

Effect of Coulomb interaction in quasifree scattering and quasifree reactions in three body breakup processes

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The effect of Coulomb interaction in quasifree scattering and quasifree reactions has been investigated under different kinematical conditions for the processes involving up to two charges per particle and a total of six nucleons in the incoming channel, and three particles in the outgoing channel. The differential cross section was calculated using the plane wave impulse approximation. The influence of the Coulomb interaction on the magnitude and shape of these spectra were determined and compared with experimental data. It has been found that for the quasifree peak of the Coulomb effect within this model depends only on the outgoing particles in quasifree vertex and on energy between them. At low energies the influence of the Coulomb interaction cannot account for the discrepancy between the quasifree data and the impulse approximation predictions, although it can produce effects of several orders of magnitude, depending on the energy and the charges of particles. The Coulomb interaction changes only the magnitude, and has a very weak influence on the shape of the quasifree spectra. At high energies the Coulomb influence on the maxima of the spectra is in the 5–20% range. When those Coulomb effects are taken into account and when a proper wave function for the spectator-transfer particle system is used, then at energies above 200 MeV this model agrees with data.

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I. INTRODUCTION

Quasifree scattering (QFS) [1–4] and quasifree reactions (QFR) [5,6] can be described by the spectator model, using the plane or distorted wave impulse approximation (PWIA) [7–9]. It is known [10] that PWIA in general fits the shape of energy spectra (better fit can be obtained by, e.g., introducing a cutoff radius), but at incident energies below 100 MeV PWIA cross section is much larger than the magnitude of the experimental data. A normalization factor N is introduced:

$$N = \frac{d^3\sigma_{\text{exp}}}{d^3\sigma_{\text{pwia}}} \quad (1)$$

and its value depends on the quasifree (QF) process, it decreases when the energy of the incoming particle is reduced [11–15] and it is always smaller than one, e.g., for ${}^2\text{H}(p, 2p)n$ at $E_{\text{in}} = 15$ MeV, $N = 0.1$ [16], while at 85 MeV, $N = 0.72$ [17], for ${}^3\text{He}({}^3\text{He}, pt)2p$ at $E_{\text{in}} = 65$ MeV, $N = 0.07$ [18]. This discrepancy between theory and experiment cannot be explained completely by uncertainties in the target wave function, nor by the half-shell cross section [9].

Many papers [19–26] pointed out the importance of the Coulomb effect in quasifree processes. Kok and van

Haeringen have shown [27] that the relation

$$\sigma_{\text{half}} = \sigma_{\text{on}}(E, q^2)C_0^2(\gamma)D^2(p, k) \quad (2)$$

holds true for the case when, in addition to Coulomb, there is also a short-range potential. Here \mathbf{k} and \mathbf{p} are final and initial relative momenta and $\mathbf{q} = \mathbf{p} - \mathbf{k}$ is the momentum transfer. The quantity $D(p, k)$ is given by

$$D(p, k) = \begin{cases} 1 & p > k, \\ e^{\pi\gamma} & p < k, \end{cases} \quad (3)$$

where $\gamma \equiv ze^2/2k$ is the Sommerfeld parameter and

$$C_0(\gamma) = \frac{2\pi\gamma}{e^{2\pi\gamma} - 1} \quad (4)$$

is the Coulomb penetrability. It has been also shown that the difference between σ_{half} and σ_{on} can be large for low incident energies or for small momentum transfer [28]. Therefore, one must introduce the Coulomb interaction into PWIA.

Bajzer included the Coulomb effect in the quasifree processes treated as a three-body process with two or three charged bodies in the final state [29]. His approach is based on the exact three-body treatment [30–34] and the factorization of the exact breakup amplitude T_{0i} into a directly calculable Coulomb interaction dependent factor and the part which requires the solution of the Alt-Grassberger-Sandhas (AGS) equations:

$$|T_{0i}|^2 = |\Gamma(1 - i\eta)\Gamma(1 - i\Theta_i)|^{-2} \times \lim_{\varepsilon \rightarrow 0} |(\mathbf{p}'_j, \mathbf{q}'_j | U_{0i}(E'_j + i\varepsilon) \Psi_i \mathbf{q}_i) |_{E'_j = E_i}. \quad (5)$$

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Here index $i \in \{1, 2, 3\}$ denotes the initial channel and Ψ_i , E_i are the bound-state wave function and the total energy in this channel. Index 0 denotes the final (breakup) channel. \mathbf{p}_i and \mathbf{q}_i are relative momentum variables invariant to the choice of coordinate system, defined as

$$\begin{aligned} \mathbf{p}_i &= \epsilon_{jk}(m_j \mathbf{k}_k - m_k \mathbf{k}_j)/(m_j + m_k), \\ \epsilon_{12} &= \epsilon_{23} = \epsilon_{31} = 1, \quad \epsilon_{ij} = -\epsilon_{ji}, \\ \mathbf{q}_i &= [m_i(\mathbf{k}_j + \mathbf{k}_k) - (m_j + m_k)\mathbf{k}_i]/(m_i + m_j + m_k), \end{aligned} \quad (6)$$

where \mathbf{k}_i are the momenta of the particles in three body c.m. (or laboratory) system and i, j , and k are the cyclic channel indexes (having values 1, 2, 3). \mathbf{p}'_j and \mathbf{q}'_j in Eq. (5) are the final-state relative momentum variables with the corresponding kinetic energy E'_j . The gamma functions in Eq. (5) describe the long-range nature of the Coulomb interaction and

$$\eta = \sum_{i=1}^3 \eta_i(p'_i), \quad \eta_i(p'_i) = e_j e_k \mu_i / p'_i,$$

$$\Theta_i = e_i(e_j + e_k)\nu_k/q_i,$$

$$\nu_i = m_i(m_j + m_k)/(m_i + m_j + m_k),$$

$$\mu_i = m_j m_k / (m_j + m_k),$$

where e_i is the charge of the particle i . The operator U_{0i} is the breakup AGS operator [30] defined as $U_{0i}(z) = (z - H_0)(z - H)^{-1}(z - H_i)$, $\text{Im } z \neq 0$, where $H = H_0 + V_1 + V_2 + V_3$ and $H_i = H_0 + V_i$ are the total and the channel Hamiltonians of the system. [We wish to note here that in the corresponding definition for the AGS operator in paper [29] Eq. (12) is incorrect and $U_{\alpha\beta}$ should be replaced by $U_{0\alpha}$. However, this mistake is not propagated in the paper.]

For the quasifree conditions, Bajzer applied the impulse approximation to formula (5) and found that the Coulomb effect, besides the already mentioned Kok-van Haeringen factor, contains also an additional factor resulting from the Coulomb interaction in the incoming channel. Assuming that \mathbf{k}_0 is the laboratory momentum of the incident particle and that \mathbf{k}_1 , \mathbf{k}_2 , and $\mathbf{k}_3 \equiv \mathbf{k}_s$ (spectator particle) are the laboratory momenta of three outgoing particles with corresponding solid angles Ω_1 , Ω_2 , and Ω_3 , and E_1 is the energy of the outgoing particle 1, the following explicit formula for differential cross section was obtained [29]:

$$\frac{d^3\sigma}{d\Omega_1 d\Omega_2 dE_1} = f_{\text{PS}} \frac{d\sigma_{pt}}{d\Omega} |\Psi_1(\mathbf{k}_s)|^2 |f_{\text{KH}}|^2 f_{\text{IC}}. \quad (7)$$

Here f_{PS} is phase space factor:

$$\begin{aligned} f_{\text{PS}} &= m_3(m_1 + m_2)^2 k_2^2 k_1 / [m_2^2 |k_2(1 + m_3/m_2) \\ &\quad + k_1 \cos \theta_{12} - k_0 \cos \theta_2|], \end{aligned} \quad (8)$$

where $\cos \theta_{12} = \hat{\mathbf{k}}_1 \cdot \hat{\mathbf{k}}_2$, $\cos \theta_2 = \hat{\mathbf{k}}_0 \cdot \hat{\mathbf{k}}_2$, and $\Psi_1(\mathbf{k}_s) = \langle \mathbf{k}_s | \Psi_1 \rangle$ is the bound-state wave function of the spectator particle “s” and the transfer particle “t” calculated at the final momentum of the spectator particle. In Eq. (7) the $d\sigma_{pt}/d\Omega$ is a two body differential cross section at the energy E and at the scattering angle θ between the transfer particle and projectile “p,”

$$\begin{aligned} E &= P^2(m_1 + m_2)/(2m_1 m_2), \quad \cos \theta = \hat{\mathbf{Q}} \cdot \hat{\mathbf{P}}, \\ \mathbf{P} &= \mathbf{p}'_3 = (m_1 \mathbf{k}_2 - m_2 \mathbf{k}_1)/(m_1 + m_2), \\ \mathbf{Q} &= -\mathbf{k}_0 + m_1(\mathbf{k}_1 + \mathbf{k}_2)/(m_1 + m_2), \end{aligned} \quad (9)$$

that is, between the particles taking part in QFS. The factor f_{KH} in Eq. (7) is the Kok-van Haeringen factor [see (3) and (4)]:

$$f_{\text{KH}}(P, Q) = C_0(\eta_3)D(P, Q), \quad \eta_3 = e_1 e_2 \mu_3 / P, \quad (10)$$

which together with Coulomb distortion factor in incident channel f_{IC} ,

$$f_{\text{IC}}(k_0) = |\Gamma(1 - i\Theta_1)|^{-2} = \sinh \pi \Theta_1 / \pi \Theta_1, \quad (11)$$

$$\Theta_1 = e_1 e_2 m_1 / k_0,$$

represents the whole Coulomb distortion factor,

$$f_{\text{C}} = |f_{\text{KH}}(P, Q)|^2 f_{\text{IC}}(k_0). \quad (12)$$

Both f_{IC} and f_{KH} appear as multiplicative factors in Eq. (7) for the QF cross section. The factor f_{IC} is greater than 1 and it tends to 1 when incident energy increases. The factor f_{KH} is smaller than 1.

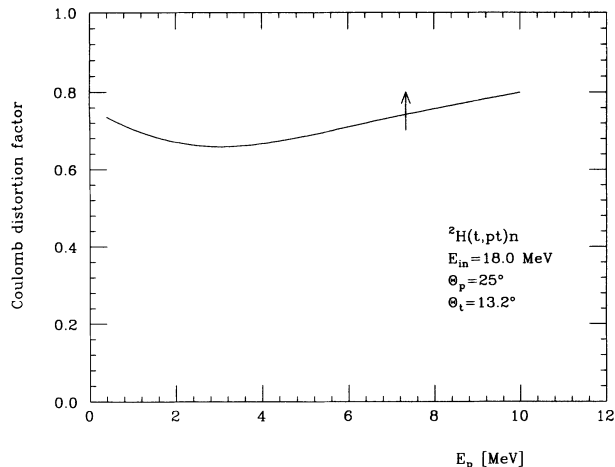
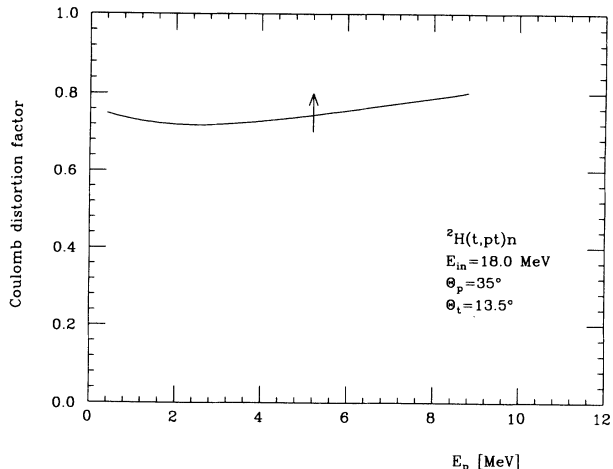
The aim of this work is to determine the Coulomb effects using Eqs. (12) and (7) for many different processes and a broad set of kinematical conditions. The purpose is to determine under which conditions this inclusion of Coulomb effects tends to eliminate the discrepancy between the PWIA and the experimental cross section.

As it has been emphasized by Bajzer [29] this treatment of the Coulomb effect is expected to be applicable for QF processes with a neutral particle as a spectator. If the spectator particle is charged, possibly important Coulomb effects are not taken into account by the presented formulae. However, for comparison we included several such processes in our considerations.

II. RESULTS AND DISCUSSION

We restricted our study to energies less than 600 MeV. At energies higher than 600 MeV, the PWIA explains the magnitude of the QFS data quite well [17,35,36] and Coulomb effects are negligible. Our interest here is primarily on the few-body systems and consequently we limited ourselves to systems with less than 6 nucleons.

Our model has been developed for QFS processes and it is strictly valid only when the spectator particle is neutral and therefore, there is no Coulomb interaction between the QFS pair and the spectator. Nevertheless, we will apply our model also to those QF processes where

FIG. 1. Coulomb distortion factor for ${}^2\text{H}(t,pt)n$ QFS.FIG. 2. Coulomb distortion factor for ${}^2\text{H}(t,pt)n$ QFS.

the spectator is a charged particle, and also to QFR. We will show later that our model is about equally successful for true QFS processes as well as for those with a charged spectator and for QFR processes. This result, maybe, provides an *a posteriori* justification for applying our model so broadly.

Differential cross section for QF processes is given by Eq. (7), and specifically, the Coulomb correction is given by two factors f_{KH} and f_{IC} . The absolute square of the two body amplitude is replaced by an experimental two body cross section $\frac{d\sigma_{pt}}{d\Omega}$ at the energies and angles given

by formula (9). Experimental two-body cross sections are those from [37].

From Eqs. (7)–(12) one sees that the Coulomb correction increases as the incident energy decreases (i.e., factor f_C becomes smaller and smaller) and that it depends only on the outgoing particles at the QF vertex and on energy T_{12} between them (see Table I). For example, the Coulomb correction factors f_C for the processes ${}^4\text{He}(p,pt){}^1\text{H}$ and ${}^3\text{H}({}^2\text{H},pt)n$ with T_{12} energies (relative energy of two detected particle 1 and 2 in the exit channel) of 5 MeV for the two QF particles, are identical,

TABLE I. Coulomb distortion factor at energies 5, 10, 20, and 40 MeV for various QFS and QFR.

Quasifree process	$T_{12} = 5$ (MeV)	$T_{12} = 10$ (MeV)	$T_{12} = 20$ (MeV)	$T_{12} = 40$ (MeV)
${}^2\text{H}(p,pp)n$	0.853	0.895	0.925	0.947
${}^3\text{H}(p,pp)2n$	0.853	0.895	0.925	0.947
${}^3\text{He}(p,pp){}^2\text{H}$	0.853	0.895	0.925	0.947
${}^4\text{He}(p,pp){}^3\text{H}$	0.853	0.895	0.925	0.947
${}^2\text{H}({}^2\text{H},pd)n$	0.832	0.879	0.914	0.939
${}^3\text{H}({}^2\text{H},pd)2n$	0.832	0.879	0.914	0.939
${}^3\text{He}({}^2\text{H},pd){}^2\text{H}$	0.832	0.879	0.914	0.939
${}^4\text{He}({}^2\text{H},pd){}^3\text{H}$	0.832	0.879	0.914	0.939
${}^4\text{He}(p,pd){}^2\text{H}$	0.832	0.879	0.914	0.939
${}^3\text{He}(p,pd){}^1\text{H}$	0.832	0.879	0.914	0.939
${}^3\text{H}(p,pd)n$	0.832	0.879	0.914	0.939
${}^4\text{He}(p,pt){}^1\text{H}$	0.821	0.872	0.909	0.935
${}^3\text{H}({}^2\text{H},pt)n$	0.821	0.872	0.909	0.935
${}^2\text{H}(t,pt)n$	0.821	0.872	0.909	0.935
${}^3\text{H}(t,pt)2n$	0.821	0.872	0.909	0.935
${}^3\text{He}(t,pt){}^2\text{H}$	0.821	0.872	0.909	0.935
${}^4\text{He}({}^2\text{H},pt){}^2\text{H}$	0.821	0.872	0.909	0.935
${}^4\text{He}(t,pt){}^3\text{H}$	0.821	0.872	0.909	0.935
${}^4\text{He}(p,p{}^3\text{He})n$	0.667	0.756	0.823	0.873
${}^2\text{H}({}^3\text{He},p{}^3\text{He})n$	0.667	0.756	0.823	0.873
${}^3\text{H}({}^3\text{He},p{}^3\text{He})2n$	0.667	0.756	0.823	0.873
${}^3\text{He}({}^3\text{He},p{}^3\text{He}){}^2\text{H}$	0.667	0.756	0.823	0.873
${}^4\text{He}({}^3\text{He},p{}^3\text{He}){}^3\text{H}$	0.667	0.756	0.823	0.873

TABLE II. Coulomb distortion factor at various T_{12} for different QF processes.

QFS	$T_{12} = 0.1$ (MeV)	$T_{12} = 0.5$ (MeV)	$T_{12} = 1.0$ (MeV)	$T_{12} = 5.0$ (MeV)	$T_{12} = 10$ (MeV)	$T_{12} = 20$ (MeV)	$T_{12} = 40$ (MeV)	$T_{12} = 50$ (MeV)
$p-p$	0.272	0.589	0.694	0.853	0.895	0.925	0.947	0.952
$p-d$	0.215	0.536	0.651	0.832	0.879	0.914	0.939	0.945
$p-t$	0.192	0.513	0.632	0.821	0.872	0.909	0.935	0.942
$d-d$	0.143	0.460	0.588	0.798	0.854	0.895	0.925	0.933
$d-t$	0.115	0.422	0.555	0.778	0.839	0.885	0.918	0.926
$t-t$	0.0847	0.377	0.514	0.755	0.822	0.872	0.909	0.918
$p-^3\text{He}$	0.024	0.236	0.377	0.667	0.756	0.823	0.873	0.886
$d-^3\text{He}$	0.0072	0.151	0.282	0.596	0.700	0.780	0.841	0.857
$t-^3\text{He}$	0.0036	0.116	0.238	0.559	0.671	0.758	0.824	0.842
$^3\text{He}-^3\text{He}$	3.0×10^{-6}	0.008	0.040	0.290	0.435	0.568	0.677	0.706
$^4\text{He}-^4\text{He}$	7.0×10^{-7}	0.005	0.034	0.271	0.406	0.533	0.642	0.673

TABLE III. Influence of the Coulomb factor f_C on the normalization factor N for different QF processes and different kinematical conditions.

QF process	E_{in} (MeV)	θ_1	θ_2	f_C	N	$N' = N/f_C$
$p-p$ QFS	14.45	30.0	30.0	0.814	0.206	0.253
$^2\text{H}(p, 2p)n$	65.00	43.6	43.6	0.938	0.560	0.597
	85.00	43.6	43.6	0.946	0.720	0.761
	100.0	43.6	43.6	0.951	0.770	0.810
$p-d \rightarrow p-d$ QFS	19.8	32.0	23.9	0.821	0.232	0.283
$^2\text{H}(^2\text{H}, pd)n$	24.8	25.0	25.0	0.844	0.260	0.308
	24.8	32.0	25.2	0.844	0.255	0.302
	30.6	32.0	26.0	0.864	0.223	0.258
$d-d$ QFS	35.0	32.0	32.0	0.837	0.170	0.203
$^3\text{H}(^2\text{H}, dd)n$	35.0	34.0	43.3	0.862	0.160	0.186
$^3\text{He}(^2\text{H}, dd)^1\text{H}$	35.0	32.0	32.0	0.838	0.130	0.155
$d-d \rightarrow p-t$ QFR	35.0	34.0	43.3	0.862	0.060	0.070
$^3\text{H}(^2\text{H}, pt)n$						
$^3\text{He}-p$ QFS	50.0	20.0	20.0	0.735	0.078	0.106
$^3\text{H}(^3\text{He}, p^3\text{He})2n$						
$^3\text{He}-d$ QFS	50.0	37.5	30.0	0.737	0.160	0.217
$^3\text{H}(^3\text{He}, d^3\text{He})n$						
$^3\text{He}-n \rightarrow p-t$ QFR	50.0	37.0	18.18	0.762	0.530	0.690
$^2\text{H}(^3\text{He}, pt)^1\text{H}$						
$^3\text{He}-d \rightarrow p-\alpha$ QFR	50.0	37.5	30.0	0.768	0.100	0.130
$^3\text{H}(^3\text{He}, p\alpha)n$						

TABLE IV. Normalization factors [17,35,36,39] N and N' with and without factor f_C , respectively, and factor N'' that includes D state of deuteron.

E_{in} (MeV)	$^2\text{H}(p, 2p)n$ QFS			N'' obtained after correction of N' by 7% D state
	N	f_C	$N' = \frac{N}{f_C}$	
65	0.56	0.938	0.60	0.64
85	0.72	0.946	0.76	0.81
100	0.77	0.950	0.81	0.87
145	0.82	0.960	0.85	0.91
200	0.86	0.965	0.89	0.95
600	0.92	0.980	0.94	1.00

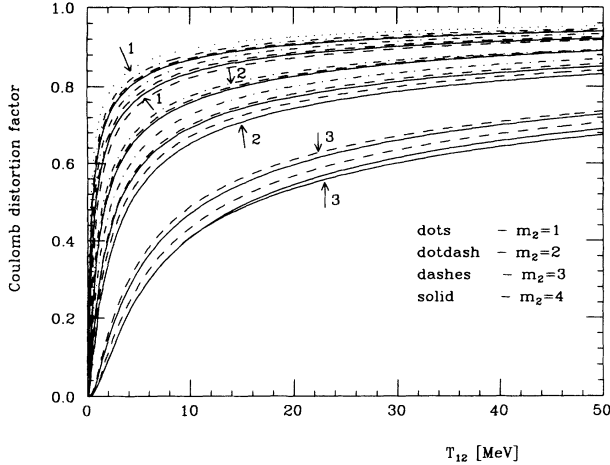


FIG. 3. Coulomb distortion factor as a function of the relative energy between particles 1 and 2. Several distinct families correspond to charges e_1 and e_2 of particles 1 and 2 in the exit channel: $(e_1, e_2) = (1, 1)$, $(1, 2)$, and $(2, 2)$, from top to bottom. Arrows with labels 1, 2, and 3, respectively, encompass each family. Each family is composed of various mass combinations $[m_1, m_2]$. For family (1,1) charge combination, $[m_1, m_2] = [1, 1], [1, 2], [1, 3], [1, 4], [2, 2], [2, 3], [2, 4],$ and $[3, 3]$. For family (1,2) charge combination, $[m_1, m_2] = [1, 2], [1, 3], [1, 4], [2, 2], [2, 3], [2, 4], [3, 3],$ and $[3, 4]$, and for family (2,2) charge combination, $[m_1, m_2] = [2, 3], [2, 4], [3, 3], [3, 4],$ and $[4, 4]$.

$f_C = 0.821$, though the two QF processes are totally different: one is p - t elastic scattering and the other is a ${}^2\text{H}(d, p){}^3\text{H}$ reaction (see Table I). Also, the factor f_C for kinematics conditions corresponding to the QF peak (i.e., zero momentum transfer for the spectator) does not depend on angles θ_1 and θ_2 of the two outgoing detected particles. In Figs. 1 and 2 we show the factor f_C as a function of detected particle energy for the reaction ${}^2\text{H}({}^3\text{H}, pt)n$ at two different angles. Although the factor

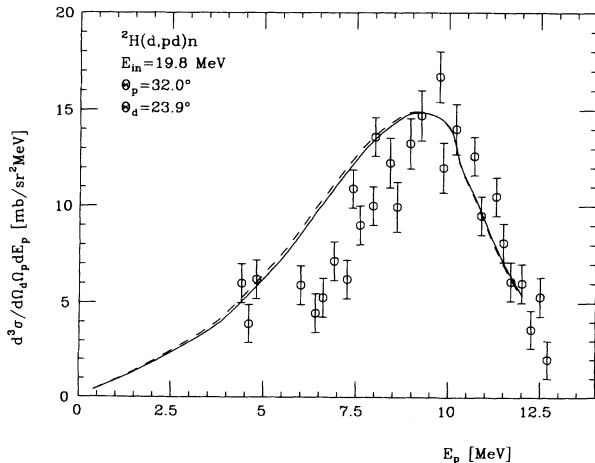


FIG. 4. Comparison between experimental data, PWIA $\times N$ data ($N = 0.232$ dotted line) and PWIA $\times N \times f_C$ data ($f_C = 0.82$, $N' = 0.282$ solid line).

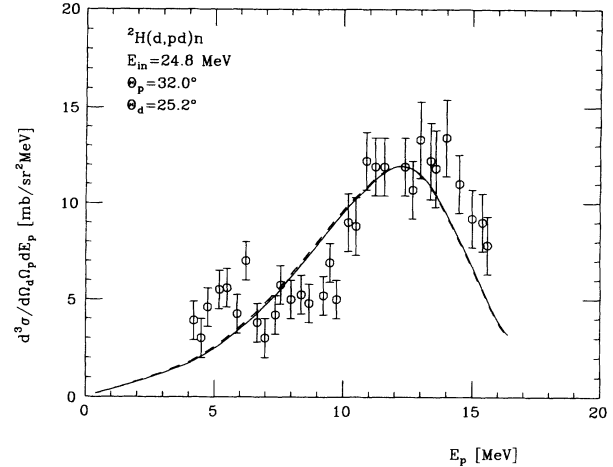


FIG. 5. Comparison between experimental data, PWIA $\times N$ data ($N = 0.255$ dotted line) and PWIA $\times N \times f_C$ data ($f_C = 0.84$, $N' = 0.303$ solid line).

f_C varies with the detected particle energy we see that at the exact QF conditions labeled by the arrow, the f_C factor has the same value for both cases.

Since at the quasifree peak the Coulomb contribution factor f_C depends only on the particles in the exit channel and energy between them, we present f_C in Fig. 3 as a function of T_{12} . Several distinct families of f_C vs T_{12} curves can be seen in Fig. 3, depending on the charges in the incident channel. Each family is composed of the curves describing f_C for various masses in the exit channel (see also Tables I and II). Figures 1 and 2 show that f_C varies with relative energy of particles 1 and 2 and therefore it is possible that f_C appreciably modifies the theoretically predicted spectra, not only their magnitude, but also their shape. However, the dependence of f_C on the relative momentum of the outgoing particles in all cases close to the QF process is almost constant as can be seen in Figs. 4 and 5. At energies higher than 100 MeV, factor f_C changes so slowly with the energy of a detected particle that it does not influence the spectra at all.

Table III summarizes the effect of the Coulomb correction factor f_C on the PWIA predicted cross section. The inclusion of the factor f_C changes the overall normalization factor N that has to be used to match the calculation to the data. Two columns N and N' give values without and with the factor f_C for several QFS and QFR and for various incident energies. One concludes from Table III that the inclusion of the factor f_C always improves the

TABLE V. Energy dependency of the Coulomb factor f_C for the ${}^2\text{H}({}^3\text{He}, p{}^3\text{He})n$ QFS.

${}^2\text{H}({}^3\text{He}, p{}^3\text{He})n$ QFS	
E_{in} (MeV)	f_C
100	0.832
200	0.883
400	0.918
600	0.934

TABLE VI. Coulomb factor f_C for different QF processes at energy projectile $E_{in} = 600$ MeV.

QF Process	f_C at $E_{in} = 600$ MeV
${}^3\text{H}(p, pd)n$	0.981
${}^3\text{H}({}^3\text{He}, p\alpha)n$	0.947
${}^3\text{He}(p, p{}^2\text{He})n$	0.963
${}^3\text{He}({}^3\text{He}, p{}^3\text{He}){}^2\text{H}$	0.934
${}^4\text{He}(H, p{}^3\text{He})n$	0.963
${}^4\text{He}({}^3\text{He}, p{}^3\text{He}){}^3\text{H}$	0.930

agreement between the PWIA and the data. However, at energies below 100 MeV the multiple scattering series does not converge and the PWIA is a very poor description of the data, and consequently the correction f_C does not bring the calculation close enough to the data.

At energies higher than 100 MeV, the assumptions built into the PWIA are more and more satisfied [7,38]. Therefore, it is of interest to study how well the PWIA with Coulomb factors f_C describes the data and whether the Coulomb correction f_C eliminates all differences between the PWIA and the experiment. For this purpose we considered the QF processes where for the target we chose ${}^2\text{H}$, ${}^3\text{H}$, ${}^3\text{He}$, ${}^4\text{He}$ and for the projectile p and ${}^3\text{He}$ with energies from 65 (or 100 for ${}^3\text{He}$) to 600 MeV. Table IV lists Coulomb and normalization factors for ${}^2\text{H}(p, 2p)n$ QFS from 65 to 600 MeV, Table V lists factor f_C for the reaction ${}^2\text{H}({}^3\text{He}, p{}^3\text{He})n$ from 100 to 600 MeV and Table VI for reactions with ${}^3\text{H}$, ${}^3\text{He}$, and ${}^4\text{He}$ targets at 600 MeV energy. At energies higher than 100 MeV the PWIA almost accounts for the data, i.e., the predicted cross section is only 10–22% larger than the measured one [17,38,39]. Coulomb factors f_C account for a part of this difference 2–7%, the rest of difference is related to

the fact that the ground state of the nucleus containing the spectator is not only the S state, but it contains the D state: 4–7% in ${}^2\text{H}$, 4–11% in ${}^3\text{H}$, and 5–15% in ${}^3\text{He}$ and ${}^4\text{He}$, depending on the potential for which the wave function is being calculated. When the correct ground-state function is used then the PWIA together with the present Coulomb correction factor f_C fully accounts for the data at higher energies $N'' = 1$ at 600 MeV for the process ${}^2\text{H}(p, 2p)n$ (see Table IV).

III. CONCLUSION

We conclude that at low energies the Coulomb correction factor f_C can change the spectator model predicted values even for several orders of magnitude without, however, appreciably changing the shapes of the spectra. For the QFS conditions, the factor f_C will depend only on the incident particle energy and on the outgoing particles in the QF vertex. The Coulomb effect cannot explain the entire discrepancy between the spectator model and the experiment at energies below 100 MeV. At higher energies the inclusion of the Coulomb corrections brings the spectator model into full agreement with the experiment if the proper partial wave components of the wave function (describing the spectator-transfer particle system) are taken into account.

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