Pion-nucleon partial-wave analysis with fixed-t dispersion relation constraints

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We have performed a set of partial-wave analyses of the pion-nucleon elastic scattering data to 2 GeV employing rigorous constraints from simultaneous forward and fixed-t dispersion relations. Constraints were generated from the forward $C^{\pm}$ amplitudes and the invariant amplitudes A and B at fixed t in the range 0 to -0.3 GeV^2. Solutions were generated for a range of pion-nucleon coupling constants ($g^2/4\pi$) and isoscalar scattering lengths ($a^{(+)}$). A chi-squared map for these ($g^2/4\pi, a^{(+)}$) solutions exhibits a clear minimum near $g^2/4\pi = 13.75$ ($f^2/4\pi = 0.076$) for both the fits to the data and the dispersion relations. While favoring a particular $g^2/4\pi$, this work shows that it is possible to obtain good, stable fits for nearby values, but at the cost of increased chi squared. Consequently, this approach provides a criterion for defining the preferred value and uncertainty of $g^2/4\pi$ from pion-nucleon scattering data. On this basis, we conclude that $g^2/4\pi = 13.75 \pm 0.15$ ($f^2/4\pi = 0.076 \pm 0.001$).

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I. INTRODUCTION

There is an ongoing controversy in pion-nucleon and nucleon-nucleon circles regarding claims for a revised value of the pion-nucleon coupling constant substantially lower than the one proposed by the Karlsruhe-Helsinki (KH80) group in 1980 [1], which has come to be regarded as "canonical." Since our contribution to the debate has evolved over the years, a brief review would be instructive to the reader.

Our foray into this arena was sparked by two events which can be traced to the Few Body conference [2] held in Vancouver in 1989. The Nijmegen group had reported a value ($f^2/4\pi = 0.075 \pm 0.001$) for the neutral-pion-nucleon coupling, considerably lower than the "canonical" charged-pion result of KH80 ($f^2/4\pi = 0.079 \pm 0.001$), implying (at that time) a large charge-independence-breaking (CIB) effect. Then Höhler displayed the poor agreement of VPI solution FA87 with the fixed-t dispersion relation (FTDR) for the invariant amplitude B. A subsequent VPI solution (SM90) [3] was used to check Höhler's claims. We found an improved behavior near $t = 0$ and a much greater consistency away from $t = 0$ (the older FA87 solution was also found to be "reasonable" away from this point). The resulting pion-nucleon coupling constant ($f^2/4\pi = 0.0735 \pm 0.0015$) [3] was compatible with the neutral-pion result from Nijmegen, i.e., considerably below the KH80 result. Despite our improved consistency with the FTDR, and the removal of the once-suggested large CIB, the revised value has not been well received.

As a result of the controversy, much of the past 2 years has been spent in checking our results and techniques. These efforts have focused on (a) addressing some concerns raised by Höhler and Bugg about our energy-dependent parametrization of partial waves, (b) investigating the use of various forward and FTDR constraints (DRC), and (c) developing a strategy for determining the "best" values of the pion-nucleon coupling constant and S-wave scattering lengths from elastic scattering data. Concerning point (a), a problem of unphysical structures near threshold in some partial waves was traced to the Chew-Mandelstam representation used in our analyses. We found that these unphysical structures could be removed if relatively minor changes were made in the parametrization. Further questions regarding this parametrization and our choice of charge-correction scheme are still under study, however, there are strong indications that future modifications (if any) will not significantly alter the conclusions presented here.

In previous publications [3–6] we have used dispersion relations in order to extract a value for the pion-nucleon coupling constant. However, the analyses presented here are the first in which we have employed DRC in the fitting procedure. For this work, we chose the forward $C^{\pm}(\omega)$ and FTDR. The forward dispersion relations were chosen because of the link to well-measured total cross sections via the optical theorem, and the Goldberger-Miyazawa, Oehme (GMO) sum rule [6]. The FTDR offers an interesting graphical visualization which can easily be used to (a) determine constraints required in the analysis, (b) evaluate how well the π^+ and π^- amplitudes satisfy the dispersion relations, and/or (c) extract the coupling constant (see, e.g., [1,3,7]).

Höhler has commented on the difficulty of estimating an uncertainty for the coupling extracted from the FTDR [8]. Errors derived from straight-line fitting procedures usually severely underestimate the uncertainty since it is notoriously difficult to obtain an error estimate for the amplitudes used in the dispersion relations arising from uncertainties in the data. Consequently, in most cases these uncertainties are ignored. He suggests that a better estimate comes from gauging the self-consistency of the dispersion relations at various values of momentum transfer. The latter suggestion is a necessary ingredient in any realistic error estimate, but is not sufficient.

The trouble with obtaining the coupling constant from a dispersion relation constrained analysis is that first one must *assume* a value of the coupling in order to calculate the dispersion relations and hence to determine the constraints used to make the amplitudes satisfy them. One would not be surprised if, after the analysis was complete, a coupling determined from the FTDR was equal to the value assumed initially. Nonetheless, this is exactly what was done in the KH80 analysis [1], where the value $f^2/4\pi = 0.079$ used in all their dispersion relations was the value they ultimately "determined" from their analysis. The analysis was not attempted with other values. Höhler has countered [9,10] that in these analyses [1,9], the final step was a fit to the data, and so the data could have "forced" a different value of the coupling constant. To check this claim, we performed our analysis for a range of input couplings, and found that stable, consistent solutions could be obtained for all reasonable values of the input, and in these cases, the value extracted via the FTDR was always the same as the input. Moreover, the chi-squared goodness of fit ($\mathcal{X}^2$) varied smoothly with the coupling constant, yielding a single pronounced minimum. This fact illuminates what we feel is the most natural criterion for establishing the "best" value of the coupling constant and its uncertainty. The "best" value gives the smallest $\mathcal{X}^2$, and its uncertainty can be estimated from the depth of this minimum, as well as from the self-consistency of the dispersion relations.

In the following, we describe the steps followed and the techniques used in our investigation of the effect of imposed DRC on our partial-wave solutions and the determination of the pion-nucleon coupling constant. Section II will describe the form in which the DRC were implemented. Details of the analyses are discussed in Sec. III. In Sec. IV, we give our results. Finally, in Sec. V, we summarize our findings in the context of other recent results, and consider what further work is required.

II. DISPERSION RELATION CONSTRAINTS

We have carried out a number of analyses using different subsets of the DRC described below. The FTDR, which was used in our initial extraction of the πNN coupling [3], has been used to constrain our analyses at several fixed values of momentum transfer t. From relations for the $\pi^{\pm}p$ invariant amplitudes in terms of the crossing even and odd amplitudes, $B_{\pm}(\nu,t) = B^{+}(\nu,t) \mp B^{-}(\nu,t)$ the following dispersion relation can be constructed

$$(\nu_B \pm \nu)\left\{\mp \mathrm{Re}B_{\pm}(\nu,t) \pm \frac{\nu}{\pi}\int_{\nu_{\rm th}}^{\infty}\left[\frac{\mathrm{Im}B_{+}}{\nu' \mp \nu} + \frac{\mathrm{Im}B_{-}}{\nu' \pm \nu}\right]\frac{d\nu'}{\nu'}\right\} = \frac{g^2}{M} + \tilde{B}(0,t)(\nu_B \pm \nu) \ , \tag{1}$$

with

$$\tilde{B}(0,t) = \frac{2}{\pi}\int_{\nu_{\rm th}}^{\infty}\frac{\mathrm{Im}B^{-}(\nu',t)}{\nu'}d\nu' \ , \tag{2}$$

where s, t, and u are the usual Mandelstam variables, $\nu = (s-u)/4M$, $\nu_B = (t-2\mu^2)/4M$ is the nucleon pole, and $\nu_{\rm th} = \mu + t/4M$, μ and M being the charged-pion and proton masses, respectively. [The pseudoscalar and pseudovector coupling constants are related by $g^2 = (2M/\mu)^2 f^2$.] This dispersion relation has a contribution from the integral in the unphysical region between $\nu_{\rm th}$ and the point where $\cos(\theta) = -1$. $\mathrm{Im}B^{\pm}$ in this region was determined at each iteration in the analysis by an analytic continuation from the physical region using the current solution.

From the right-hand side of Eq. (1), it is evident that this relation should be linear in $\nu_B \pm \nu$ with an intercept corresponding to g^2/M. Deviations from linearity were corrected by introducing constraint values of $\mathrm{Re}B_{\pm}(\nu)$ which were to be included as "quasidata" in the next iteration of the fitting procedure.

The forward isospin-odd amplitude $C^{-}(\omega)$ was used to link the $a^{(-)}$ scattering length with $g^2/4\pi$. As the GMO integral [11] is slowly convergent, we chose to use the subtracted form to constrain our solutions:

$$a^{(-)} = \frac{1}{4\pi}\frac{\mu}{\omega}\frac{M}{M+\mu}\left\{\mathrm{Re}C^{-}(\omega) + \frac{\mu^2}{2M^2}\frac{g^2k^2\omega}{(\omega^2-\omega_B^2)(\mu^2-\omega_B^2)} - I^{(-)}(k)\right\} \ , \tag{3}$$

with k equal to the pion laboratory momentum, $\omega_B = \nu_B(t=0)$, and

$$I^{(-)}(k) = \frac{k^2\omega}{\pi}\int_0^{\infty}\frac{\sigma_{\pi^-p}(k') - \sigma_{\pi^+p}(k')}{k'^2-k^2}\frac{dk'}{\omega'} \ . \tag{4}$$

The subtraction reduced our reliance on high-energy contributions. Here $\sigma_{\pi^{\pm}p}$ denotes the hadronic total cross section (i.e., all Coulomb contributions removed).

The forward C^{+} dispersion relation was used in the following once-subtracted form:

$$a^{(+)} = \frac{1}{4\pi}\frac{M}{M+\mu}\left\{\mathrm{Re}C^{+}(\omega) + \frac{g^2}{M}\frac{k^2\omega_B^2}{(\omega_B^2-\omega^2)(\mu^2-\omega_B^2)} - I^{(+)}(k)\right\} \ , \tag{5}$$

with

$$I^{(+)}(k) = \frac{k^2}{\pi}\int_0^\infty \frac{\sigma_{\pi^-p}(k') + \sigma_{\pi^+p}(k')}{k'^2 - k^2} dk' \ . \qquad (6)$$

The once-subtracted form was chosen over the more convergent twice-subtracted form in order to avoid the introduction of the subthreshold constant $C^{(+)}(0,0)$ which is not known *a priori*. The penalty is that some assumption on the high-energy behavior of the integral must be made.

These expressions were constructed such that constant values are expected when the right-hand sides of Eqs. (3) and (5) are evaluated. (At threshold, these relations reduce to identities for the $a^\pm$ scattering lengths.) One can choose to fix values for $a^\pm$, or to select an "average" value after each iteration from the expressions on the right-hand sides. This constant was used to constrain our analyses. Analogously to the FTDR, deviations were corrected through constraints imposed on $\mathrm{Re}C^\pm$ in subsequent iterations.

After much of this work was completed, it was pointed out [12] that the $B_\pm$ dispersion relations used do not effectively constrain the S waves since they are suppressed by a kinematical factor, and are dominated by the P_{33} in the important (3,3) resonance region. Consequently, as a consistency check, we calculated for each final solution the following subtracted $A^\pm$ dispersion relations

$$A^+(0,t) = \mathrm{Re}A^+(\nu,t) - \frac{\nu}{\pi}\int_{\nu_{\mathrm{th}}}^\infty \frac{d\nu'}{\nu'}\left\{\frac{\mathrm{Im}A^+(\nu',t)}{\nu'-\nu} - \frac{\mathrm{Im}A^+(\nu',t)}{\nu'+\nu}\right\} \qquad (7)$$

and

$$\frac{2}{\pi}\int_{\nu_{\mathrm{th}}}^\infty \frac{d\nu'}{\nu'^2}\mathrm{Im}A(\nu',t) = \frac{\mathrm{Re}A^-(\nu,t)}{\nu} - \frac{\nu}{\pi}\int_{\nu_{\mathrm{th}}}^\infty \frac{d\nu'}{\nu'^2}\left\{\frac{\mathrm{Im}A^-(\nu',t)}{\nu'-\nu} - \frac{\mathrm{Im}A^-(\nu',t)}{\nu'+\nu}\right\} , \qquad (8)$$

where the crossing-odd amplitude A^- was divided by ν to make it crossing even so that it does not vanish at $\nu = 0$. The right-hand sides of Eqs. (7) and (8) were evaluated and plotted as a function of ν, where again the contributions from the unphysical region were taken from an analytic continuation using the current solution. If this dispersion relation is to be satisfied for each fixed value of t, a constant value is required.

III. THE PARTIAL-WAVE ANALYSES

In practice, the integrals of Sec. II were evaluated up to some k_{max} which ensured sufficient accuracy for the analyses, except for the case of the forward C^+ dispersion relation, where the integral was taken up to infinity. Two different high-energy parametrizations for this integral were tested [7,13] for values of pion laboratory kinetic energy above 10 GeV, whereas the piece from 4 to 10 GeV was taken from Höhler's table of forward amplitudes [7]. The results were found to be insensitive to the choice of high-energy parametrization. The fixed-t and subtracted C^- dispersion relation were evaluated up to a pion laboratory kinetic energy of 4 GeV, the contribution from 2 to 4 GeV coming from the Karlsruhe solution [9]. The $C^\pm$ dispersion relation constraints were imposed at 25 MeV intervals, from a T_{lab} of 25–600 MeV. The FTDR constraints covered different ranges of T_{lab} depending upon the value of t — which extended from 0 to $-0.3\ \mathrm{GeV}^2$.

Analyses were performed with the value of $g^2/4\pi$ constrained to selected values between 13.0 and 14.5. The value of $a^{(-)}$ was determined in two ways. In one approach, $a^{(-)}$ was obtained from the GMO sum rule [11]. In this way, the scattering length was essentially fixed by the value of $g^2/4\pi$. In another, an average value of $a^{(-)}$ was calculated from the right-hand side of Eq. (3) over the range of ω values extending from 25 to 600 MeV. We found that these two methods produced essentially identical results. The second method was retained in our final set of analyses.

Since the very small scattering length $a^{(+)}$ arises from the cancellation of two relatively large terms, it was obvious that attempting to constrain to an "average" value from the right-hand side of Eq. (5) would not be the most reliable way of implementing that dispersion relation. Consequently, it was decided that the value of $a^{(+)}$ would be varied over some range for each value of the coupling. This resulted in a grid of solutions, one for each $(g^2/4\pi, a^{(+)})$ combination.

As mentioned in Sec. II, deviations from our DRC were corrected by determining values of $\mathrm{Re}B_\pm$ and $\mathrm{Re}C^\pm$ to be fit in conjunction with the experimental data. After several search cycles, the constraints were regenerated from the resulting solution, and the process was repeated. This was continued until the overall chi-squared stabilized. We found that great care was required in determining a global minimum for these solutions. Numerous local minima were encountered. Solutions caught in local minima were often found by plotting $\mathcal{X}^2$ versus $g^2/4\pi$ for the full set of analyses. These offending solutions generally deviated from the parabolic behavior exhibited by consistent "families" of analyses. Plots of $\mathcal{X}^2$ versus $g^2/4\pi$ are given in Fig. 1 for different choices of $a^{(+)}$.

IV. NUMERICAL RESULTS

Our initial analysis consisted of a grid of 12 solutions comprised of coupling values of 13.0, 13.5, 14.0, and 14.5, and scattering length values of $-0.025\mu^{-1}$, $-0.050\mu^{-1}$, or $-0.075\mu^{-1}$. After generating our first solutions, it became obvious that 13.0 and 14.5 were heavily dis-

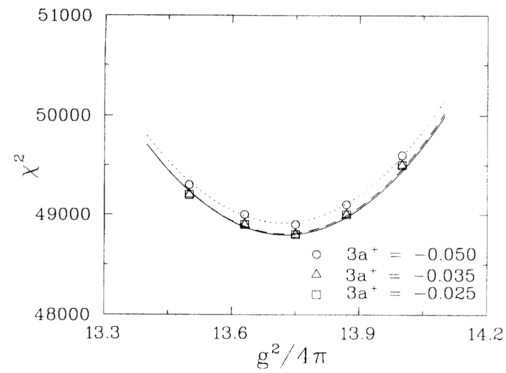

FIG. 1. Quadratic fit to total $\mathcal{X}^2$ (data and constraint) for sets of solutions with $3a^+ = -0.025\mu^{-1}$ (squares), $3a^+ = -0.035\mu^{-1}$ (triangles), and $3a^+ = 0.050\mu^{-1}$ (circles).

favored values for $g^2/4\pi$, as was $-0.075\mu^{-1}$ for $3a^{(+)}$. We eventually focused on a grid of 15 solutions which straddled the best fit suggested by the first grid of solutions: $[g^2/4\pi = 13.5, 13.63, 13.75, 13.87$, and $14.00; 3a^+ = (-0.025, -0.035, -0.050)\mu^{-1}]$. The resultant $\mathcal{X}^2$ "map" derived from these solutions was fitted with a six-parameter, biquadratic function (as shown in Fig. 1). The individual $\mathcal{X}^2$ values for each solution are given in Table I. A deep minimum was revealed near $g^2/4\pi = 13.7$ and $3a^+ = -0.03\mu^{-1}$. It is significant that the same minimum is seen in both the constraint "data" and the experimental scattering data. This means that the dispersion relations themselves are better satisfied for that particular value of coupling. While it was again satisfying to see that the $\pi^{\pm}p$ components of the database data gave the same "best value" for $g^2/4\pi$, a problem was seen in the charge-exchange (CXS) data. We do not yet have an explanation for this disparity. Fortunately, the CXS data add a comparably small contribution to the data $\mathcal{X}^2$, and so the essential results should persist once this anomaly has been sorted out [14].

The errors chosen [15] for the dispersion relation constraints used in our fits resulted in a $\mathcal{X}^2$/constraint close to unity for near-optimal combinations of couplings and scattering lengths. Nonetheless, to check for effects due to the strength of these constraints, we cut these constraint errors in half and regenerated the full set of solutions. Here the optimal value of $g^2/4\pi$ increased to about 13.8, with a reduced discrepancy between the minima found from the different charge channels and from the constraints. (The $\pi^{\pm}p$ and charge-exchange results differ by less than 0.15 in this case.) The total $\mathcal{X}^2$ increased by approximately 1000 in these solutions. We have also made comparisons with the Karlsruhe solution KA84 [16]. In general, our solutions with both "soft" and "hard" constraints were in good agreement with the

TABLE I. $\mathcal{X}^2$ values for solutions with soft constraints (see text). The values of $g^2_{\rm min}/4\pi$ are found from quadratic fits.

Solution	$g^2/4\pi$	Data (21 078)	Constraints (496)	π^+ (10 106)	π^- (9 304)	CXS (1 668)
S352	13.50	48 738	493	23 080	20 590	5 068
S362	13.63	48 467	407	22 973	20 527	4 967
S372	13.75	48 414	386	22 979	20 530	4 905
S382	13.87	48 555	405	23 079	20 632	4 844
S402	14.00	48 981	478	23 395	20 784	4 802
$g^2_{\rm min}/4\pi$		13.72	13.76	13.68	13.66	14.25
$(3a^+ = -0.025)$		±0.02	±0.04	±0.02	±0.03	±0.06
S353	13.50	48 745	481	23 100	20 578	5 067
S363	13.63	48 490	398	23 002	20 517	4 971
S373	13.75	48 421	392	22 983	20 525	4 913
S383	13.87	48 578	400	23 104	20 637	4 837
S403	14.00	48 976	490	23 395	20 781	4 800
$g^2_{\rm min}/4\pi$		13.72	13.74	13.68	13.66	14.36
$(3a^+ = -0.035)$		±0.02	±0.04	±0.02	±0.03	±0.07
S355	13.50	48 849	471	23 173	20 627	5 049
S365	13.63	48 588	411	23 049	20 560	4 979
S375	13.75	48 506	411	23 017	20 566	4 923
S385	13.87	48 695	410	23 175	20 694	4 826
S405	14.00	49 078	521	23 454	20 829	4 795
$g^2_{\rm min}/4\pi$		13.71	13.72	13.68	13.65	15.42
$(3a^+ = -0.050)$		±0.02	±0.04	±0.02	±0.03	±0.11
$\mathcal{X}^2_{\rm min}$		48 405	383	22 950	20 512	4 753
$g^2/4\pi$		13.72	13.75	13.68	13.67	14.32
$\Delta(g^2/4\pi)$		±0.01	±0.03	±0.02	±0.02	±0.05
$3000a^+$		−26.9	−32.3	−18.0	−32.3	
$\Delta(3000a^+)$		±2.2	±5.1	±3.8	±2.6	

dispersion relations considered above.

In order to assess the "cost" (in $\mathcal{X}^2$) of imposing these DRC, we generated a solution with all fixed-t constraints removed but with forward constraints retained. The data $\mathcal{X}^2$ for an "unconstrained" fit were then compared to that for the fit with $g^2/4\pi = 13.75$ and $3a^+ = -0.025\mu^{-1}$. As the strength of the constraints was increased, the overall $\mathcal{X}^2$ increased by 800 from a baseline of about 48 000. We feel that this cost is quite modest in order to satisfy the dispersion relations.

Our estimate of the uncertainty in $g^2/4\pi$ was based upon the spread of values from the FTDR, from the consistency of these and the forward dispersion relations, as well as the depth of the $\mathcal{X}^2$ minimum. From these considerations, and the results of our fits with increased constraints, we conclude that $g^2/4\pi = 13.75 \pm 0.15$. The uncertainty estimated from our $\mathcal{X}^2$ mapping was only a small fraction of the value quoted above. The uncertainty was determined mainly from the span of values found in linear fits to the FTDR given in Eq. (1). For comparison's sake, the rms deviations [4] for $g^2/4\pi$ taken from the Hamilton B^+ dispersion relation (which was not used in this analysis) were around 0.04, far smaller than our estimate of 0.15.

After generating our solutions, we checked their consistency with the $A^\pm$ DRC displayed in Eqs. (7) and (8). Though these DRC were not considered in our analyses, they are both very well satisfied. In fact, the consistency is superior to that displayed by KA84.

As a final check, we have extracted the coupling constant using charge-corrected amplitudes, instead of the uncorrected, or hadronic, amplitudes. Note that our Coulomb correction scheme employs the same direct Coulomb and Coulomb rotation terms used in KH80, differing only in the Coulomb barrier piece [3]. In the FTDR, we examined the change in extracted couplings when the Coulomb rotation and barrier were turned on. The difference was comparable to the error we have quoted. Consequently, differences between our Coulomb barrier scheme and that used by KH80 are not expected to significantly alter our findings.

V. SUMMARY AND DISCUSSION

The generation of these fits, with various combinations of constraints, was a very lengthy procedure. These tests would not have been feasible without access to a number of fast UNIX workstations. In some cases, the convergence of our iterative procedure was slow. This was particularly true for unfavorable combinations of the coupling constant and scattering lengths, and for solutions with tighter DRC. In the beginning, we were not certain that our iterative algorithm would lead to convergence.

By generating a grid of solutions, we can immediately see the sensitivity of the data to different choices of pion-nucleon coupling constant and scattering lengths. This approach is clearly the most natural way of determining the "best" value and error estimate for $g^2/4\pi$. One scattering length combination, $a^{(-)}$, is intimately connected to the value of $g^2/4\pi$ through the GMO sum rule. It is interesting to note that our results were much less sensitive to the value of $a^{(+)}$, as can be seen, for example, in Fig. 1. This too is important. Although our "near optimal" solution (S372) gives values for $3a^{(-)}$ $(0.264\mu^{-1})$ and $a_{\pi^- p}$ $(0.079\mu^{-1})$ consistent with determinations from the Panofsky ratio and pionic hydrogen measurements, respectively [17,18], we can easily accommodate any "reasonably similar" value for the scattering length combination $a_{\pi^- p}$ coming from a recently completed improved measurement [19]. The solution S372 is also consistent with the GMO sum rule, if one takes into account the uncertainties of $f^2/4\pi, a^{(-)}$, and the integral over total cross sections.

The results from other groups have continued to evolve and now seem to be converging on a mutually agreeable value for the coupling constant. The uncertainty on the Karlsruhe value for $f^2/4\pi$ has been enlarged from 0.001 to a value between 0.002 and 0.003 [8]. Bugg has recently claimed [12] a value of $f^2/4\pi = 0.0771 \pm 0.0014$. The Nijmegen value remains [20] near $f^2/4\pi = 0.075$. We should also mention that the effect of a reduced pion-nucleon coupling has been extensively studied in the deuteron system [21]. The most recent results [21] have confirmed a neutral-pion coupling near 0.075. The charged-pion coupling was less certain in this study.

We are currently considering a question, raised by Höhler, regarding the flexibility of our parametrization to accommodate structures found in the Karlsruhe solution, but not in our previous analyses. Other questions regarding the form of charge corrections, and the threshold parametrization of P-wave amplitudes are also under study [12].

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