Effects of the three-body force in three-nucleon systems

Shinsho Oryu and Hiroshi Yamada*
Department of Physics, Faculty of Science and Technology, Science University of Tokyo, Noda, Chiba 278, Japan
(Received 3 August 1993)

The effects of the three-body force are investigated in the three-nucleon systems. The three-body Faddeev equation is completely solved with a phenomenological three-body force which can reproduce the triton binding energy. The adopted two-body force is the PEST potential (or the Ernst-Shakin-Thaler's separable expansion of the Paris potential) up to $J = 2$. Calculated results of the differential cross section, and the values of the doublet and the quartet n-d scattering lengths, agree very well with the experimental data. The calculated three-body-force effects on the N-d scattering observables A_y, iT_{11}, T_{20}, T_{21}, T_{22} are also discussed together with the Doleschall-type Coulomb correction.

PACS number(s): 21.30.+y, 21.45.+v, 25.10.+s

The necessity of the three-body force has been claimed by a number of investigations. Phillips observed a strong correlation among the theoretically calculated n-d spin-doublet scattering length ${}^2a_{nd}$ and the triton binding energy [1]. Slaus *et al.* claimed that the three-body force is necessary to obtain consistent values of the a_{nn} scattering length from the reactions ${}^2\mathrm{H}(n,p)nn$ and ${}^2\mathrm{H}(\gamma,\pi^-)nn$ [2,3]. It was also mentioned that the discrepancy between the experimental and the theoretical triton binding energies and other observables which are related to the triton wave function can be removed by the three-body force [4–8]. In scattering problems, a few theoretical treatments have been carried out on the cross sections and other related properties [9,10]. Non-Faddeev-type approach to three- and four-nucleon calculations at negative energies has been extensively used by some authors until the 1970s [11].

In this paper, we propose a new systematic treatment to obtain the full calculation with the Faddeev equations using a three-body foce. Generally, the three-body Faddeev equations are modified by the three-body force amplitude (3BFA) which is obtained by the two-variable Lippmann-Schwinger (LS) type equation with the three-body force (3BF) [12–16]. Our treatment will not only represent some well-known three-body equations but also the probable interference between three-body force and the Coulomb force will be foreseen. Therefore, in the following, we will dare to trace them from the beginning.

Let us introduce modified Faddeev equations from the three-body Hamiltonian with two-body forces $V = V_\alpha + V_\beta + V_\gamma$, and the three-body force W,

$$H = H_0 + V + W. \tag{1}$$

The Schrödinger equation is given by

$$(E - H_0 - W)\psi = V\psi. \tag{2}$$

The formal solution of this equation satisfies the three-body Lippmann-Schwinger (LS) equation,

$$\psi = \varphi + G_3 V\psi = \varphi + G_3 T\varphi \tag{3}$$

with

$$G_3 = (E - H_0 - W)^{-1}, \tag{4}$$

where the boundary condition is given by

$$(E - H_0 - W)\varphi = 0. \tag{5}$$

The solution of this equation satisfies another LS equation by using the three-body free Green's function $G_0 = (E - H_0)^{-1}$. This equation is

$$\varphi = \varphi_0 + G_0 W\varphi = \varphi_0 + G_0 T_0\varphi_0 = \omega\varphi_0, \tag{6}$$

where ω is the Møller operator, and T_0 is the 3BFA defined by

$$\begin{aligned} T_0 &= W + WG_3W = W + WG_0T_0 \\ &= W(1 + G_0T_0) = W\omega \\ &= (1 + T_0G_0)W = \omega^*W, \end{aligned} \tag{7}$$

where G_3 is the modified Green's function or the resolvent with 3BF which satisfies also

$$G_3 = G_0 + G_0WG_3 = G_0 + G_0T_0G_0 = \omega G_0 = G_0\omega^*. \tag{8}$$

Now, the three-body free to three-body free (or $\{3\} \to \{3\}$) transition matrix is defined by

$$\langle\varphi, V\psi\rangle = \langle\varphi, T\varphi\rangle = \langle\varphi_0, \omega^*T\omega\varphi_0\rangle. \tag{9}$$

From Eqs. (3), (6), and (9), one obtains an operator equation

$$\omega^*T\omega = \omega^*V\omega + \omega^*VG_3T\omega. \tag{10}$$

Here, $\omega^*T\omega$ corresponds to the $\{3\} \to \{3\}$ transition operator. On the other hand, the elastic and the rearrangement amplitudes are given by the $\{2\} \to \{2\}$ operator, and the breakup amplitudes are given by the $\{2\} \to \{3\}$ operator. In order to obtain the $n + d \to n + d$ scattering amplitude, one should adopt the three-body operator T instead of $\omega^*T\omega$. Separating the Møller operators from

*Present address: Nakabaru Special School of Saga, Nakabaru, Miyaki, Saga 849-01, Japan.

both sides of Eq. (10), one obtains the three-body LS equation

$$T = V + VG_3T. \tag{11}$$

By the separation of the three-body amplitude $T = T^\alpha + T^\beta + T^\gamma$, the Faddeev equation is given by

$$T^\alpha = T_\alpha + T_\alpha G_3(T^\beta + T^\gamma) + T_\alpha(G_3 - G_0)T^\alpha. \tag{12}$$

Upon further expanding the amplitudes $T^\alpha = \sum_\beta T_{\alpha\beta}$, one obtains the following modified Faddeev equation in matrix form:

$$T_{\alpha\beta} = T_\alpha\delta_{\alpha\beta} + \sum_{\gamma\neq\alpha} T_\alpha G_3 T_{\gamma\beta} + T_\alpha G_0 T_0 G_0 T_{\alpha\beta}. \tag{13}$$

In order to obtain connected equations, let us introduce operators $U_{\alpha\beta}$ and $u_{\alpha\beta}$, which are defined by

$$U_{\alpha\beta} = T_{\alpha\beta} - T_\alpha\delta_{\alpha\beta} = T_\alpha G_0 u_{\alpha\beta} G_0 T_\beta. \tag{14}$$

By using Eqs. (12) and (13), it is seen that the operator satisfies the following equations:

$$U_{\alpha\beta} = T_\alpha G_0 T_\beta \overline{\delta_{\alpha\beta}} + T_\alpha G_0 T_0 G_0 T_\beta \sum_\gamma T_\alpha G_0 \overline{\delta_{\alpha\gamma}} U_{\gamma\beta} + \sum_\gamma T_\alpha G_0 T_0 G_0 U_{\gamma\beta} \tag{15}$$

with $\overline{\delta_{\alpha\beta}} = 1-\delta_{\alpha\beta}$. Here, the second and the fourth terms of the right hand side are extra operators which should be emphasized in this paper. Introducing the separable expansion of the two-body amplitude by $T_\alpha = g_\alpha\tau_\alpha g_\alpha$, one finds that the operators $U_{\alpha\beta}$ can be written as follows:

$$U_{\alpha\beta} = g_\alpha\tau_\alpha X_{\alpha\beta}\tau_\beta g_\beta. \tag{16}$$

Here $X_{\alpha\beta}$ satisfies the Amado-Lovelace-Mitra (ALM) equations [17] which are modified by 3BFA, i.e.,

$$X_{\alpha\beta} = Z_{\alpha\beta} + \sum_\gamma Z_{\alpha\gamma}\tau_\gamma X_{\gamma\beta}, \tag{17}$$

where the three-body potentials $Z_{\alpha\beta}$ are given by

$$Z_{\alpha\beta} = Z^0_{\alpha\beta} + Z'_{\alpha\beta} \tag{18}$$

with $Z'_{\alpha\beta} = g_\alpha G_0 g_\beta \overline{\delta_{\alpha\beta}}$ and $Z^0_{\alpha\beta} = g_\alpha G_0 T_0 G_0 g_\beta$. Equation (17) with (18) as a simple reduction of our operator equation (10) is the same one as Eq. (3.3) of Phillips [1] which was also rediscovered by some authors later on [10]. Likewise, the Alt-Grassberger-Sandhas equations are written by using Eqs. (14) and (15) [18]

$$u_{\alpha\beta} = (G_0^{-1}\overline{\delta_{\alpha\beta}} + T_0) + \sum_\gamma (G_0^{-1}\overline{\delta_{\alpha\gamma}} + T_0) G_0 T_\gamma G_0 u_{\gamma\beta}. \tag{19}$$

In our formalism, 3BFA (T_0) is obtained from 3BF (W) by solving Eq. (7) which is a two-variable integral equation. For the purpose of seeing the three-body-force effects in the quickest way, we assume in this paper a simple phenomenological 3BFA instead of solving Eq. (7):

$$T_0(\mathbf{p},\mathbf{q};\mathbf{p}',\mathbf{q}';E) = A_0 \exp\{-a(p^2+p'^2) - b(q^2+q'^2)\}. \tag{20}$$

Because of this 3BFA must be strictly distinguished from 3BF. For the sake of three-particle exchange symmetry, we choose $a = 4b/3$. Then, the unknown parameters are A_0 and b. Furthermore, the range parameter b is related to the well-known cutoff parameter $\Lambda = 800$ MeV [4–6] or

$$b = \frac{1}{\Lambda^2} = 0.0567 \text{ fm}^2. \tag{21}$$

^3H binding energy. The remaining parameter A_0 is fitted to obtain the experimental triton binding energy ^{3}H(BE). To this end we adopt the PEST1 $^1S_0$, $^3S_1$-$^3D_1$ potentials [19] for the Faddeev calculation, which gives -7.46 MeV without 3BFA. Here, the calculation was done by using the three-body generalized-separable-expansion (GSE) method with 30 Gaussian points [17]. Results with and without 3BFA are shown in Table I. We find that the use of $A_0 = -69.98$ fm^5 yields the experimental value of -8.48 MeV.

n-d scattering length. The n-d scattering length has been calculated by many authors [1,4,20–26]. The quartet state n-d scattering length was well represented; however, the doublet-state result did not fit the experiment except with the introduction of a sophisticated unphysical pole near the threshold-energy region [21,23,24] or three-body forces [1,4]. We confirm that our 3BFA can reproduce both of them by the same parameter as in the binding-energy case. Results are also listed in Table I. The quartet-state scattering length is not strongly affected by the 3BFA. On the other hand, the situation in the doublet-state case is quite different; here the scattering length is strongly influenced by the 3BFA. (See Fig. 1.)

Differential scattering cross section. The differential scattering cross sections are calculated at neutron laboratory energies of 2.5 and 5 MeV. Here, the channels and potentials used are the same as in the ^{3}H binding-energy case. It is well known that the elastic cross section is dominated by the quartet state [27]. However, our 3BFA has a strong influence in the doublet state, but not in the quartet state. Thus, the total cross sections are hardly affected by 3BFA, although it can be seen from Fig. 2

TABLE I. The triton binding energy and the scattering lengths $^2a_{nd}$ and $^4a_{nd}$ of the n-d spin-doublet and spin-quartet states. The calculations are carried out by using the GSE method. The PEST1 potentials for $^1S_0$ and $^3S_1$-$^3D_1$ states are adopted. The results are obtained by taking into account the three-body force amplitude (3BFA) with parameters A_0 and $b = 3a/4 = 0.0567$ fm^2 in Eq. (20).

A_0 (fm^5)	^{3}H (MeV)	$^2a_{nd}$ (fm)	$^4a_{nd}$ (fm)
0.0	−7.46	1.56	6.21
−69.98	−8.48	0.662	6.21
exp	−8.48	0.65±0.04	6.35±0.02

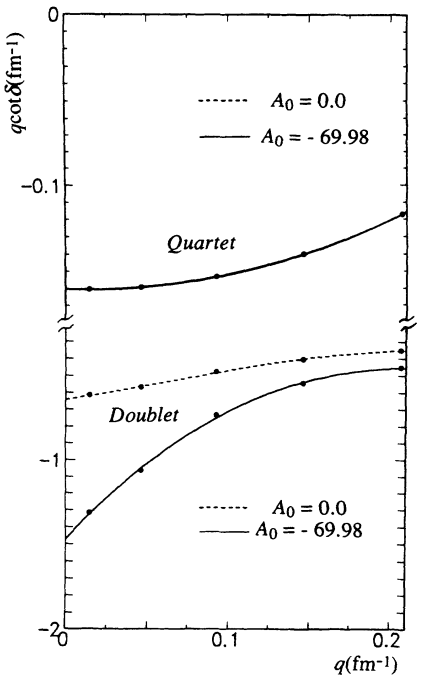

FIG. 1. The values $q\cot\delta$ for spin-doublet and spin-quartet states. The dashed lines represent values without the three-body-force amplitudes (3BFA), while the solid lines represent values with 3BFA. For the quartet state, the dashed and solid lines overlap.

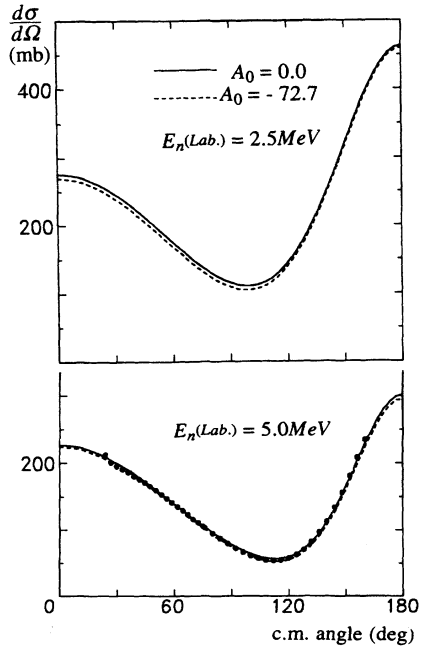

FIG. 2. Differential cross sections of n-d elastic scatterings at $E_n(\mathrm{lab}) = 2.5$ and 5.0 MeV. Solid lines are obtained without 3BFA, while the dashed lines are obtained with 3BFA. Calculations are done by using $^1S_0$, $^3S_1$-$^3D_1$ PEST1 potentials. Dotted line represents the experimental data given in Ref. [28].

that our results are in very good agreement with experimental values [28].

Polarizations. We are also interested in examining the effects of 3BFA on other scattering observables such as the nucleon polarization A_y, the deuteron polarization iT_{11}, and the deuteron analyzing powers T_{20}, T_{21}, and T_{22}. It is well known that A_y, iT_{11}, T_{20}, and T_{21} are greatly affected by the p-wave nucleon-nucleon forces [29]. In the present work, we adopted PEST1 potentials for $^1S_0$, $^3S_1$-$^3D_1$ states, PEST2 for $^1P_1$, $^3P_0$, $^3P_1$, $^1D_2$, $^3D_2$ states, and PEST3 for $^2P_2$-$^3F_2$ states. It is clear from Figs. 3–5 that 3BFA strongly affects the quantities A_y, iT_{11}, and T_{22}. Here, we have partly changed the value of the parameter A_0 (-69.98 fm^5 and/or -72.7 fm^5) to see the sensitivity to the ^{3}H binding energy; however, the difference was within the drawing accuracy in Figs. 2–4. Further one should expect that the p-wave N-N force will also be coupled with higher partial waves of a realistic 3BFA.

$p + d$ scattering with the Coulomb force. In the $p + d$ scattering, the above-mentioned formalism from Eq. (1) to Eq. (11) are also available, because the potential W is replaced by $\mathcal{W}(= W + V_c)$, and also T_0 by $\mathcal{T}(= T_0 + T_c = T_0 + \omega^* t_c \omega = \mathcal{W} + \mathcal{W}G_0\mathcal{T} = \mathcal{W}\Omega = \Omega^*\mathcal{W})$, where $t_c = V_c + V_c\mathcal{G}_3 t_c$ with $\mathcal{G}_3 = (E - H_0 - \mathcal{W})^{-1}$, and the Møller operators are given by $\Omega = (1 + G_0\mathcal{T})$ and

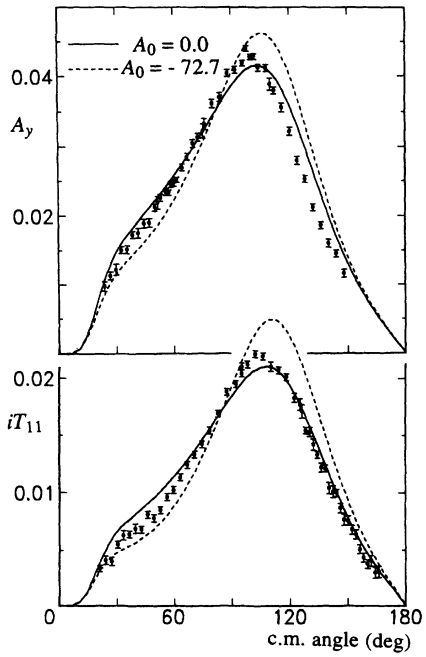

FIG. 3. The upper half is the proton polarization A_y at $E_p(\mathrm{lab}) = 2.5$ MeV. The solid line is the calculated results by using PEST1 NN potentials for $^1S_0$, $^3S_1$-$^3D_1$ states, PEST2 for $^1P_1$, $^3P_0$, $^3P_1$, $^1D_2$, $^3D_2$ states, and PEST3 for $^3P_2$-$^3F_2$ states, but without 3BFA. Dashed line is the value obtained by using the same N-N forces plus 3BFA. The Coulomb corrections were done by using Doleschall's technique. The experimental values are taken from Ref. [28]. The lower half is the deuteron polarization iT_{11} at $E_d(\mathrm{lab}) = 5.0$ MeV. The other explanations are the same as those for A_y.

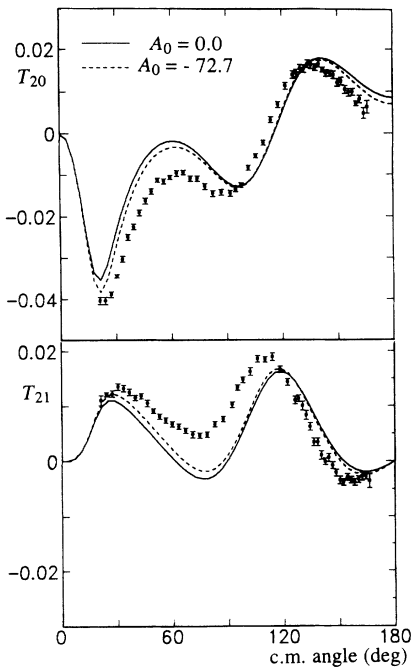

FIG. 4. Deuteron analyzing powers T_{20} and T_{21} at E_d(lab) = 5.0 MeV. The explanations are the same as those in Fig. 3.

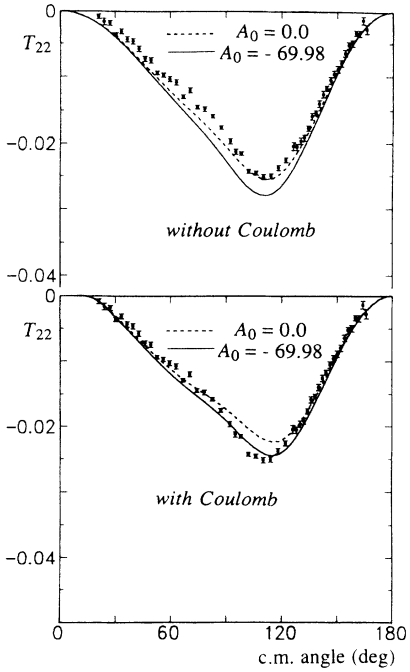

FIG. 5. The deuteron analyzing power T_{22} at E_d(lab) = 5.0 MeV. The calculations shown in the upper half plane are done by using $^1S_0$, $^3S_1$-$^3D_1$ PEST1 N-N potentials, and also with 3BFA (solid line), and without 3BFA (dashed line). The calculations shown in the lower half plane are carried out by using the same N-N forces plus the Coulomb force. The dashed line is obtained without 3BFA, and the solid line is with 3BFA. The experimental data were taken from Ref. [28].

$\Omega^* = (1 + \mathcal{T}G_0)$. Therefore, in Eq. (10), $\omega^* T\omega$ is substituted by $\Omega^* \mathsf{T} \Omega$, where T satisfies Eq. (11) with the modified Green's function $\mathcal{G}_3$. Therefore, parameters of the 3BFA will be fitted to the binding energy of ^{3}He in the framework of modified form of Eq. (18) for T. Such treatments may be carried out by an appropriate Coulomb correction method with a screened Coulomb potential. However, if we assume that Coulomb effects in the term $\omega^* t_c \omega$ could be separated into the long-range part and the short-range one, the short-range part may decrease the three-nucleon binding energy, while the long-range part modify the phase shift in the initial and the final state. The former suggests that the outcome may be given by using small modified parameters in 3BFA without the rigorous three-body Coulomb amplitude. For the latter assumption, an *ad hoc* approximation for the long-range Coulomb parts was given by Doleschall *et al.*, [30,31], although genuine n-d scattering amplitude was used in their method. In the present work we adopted the Doleschall approximation for simplicity (see Fig. 5). Due to the replacement of the potential by $\mathcal{W} = W + V_c$, it seems that the cancellation between the Coulomb force and 3BFA could appear in our formalism. One may see it in the results of T_{22}.

The aim of this paper is not to obtain a good phenomenological 3BFA with many fitting parameters, but to call our attention to the effects of the 3BFA in the scattering region. For our conclusions, it should be stressed that genuine 3BFA should be obtained by solving Eq. (7) with a three-body force. Nevertheless, our simple two-parameter 3BFA is working well in the $3N$ calculations and can represent the experimental data systematically. It is clear from the present work that the 3BFA affects $3N$ systems not only in the ^{3}H binding energy but also in the scattering properties. Our modified Faddeev equations have diagonal terms in the kernels ($Z_{\alpha\alpha} = Z^0_{\alpha\alpha} \neq 0$). Such terms are not allowed in the genuine Faddeev equations because of the compactness property of the kernel. More precise calculations with Coulomb force and realistic three-body forces will also be carried out in the scattering region.

This work was done by using IBM 9121/320 of Science University of Tokyo, FACOM M1800/20 computer of Kyushu University, FACOM M780/10S of INS (Tokyo), and also FACOM M380R of RCNP (Osaka).

[1] A. C. Phillips, Phys. Rev. **142**, 984 (1966); Nucl. Phys. **A184**, 337 (1972); **A107**, 209 (1968); Rep. Prog. Phys. **40**, 905 (1977).

[2] Ivo Slaus, Y. Akaishi, and H. Tanaka, Phys. Rev. Lett. **48**, 994 (1982); Ivo Slaus, IEEE Trans. Nucl. Sci. **NS-30**, 1128 (1983).

[3] R. J. Slobodrian, Phys. Lett. **163B**, 287 (1985); Phys. Rev. Lett. **49**, 300 (1982).

[4] C. R. Chen, G. L. Payne, J. L. Friar, and B. F. Gibson, Phys. Rev. C **33**, 401 (1986); Phys. Rev. Lett. **55**, 374 (1985); J. L. Friar, B. F. Gibson, G. L. Payne, and C. R. Chen, Phys. Rev. C **30**, 1121 (1984); J. L. Friar, B. F. Gibson, and G. L. Payne, *ibid.* **28**, 983 (1983); Phys. Lett. **124B**, 287 (1983); Annu. Rev. Nucl. Part. Sci. **34**, 403 (1984).
[5] S. Ishikawa and T. Sasakawa, Few Body Syst. **1**, 143 (1986); T. Sasakawa and S. Ishikawa, *ibid.* **1**, 3 (1986).
[6] Yong Wu, Ph.D. thesis, Tohoku University, 1992.
[7] A. Bömelburg, Phys. Rev. C **28**, 403 (1983); A. Bömelburg and W. Glöckle, *ibid.* **28**, 2149 (1983); A. Bömelburg, *ibid.* **34**, 14 (1986).
[8] H. T. Coelho, T. K. Das, and M. R. Robilotta, Phys. Rev. C **28**, 1812 (1983).
[9] J. Torre and B. Goulard, Phys. Rev. C **28**, 529 (1983); J. Torre, J. J. Benayoun, and J. Chauvin, Z. Phys. A **300**, 319 (1981).
[10] W. Meier and W. Glöckle, Phys. Lett. **138B**, 329 (1984).
[11] H. Tanaka, Nucl. Phys. **A328**, 454 (1979).
[12] Muslim, Y. E. Kim, and T. Ueda, Nucl. Phys. **A393**, 399 (1983).
[13] S. A. Coon, M. D. Scadron, P. C. McNamee, B. R. Barrett, D. W. E. Blatt, and B. H. J. McKellar, Nucl. Phys. **A317**, 242 (1979).
[14] S. A. Coon and W. Glöckle, Phys. Rev. C **23**, 1790 (1981).
[15] B. H. J. McKellar and W. Glöckle, Nucl. Phys. **A416**, 435c (1984).
[16] A. Stadler and P. U. Sauer, Phys. Rev. C **46**, 64 (1992).
[17] S. Oryu, Prog. Theor. Phys. Suppl. **61**, 180 (1977).
[18] W. Glöckle and R. Brandenburg, Phys. Rev. C **27**, 83 (1983).
[19] J. Haidenbauer and W. Plessas, Phys. Rev. C **30**, 1822 (1984); **32**, 1424 (1985).
[20] W. Dilg, L. Koester, and W. Nistler, Phys. Lett. **36B**, 208 (1971).
[21] J. Arvieux, Nucl. Phys. **A221**, 253 (1974).
[22] W. T. H. Van Oers and K. W. Brockman, Jr., Nucl. Phys. **A92**, 561 (1967).
[23] J. S. Whiting and M. G. Fuda, Phys. Rev. C **14**, 18 (1976).
[24] H. Zankel and L. Mathelitsch, Phys. Lett. **132B**, 27 (1983).
[25] E. Huttel, W. Arnold, H. Baumgart, H. Berg, and G. Clausnitzer, Nucl. Phys. **A406**, 443 (1983).
[26] L. Tomio, A. Delfino, and S. K. Adhikari, Phys. Rev. C **35**, 441 (1987); S. K. Adhikari, A. Delfino, and L. Tomio, in *Few-Body Approaches To Nuclear Reactions In Tandem and Cyclotron Energy Regions*, edited by S. Oryu and T. Sawada (World Scientific, Singapore, 1987), p. 52.
[27] W. M. Kloet and J. A. Tjon, Ann. Phys. (N.Y.) **79**, 407 (1973).
[28] S. Shimizu, K. Sagara, K. Maeda, H. Nakamura, T. Miwa, N. Nishimori, S. Ueno, and T. Nakashima, in *Book of Contribution XIII International Conference on Few-Body Problems in Physics*, edited by I. R. Afnan and R. T. Cahill (Adelaide, Australia, 1992), p. 308; K. Sagara, H. Oguri, S. Shimizu, K. Maeda, H. Nakamura, and T. Nakashina, *ibid.*, p. 284.
[29] Y. Koike and J. Haidenbauer, Nucl. Phys. **A463**, 365c (1987); J. Haidenbauer and Y. Koike, Phys. Rev. C **34**, 1187 (1986).
[30] P. Doleschall, W. Grüebler, V. König, P. A. Schmelzbach, F. Sperisen, and B. Jenny, Nucl. Phys. **A380**, 72 (1982).
[31] H. Yamada and S. Oryu, in [28], p. 310.