where $\mathcal{N}$ (a normalization factor), P, and Q are defined as

$$\begin{aligned} \mathcal{N} &= \left(\textstyle\sum_{\mathbf{p}}^{\Lambda} \frac{1}{Ep^2}\right)^{-1/2}, \quad P = i\mathcal{N} \textstyle\sum_{\mathbf{p}}^{\Lambda} \frac{\sigma\cdot\mathbf{p}}{p^2}(\tilde{\beta}_{\mathbf{pp}} - \tilde{\beta}^{*}_{\mathbf{pp}}), \\ Q &= \mathcal{N} \textstyle\sum_{\mathbf{p}}^{\Lambda} \frac{\sigma\cdot\mathbf{p}}{2Ep^2}(\tilde{\beta}_{\mathbf{pp}} + \tilde{\beta}^{*}_{\mathbf{pp}}) \ . \end{aligned} \tag{31}$$

These equations have the solution $\omega = 2m$. Since P and Q under the parity transformation are invariant, the modes are scalar.

In the normal modes, the RPA collective Hamiltonian assumes the diagonal form

$$H^{M}_{\mathrm{RPA}} = \frac{\pi^2}{2\mathcal{J}} + P^2 + 4m^2Q^2 + \sum_{\mathbf{p}} \omega_{\mathbf{p}} O^{*}_{\mathbf{p}} O_{\mathbf{p}} \ , \tag{32}$$

where $O_{\mathbf{p}}$ denotes the continuum mode, $\omega_{\mathbf{p}}$ is the eigenenergy, and $\mathcal{J}$ is the inertia parameter of the chiral rotation energy. It is well known that the RPA treatment results in a special eigenmode with zero energy [11]. The conjugate coordinate α of π is defined as

$$\alpha = \sum_{\mathbf{p}} \alpha_{\mathbf{p}}(\tilde{\beta}_{\mathbf{pp}} - \tilde{\beta}^{*}_{\mathbf{pp}}) \ , \tag{33}$$

which satisfied the equations

$$[\alpha, \pi]_P = i, \quad [H_{\mathrm{RPA}}, \alpha]_P = -i\frac{\pi}{\mathcal{J}} \ . \tag{34}$$

The mesons are considered to be the collective modes of the time-dependent mean fields, and the ideal quarks are the dressed particles which satisfy the commutation relations and differ from the conventional fermions. Then, the Hamiltonian (15) including the ideal quarks is given as

$$H' = H^{M}_{\mathrm{RPA}} + H_{\mathrm{quark}} + H_{\mathrm{coupl}} \ , \tag{35}$$

where H_{quark} is the ideal quark Hamiltonian

$$\begin{aligned} H_{\mathrm{quark}} = &\sum_{\mathbf{p}s} E_{\mathbf{p}}(a^{*}_{\mathbf{p}s}a_{\mathbf{p}s} - c^{*}_{\mathbf{p}s}c_{\mathbf{p}s}) \\ &+\text{quadratic terms at ideal quarks} \ , \end{aligned} \tag{36}$$

and H_{coupl} is the coupling Hamiltonian between the mesons and the ideal quarks.

Thus, we derive the meson Hamiltonian interacting with the ideal quarks in the RPA order. As seen from (32), the meson Hamiltonian H^{M}_{RPA} includes the continuum modes $O_{\mathbf{p}}$. The ideal-quark Hamiltonian H_{quark} consists of the single ideal-quark Hamiltonian and the ideal-quark–ideal-quark two-body interaction. The Hamiltonian (35) has the chiral symmetry through the chiral rotation (Nambu-Goldstone mode) in RPA order. If we neglect the continuum mode and the quadratic terms at the ideal quarks in the Hamiltonian (35), it may correspond to the Hamiltonian obtained from the linear σ model [5].

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