PHYSICAL REVIEW C VOLUME 49, NUMBER 5 MAY 1994

Looking for quark droplets in ultrarelativistic collisions

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(Received 8 December 1993)

Heavy ion collisions at ultrarelativistic energies are expected to provide an environment where quarks and gluons replace hadrons as the appropriate degees of freedom. As the excited region expands and cools, the transition to the hadronic state might be characterized by phase separation with hadrons being emitted from dense droplets of quark-gluon matter. Here we study four techniques to search for such droplets: rapidity correlations, identical kaon correlations, ϕ meson production, and proton correlations. We conclude that rapidity correlations are the clearest signal of such fluctuations, and that proton correlations and ϕ production can also be strongly affected by drop formation.

PACS number(s): 25.75.+r, 25.70.Pq

Early in the next decade two heavy-ion accelerators, the Relativistic Heavy Ion Collider (RHIC) and the Large Hadron Collider (LHC), will create highly excited regions, similar to heavy ions in size and with temperatures exceeding 200 MeV. Early in the collision, average particle separation should be smaller than the radius of a hadron, making description in terms of hadrons irrelevant. At this point the relevant degrees of freedom become those of quarks and gluons. The hot region will then expand and cool undergoing a transition from quark-gluon plasma into hadrons.

Since the size of the excited region is an order of magnitude larger than usual hadronic scales and the particle number in the thousands, one can use the macroscopic language of thermodynamics and phase transitions to describe the reaction's evolution. Through thoughtful measurements and phenomenology one can gain insight regarding fundamentals of the phase transition from our hadronic world to the quark-gluon plasma. The most basic challenge regarding this phase transition is determining the order of the transition. Interpretations of lattice gauge calculations have not yet settled on this issue as opinions range from first order with a large latent heat, several GeV/fm^3, to no phase transition at all [1,2]. A second question regarding the phase transition is the surface energy between coexisting phases. Although not performed with the correct number of quark flavors, lattice gauge calculation points to a modest or small surface energy [3,4]. The surface energy is important as it determines the nature of the mixed phase and the dynamics of the transition. A large surface energy would promote large super-cooling and the creation of large inhomogeneities in the energy density during the mixed phase. If surface energies are small, there are no impediments to the formation of many small drops or bubbles [5], leading to energy density profiles which are smooth when viewed on a scale of more than two or three fermi.

Signatures of a large latent heat or an absence thereof should be straightforward to find. A large latent heat brings with it very low pressure relative to the energy density, and therefore little collective expansion, which can be determined from singles spectra [6] as well as from interferometry [7]. Density inhomogeneities are more difficult to measure. Spatial information can only be obtained through many-body observables. Two-particle interferometry has been a useful tool for ascertaining average spatial and temporal sizes for nuclear reactions at a variety of energy regions. However, fluctuations in the spatial and temporal characteristics of a reaction are beyond the usual interferometric formalisms.

Two techniques have been proposed for identifying such fluctuations. Seibert [8,9] discussed how rapidity correlations can reveal the existence of drops of hot plasma. This method makes use of the fact that due to incomplete stopping, hadrons should be emitted over several units of rapidity. Hadrons emitted from a quark-matter droplet would lie within one unit of rapidity of the droplet. Thus if a large fraction of hadrons are emitted from drops and the number of drops is not too large, measureable fluctuations in the rapidity distribution should ensue, which can be quantified with rapidity correlations. Several experiments have searched for such effects [10–12]. A second method is to make use of the fact that droplet formation increases the probability of two hadrons being emitted very close to one another in coordinate space [13]. If two hadrons interfere with one another, for example Bose-Einstein interference for identical kaons, that interference will be magnified by the existence of drops. This interference is quantified through the two-particle correlation function. Other sources of interference would be the interaction of the hadrons through a resonance, such as two oppositely charged kaons which can create a ϕ meson or two protons which can form the isospin-one version of the deuteron just above threshold.

The purpose of this paper is to assess the merits of four methods of searching for quark-matter droplet formation. The four methods are: rapidity correlations, proton-proton interfermometry, the size of the ϕ peak relative to the K^+-K^- background, and identical kaon interferometry. To do this we first construct a simple model of particle emission with fluctuations in emission probabilities, where we can vary the number of fluctuations or droplets. We then construct the relevant observables for each of the four methods, trying to determine at which point the droplet number becomes so large that the signal is washed out. Aside from exploring whether the latter three methods can signal drop formation, we need to understand whether usual interpretations of correlation meausurements are significantly distorted by the presence of drops.

Modifiying Bjorken's thermal picture [14] can give a simple model that includes random emission, collective dynamics, and droplet emission. Bjorken's model assumes an ensemble of thermal sources, uniformly spread in rapidity, dissolve at a fixed proper time $\tau = \sqrt{t^2 - z^2}$. The sources move with a collective velocity $v_z = z/t$. We modify Bjorken's picture in two ways. First, we give the sources a Gaussian distribution in rapidity. Letting η_s refer to the rapidity of the source,

$$P(\eta_s) = e^{-\eta_s^2/2\Delta^2}. \tag{1}$$

Each source emits with a relativistic Boltzmann distribution, determined by the temperature T. If p' is the momentum as measured in the frame of the source,

$$\frac{dP(p')}{d^3p'} = e^{-E(p')/T}. \tag{2}$$

We generate a distribution by first generating a thermal distribution in the frame of the source, then boosting the distribution along the beam axis according to the rapidity of the source. The positions and times of the particle's emissions are also needed to calculate interference effects. For a source moving with a rapidity η_s, the source will emit at a z coordinate

$$z = \tau \sinh(\eta_s). \tag{3}$$

The proper time of the emission is given by an exponential distribution characterized by the lifetime of the source τ_s and the turn-on time τ_0:

$$P(\tau) = \exp(-\tau/\tau_s)\theta(\tau - \tau_0). \tag{4}$$

The transverse spatial coordinate is chosen according to the distribution

$$P(x_s, y_s) = \exp\left(-\frac{x_s^2 + y_s^2}{2R^2}\right). \tag{5}$$

If the source is a droplet, the coordinate of the emission is spread out by the extent of the drop, R_d:

$$P(\mathbf{r} - \mathbf{r}_s) = \exp\left(-\frac{(\mathbf{r} - \mathbf{r}_s)^2}{2R_d^2}\right). \tag{6}$$

The spreading out is done in the center-of-mass of the drop to be relativistically consistent. Phase-space coordinates are generated as follows.

(1) First choose a rapidity of the source according to Eq. (1).

(2) Choose a momentum for the particle according to a relativistic Boltzmann distribution, Eq. (2).

(3) Generate a time according to the expression for the proper time in Eq. (4).

(4) For nondroplet emission, choose $z = 0$ and x and y according to Eq. (5). For emission from drops, modify x, y, and z through Eq. (6).

(5) Boost both the spatial and momentum coordinates according to the rapidity of the source. For example,

$$\begin{aligned} t' &= \cosh(\eta_s)t - \sinh(\eta_s), \\ z' &= \cosh(\eta_s)z - \sinh(\eta_s)t. \end{aligned} \tag{7}$$

Rather arbitrarily, we choose the drop size R_d to be 1.0 fm, the transverse source size, R, to be 5.0 fm, and the temperature, T, to be 175 MeV. We also needed to choose the fraction of particles emitted from drops as opposed to the uniform background to be $F = 0.5$. The rapidity spread of the source, Δ, was chosen to be 1.5 corresponding to a spread of three units of rapidity. None of the predictions in this paper depends noticeably on Δ, and since no experiment is likely to cover more than three units of rapidity, this should be sufficient. The number of drops in a collision, N_d, is a variable which we vary. Since, most of the forthcoming predictions depend only on the number of drops per unit rapidity, one should divide N_d by three to get a feel for how many drops per unit rapidity this represents. The lifetimes, τ_s and τ_0, used for the model are discussed later in the paper. The probability p of any given pair coming from the same drop is

$$p = \frac{F^2}{N_d}. \tag{8}$$

Given the probabilities described above, one can generate pairs of particles statistically. First we look at rapidity correlations. For the results reported below, only thermal pions were used. More detailed calculations have been performed which include effects of decays and other charged particles, but the difference from the direct-pion case is not substantial. The correlation function $C(y)$ is defined

$$C(y) = \frac{P_{\mathrm{same}}(y)}{P_{\mathrm{mixed}}(y)}. \tag{9}$$

The distributions P_{same} and P_{mixed} are found by binning pairs of particles according to their relative rapidity for two particles from the same event or by mixing two particles from different events. The results are shown in Fig. 1 for $N_d = 50$, 100, and 200. The height of the correlation scales as the inverse of the number of drops per unit rapidity and is also proportional to the square of the fraction of particles originating from drops. There is nothing new in these graphs as similar calculations were done by Seibert and collaborators in the formalism of factorial moments [8,9]. It has been shown that other causes of correlation such as Bose-Einstein and resonant decays, also contribute to the correlation [15]. But these other causes of correlation can easily be identified and their contributions subtracted from the correlation function. For collisions at RHIC where the multiplicity will be several hundreds per unit rapidity, the contribution from droplet formation should be identifiable if it exceeds one

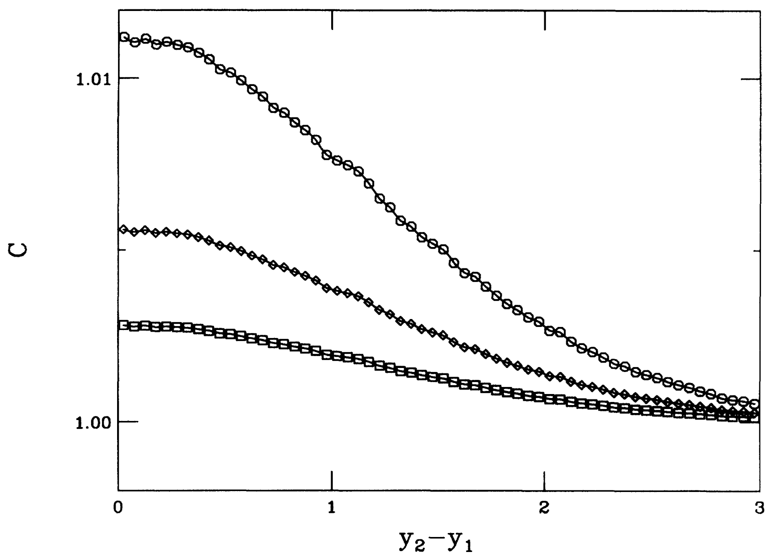

FIG. 1. Rapidity correlations for pions. In order of the peak height, correlations for 50 drops, 100 drops, and 200 drops are represented by circles, diamonds, and boxes. Even if drops are as numerous as 50 per unit rapidity, a measurable signal results.

or two tenths of a percent. Thus if the number of drops per unit rapidity is less than 100, and half the particles originate from drops, the drops should be unmistakably manifest in the rapidity-correlation measurement.

Two-proton emission is enhanced for small relative momenta due to the resonantlike interaction at threshold. This leads to an enhancement in the correlation for relative momenta near 25 MeV/c. The size of this enhancement scales as the inverse volume of the proton emission. Thus, measurement of the peak in the correlation function has often served as a means to extract a source size. More accurately stated, the peak in the p-p correlation function is proportional to the probability that two protons are emitted within one or two fermi of one another. If a source contains large density inhomogeneities, the probability that two protons are emitted in close proximity to one another is enhanced relative to the case where emission is spread out uniformly.

The correlation function for two protons can be calculated by first simulating proton pairs with the model described above, and then binning the pairs according to the relative momenta. But instead of adding a constant to each bin when a pair has the appropriate relative momenta, one adds the square of the relative wave function which depends on the relative momenta and the relative position at the time the latter proton is emitted. We refer to this two-particle emission weight as $w_{1,2}$. This same technique will be used for the same-sign and opposite-sign kaon correlations to be discussed later.

$$w_{1,2} = \left|\phi\left(p_1 - p_2, r_1 - r_2 + \frac{v_1 + v_2}{2}(t_1 - t_2)\right)\right|^2 . \quad (10)$$

This weighting is the standard prescription in correlation analysis. Fig. 2(a) shows the correlation functions for the cases of $N_d = 10$, $N_d = 25$, and $N_d = \infty$ for drops which emit instantaneously at $\tau = 20$ fm/c. The enhancement of the correlation function for small drop number can be considered as a signal for the formation of drops. But the assumption of instantaneous emission is unfortunately of crucial importance. If droplets emit over a long time, protons from a given drop will be well separated and not interfere as often. In fact, if the lifetime of the drops is longer than the lifetime of the uniform background [16], as expected, proton pairs from drops might even be less correlated than those from the background and the signal would be erased. Fig. 2(b) shows the correlation function for the case where the background emits according to equation 2 with $\tau_s = 10$ fm/c for particles from the background and $\tau_s = 20$ fm/c for particles from drops. For all particles the minimum time τ_0 was chosen to be 5 fm/c. From viewing Fig. 2(b) we conclude that the signal washes out when N_d exceeds about 30, corresponding to 10 drops per unit rapidity. By comparing Fig. 2(a) to Fig. 2(b) one can see how the uncertainty in lifetimes affects any conclusion one might draw regarding the clustering of emitted protons. If both background and droplet emission were characterized by the same lifetime, that lifetime could be understood from other correlation measurements and much firmer conclusions could be reached regarding the p-p measurements discussed above.

The conclusions are also muddled by the uncertainty regarding drop size. The choice of one fermi is rather arbitrary. Given the number of drops and percentage of particles originating from drops, one could determine the drop size if the energy density of the drops was known. If the maximum energy density of a drop with a Gaussian distribution characterized by 1 fermi was 2 GeV/fm^3 the amount of energy in a single drop would be 30 GeV which would account for approximately 50 hadrons. Thus if 1000 hadrons are emitted per unit rapidity it is not inconceivable that a significant fraction would originate from several drops of 1 fermi size or smaller. If a smaller size were chosen the signal would be stronger, although it is unreasonable to choose a size smaller than a typical hadronic size of one-half fermi.

Another example of two hadrons which interact through a resonance near threshold is two oppositely charged kaons which can form a ϕ meson. The ϕ has been measured in heavy-ion collisions at the AGS [17]. Here the interpretation is a bit more shaky. K^+ and

K^- are antiparticles of one another, hence they may be correlated due to the fact that when a strange quark is produced an anti-strange quark is produced nearby. In order to apply the correlation formalism, one must assume that the ϕ is in local equilibrium with the K^+-K^- interaction. For this to be the case, strangeness must be thoroughly dissolved in the reaction volume. For such equilibrated emission the weighting factor $w_{1,2}$ can be chosen to be

$$w_{1,2} = 1 + c \exp\left(-\frac{r^2}{2a^2}\right) \frac{1}{(M - M_\phi)^2 + \Gamma^2}. \qquad (11)$$

Here M is the invariant mass of the kaon pair which depends on the relative momentum, Γ is the width of the ϕ meson, a is an arbitrary small size, and c is a constant which is chosen such that the amount of extra weight integrated over all relative momenta and relative position corresponds to the correct amount of phase space:

$$\int \frac{d^3k d^3r}{(2\pi)^3}(w_{1,2} - 1) = \frac{3}{2}. \qquad (12)$$

The factor 3/2 is due to the three spin states of the ϕ and the 50% probability that a ϕ decays into charge kaons. Results for opposite-charged kaon pairs are shown in Fig. 3(a) and Fig. 3(b) for the same circumstances as the proton pairs in Fig. 2. The ability to resolve droplet structures with charged kaons appears somewhat stronger than the resolving power of protons. The ϕ yields a strong peak because it is a sharp resonance near threshold, but not so near threshold that the Coulomb interaction would dampen its peak. The ϕ resonance resides 32 MeV above the K^+-K^- threshold. Other resonances one might consider are located even more above threshold. The height of the peaks in a correlation function will scale inversely with the density of free states which means that resonances which are located further above threshold such as the delta resonance or the K^*

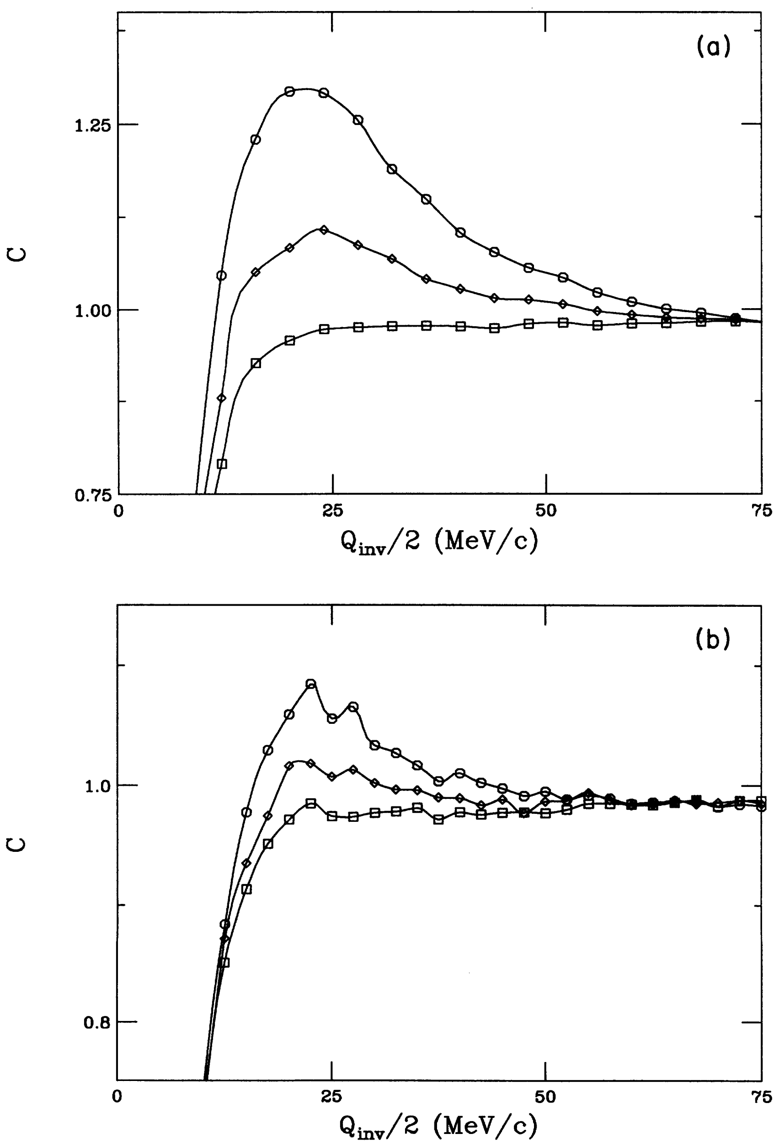

FIG. 2. Correlations for two protons, assuming the protons were emitted at a proper time of τ = 20 fm/c, are shown in the upper panel. Correlations, assuming protons were emitted exponentially with the times described in the text, are shown in the lower panel. In order of the peak height, correlations for 10 drops, 25 drops, and an infinite number of drops are represented by circles, diamonds, and boxes.

become increasingly difficult to extract from statistical noise.

Another source of interference among particles which depends on their relative positions is the Bose-Einstein interference between identical particles. Here we consider same-sign kaons. Kaons' lower thermal velocities makes them somewhat more correlated to a distinct spatial region thus reducing the number of drops which could contribute to the emission of two kaons with the same momentum. Thus two-kaon interferometry is slightly more distorted by drops than is two-pion interferometry. For this calcuation we use the Coulomb-corrected symmetrized wave function for the scattering wave and weight the pairs as is shown in Eq. 10. Since there are three uncertainty relations, one can investigate the three components of the relative coordinate of emitted kaons. Trying to reduce the effects of lifetime, we choose to sample pairs whose relative momentum is perpendicular to both the beam axis and the momentum of the kaon pair. This is referred to as the sidewards direction. Figs. 4(a) and 4(b) present Gamow-corrected correlation functions. The width of the Bose-Einstein correlation is inversely proportional to the source size, and droplet substructure manifests itself through a broadening of the correlation function at large relative momentum.

By inspection of Fig. 4 one sees that Bose-Einstein correlations are the least affected by droplet formation or phase inhomogeneities. This is not a disappointing result as this allows us to trust Bose-Einstein results for yielding source sizes using single-particle distributions and neglecting many-body density fluctuations. Only in extreme cases of emission from just a few drops and severely restricting the direction of the relative momentum, is the correlation strongly distorted by droplet formation. In these cases the inhomogeneities should be already apparent in the opposite-charged kaon correlations as well as in p-p correlations.

Despite the fact that simple models were used with somewhat arbitrary assumptions, we can come to several useful conclusions. Most importantly, the most power-

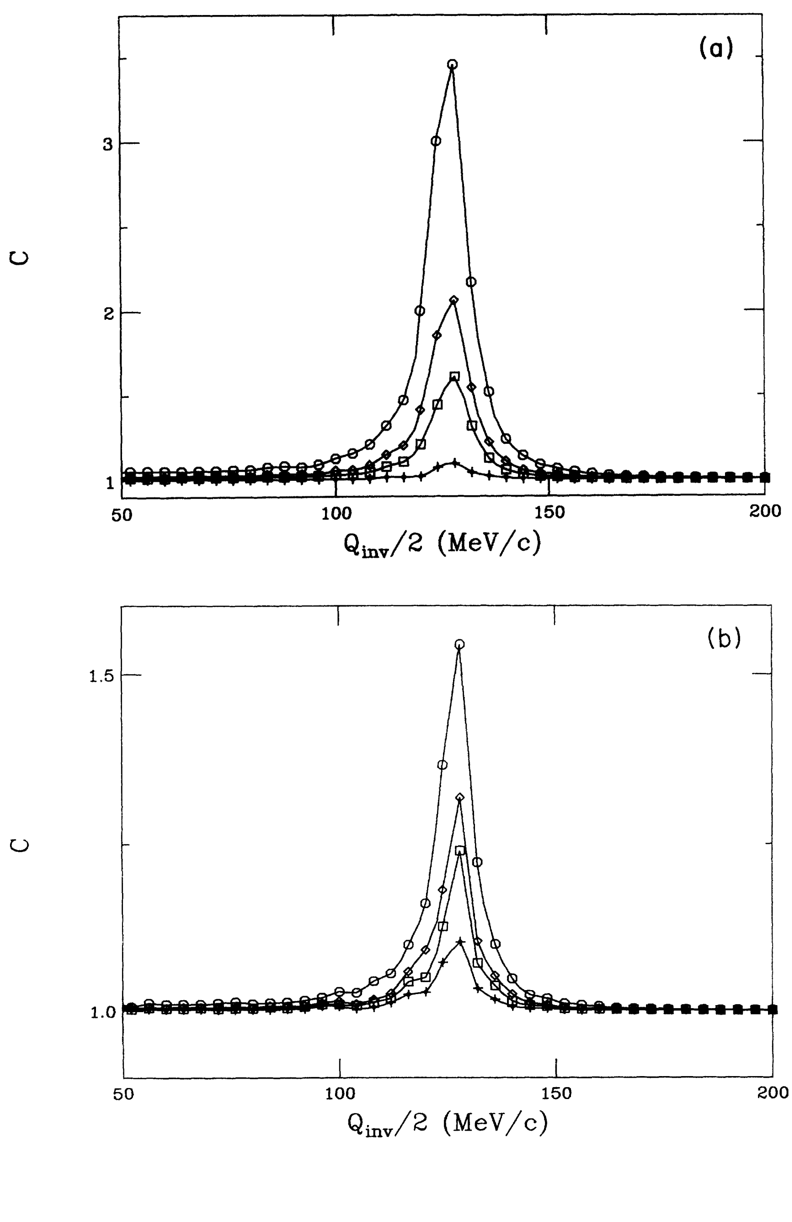

FIG. 3. Correlations for K^+, K^- pairs in the vicinity of the ϕ resonance, assuming emission at a proper time $\tau = 20$ fm/c, are shown in the upper panel. Correlations, assuming exponential emission with the times described in the text, are shown in the lower panel. In order of the peak height, correlations for 10 drops, 25 drops, 50 drops, and an infinite number of drops are represented by circles, diamonds, boxes, and crosses. Even if the fluctuations are numerous, but less than about 20 per unit rapidity, a noticeable distortion ensues.

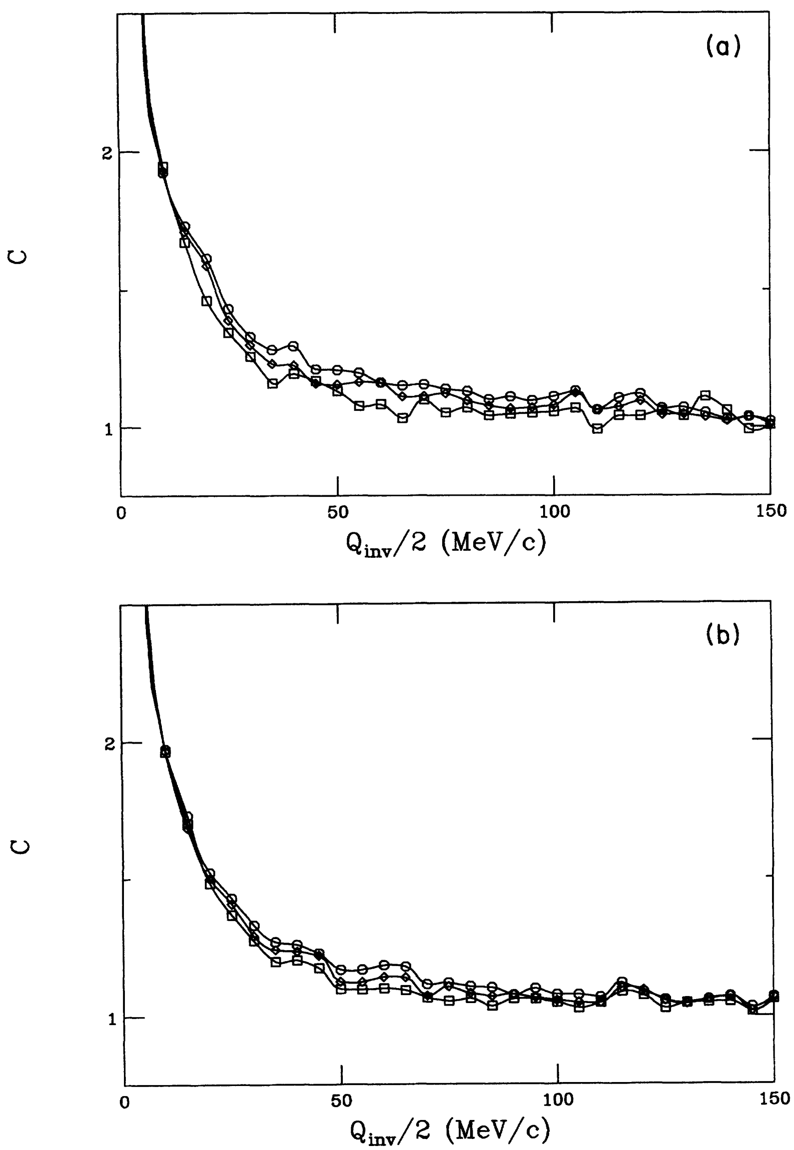

FIG. 4. Correlations for identical kaons, assuming emission at a proper time $\tau = 20$ fm/c, are shown in the upper panel. Correlations, assuming exponential emission with the times described in the text, are shown in the lower panel. In order of the peak height, correlations for 5 drops, 10 drops, and an infinite number of drops are represented by circles, diamonds and boxes. The drops affect the tail of the correlation function, but are not noticeable for large drop numbers.

ful method to view droplet formation is through rapidity correlations. If the number of droplets per unit rapidity is less than about 100 and half the particles are from drops, an experimental signature should exist. If a smaller fraction, for instance one fourth, of the particles originate from drops, the number of drops per unit rapidity which would yield a measurable signal would be about 25. Other methods in order of their resolving power are opposite-sign-kaon correlations near the ϕ peak, proton-proton correlations, and same-sign-kaon correlations due to Bose-Einstein interference.

Opposite-sign-kaon correlations are plagued by the assumption that strangeness has dissolved throughout the plasma and that kaons are uncorrelated in coordinate space aside from their interaction through the ϕ. In fact if rapidity correlations demonstrate the lack of spatial inhomogeneities, the ϕ peak could be used to gain insight into the dynamics of emitting strange mesons and in particular whether strange and anti-strange mesons are uncorrelated. To do this one would compare source sizes extracted from identical-kaon interferometry with source sizes extracted from opposite-sign-kaon measurements.

All four methods have the same scaling with respect to the fraction of particles from drops and the number of drops per unit rapidity, F^2/N_d. In order to determine the fraction of particles emitted from drops F and the number of drops N_d separately, one would have to use a three-body measurement such as a three-body rapidity correlation. Three-body correlations scale as F^3/N_d^2. The last three methods depend on droplet size, so there is reason to hope that some information about substructure could be extracted. Finally, since the distortion of identical-particle Bose-Einstein correlations is small and confined to the tail of the peak, it allows us to safely extract source sizes from correlation functions without considering the effects of density inhomogeneities.

ACKNOWLEDGMENTS

This work was supported by W. Bauer's Presidential Faculty Fellow Award, NSF Grant No. PHY-925355, and from NSF Grant No. PHY-9017077.

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