Nucleon-nucleon interaction with consistently soft form factors for one and two pion exchange

J. Haidenbauer,[1,2] K. Holinde,[1] and A. W. Thomas[2]
[1] *Institut für Kernphysik (Theorie), Forschungszentrum Jülich GmbH, D-52428 Jülich, Germany*
[2] *Department of Physics and Mathematical Physics, University of Adelaide, Adelaide, South Australia 5005, Australia*
(Received 3 August 1993)

It has recently proven possible to construct an acceptable one-boson-exchange potential in which the $NN\pi$ form factor is quite soft. In order to do so it was necessary to add an effective π' meson which increased the short-range tensor force. It is much less obvious that one can use such soft form factors if the two-pion-exchange diagrams are calculated explicitly rather than being parametrized as scalar meson exchange. Here we show that it is indeed possible to construct a model which describes nucleon-nucleon data with comparable quality as the original full Bonn potential, but in which the two pion exchange is included explicitly with consistently soft $NN\pi$ and $\Delta N\pi$ vertices.

PACS number(s): 21.30.+y, 13.75.Cs

I. INTRODUCTION

A microscopic understanding of the nucleon-nucleon (NN) force is an essential ingredient of modern theories of nuclear structure. Traditionally one tends to model this force from the outside in. That is, one begins with one pion exchange at the largest separations and works in through two and three pion exchange to some phenomenological parametrization at very short distance. Rather than deal explicitly with correlated two and three pion exchange it has proven very useful to work with the so-called one-boson-exchange (OBE) models. These can provide a reasonable description of NN data in terms of relatively few parameters, while being simple enough to be applied in three-, four-, and many-body systems.

From a theoretical point of view the main defect of the OBE potentials is that, in addition to the observed vector mesons (ρ and ω), one must include an isoscalar scalar meson ($\sigma_{\rm OBE}$), which definitely does not exist at low mass, but should be considered as an effective parametrization mainly of correlated and uncorrelated 2π-exchange processes in the scalar-isoscalar channel. Given the crucial role of the $\Delta(1232)$ in low-energy pion physics that is not too surprising. A large part of the phenomenological scalar exchange arises from two-nucleon irreducible diagrams involving the excitation of a virtual Δ. This was clearly demonstrated by the Paris and Stony Brook groups [1,2] through their construction of a complete two-pion-exchange (TPE) potential using dispersion relations—see also Ref. [3].

The full Bonn potential [4] was constructed to preserve as much as possible the simplicity of the OBE description while making the role of the coupling to virtual ΔN (and $\Delta\Delta$) states explicit. This permits one, for example, to examine medium modifications of the longest range TPE force. As expected the significance of the phenomenological σ exchange decreased considerably (by about 30%) once the explicit ΔN component of the TPE force was included; the remaining part is mainly built up by contributions from correlated 2π exchange, which in the (full) Bonn potential is still parametrized in terms of an effective scalar exchange (called σ' in Ref. [4]).

We have been forced to reexamine the full Bonn potential in the light of recent developments concerning one-pion exchange (OPE) which was previously considered well known. In particular, there is now overwhelming evidence that the $NN\pi$ vertex form factor is much softer than has been conventionally assumed in NN models [5–8]. In the full Bonn potential the cutoff mass $\Lambda_{NN\pi}$ was taken to be 1.3 GeV (in a monopole form-factor parametrization) in order to be able to reproduce the properties of the deuteron. On the other hand, all indications from systems other than NN indicate a value less than 1 GeV.

Within the pure OBE framework we showed recently [9] that this problem could be resolved by the introduction of a heavy π' meson (with a mass of 1.2 GeV). That is, one can obtain a comparable fit to NN data with a soft $NN\pi$ form factor as long as there is some additional short-range, isovector, tensor force. The physical interpretation of the π' is unclear as yet, but mechanisms considered include short-distance quark exchange [10] (with gluons or pions), dressing of the pion propagator [11] and correlated π-ρ exchange [12,13].

At this point one remark is in order: It was advocated in Ref. [4] that it does not make much sense to include meson exchanges with masses larger than the smallest cutoff mass, i.e., to include meson contributions explicitly which are shorter ranged than the form-factor parametrization. However, this argument holds only if the form factor is phenomenologically adjusted (as in Ref. [4]). If it is treated dynamically, representing in one possible scenario (a), the hadronic core extension or, alternatively (b), arising from explicit meson exchange diagrams (cf. Ref. [13]) then it is perfectly legitimate to include shorter-ranged mechanisms like explicit quark-exchange diagrams in case of (a) or correlated $\pi\rho$ exchange in case of (b).

In retrospect it is not so surprising that one can live with a soft form factor in an OBE model if π' exchange is added. What is less obvious is whether this is also possible in a more extended treatment like in the (full) Bonn potential, in which a good part of the effective scalar exchange is described explicitly in terms of (uncorrelated) 2π-exchange diagrams. Obviously, these terms are influenced by the choice of $NN\pi$ and $N\Delta\pi$ form factors whereas in OBE models, the effective σ exchange is assumed to be unchanged.

The purpose of the present paper is to show that the full Bonn potential can also accommodate a soft $NN\pi$ and $N\Delta\pi$ form factor provided the $N\Delta\pi$ coupling constant is increased to a value which in fact appears more realistic.

There is another at least as interesting possibility with regard to the one-pion-exchange potential; namely, there are claims by several groups [14,15] that the πNN coupling constant $g^2_{NN\pi}$ is considerably smaller than previously assumed (about 13.5 vs 14.4). We decided to still use the "old" value, for two reasons: First, the modification advocated in Refs. [14,15] is not universally accepted (see e.g., the review by Ericson [16]). Second, our qualitative conclusion (namely, that soft $NN\pi$ and $N\Delta\pi$ form factors are not ruled out by NN data) should not be influenced by the precise value of $g^2_{NN\pi}$. We will provide corresponding arguments later.

The structure of the paper is as follows. In Sec. II we briefly recap the main ingredients of the full Bonn potential. The parameters of the new version and the corresponding phase shifts and deuteron properties are then presented and discussed. In Sec. III we make some concluding remarks.

II. CALCULATION AND RESULTS

The full Bonn potential is based upon the solution of the relativistic Lippmann-Schwinger (LS) equation

$$T(E) = V + V(E^+ - h_0)^{-1}T(E). \quad (1)$$

The potential V is energy dependent because of the inclusion of retardation. In the present version we include

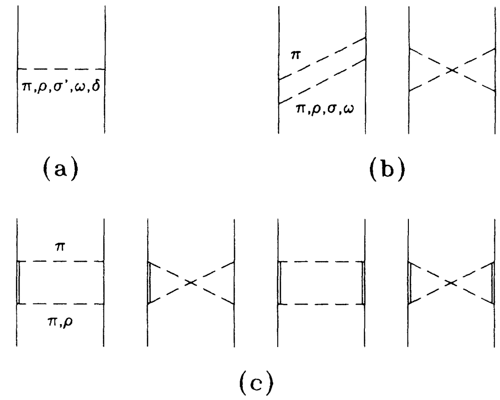


FIG. 1. Diagrams included in the full Bonn NN interaction model. (a) One-boson-exchange contributions. (b) Irreducible (stretched- and crossed-box) parts of the two-boson-exchange contributions involving nucleons in intermediate states. (c) Two-boson-exchange contributions involving the Δ isobar in intermediate states.

in V the exchange of π,π',ρ, and ω mesons as well as σ' and δ. However, the essential feature of the model is the inclusion of explicit TPE involving the Δ—see Fig. 1. In the present work we constrain the form factors at all the $NN\pi$ and $\Delta N\pi$ vertices to have a (monopole) cutoff mass of 0.9 GeV.

In order to evaluate the NN phase shifts we solve the R-matrix equation:

$$R = V + V\frac{P}{E - h_0}R \quad (2)$$

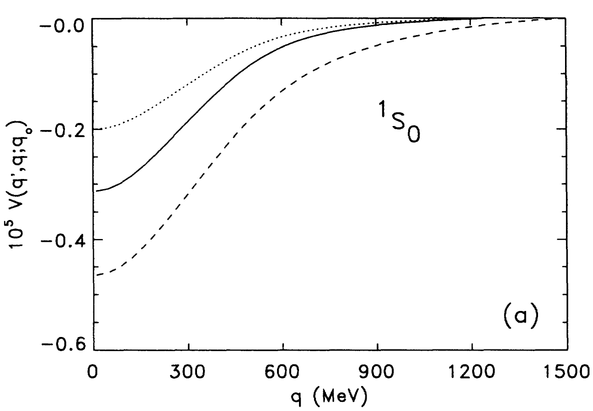
[where $T = R - i\pi R\delta(E - h_0)T$] in the helicity representation. Full details of the construction of the potential in the helicity representation, and the relation between the on-shell R matrix and the bar phase shifts may be found in Ref. [4]. For the evaluation of deuteron properties we likewise refer the reader to Ref. [4].

TABLE I. Meson parameters applied in the model with soft pion form factor. Numbers in round brackets denote corresponding values of the full Bonn potential, when different. Nucleon mass m=0.938 926 GeV; mass of Δ-isobar=1.232 GeV.

Vertex	$I(J^P)$ of meson	Meson mass m_α (GeV)	$g^2_\alpha/4\pi$; $[f_\alpha/g_\alpha]$	Cutoff mass Λ_α (GeV)	n_α
$NN\pi$	$1(0^-)$	0.138 03	14.4	0.9 (1.3)	1
$NN\pi'$	$1(0^-)$	1.2	70.0	2.0	1
$NN\rho$	$1(1^-)$	0.769	0.88 [6.7] (0.84)([6.1])	1.25 (1.4)	1
$NN\omega$	$0(1^-)$	0.7826	20.0	1.5	1
$NN\delta$	$1(0^+)$	0.983	3.7644 (2.8173)	2.0	1
$NN\sigma'$	$0(0^+)$	0.600 (0.550)	7.4466 (5.6893)	1.7	1
$N\Delta\pi$	$1(0^-)$	0.138 03	0.35 (0.224)	0.9 (1.2)	1 1
$N\Delta\rho$	$1(1^-)$	0.769	25.20 (20.45)	1.425 (1.4)	2

In Table I we present the new meson parameters resulting from a best fit to the NN scattering phase shifts, in comparison with those used in the full Bonn potential. Compared with the corresponding OBE model of Ref. [9], the required π' contribution is now reduced by 30% (from 100 to 70). The main reason for this is that the $NN\pi$ cutoff has now been fixed as 0.9 GeV, rather than 0.8 GeV, as in Ref. [9]. On the other hand, the softening of both the $NN\pi$ and $\Delta N\pi$ form factors now suppresses further higher-order diagrams—especially the important iterative diagrams involving $\pi\pi$ and $\pi\rho$ exchange with $N\Delta$ and $\Delta\Delta$ intermediate states. Therefore it should be of no surprise that the $\Delta N\pi$ coupling constant now has to be increased substantially—from 0.224 to 0.35—in order to obtain a comparably good fit. It is interesting to note that such an increase would improve the predictions of the Bonn model for pion production. According to the calculations of Elster *et al.* [17] the use of $g^2_{\Delta N\pi} = 0.224$ underestimated the inelastic NN cross sections considerably.

Of course, making the $NN\pi$ and $\Delta N\pi$ form factors softer while increasing the $\Delta N\pi$ coupling constant leads to a net effect quite different in various partial waves. For example, form-factor modifications affect the lower partial waves only. This is clearly demonstrated in Fig. 2, which shows the half-shell potential $V(q',q;q_0)$ generated by the $N\Delta$ iterative two-pion-exchange diagram for $q' = q_0 = 250$ MeV/c in both the $^1S_0$ and $^1G_4$ partial waves. Whereas the change in the form factor leaves the potential in $^1G_4$ for the relevant momenta essentially unaltered, the contribution in $^1S_0$ is reduced by more than a factor of two. Even with the increase in the $\Delta N\pi$ coupling constant, noted above, it remains some 30% smaller, whereas the combined modification results in a considerably increase in $^1G_4$. In order to compensate for this effect, we had to give the σ' contribution (parametrizing correlated 2π-exchange processes in the scalar t channel) a somewhat shorter range, i.e., increase the σ' mass by 50 MeV, and at the same time increase the coupling constant by about 20% to keep the overall σ' strength. Also some other parameters had to be changed slightly in order to obtain the same quantitative description as before.



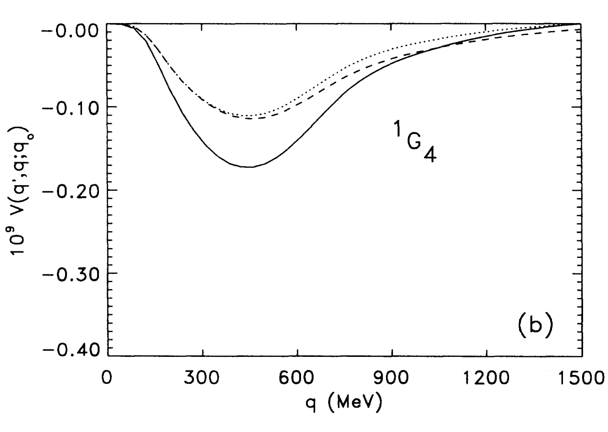


FIG. 2. Half-shell potential $V(q',q;q_0)$ generated by the $N\Delta$ iterative two-pion-exchange diagram for $q' = q_0 = 250$ MeV/c in the (a) $^1S_0$ and (b) $^1G_4$ partial waves. The dashed curve corresponds to the full Bonn NN model [4]. The dotted curve is obtained by replacing the (original) $NN\pi$ and $\Delta N\pi$ cutoff masses by 0.9 GeV. For the solid line in addition the $\Delta N\pi$ coupling constant is increased from 0.224 to 0.35.

In Fig. 3 we show the resulting phase shifts for total angular momentum $J = 0$–2 and for both the present model and the full Bonn potential. In all cases but one the results are quite comparable. The only exception is the ϵ_1 mixing parameter. Here the new model leads to a strong rise with energy which can be traced to appreciably more high-momentum tensor force components—due to the sizable π' contribution. The increase appears to be a little too strong compared with experiment and thus indicates that the mass of the effective π' has been chosen somewhat too high. Indeed, in the πNN form-factor calculation of Ref. [13], the spectral function generated essentially by correlated $\pi\rho$ exchange has its maximum very near the mass of the π'.

Nevertheless it is well known that, if such a spectral function is approximated by a single, sharp mass exchange, this mass should be considerably smaller than the maximum of the spectral function in order to take into account the comparably stronger effect of the left (longer ranged) tail. Clearly it will be quite interesting to see the outcome if the phenomenological π' is replaced by a realistic treatment of correlated $\pi\rho$ exchange; such calculations are presently under way in our group.

We mention here that a slight decrease of the πNN coupling constant (by at most 10%) as advocated in Refs. [14,15] would not influence the qualitative conclusions of the present paper: The reason is that the modification of the interaction induced by such a change is likewise about 10% and thus much smaller than the modification due to the change of the πNN cutoff mass discussed in this paper, which is more than a factor of 2 in low partial waves, cf. Fig. 2. Consequently, a comparable description should be obtained, possibly with a slight increase of the $\Delta N\pi$ and π' coupling constant.

Finally in Table II we show the deuteron and low-energy scattering parameters predicted by the new model. Again for comparison we show the values obtained with the full Bonn potential. The increase of the short-range tensor force in the present model manifests

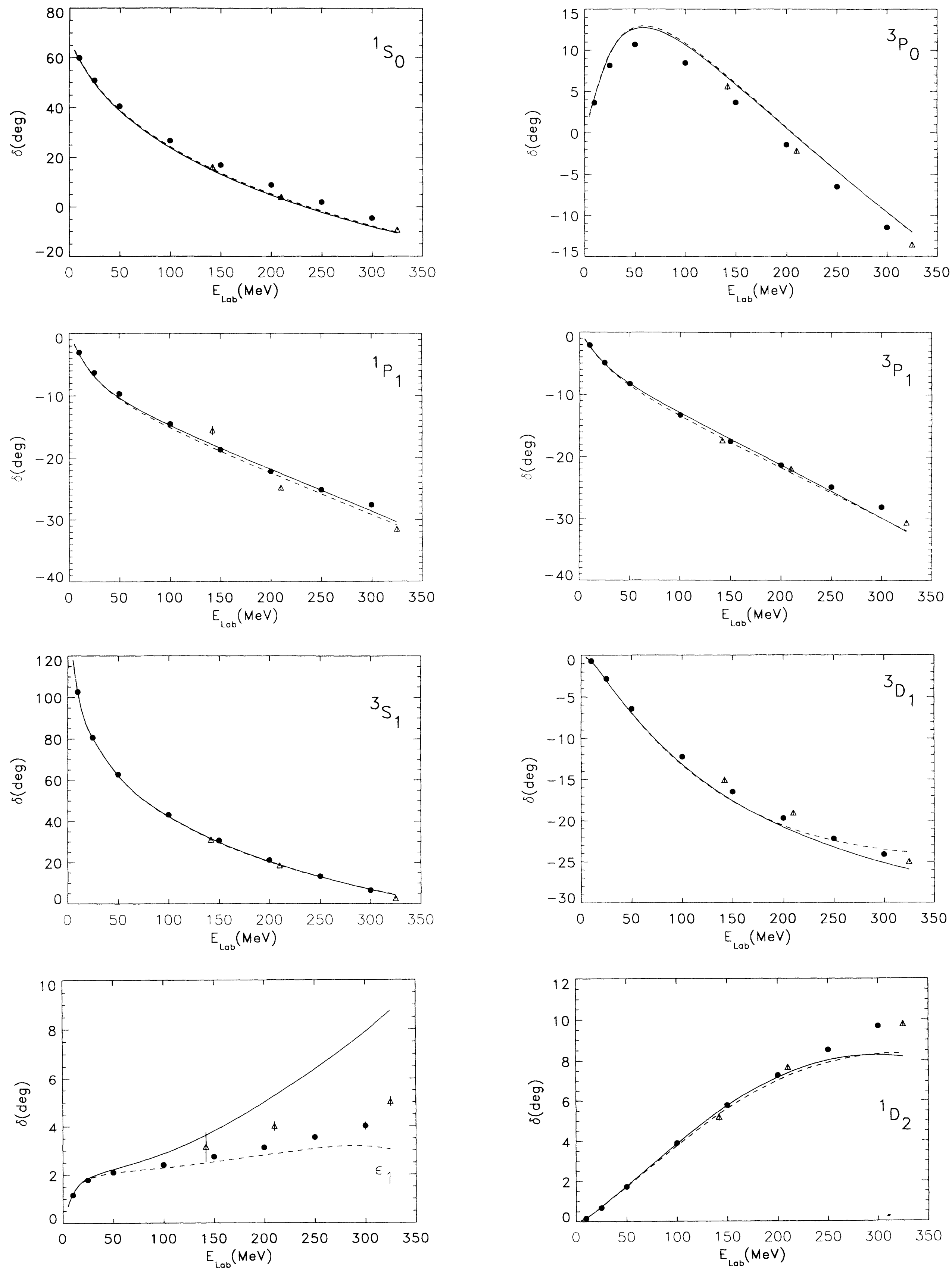


FIG. 3. *np* phase shifts for partial waves $J \leq 2$. The solid curve is the result for the model with soft pion form factor; the dashed curve is the full Bonn potential [4]. Experimental phase shifts are from Bugg and Bryan [18] and Stoks *et al.* [19].

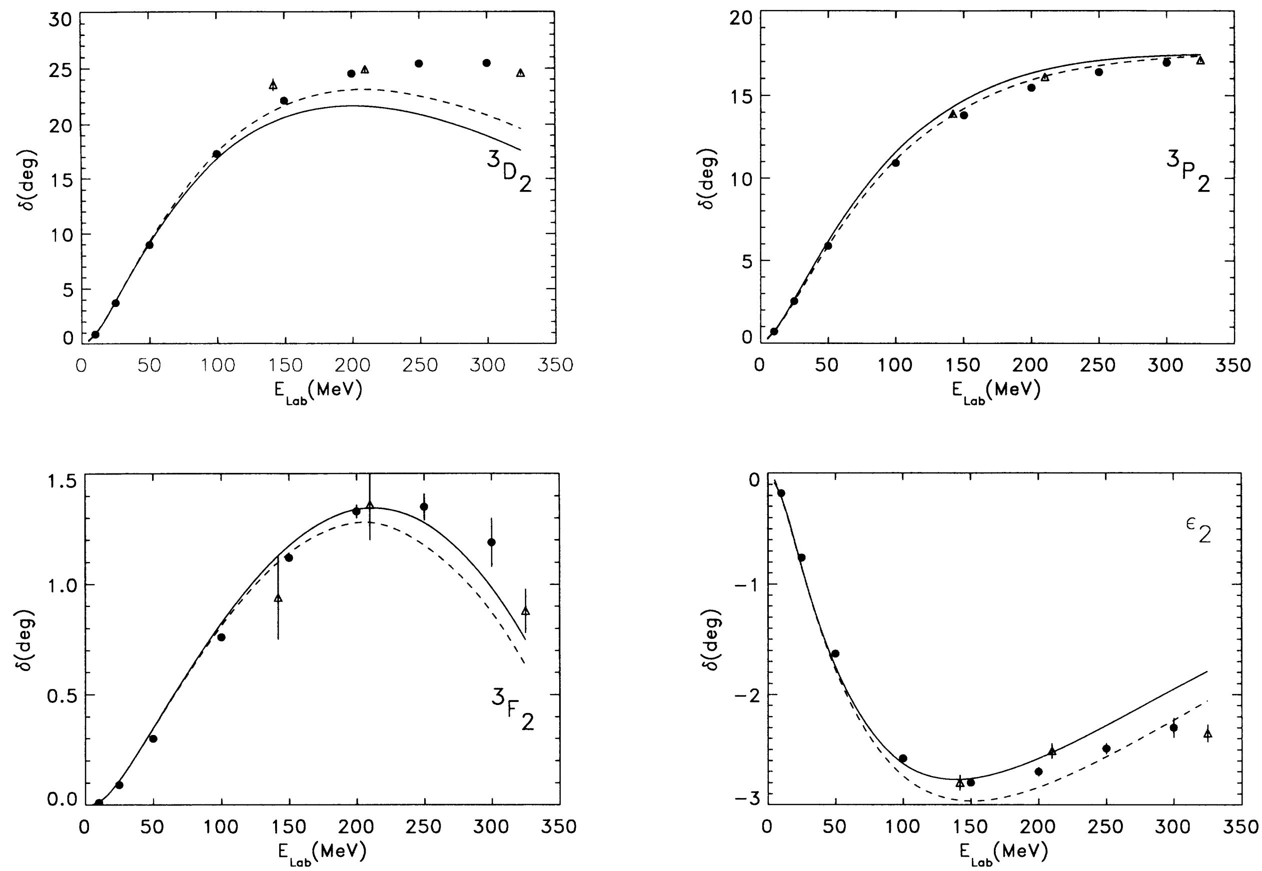


FIG. 3. (*Continued*).

TABLE II. Deuteron and low-energy scattering parameters predicted by the model with soft pion form factor and by the full Bonn potential compared with experiment. The experimental values are taken from the references indicated in the footnotes; low-energy scattering data are from Dumbrajs *et al.* [24]. Note that the deuteron properties are calculated from the nucleonic wave functions normalized to unity.

	Present model	Full Bonn (Ref. [4])	Expt.
Deuteron:			
binding energy ϵ_d (MeV)	2.2247	2.2247	2.224 575[a]
D-state probability P_d (%)	5.01	4.25	
Quadrupole moment Q_d (fm^2)	0.2817	0.2807	0.2859 ± 0.0003[b]
Asymptotic S-state A_s ($fm^{-1/2}$)	0.9039	0.9046	0.8846 ± 0.0016[c]
Asymptotic-D/S state D/S	0.0266	0.0267	0.0256 ± 0.0004[d]
$\Delta\Delta$ probability $P_{\Delta\Delta}$(%)	0.39	0.62[e]	
Neutron-proton low-energy scattering (scattering length a, effective range r):			
$^1S_0$: a_s (fm)	−23.75	−23.75	-23.758 ± 0.010
r_s (fm)	2.782	2.766	2.75 ± 0.05
$^3S_1$: a_t (fm)	5.426	5.427	5.424 ± 0.004
r_t (fm)	1.754	1.755	1.759 ± 0.005

[a]Reference [20].
[b]Reference [21].
[c]Reference [22].
[d]Reference [23].
[e]This value is more accurate than the one given in Ref. [4].

itself in the larger D-state probability. (Note that although in both cases it is quite small this is a well-known feature of the energy dependent interaction, not a sign of a weak tensor force. For the same reason, due to the lack of wave-function renormalization in these calculations, the resulting values of A_s are somewhat larger than the experimental value.)

III. CONCLUDING REMARKS

The purpose of the present paper was to show that a meson exchange model with consistently soft form factors for one- and two-pion exchange can be constructed, which provides a reasonable description of low energy NN data. This is important since there is considerable evidence favoring the use of relatively soft $NN\pi$ and $\Delta N\pi$ form factors in making a three-body force [25]. The same is true of pion-nucleon scattering [26]. However, this model in the present stage is not meant to replace the full Bonn potential [4]. Considerable work is still required in order to replace the purely fictitious π' by a realistic evaluation of correlated $\pi\rho$ exchange processes; also, the model should then be confronted in more detail with the empirical data. We believe, however, that the results presented so far indicate already the possibility for a satisfying consistency in that one can model forces in the one, two, and three nucleon systems in terms of the safe soft form factors.

ACKNOWLEDGMENTS

One of us (A.W.T.) would like to acknowledge the hospitality of J. Speth and the theory group at IKP Jülich where this work was completed. The work was supported in part by the Alexander von Humboldt Foundation, the Australian Research Council, and the Deutsche Forschungsgemeinschaft (Az: Schu 439/7-1).

[1] M. Lacombe, B. Loiseau, J.-M. Richard, R. Vinh Mau, P. Pirès, and R. de Tourreil, Phys. Rev. D **12**, 1495 (1975); M. Lacombe, B. Loiseau, J.-M. Richard, R. Vinh Mau, J. Côté, P. Pirès, and R. de Tourreil, Phys. Rev. C **21**, 861 (1980).
[2] A. D. Jackson, D. O. Riska, and B. Verwest, Nucl. Phys. **A249**, 392 (1975); G. E. Brown and A. D. Jackson, *The Nucleon Nucleon Interaction* (North-Holland, Amsterdam, 1976).
[3] A. M. Green, Rep. Prog. Phys. **39**, 1109 (1976).
[4] R. Machleidt, K. Holinde, and Ch. Elster, Phys. Rep. **149**, 1 (1987).
[5] A. W. Thomas, Phys. Lett. **126B**, 97 (1983).
[6] S. A. Coon and M. D. Scadron, Phys. Rev. C **23**, 1150 (1981).
[7] L. L. Frankfurt, L. Mankiewicz, and M. I. Strikman, Z. Phys. A **334**, 343 (1989).
[8] A. W. Thomas, in *Proceedings of the International Conference on Nuclear Physics*, edited by M. S. Hussein *et al.* (World Scientific, Singapore, 1990), p. 235.
[9] K. Holinde and A. W. Thomas, Phys. Rev. C **42**, R1195 (1990).
[10] G. Q. Liu, M. Swift, A. W. Thomas, and K. Holinde, Nucl. Phys. **A556**, 331 (1993).
[11] T. Y. Saito, Phys. Rev. C **47**, 69 (1993).
[12] T. Ueda, Phys. Rev. Lett. **68**, 142 (1992).
[13] G. Janßen, J. W. Durso, K. Holinde, B. C. Pearce, and J. Speth, Phys. Rev. Lett. **71**, 1978 (1993).
[14] J. R. Bergervoet, P. C. van Campen, R. A. M. Klomp, J.-L. de Kok, T. A. Rijken, V. G. J. Stoks, and J. J. de Swart, Phys. Rev. C **41**, 1435 (1990); V. G. J. Stoks, R. Timmermanns, and J. J. de Swart, *ibid.* **47**, 512 (1993).
[15] R. A. Arndt, Z. Li, L. D. Roper, and R. L. Workman, Phys. Rev. D **44**, 289 (1991).
[16] T. E. O. Ericson, Nucl. Phys. **A543**, 409c (1992).
[17] Ch. Elster, K. Holinde, D. Schütte, and R. Machleidt, Phys. Rev. C **38**, 1828 (1988).
[18] D. V. Bugg and R. A. Bryan, Nucl. Phys. **A540**, 449 (1992).
[19] V. G. J. Stoks, R. A. M. Klomp, M. C. M. Rentmeester, and J. J. de Swart, Phys. Rev. C **48**, 792 (1993).
[20] C. van der Leun and C. Alderliesten, Nucl. Phys. **A380**, 261 (1982).
[21] D. M. Bishop and L. M. Cheung, Phys. Rev. A **20**, 381 (1979).
[22] T. E. O. Ericson and M. Rosa-Clot, Nucl. Phys. **A405**, 497 (1983); T. E. O. Ericson, *ibid.* **A416**, 281c (1984).
[23] N. L. Rodning and L. D. Knutson, Phys. Rev. C **41**, 898 (1990).
[24] O. Dumbrajs *et al.*, Nucl. Phys. **B216**, 277 (1983).
[25] T. Sasakawa, S. Ishikawa, Y. Wu, and T.-Y. Saito, Phys. Rev. Lett. **68**, 3503 (1992).
[26] S. Théberge, A. W. Thomas, and G. A. Miller, Phys. Rev. D **22**, 2838 (1980).