Total neutron-nucleus cross sections and color transparency

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The neutron-nucleus cross section at Fermi lab energies is computed using Glauber-Gribov multiple scattering theory. The effects of higher moments in the cross section fluctuations are included and their physical origin discussed. The validity of the frozen approximation is critically examined. These studies of the nucleon-nucleus total cross sections provide a test of the $pp \rightarrow Xp$ diffractive amplitudes used in calculations of color transparency effects.

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Color transparency has been the subject of much recent experimental [1-3] and theoretical attention, see e.g., Refs. [4-9]; for reviews see Refs. [10, 11]. Color transparency is the suppression of initial- or final-state interactions of a hadron in high momentum transfer exclusive reactions on a nuclear target. This novel effect is a consequence of the proposition that a small object (ejected wave packet or ejectile) is produced in a high momentum transfer reaction.

Color transparency effects were first postulated [12, 13] and evaluated [5] using a quark basis, but can also be expressed in terms of a hadronic basis [8, 9]. In this case, one treats the ejectile as a coherent sum (or integral) of baryonic states. The ejectile-nucleus interaction then depends on the baryon-nucleon amplitudes for diffractive dissociation (DD) $(pp \rightarrow Xp)$. Such cross sections have been measured (see e.g., the review [14]), and have been used as inputs in calculations [9, 15] of color transparency effects. In particular, a recent calculation [15] using the measured DD cross sections has had reasonable success in reproducing the measured (p, pp) data of Ref. [1]. Even so, the need to consider off-shell extrapolations and unmeasured phases causes uncertainties in calculations of color transparency effects. Thus it is worthwhile to search for further tests of the inputs to such calculations.

The total nucleon-nucleus cross sections provide another set of observables that are sensitive to the effects of these very same diffractive dissociation cross sections. This sensitivity arises from the importance of inelastic shadowing processes [16] in which the incident nucleon can be converted to an excited or resonant state, N^{*}, by its interaction with a nucleon in the target and deexcited by another. The forward diffractive dissociation cross sections are used as input in computations of the effects of inelastic shadowing, Ref. [17]. Thus the total nucleon nucleus cross section can provide a test of the inputs and (as we shall see) the approximations used in color transparency calculations. The inelastic shadowing corrections to the neutronnucleus total cross section are also of interest in their own right. Questions related to the convergence of the moment expansion and the validity of the frozen approximation can be examined.

Our purpose here is to study the various frequently used approximations and show that the neutron-nucleus total cross sections can indeed be described in a model that is consistent with our color transparency calculations [15, 9]. However, the standard nucleonic multiple scattering series gives the dominant contribution to the total cross section, so the present comparison with data is not a very severe test.

Our approach is based on combining the standard Glauber multiple-scattering contribution with the inelastic shadowing term of Karmanov and Kondratyuk [17]. Thus, the present calculation starts with an approach very similar to the work of Murthy *et al.* [18] except that we use modern versions of the input data: nuclear densities, nucleon-nucleon forward scattering amplitude, and diffraction dissociation cross sections. These changes in input cause substantial changes in the results, so it is necessary to recheck the original conclusion that inelastic shadowing terms are necessary. We also extend the model by allowing the N^*N total cross section.

There is a more recent calculation of neutron-nucleus scattering by Nikolaev [19], claiming that the total cross section data cannot be reproduced without allowing the nucleon-nucleon total cross section to be significantly (5– 15%) larger for bound target nucleons than for free target nucleons. In contrast, we find that such an enhancement does not seem to be required. We comment more on this below.

We now describe our calculation in detail. We include two contributions to the total cross section. The first is the ordinary Glauber term σ_G which includes the effects of multiple scattering series of nucleons:

$$\sigma_G = 4\pi \operatorname{Re} \int_0^\infty \left\{ 1 - \left(1 - \frac{1}{2} (1 - i\alpha) \sigma_T / A \int dz \rho(r) \right)^A \right\} b \ db. \tag{1}$$

The above expression is obtained in the first-order optical potential approximation in which the effects of correlations between bound nucleons are neglected. The density ρ is obtained by removing the effects of the nucleon charge densities from the nuclear charge distributions measured in low energy electron nuclear scattering [20] and then performing a convolution integral between the resulting density and the range of the interaction, represented by the size of the scattering neutron-nucleon system. This size was obtained from the t dependence of the proton-proton elastic scattering cross section and is energy dependent. The total proton-nucleon cross section σ_T is taken from the parametrized form in the Particle Data Book [21] and the ratio of real to imaginary part of the forward amplitude α was taken from Block *et al.* [22]. At high energies α goes to ≈ 0.2 . Murthy et al. [18] took the ratio of real to imaginary part of the forward amplitude to go to zero for high energy (s) while there is some evidence that it changes sign and remains finite [21, 22]. The computed value of σ_G is sensitive to the value of α . For example, setting α to zero at our lowest

The integrand in Eq. (1) is frequently replaced by the exponential approximation $1 - \exp[-(1 - i\alpha)\sigma/2\int dz\rho(r)]$. The use of the exponential expression can cause up to 5% errors for light nuclei. This is significant at the precision we are working in this calculation but is probably not significant for the color transparency calculations which currently require less accuracy.

The second contribution to the total cross section comes from the inelastic scattering correction Δ_{inel} in which the projectile nucleon is diffractively excited in its interaction with a target nucleon and then de-excited in a second scattering. We find that the total nuclear cross section $\sigma_{\text{tot}}(A)$ is then given by

$$\sigma_{\rm tot}(A) = \sigma_G - \Delta_{\rm inel} \tag{2}$$

with:

$$\begin{aligned} \Delta_{\rm inel} &= \frac{(4\pi)^2}{4} \int db^2 dz_1 dz_2 \rho(b, z_1) \rho(b, z_2) \int dM_x^2 \frac{d^2 \sigma(t=0)}{dM_x^2 dt} \\ &\times [\Theta(z_1 - z_2) e^{i(p_m - p_{\rm lab})z_1} e^{i(p_{\rm lab} - p_m)z_2} e^{-\int_{-\infty}^{z_2} dz' \rho(b, z')\sigma_T/2} e^{-\int_{z_1}^{\infty} dz' \rho(b, z')\sigma_T/2} e^{-\int_{z_2}^{z_1} dz' \rho(b, z')\sigma_r/2} \\ &+ \Theta(z_2 - z_1) e^{i(p_m - p_{\rm lab})z_2} e^{i(p_{\rm lab} - p_m)z_1} e^{-\int_{-\infty}^{z_1} dz' \rho(b, z')\sigma_T/2} e^{-\int_{z_2}^{\infty} dz' \rho(b, z')\sigma_T/2} e^{-\int_{z_1}^{z_2} dz' \rho(b, z')\sigma_r/2}], \end{aligned}$$

$$\end{aligned}$$

$$\end{aligned}$$

$$\end{aligned}$$

where b is the impact parameter, and $d^2\sigma(t=0)/dM_x^2 dt$ is the diffractive differential cross section for the process $N + N \rightarrow N + X$ evaluated at t = 0. The quantity σ_r represents total cross section for excited statenucleon scattering and can depend on M_x^2 . It is useful to define the difference between the two cross sections as $\Delta\sigma = \sigma_r - \sigma_T$. In the limit $\Delta\sigma = 0$, Δ_{inel} reduces to the expression given by Karmanov and Kondratyuk [17]:

$$\Delta_{\text{inel}} = 4\pi \int d^2b \int dM^2 \frac{d^2\sigma(t=0)}{dM^2 dt} \\ \times \exp\left(-\frac{1}{2}\sigma_T T(b)\right) |F(q_L,b)|^2 ,$$
(4)

where $T(b) = \int_{-\infty}^{\infty} \rho(b, z) dz$, $q_L = p_{\text{lab}} - \sqrt{M^2 - M_x^2 + p_{\text{lab}}^2} \approx (M_x^2 - M^2) M/s$ is the longitudinal momentum transfer in the production of mass M_x (the nucleon mass is M) and F is the form factor $F(q_L, b) = \int \rho(b, z) \exp(iq_L z) dz$. The approximate form of q_L was found not to be of sufficient accuracy for low values of the laboratory momentum p_{lab} .

The density used in Eq. (3) is the same as for the Glauber contribution. The differential cross-section $d^2\sigma(t=0)/dM_x^2 dt$ was taken from the collection of Goulianos [14]. This consists of $pp \to Xp$ data taken at five energies. We interpolated the total cross section to obtain results at other energies. These data are newer than those used by Murthy and are significantly different. In particular, there is a strong S dependence; Murthy, in contrast, used a $d^2\sigma(t=0)/dM_x^2 dt$ that was independent of energy. This is an important point because a successful calculation should reproduce the energy dependence of the observed total cross-section data. The Goulianos cross sections are used as input in computing color transparency effects in [15], so determining if their use would produce the observed value of Δ_{inel} is an important test of that calculation. Indeed an earlier version of the color transparency calculation [8] used a single excited state which yields values of Δ_{inel} that are too large by more than a factor of 4 at the higher energies.

The data given by Goulianos [14] is for $t = 0.042 \text{ GeV}^2$, not for $t = 0 \text{ GeV}^2$. To extrapolate to $t = 0 \text{ GeV}^2$ we have used the t dependence given by Goulianos [14]. Thus the value of $d^2\sigma(t=0)/dM_x^2 dt$ is 31% higher than shown in the figures of Goulianos. The differential cross section given is not just the diffractive cross section but rather the total contribution to the $pp \to pX$ cross section. Since the inelastic correction needs the diffractive cross-section we have extrapolated from the $\approx 1/M_x^2$ region with a $M_x^{2.16}$ fall-off instead of using the data for $pp \rightarrow pX$. For those lower energies where the $\approx 1/M_x^2$ behavior is not obtained we have extrapolated from the last point given. Whenever the value of M_x^2 is greater than about 0.15 S, there is a possibility of contamination by nondiffractive contributions.

An examination of Eq. (2) shows that the nuclear form factors $F(q_L, b)$ play an essential role in reducing the contributions from large M_x^2 ; $M_x^2 > s/(5R_A/Fm)$. Indeed if we assume that the diffractive cross section falls as M_x^{-2} , then it is the form factors that make the M_X^2 integral in Δ_{inel} convergent. Even with realistic cross sections that fall-off slightly faster $(M_x^{2.16})$ the form factors play a large role and cannot be set to unity unless another cutoff mechanism is found. Setting the nuclear form factors to unity, also known as the frozen approximation, implies $q_L \approx 0$ and assumes that all the important nucleon excited states are at the same energy.

The frozen approximation is of considerable interest as it is frequently used to simplify calculations of high energy nuclear reactions. Furthermore, its approximate validity is one of the three requirements for color transparency to occur [13, 12, 8, 5]. In the frozen approximation the general expression (3) for the inelastic contribution reduces to:

$$\Delta_{\text{inel}} = -4\pi \int d b^2 dM_x^2 e^{-T(b)\sigma_T/2} \frac{d^2\sigma}{dt \, dM_x^2} \left(\frac{2}{\Delta\sigma}\right)^2 \\ \times \left[e^{-T(b)\Delta\sigma/2} - 1 + \frac{\Delta\sigma}{2}T(b)\right].$$
(5)

Note that the integral over M_x^2 can be performed immediately so that Δ_{inel} is proportional to

$$\frac{d\sigma(t=0)}{dt} \equiv \int dM_x^2 \frac{d^2\sigma(t=0)}{dM_x^2 \, dt}.$$
(6)

This quantity has been measured and can be interpreted using the diffractive-eigenstate formalism of Feinberg and Pomeranchuk [23] and Good and Walker [24], to account for coherence effects arising at high energies. In particular, the projectile can be treated as a coherent superposition of scattering eigenstates, each with an eigenvalue σ . The probability that a given configuration interacts with a nucleon with a total cross-section σ is $P(\sigma)$. Then $\sigma_T = \int d\sigma \sigma P(\sigma) \equiv \bar{\sigma}$ and [25–27]

$$\frac{d\sigma(t=0)}{dt} = \frac{1}{16\pi} \int d\sigma P(\sigma)(\sigma - \bar{\sigma})^2.$$
(7)

The right-hand side is measured and we use the value $0.25 \sigma_T^2$ of Ref. [25, 26]. The left-hand side is determined by integration of $d^2\sigma(t=0)/dt dM_x^2$ up to a value of M_x^2 such that the left- and right-hand sides of the equation are equal. This leads to values of the maximum $M_x^2(\max) \approx 0.2s$. This is actually fairly close to the sharp-cut off used in the color transparency calculations[9] but less strongly S dependent. With this upper limit, the use of the frozen approximation causes an error in the computed value of Δ_{inel} of about 5 to 10% for light nuclei and about 25% for heavy nuclei even at $p_{\text{lab}} = 256 \text{ GeV}/c$. Such errors are a very small percentage of the measured total cross section. The imperfect accuracy

of the frozen approximation occurs because $M_x^2(\max)$ is proportional to s. As s gets larger, larger values of M_x^2 become important and q_L is not small. [Using lower values of $M_x^2(\max)$ would increase the accuracy of the frozen approximation.]

In the diffraction eigenstates formalism (which requires the frozen approximation) the inelastic shadowing correction is the difference between the total cross section evaluated as an integral over σ and the integrand evaluated at $\sigma = \bar{\sigma}$ [28, 29]:

$$\Delta_{\text{inel}} = -\int d\sigma P(\sigma) \left[e^{-T(b)(\sigma-\bar{\sigma})/2} - 1 \right] e^{-\bar{\sigma}T(b)}.$$
 (8)

The equations (5) and (8) represent two expressions for the same quantity. Thus one may relate σ_r to moments of σ . This is done by making power series expansion in T(B). Equating powers of T(B) leads to the result:

$$\int dM_x^2 \frac{d^2 \sigma_{\text{diff}}}{dt \, dM_x^2} \Delta \sigma^N = \frac{1}{16\pi} \int d\sigma P(\sigma) (\sigma - \bar{\sigma})^{N+2}.$$
(9)

Thus we see that choosing a nonzero value of $\Delta \sigma$ allows one to reproduce the effects of the higher moments on Δ_{inel} . The values of σ_r necessary to reproduce the higher moments in the cross-section distribution used by [29] are presented in Table I. In most cases σ_r is between 3 and 4 σ_T . Presumably even more moments could be reproduced by allowing σ_r to depend on M_x^2 , but we shall show that Δ_{inel} is very insensitive to the values of σ_r between 3 and 4 σ_T . Thus we treat σ_r as independent of M_x^2 .

The above gives us an idea about the value of σ_r and the value of $M_x^2(\max)$. In Fig. 1, we present results using $\sigma_r = 3\sigma_T$ obtained from Eqs. (1) and (5) by integrating over M_x^2 to $M_x^2(\max)$. The data are from Refs. [18, 30, 31]. The solid curve is shows the Glauber approximation Eq. (1) with no inelastic correction. The dashed and dot-dashed curves show the effects of including inelastic shadowing using either all of the moments (dashed) or up to the fourth moment (dot-dashed). The Glauber approximation Eq.(1) lies above the data. Most of that difference is accounted for by including the effects of Δ_{inel} , but some discrepancy between the theory and the data remains. The error due to using the moment expansion increases for heavier nuclei. Although the error is relatively large for Δ_{inel} it causes little effect in the total cross section since the total inelastic correction is small compared to the total cross section.

TABLE I. The values of the σ_r/σ_t need to fit the various moments of the cross section distributions. The three columns correspond to three different momentum distributions used in Ref. [29].

Moment fit	σ_{r1}/σ_t	σ_{r2}/σ_t	σ_{r3}/σ_t
Third	4.2	4.0	4.0
Fourth	3.7	3.5	3.5
Fifth	3.5	3.3	3.3
Sixth	3.3	3.2	3.2



FIG. 1. Total cross sections for an 27 Al target in the frozen approximation. The data are from Refs. [18,30,31]. The solid curve includes only the Glauber contribution of Eq. (1). The dashed (dot-dashed) curve includes also the effects of inelastic shadowing of Eq. (5) with all the moments (up to the fourth moment) generated by using $\sigma_r = 3\sigma_T$.



FIG. 2. Total cross sections for an ²⁷Al target. The data are from Refs. [18, 30, 31]. The solid curve includes only the Glauber contribution of Eq. (1). The dot-dashed curve includes also the effects of inelastic shadowing of Eq. (5) with $\sigma_r = \sigma_T \ (\sigma_r = 3\sigma_T, \ \sigma_r = 4\sigma_T)$.



FIG. 3. Final calculation of the total cross sections for Be, C, Al, Cu, and Pb targets. The data are from Refs. [18, 30, 31]. The solid curve includes only the Glauber contribution of Eq. (1). The dot-dashed curve includes also the effects of inelastic shadowing of Eq. (3) with $\sigma_r = \sigma_T$ ($\sigma_r = 3\sigma_T$, $\sigma_r = 4\sigma_T$).



FIG. 3 (Continued).

We wish to clarify a significant difference between the assumptions used in deriving Eqs. (8) and (5). In the derivation of Eq. (5) we have assumed that the nucleon is excited to a given state, stays in that state (while possibly interacting with the nucleus through σ_r) and is then de-excited into a nucleon. Multistep processes in which an intermediate resonant state is diffractively excited into another excited state before being de-excited are not included. On the other hand Eq. (8) includes all such multistep process. By taking σ_r different from the free resonance-nucleon scattering cross section we can approximately take such effects into account — at least in the frozen limit. Thus a σ_r empirically determined from total neutron-nucleus scattering may not correspond to the real resonance-nucleon scattering cross section.

To the extent that the higher moments for the lefthand side of Eq. (9) are known we can estimate the importance of multistep processes. If we take the higher moments from Ref. [29] and use our result that the total cross section is insensitive to variations in σ_r given in the table we can conclude that the multistep processes are not very important. The downside of this is that the contributions of higher moments cannot be determined very well from the total cross-section data.

The sensitivity to σ_r is shown in Fig. 2. Adding the

inelastic contribution with $\sigma_r = \sigma_T$ gives the dotted line which tends to lie below the data. The long dasheddotted line is obtained with $\sigma_r = 3\sigma_T$ while the short dashed-dotted line uses $\sigma_r = 4\sigma_T$. These last two lines are close together in all cases. The exact choice of σ_r is, thus, not crucial.

Having determined σ_r from the consideration of the moment expansion in the frozen approximation we can use it in the general expression Eq. (3). Note, however, that our final result does not use the frozen approximation and includes all moments. This is our best calculation. The agreement shown in Fig. 3 is good except for carbon. For lead the energy dependence appears wrong but the inelastic total correction is comparable to the experimental error bars. But the differences between Figs. 2 and 3 are small. In either case, it seems that relatively little physics is missing. A reasonable description of the total cross-section data (except perhaps for carbon) is obtained without exotic effects.

We next compare our approach to that of Ref. [19]. It is assumed there that the total projectile-nucleon cross section is enhanced for bound nucleon targets. Then any small differences between the standard calculation (nucleon multiple-scattering plus inelastic shadowing) are ascribed to such enhancements. But the total neutronnucleus cross section is not very sensitive to the total nucleon cross section, so that relatively large medium effects are required. We find fairly good (but not perfect) agreement without such effects. The small differences between our calculations and the data could be due to any number of other effects such as the M_X^2 dependence of σ_r or the effects of nucleon-nucleon correlations.

Our final comment concerns color transparency (Ct). We note that the size of Ct and inelastic shadowing effects depends on similar inputs. The size of the DD amplitudes and value of $M_x^2(\max)$ used in our two calculations are found to be similar; this is reassuring. Here, as in Ct calculations, the validity of the frozen approximation depends on an imprecisely determined value of $M_x^2(\max)$. The good agreement between computed and measured total neutron-nucleus cross sections obtained here gives us some confidence in using the $d^2\sigma(t=0)/dM^2 dt$ of Ref. [14] to compute color transparency effects.

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