$K^{\pi}=1^{+}$ pairing interaction and moments of inertia of superdeformed rotational bands in atomic nuclei

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The effect of the pairing interaction coming from the rotationally induced $K^{\pi}=1^{+}$ pair-density on the nuclear moments of inertia is studied. It is pointed out that, contrary to the situation at normal deformations, the inclusion of the $K^{\pi}=1^{+}$ pairing may appreciably modify the frequency dependence of the moments of inertia at superdeformed shapes.

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The influence of large shape elongations on the pairing field in rotating nuclei is still not well understood. In the $A\sim150$ mass region, the lowest spins of known superdeformed (*sd*) bands are estimated to be around $24\hbar$ and the rotatonal frequencies are $\hbar\omega>300$ keV [1]. Due to the low level density of single-particle states in *sd* minima and large rotational frequencies, the superfluid-type correlations in nuclei around ^{152}Dy are expected to be seriously quenched and mainly of a dynamical character [2,3]. Some limited evidence for static pairing comes from the data on ^{150}Gd [4] and ^{149}Gd [5], where the large increase of the dynamical moment of inertia, $\mathcal{J}^{(2)}$, in the lower part of the *sd* band has been interpreted as a *paired* band crossing associated with an alignment of the $N=7$ neutron pair [2].

Contrary to superdeformed Gd and Dy nuclei, the *sd* bands in the $A\sim190$ nuclei are known in the low-ω region [6] where pairing correlations can play an important role. Indeed, a steady increase of $\mathcal{J}^{(2)}$ in the *sd* bands around ^{192}Hg has been attributed [7,8] to the Coriolis-antipairing effect followed by the alignment of $N=7$ quasi-neutrons and $N=6$ quasi-protons. Calculations without pairing yield fairly constant moments of inertia [9,10]. On the other hand, there are many pieces of evidence indicating that pairing correlations are rather weak also in the $A\sim190$ region. Namely, (i) the moments of inertia of *sd* bands in this region are very similar and blocking effects are very weak [11], and (ii) even a strongly reduced pairing field yields too strong a quasiparticle alignment; i.e., contrary to the particle-number projected cranking calculations including monopole pairing [8,12], the dynamic moments of inertia were found to increase continuously with rotational frequency. For instance, in the nucleus ^{192}Hg $\mathcal{J}^{(2)}$ keeps rising in the whole range of observed frequencies, i.e., $\hbar\omega<450$ keV [13].

In the low-ω region the dynamic moment of inertia, $\mathcal{J}^{(2)}\equiv dI/d\omega$, can be expressed by a two-parameter Harris expansion:

$$\mathcal{J}^{(2)}=\alpha_J+\beta_J(\hbar\omega)^2. \qquad (1)$$

For the nuclei around ^{192}Hg, the typical values of expansion parameters in (1) are $\alpha_J\approx$86–88 $\hbar^2$/MeV and $\beta_J\approx$280–360 $\hbar^2$/MeV^3 (see, e.g., Ref. [14]). The moments of inertia calculated with the pairing self-consistent approaches (see, e.g., Refs. [8,15]) yield β_J values which are too large. The (too) fast rise of $\mathcal{J}^{(2)}$ remains also in the calculations with constant (ω-independent) values of pairing gaps and in the particle-number-projected calculations. (The actual value of β_J is very sensitive to the ω dependence of pairing field.) The parameter β_J is reduced only for very weak pairing strengths, but then the alignment of high-N nucleons appears too fast. Consequently, cranking calculations with monopole pairing interaction have not been able to reproduce both the low- and high-ω dependence of $\mathcal{J}^{(2)}$ at the same time.

Contrary to the particle-hole channel, where great attention has been paid to optimize the effective interactions, pairing interaction is usually approximated by a simple state-independent seniority force. Often, a parametrization of the pairing interaction is done based on the multipole expansion. Since at *sd* shapes there appear high-N, high-j single-particle orbits which are not present around the Fermi level at normal deformations, the state-dependent pairing interaction should play an important role [16].

In the limit of small ω, the moment of inertia of doubly even nuclei is given by $\mathcal{J}=\mathcal{J}_C+\mathcal{J}_M$ [17], where

$$\mathcal{J}_C=\sum_{\alpha,\beta}\frac{|\langle\alpha|j_x|\beta\rangle|^2}{E_\alpha+E_\beta}\,(u_\alpha v_\beta-v_\alpha u_\beta)^2 \qquad (2)$$

is the standard cranking-model moment of inertia derived by Beliaev [18] (E_α is a BCS quasiparticle energy and v_α, u_α are the BCS transformation coefficients) and

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$$\mathcal{J}_M = (-2) \sum_{\alpha\beta\gamma\delta} \frac{\langle\alpha|j_x|\beta\rangle}{E_\alpha + E_\beta} \mathcal{V}_{\alpha\bar{\beta}\gamma\bar{\delta}} (u_\alpha v_\beta - v_\alpha u_\beta)(u_\alpha u_\beta + v_\alpha v_\beta)(u_\gamma u_\delta + v_\gamma v_\delta) Z_{\gamma\delta}/\omega \tag{3}$$

is the Migdal term. In Eq. (3) $\mathcal{V}_{\alpha\bar{\beta}\gamma\bar{\delta}}$ is a matrix element of the two-body pairing interaction and $Z_{\alpha\beta}$ is a Thouless matrix of the Hartree-Fock-Bogoliubov (HFB) transformation between nonrotating and rotating quasiparticles (see Refs. [19,20]). It is seen that for the separable pairing interaction, e.g., $\mathcal{V}_{\alpha\bar{\beta}\gamma\bar{\delta}} = -G\langle\alpha|V|\beta\rangle\langle\gamma|V|\delta\rangle$, the Migdal term involves the product $\langle\alpha|j_x|\beta\rangle\langle\alpha|V|\beta\rangle$. Consequently, only the $K^\pi=1^+$ (e.g., Y_{21}) part of pairing interaction contributes to $\mathcal{J}_M$.

Taking the Y_{21} pairing with the coupling constant (G) obtained from the expansion of a zero-range interaction, the upper part of Fig. 1 shows an estimate of the pairing and the deformation dependence of the ratio $\mathcal{J}_M/\mathcal{J}_C$ for ^{194}Hg at ω=0 as a function of the quadrupole deformation ϵ. The calculations were carried out by means of the deformed oscillator model [17] and the modified oscillator model [19]. (The parameters of the modified oscillator model are taken from Ref. [21].) In the deformed oscillator model with frequencies $\omega_\perp$ and ω_z, there exist simple expressions for $\mathcal{J}_C$ and $\mathcal{J}_M$ [17]. In particular,

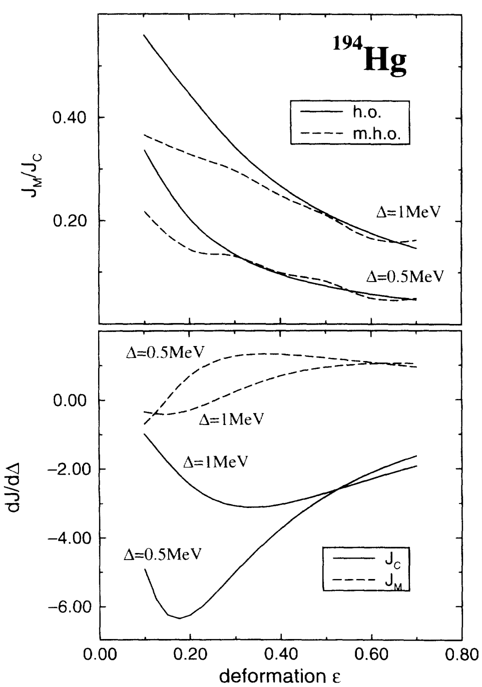

FIG. 1. Upper part [on α_J coefficient in Eq. (1)]: ratio of the Migdal to the cranking moment of inertia, $\mathcal{J}_M/\mathcal{J}_C$ as a function of quadrupole deformation ϵ calculated in the deformed oscillator model, Eq. (4), and modified oscillator model for the two values of the monopole pairing gap, Δ=0.5 MeV and 1 MeV. Lower part [on β_J coefficient in Eq. (1)]: $d\mathcal{J}/d\Delta$ values (in units of $\mathcal{J}_{\rm rigid}/\hbar\omega_{\rm osc}$) estimated using the deformed oscillator model. Note that at sd shapes ($\epsilon\sim$0.5, Δ=0.5 MeV) $(d\mathcal{J}_M/d\Delta)/(d\mathcal{J}_C/d\Delta)\sim-1/3$, while at normal deformations ($\epsilon\sim$0.25, Δ=1 MeV) $d\mathcal{J}_M/d\Delta\sim0$.

$$\frac{\mathcal{J}_M}{\mathcal{J}_C} = \frac{(g_1+g_2)^2 x_1^2 x_2^2}{(x_1^2+x_2^2-g_2x_1^2-g_1x_2^2)(g_1x_1^2+g_2x_2^2)}, \tag{4}$$

where $g(x)$=arg sh$(x)/x\sqrt{1+x^2}$, $x_1=(\omega_\perp-\omega_z)/2\Delta$, and $x_2=(\omega_\perp+\omega_z)/2\Delta$. Due to the presence of the $\boldsymbol{l}\cdot\boldsymbol{s}$ and $\boldsymbol{l}^2$ terms, the ratio calculated in the modified oscillator model deviates appreciably at smaller deformations from that in the harmonic oscillator model. However, it is seen that the calculated ratio in the two models becomes very similar at larger values of ϵ.

In the present work we study the dependence of dynamical moments of inertia on rotational frequency, taking into account the Y_{21} pairing which should naturally be included due to the presence of the $K^\pi=1^+$ pair density in a rotating system. It is known [19] that the presence of the Y_{21} pairing leads to an effective attenuation of Coriolis matrix elements. Since it has been recognized [22] that at superdeformation the variation of dynamical moments of inertia as a function of rotational frequency comes primarily from the contributions of the particles in "high-j, high-N" orbitals (in the A =190 region these are the N=6 protons and the N=7 neutrons), we diagonalize the HFB cranking Hamiltonian for a single-j shell [23,24]. In the restricted space of a single-j shell, it is not possible to calculate the strength of the pair field in a self-consistent way. Thus, for simplicity, for a given monopole pairing gap, Δ, the Y_{21} pairing field

$$\Delta_{21} \equiv G_{21} \left\langle \sum_{i,j} (Y_{21}+Y_{2-1})_{i,\bar{j}} \left(a_i^+ a_{\bar{j}}^+ + a_{\bar{j}} a_i\right) \right\rangle_{\rm HFBC} \tag{5}$$

is assumed to be proportional to the rotational frequency. The approximate proportionality was previously obtained [24,25] at normal deformation for rotational frequencies before the first band crossing. The proportionality constant is determined so that within the single-j shell space the contribution from the Y_{21} pairing to the moment of inertia, J_M, at superdeformed shapes and ω=0 is 10% of the cranking moment of inertia, irrespective of the position of the Fermi level inside the j shell (see Fig. 1).

Figure 2 displays calculated dynamical moments of inertia for a single-j (=13/2) shell, $\mathcal{J}_C^{(2)}$ (upper portion) and $\mathcal{J}_M^{(2)}$ (lower portion), as a function of ω. We note that the ω dependence of the moments of inertia calculated in the single-j-shell model can be compared qualitatively with experimental alignment patterns in sd bands, while the absolute magnitudes of calculated moments of inertia are not meaningful due to the very restricted configuration space. Namely, at sd shapes particles in orbitals other than high-N are expected to contribute weakly to the frequency-dependent part of the moment of inertia. In the case of a single-j shell the energies can be scaled by a single parameter κ [23], which is proportional to the quadrupole deformation (at superdeformation $\kappa\sim$5–

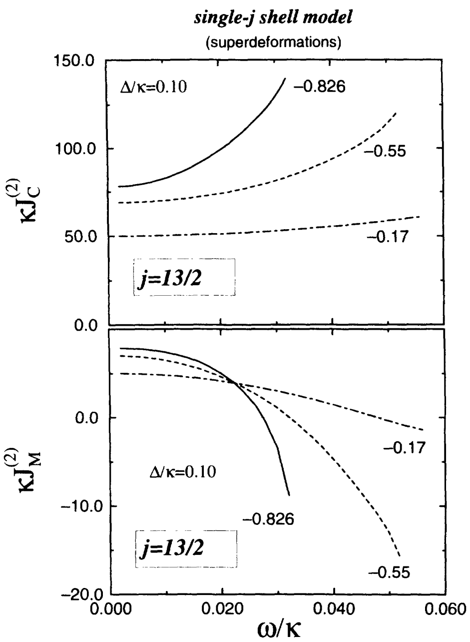

FIG. 2. Lower (upper) part: Migdal (cranking) dynamical moments of inertia calculated within a single-j shell model as a function of cranking frequency for Δ/κ=0.1. The values of λ/κ=-0.826, -0.55, and -0.17 correspond to the filling of 4, 6, and 8 particles, respectively, in the j=13/2 shell at ω=0 and Δ=0. At superdeformation the energy unit κ should be taken to be 5–6 MeV. See text for the details.

6 MeV). Thus, the monopole pairing strength used in Fig. 2 is the strength which is approximately expected for the *sd* bands in ^{192,194}Hg [4]. As numerical examples, three positions of the Fermi level are chosen, which correspond to the filling of 4, 6, and 8 particles (λ/κ= -0.826, -0.55, -0.17, respectively) in the j shell at ω=0 and Δ=0.

It is known that for the ground band of even-even nuclei the coefficient β_J in the Harris expansion (1) is positive [26]. From Fig. 2 it is seen that the coefficient $\beta_{J,M}$ is negative and its magnitude decreases with filling the j shell. In contrast, the coefficient $\beta_{J,C}$ is positive. (The magnitude of $\beta_{J,C}$ also is largest at the bottom of the shell.) The negative sign of $\beta_{J,M}$ means that the total β_J ($=\beta_{J,C}+\beta_{J,M}$) coefficient is reduced when the Y_{21} pairing is taken into account.

Using the calculated values of $\mathcal{J}_C^{(2)}$ and $\mathcal{J}_M^{(2)}$ at ω/κ=0.03, we obtain the ratio $\beta_{J,M}/\beta_{J,C}$ to be -0.22, -0.48, and -0.69 for λ/κ = -0.826, -0.55, and -0.17, respectively. We note that at the bottom of the shell (λ/κ = -0.826) the perturbation expansion (1) works poorly. Since the existing data on *sd* bands in the A = 190 region correspond to the Fermi level somewhere between λ/κ=-0.826 and -0.55 in the present model, we find that the contribution of the Y_{21} pairing to the ω dependence of $\mathcal{J}^{(2)}$ may be significant. One has to remark, however, that in microscopic calculations the coefficient β_J is very sensitive to the treatment of nuclear dynamics (mean-field changes, pairing changes, angular momentum, and particle number projections, etc.). Therefore, we feel that a fully self-consistent calculation, including the K^{π}=1^{+} pairing, should definitely be carried out.

The situation characteristic of ground-state deformations is displayed in Fig. 3 which shows $\mathcal{J}_M^{(2)}$ in the same j=13/2 shell model, but for a stronger strength of monopole pairing and $\mathcal{J}_M^{(2)}/\mathcal{J}_C^{(2)}$=0.25 [19]. At normal deformations, κ~2.5–3 MeV. Thus, the used value of Δ/κ=0.3 corresponds to Δ=0.8–0.9 MeV at the normal deformation. It is seen that, at least at low values of ω, the value of $\beta_{J,M}$ is small.

Another element, which leads to the important contribution from the Y_{21} pairing to the value of β_J at *sd* shapes, comes from the variation of Δ with rotation. It is known [27] that at the equilibrium point the contribution to β_J from the change of Δ due to rotational perturbation is proportional to $(d\mathcal{J}/d\Delta)^2$. The main contribution to the positive sign of β_J in the ground band of even-even nuclei is known to come from the coupling of pairing degree of freedom to rotaton, if the nucleus is not deformation-soft. In the lower part of Fig. 1 we show the $d\mathcal{J}/d\Delta$ values estimated using the deformed oscillator model. Taking ϵ=0.5, Δ=0.5 MeV as the parameters of the *sd* equilibrium, we obtain $(d\mathcal{J}_M/d\Delta)/(d\mathcal{J}_C/d\Delta)\sim -1/3$, i.e., the presence of the Migdal term reduces significantly the value of β_J. In contrast, at normal deformation (ϵ=0.25, Δ=1 MeV) we obtain $d\mathcal{J}_M/d\Delta\sim0$. Therefore, the corresponding contribution due to Y_{21} pairing is small.

In the case of ^{192}Hg and neighboring nuclei, the calculated neutron and proton band crossings occur at rotational frequencies appreciably lower than 400 keV, while the observed dynamical moments of inertia continue to increase at rotational frequencies higher than 400 keV. From the study of normal deformed bands (see, e.g., Refs. [20,25]), it is known that the Y_{20} pairing may efficiently increase the band-crossing frequency (see also Ref. [28] for calculations for *sd* nuclei). For simplicity, in the present study we have not considered the Y_{20} pairing, which is present already in a nonrotating system.

In conclusion, using a single-j shell model and a harmonic oscillator model, we have estimated the effect of the Y_{21} pairing on the ω dependence of the dynamical moments of inertia of doubly even nuclei before the first band crossing. It is found that, in spite of the fact that at ω=0 the Migdal moment of inertia is only ~10% of the cranking moment of inertia at *sd* shapes, it does exhibit

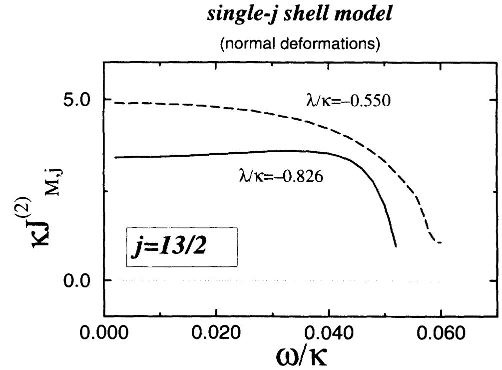

FIG. 3. Migdal dynamical moments of inertia calculated using a j=13/2 model for Δ/κ=0.3.

strong variations with rotational frequency which cancel out a significant part of the (too) strong $\mathcal{J}_C^{(2)}$ variations. Therefore, the $K^\pi=1^+$ pairing is expected to reduce appreciably the ω dependence of the moments of inertia in *sd* bands (e.g., in the Hg region), thus leading to a better agreement with experimental data.

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