Deep inelastic scattering on the deuteron in the Bethe-Salpeter formalism: Scalar meson exchange

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The deep inelastic scattering on the deuteron is considered in a fully covariant field theoretical approach. All calculations are performed in the Bethe-Salpeter formalism and the Wilson operator product expansion method within an effective meson-nucleon theory. By the operator product expansion method we obtain an explicit form of the nucleon contribution and mesonic exchange corrections to moments of the deuteron structure function F_2^D. The structure function of the deuteron is recovered by the inverse Mellin transform of the moments and is presented as the sum of two convolution terms, viz. the nucleonic (relativistic impulse approximation) and mesonic (contribution of the meson exchange currents) ones. The sum rules for the baryon number and energy-momentum of the deuteron are derived using the normalization condition of the Bethe-Salpeter amplitude and virial theorem of field theory. The numerical estimates of the nuclear effects in the deuteron structure function in the Bethe-Salpeter formalism are also presented. It is found that the binding effects of the nucleons are significant and relativistic calculations within the Bethe-Salpeter formalism are compared with the previous nonrelativistic estimates.

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I. INTRODUCTION

A. Preliminaries and motivations

The relativistic bound-state problem appears as one of the most important and interesting in the modern theory of strong interactions. In fact, the hadrons are the bound states of the quarks, relativistically moving in a confined volume. Also nuclei are the bound states of the nucleons, which while much less relativistic than quarks, may display relativistic properties in high energy processes. Since the bound-state problem in QCD is still unresolved, it is important to study the capability of the effective theories in both the quark and hadron sectors (see also recent discussion in [1]). It is feasible that an investigation of a wide class of bound systems and reactions involving bound systems within different effective theories may give some clues for the creation of the "true theory" of the strong interactions.

The relativistic description of the kinematics and dynamics of processes with bound systems may be based on methods of the covariant field theory. The most direct way to a description of the bound states in a field theory lies through a consideration of the Bethe-Salpeter equation [2,3]. The approaches based on the Bethe-Salpeter equation, or its approximations, within models using effective potentials [4,5] (for recent development see, e.g., Ref. [6]), allow a successful description of the mass spectrum and the decay widths of mesons as bound systems of two quarks (the $q\bar{q}$ system). The Bethe-Salpeter equation has also been applied to describe properties of the bound state in the two nucleon system, deuteron [7], and effective NN forces [1,8–12].

In the present context the deuteron is an exceptional object for studying the techniques for relativistic bound states. Indeed, the properties of the deuteron are well-studied experimentally, and so there is ample information to check any theoretical scheme. Besides, in the case of the deuteron, many extra difficulties of the problem of the relativistic bound state, such as the problem of the confinement in the $q\bar{q}$ system, are absent. However, by incorporating modifications relevant to the effective quark models, the methods developed for the case of deuteron then may be applied to the mesonic states, at least in the case of heavy quarks.

The present work is devoted to a relativistic theory of deep inelastic lepton scattering on the deuteron. We found the investigation of this problem to be useful and instructive. In particular, it is possible to obtain a fully consistent covariant description of the process within an effective meson-nucleon model, based only on rigorous methods of calculations and well-defined approximations. Further, the observables of this reaction, the moments of the structure functions, and its various combinations are related to rigorous theorems, sum rules, arising from the general properties of the field theory and symmetries of the interactions. Therefore the consistency of the description of the reaction may be checked explicitly.

Fully relativistic calculations have obvious advantages over the nonrelativistic approach [13,14]. First, such calculations are valid in the kinematical regions, where nonrelativistic approximations are simply meaningless. Second, a covariant description of the spin degrees of freedom is preferable in view of the relativistic nature of the spin (or total momentum of the composite system), and so we can expect some nontrivial effects in the polarization observables of reactions. However, at the same time the deuteron is, essentially, a nonrelativistic system, and it may be anticipated that the relativistic and nonrelativistic calculations will be in agreement at the boundaries of validity of the nonrelativistic approximations.

Our investigation is partially motivated by a number of the existing and forthcoming experiments on the deep inelastic scattering of leptons by deuterons (SLAC, CERN, DESY, CEBAF, etc.). A consistent relativistic theory of this process will help in the analysis of the experimental data. Of special interest are the spin-dependent structure functions and structure functions near the single-particle kinematics, as here we can expect to find differences between the nonrelativistic and relativistic descriptions.

B. Formulation of the problem

Our goal is to develop a self-consistent covariant approach to the inclusive deep inelastic scattering of leptons on the deuteron within an effective meson-nucleon theory. Below we give the technical formulation of the problem and then present a strategy for finding the solution.

The cross section of inclusive deep inelastic lepton scattering on the hadronic target can be calculated in terms of the hadronic tensor $W_{\mu\nu}$, which is usually parametrized in terms of the structure functions $F_{1,2}$, $g_{1,2}$, etc. For the unpolarized scattering, for instance, $W_{\mu\nu}$ is written as

$$W_{\mu\nu}(p,q) = F_1(p,q)\left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) + \frac{F_2(p,q)}{pq}\left(p_\mu - \frac{pq}{q^2}q_\mu\right)\left(p_\nu - \frac{pq}{q^2}q_\nu\right) . \tag{1}$$

By unitarity, the hadronic tensor is proportional to the imaginary part of the elastic amplitude of the virtual photon-hadron scattering [15–18]:

$$T_{\mu\nu}(p,q) = i\int d^4x\, e^{iqx}\langle p|T(J_\mu(x)J_\nu(0))|p\rangle , \tag{2}$$

where p is momentum of the target and q is the virtual photon momentum ($Q^2 \equiv -q^2$). Thus $W_{\mu\nu}$ is connected with the diagonal matrix element of the product of currents on the bound state. So the problem is split into two relatively independent parts: a description of the bound state and of the operator to be sandwiched between this state. Clearly, both these tasks should be considered in a self-consistent way; i.e., the approach must satisfy the general requirements of the covariant field theory and all calculations must be performed within the same field-theoretical model.

It is well known that an accurate description of both the NN interactions at energies up to ~ 1 GeV and the basic properties of the deuteron can be provided within the meson-nucleon theory [1,7,19,20]. Such a description can be either nonrelativistic, based on the Schrödinger equation, or relativistic (or almost relativistic), based on the Bethe-Salpeter equation or its various approximations [21]. Therefore we can start from the point of view that the meson-nucleon theory allows us to solve the first part of the problem, to describe the bound state of the deuteron.

As to the second part of the problem, to describe the interaction operator in (2) within the same meson-nucleon theory, one must solve the problem of the short-distance contributions. Indeed, the effective meson-nucleon theory is the theory for the long-distance phenomena, the phenomena of the nuclear scale. In the deep inelastic regime, we study, however, the short distances of the quark scale, and generally speaking, it is a nontrivial problem to describe the phenomena at these distances within a theory composed of only hadronic fields and parameters fixed at the nuclear scale. Fortunately, there is a field-theoretical method to separate and parametrize the short-distance contributions. This is the Wilson operator product expansion method [22,23]. For large momentum transfers, Q^2, the operator product expansion factorizes the amplitude into pieces depending on short- and long-distance physics. The short-distance terms, the Wilson coefficient functions, which are related to the short-distance contributions, are parametrized via the properties for the physical hadrons in this theory, while the long-distance part is exactly calculable in the framework of the meson-nucleon theory [24,25,13,14]. Therefore, combining a particular formalism for the bound state in meson-nucleon theory with the Wilson operator product expansion in the same theory, we get a consistent description of the nuclear effects in deep inelastic lepton scattering.

The idea to describe deep inelastic scattering on nuclei in terms of the effective meson-nucleon theory was first realized in Refs. [24,25] for the case of the heavy nuclei (the "Dirac phenomenology") and in [26,13,14] for the nonrelativistic limit of the theory, where the use of the Wilson operator product expansion is an essential part of the method. The attractive feature of such approaches is a full consistency of the method and the absence of free parameters, since all parameters are fixed by (i) the nuclear physics, such as the NN-scattering phase shifts and properties of the nuclei, and (ii) properties of the physical hadrons, such as structure functions of the nucleons and mesons. This allows a consistent theoretical framework to be developed to describe the phenomenon, including matrix elements of the observables, rather than simply suggesting new calculation schemes. The consistency of the approach is checked by the charge conservation laws and virial-like theorems.

In the present paper we utilize the Bethe-Salpeter formalism to describe the deuteron, a bound state of two nucleons. Further considerations are close to previous developments [24,25,13,14]. In particular, the following investigations are needed to solve the problem.

(1) To describe the deuteron as a bound state within the meson-nucleon theory and the Bethe-Salpeter formalism in the ladder approximation.

(2) To study the behavior of the deep inelastic amplitude at high momentum transfer Q^2 using the Wilson operator product expansion method in the leading twist approximation.

(3) To provide an explicit test of the consistency of the approach by the analysis of the sum rules for the baryon number and energy momentum.

(4) The derivation of the explicit formulas for the deuteron spin-averaged and spin-dependent structure functions.

(5) To calculate the deuteron structure functions in a realistic meson-nucleon theory (see, e.g., Refs. [1,11,20]).

(6) To analyze the experimental data, applying the results of calculations. Special attention should be paid to the topical problem of the extraction of the spin-averaged and spin-dependent neutron structure functions from the combined proton-deuteron data.

In the present paper we concentrate on the formalism and only some simple numerical estimates, namely, points (1)–(3) and, partially, (4) and (5). The explicit expressions for spin-averaged and spin-dependent structure functions of the deuteron and results of realistic calculations and analysis of the experimental data will be published in a separate paper.

In this paper we pay particular attention to the nonrelativistic limit of the relativistic approach and its connection with the nonrelativistic description of deep inelastic scattering on the deuteron [13,14]. It is clear that a realistic field-theoretical description of processes with the deuteron must have a well-defined nonrelativistic limit, since the deuteron is essentially a nonrelativistic system. However, the nonrelativistic reduction of the Bethe-Salpeter equation is a complex problem by itself. Here we enunciate only the basic points of our approach with regard to this question. This gives us material for comparison and understanding of the relativistic calculations.

C. Organization of the paper

This paper contains all necessary details to understand the application of the Bethe-Salpeter formalism to deep inelastic lepton scattering on the deuteron.

The paper is organized as follows.

Section II is devoted to the formalism describing deep inelastic lepton-deuteron scattering. Section II A contains a very brief review of the previous approaches to the process. Section II B presents the basic ideas and main formulas of the approach to deep inelastic scattering on the deuteron within the meson-nucleon theory. The results of Sec. II B can be used in any version of the meson-nucleon theory, such as in the nonrelativistic Tamm-Dancoff method, the Dirac phenomenology, or the Bethe-Salpeter formalism. The calculation of the observables, such as moments of the structure function $\mathsf{M}_n(F_2^D)$ and structure function F_2^D in terms of the Bethe-Salpeter amplitude is presented in Secs. II C and II D, respectively. In order to check the consistency of the formalism, we derive, in Sec. II E, the main sum rules for the deuteron structure function F_2^D in the Bethe-Salpeter formalism. It is shown that the sum rules for baryon charge and energy momentum are satisfied within the presented formalism. The last section of Sec. II is devoted to consideration of the nonrelativistic limit of the approach.

Section III presents the method to solve the Bethe-Salpeter equation for the deuteron. Our technique is different from the one used previously to solve the deuteron problem, and it is simpler and easily applicable to deep inelastic scattering. In Sec. III A we expand the spinor-spinor Bethe-Salpeter amplitude in terms of the 16 scalar amplitudes and obtain the corresponding set of equations for these amplitudes, which is equivalent to the spinor-spinor Bethe-Salpeter equation. In Sec. III B we extract the solution for the deuteron state from the full amplitude by partial wave decomposition and analysis of the quantum numbers of the partial wave amplitudes. A set of eight equations for the eight amplitudes of the deuteron state is obtained. In Sec. III C we define the Wick rotation procedure, which transforms the set of equations into a form appropriate for numerical solution. Section III D contains the formulas for the normalization condition of the Bethe-Salpeter amplitude of the deuteron and nucleonic contribution to the moments of the deuteron structure function $\mathsf{M}_n(F_2^D)$.

In Sec. IV A we present numerical estimates of the nuclear effects in the deuteron structure function within the Bethe-Salpeter formalism. The calculations are carried out in a simplified model, which represents the deuteron as a system of two spinor nucleons interacting by a scalar meson exchange. We argue that this model is sufficient to estimate, quantitatively, the nuclear corrections to the unpolarized deuteron structure function F_2^D. It is then shown that the effects of the binding of the nucleons are significant and that relativistic calculations within the Bethe-Salpeter formalism are in agreement with previous nonrelativistic estimates. In Sec. IV B we discuss the open questions in the present investigation. These are under study at this time. Finally we summarize the main results of the paper in Sec. V.

II. GENERAL APPROACH TO DEEP INELASTIC SCATTERING ON THE DEUTERON

A. Approaches to the nuclear effects in deep inelastic lepton scattering

The case of the deuteron is generically related to previous investigation of the nuclear effects in deep inelastic scattering, including the study of the famous European Muon Collaboration (EMC) effect on heavier nuclei. Here we give a brief outline of the different approaches to deep inelastic lepton scattering on nuclei; details may be found elsewhere [15–17]. The aim of this section is to explain the difference of our approach from that of others, which are related to two basic types, such as the nuclear convolution and Q^2 rescaling.

Very often calculations of the nuclear structure functions (or hadronic tensor) in both the relativistic and nonrelativistic approaches start from the so-called "convolution formula" [15–17], which assumes that the nuclear structure function can be represented in the form of a convolution of the structure functions of the physical constituent hadrons with their effective distribution functions. For example, for the nucleon contribution to the deuteron structure function F_2^D one has

$$F_2^{N/D}(x_N) = \int_{x_N}^{M_D/m} f^{N/D}(y) F_2^N(x_N/y)dy \ , \quad (3)$$

where m and M_D are the nucleon and deuteron masses, respectively, $x_N \equiv -q^2/2pq$ is the nucleon Bjorken scaling variable, $f^{N/D}(y)$ is the "nucleon distribution function" with y being the "longitudinal fraction of the deuteron momentum," and F_2^N is the structure function of the physical nucleon. In spite of its clarity and quasiparton interpretation, this formula by itself does not give a consistent method to calculate the distribution function $f^{N/D}(y)$. Intuitively, it is clear that $f^{N/D}(y)$ should be related to something like the "spectral function" $S(p)$ giving the probability to find a nucleon (or other constituent) with four-momentum p. To obtain an explicit form of the distribution $f^{N/D}(y)$, one should solve the dynamical problem for the bound state of the target nucleus. So our viewpoint is that the convolution formula may be derived as a result of detailed considerations, but should not be considered as a basis of one.

Relativistic approaches starting from the convolution formula [27–29] usually deal with the "relativistic impulse approximation," which takes into account the contribution of bound nucleons to the nuclear structure function. Yet as the picture of nucleons interacting through a potential is essentially artificial or, at least, incomplete in the relativistic case, these approaches inevitably have some internal difficulties. For instance, such approaches have problems with the normalization conditions for the distribution functions [28] and/or the recovery of the nonrelativistic limit [27].

An interesting attempt at a consistent analysis of the process is given within the light-front-dynamic approach, where the nucleus is considered as a compound system of nucleons and mesons [30]. It is shown that the structure function of the nucleus is a linear functional of the nucleon and meson structure functions if we assume that the pions and nucleons contribute incoherently, i.e., if we assume the validity of the convolution-type formula. Then the effective distribution functions are expressed in terms of the relativistic wave functions of pions and nucleons in nucleus. In practice, however, both the nucleon and meson components of the nuclear structure function are approximately calculated by using the nonrelativistic momentum densities of nucleons and mesons in the nucleus.

Within the usual nonrelativistic nuclear physics, Eq. (3) is the basis for the x-rescaling model [31,32]. The main idea of the x-rescaling model is based on the well-known fact that the properties of quasiparticle nucleons differ from those of free nucleons. In particular, the bound nucleons have an effective mass depending on the shell energy. This leads to the renormalization of the scaling variables $x_N \to (m/m^*)x_N$. The formula of the model for the deuteron case is [13,26]

$$f^{N/D}(y) = \int \frac{d\mathbf{k}}{(2\pi)^3} |\Psi_D(\mathbf{k})|^2 \left(1 + \frac{k_3}{m}\right) \times \delta\left(y - \left[1 + \frac{\varepsilon_D}{m} - \frac{\mathbf{k}^2}{2m^2} + \frac{k_3}{m}\right]\right) \ , \quad (4)$$

where $\Psi_D(\mathbf{k})$ is the deuteron wave function. All the nuclear structure effects in (4) are encoded in the definition of y. The x-rescaling approach presented by relations such as (3) and (4) is consistent with the experimental data for heavy nuclei (see, e.g., [33,34]), but to some degree is phenomenological in nature. However, it faithfully catches the essence of the EMC effect.

The main distinction of this model is that the bound nucleons carry only a part of the total momentum of the nuclear target, i.e.,

$$\langle y \rangle = (1 + \varepsilon_D/m - \langle \mathbf{k}^2 \rangle / 2m^2 + \langle k_3^2 \rangle / m^2) < 1 \ .$$

The remaining part of the total momentum is attributed to the quanta of the potential binding the nucleons in nuclei (e.g., mesons). The contribution of the mesonic degrees of freedom to the nuclear structure function within the nuclear convolution approach is calculated in terms of the nonrelativistic wave functions or densities [35,36].

In spite of the common starting point, the convolution formula, the nonrelativistic and some of the (quasi)relativistic approaches lead to different quantitative estimates of the binding effects in the nuclear structure functions (see, e.g., [27,29]). Therefore a consistent analysis of the nuclear structure functions in the covariant Bethe-Salpeter formalism with proper account of the nuclear dynamics can clarify this situation.

An alternative approach is the QCD-motivated Q^2-rescaling model [37–39], which utilizes the Wilson operator product expansion method to analyze the Compton amplitude (2). Since the problem of large distances is not solved in QCD, the operator product expansion allows one to calculate perturbatively the Wilson coefficient functions, while the nuclear matrix elements have a nonperturbative origin in QCD and consequently are connected with phenomenology. The basic idea of the Q^2-rescaling model is the change of the initial point for the Q^2 evolution of the nuclear structure function relative to the free nucleon in view of the change of the effective radius of confinement in nuclei (sometimes referred to as "swelling of the nucleon"). The model gives a reasonable estimate of the nuclear effects in the structure functions. In spite of this and its attractive fundamental basis, the Q^2-rescaling model cannot be applied to the description of the structure functions in the full kinematics region [14]. However, it is interesting to note the possibility to interrelate the QCD parameters with the nuclear ones by comparing the Q^2-rescaling model and the approaches motivated by nuclear physics [37,14]).

B. Effective meson-nucleon theory and the Wilson operator product expansion

The covariant theory of the interacting nucleons and mesons is based on a Lagrangian of the form

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{\text{int}} \ , \quad (5)$$

where the standard form of $\mathcal{L}_0$ reads

$$\mathcal{L}_0 = \frac{i}{2}[\bar{\psi}(x)\gamma_\mu \overleftrightarrow{\partial}^\mu \psi(x)] - m\bar{\psi}(x)\psi(x) + \tfrac{1}{2}\{[\partial\phi(x)]^2 - \mu^2\phi^2(x)\} , \tag{6}$$

where ψ and ϕ stand for the nucleon and meson fields, respectively, and μ is the meson mass. For realistic models with several types of mesons, one should take the sum over all mesonic fields and include vector indexes for vector mesons. However, for formal consideration it is enough to take a particular type of mesonic field. As an example, we present calculations within a theory of spinor nucleons interacting with scalar mesons, the σ meson:

$$\mathcal{L}_{\rm int} = -g_\sigma \phi(x)\bar{\psi}(x)\psi(x) . \tag{7}$$

The approach can be generalized to include other mesons [24].

It is known that to consider meson-nucleon phenomenology, using an effective Lagrangian such as (5), we should restrict ourselves to a low-order approximation. The reason is that such theory is not fundamental and calculations in high orders (heavy effective exchange masses) may be meaningless. We use the ladder approximation both for the kernel of Bethe-Salpeter equation and for all further calculations with the Bethe-Salpeter amplitude. Note that in the context of the Bethe-Salpeter equation this approximation is found to be relevant to the weakly bound and weakly relativistic system, such as the deuteron [3,40–42]. Then we postpone discussions of the vertex form factors until we deal with calculations within the realistic meson-nucleon model, and for the present we treat these vertexes as pointlike.

The deuteron state can be described within the ladder Bethe-Salpeter equation [7]. Therefore we discuss deep inelastic lepton scattering on the deuteron, i.e., the calculation of the amplitude (2), in the same approximation. A rigorous analysis of the product of the two currents at high momentum transfers is accomplished by the Wilson's operator product expansion method [23]. The operator product expansion provides a natural way for calculation of the Compton amplitude (2) within the field-theoretical formalism by using the Feynman diagram technique [43], since it presents the T product of currents in terms of the expansion on the set of local operators of the theory. The present method to calculate the deuteron structure function does not depend on any additional assumption, such as the convolution form. Moreover, the operator product expansion method allows us, in principle, to study the boundaries of validity of the convolution model.

We start with the operator product expansion of the Compton amplitude (2) in the form [15,18,13]

$$T^D_{\mu\nu}(p_D,q) = \sum_{a;n=2,4,\ldots}^{\infty} C^{(1)}_{1,n}\left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right)\frac{2^n q_{\mu_1}\cdots q_{\mu_n}}{(-q^2)^n}\langle p_D|O_a^{\mu_1\cdots\mu_n}(0)|p_D\rangle$$
$$+ \sum_{a;n=2,4,\ldots}^{\infty} C^{(2)}_{a,n}\left(g_{\mu\mu_1} - \frac{q_\mu q_{\mu_1}}{q^2}\right)\left(g_{\nu\mu_2} - \frac{q_\nu q_{\mu_2}}{q^2}\right)\frac{2^n q_{\mu_3}\cdots q_{\mu_n}}{(-q^2)^{n-1}}\langle p_D|O_a^{\mu_1\cdots\mu_n}(0)|p_D\rangle , \tag{8}$$

where $O_a^{\mu_1\cdots\mu_n}(0)$ is the set of the local operators providing the basis for the operator product expansion, a enumerates operators of different fields of the theory, and $C^{(1,2)}_{a,n}$ are the Wilson coefficient functions.

Obtaining the structure function from (8) is achieved by using the dispersion technique [15,18]. To present the results in a compact form, we define the reduced matrix elements $\mu_n^{a/D}$:

$$\langle p_D|O_a^{\mu_1\cdots\mu_n}|p_D\rangle = p_D^{\mu_1}\cdots p_D^{\mu_n}\mu_n^{a/D} \quad (a = N,\sigma,\ldots) , \tag{9}$$

which are related to the structure function (e.g., F_2^D) moments and coefficients $C^{a,n}$ by

$$\mathsf{M}_{n-1}(F_2^D) = \sum_a C^{(2)}_{a,n}\mu_n^{a/D} \quad \text{with } \mathsf{M}_n(F) = \int_0^1 F(x)x^{n-1}dx , \tag{10}$$

where x is the Bjorken scaling variable, $0 \le x \le 1$.

We are now in a position to obtain the moments of the deuteron structure function (10) in the meson-nucleon theory. First, we note that the coefficients, $C^{(1,2)}_{a,n}$, are target independent and they define moments of the physical nucleon, meson, and deuteron structure functions. By consideration of the scattering on the free nucleons and mesons, it can be shown that in the ladder approximation the coefficients $C^{(1,2)}_{a,n}$ are identical to moments of the structure function of nucleons ($a = N$) or mesons ($a = \sigma$):

$$C^{(2)}_{N,n} = \mathsf{M}_n(F_2^N), \quad C^{(2)}_{\sigma,n} = \mathsf{M}_n(F_2^\sigma) . \tag{11}$$

Second, if we define the set $O_a^{\mu_1\cdots\mu_n}(0)$ in the meson-nucleon theory, then matrix elements in Eq. (9) are calculated explicitly. For instance, it can be calculated in the nonrelativistic Tamm-Dancoff approximation [13] or expressed in terms of the Bethe-Salpeter amplitude [43].

The operators that provide a basis for an operator product expansion can be ordered by the twist τ. Assuming the deep inelastic kinematics $Q^2 \gg m^2$, we accept the *leading twist approximation.* For deep inelastic scattering the leading operators are of twist 2, $\tau = 2$, and contributions of higher twists are suppressed by factors of $(\sim x^2m^2/Q^2)^{\tau-2}$ [23,15]. For unpolarized lepton scattering within the model nucleons and scalar meson fields, the twist-2 operators are

$$O_{\psi}^{\mu_1\cdots\mu_n} = \frac{1}{2}\left(\frac{i}{2}\right)^{n-1} \mathcal{S}\{\bar{\psi}(0)\gamma^{\mu_1}\overleftrightarrow{\partial}^{\mu_2}\cdots\overleftrightarrow{\partial}^{\mu_n}\psi(0)\}\ , \tag{12}$$

$$O_{\phi}^{\mu_1\cdots\mu_n} = \frac{1}{2}\left(\frac{i}{2}\right)^{n} \mathcal{S}\{\phi(0)\overleftrightarrow{\partial}^{\mu_1}\cdots\overleftrightarrow{\partial}^{\mu_n}\phi(0)\}\ , \tag{13}$$

where $\mathcal{S}$ symmetrizes the subsequent operator and removes all traces in $\mu_1\cdots\mu_n$.

From Eqs. (9)–(13) we then get the following general form for moments of the deuteron structure function

$$\mathsf{M}_n(F_2^D) = \mathsf{M}_n(F_2^N)\mu_{n+1}^{N/D} + \mathsf{M}_n(F_2^\sigma)\mu_{n+1}^{\sigma/d}\ , \tag{14}$$

where $\mu_n^{N/D}$ and $\mu_n^{\sigma/D}$ are interpreted as moments of the effective distribution functions of the nucleons and mesons in the deuteron, respectively. Moments of the deuteron structure function in the form of a product of the "elementary" moments $\mathsf{M}_n(F_2)$ and the effective distribution moments μ_n means that the structure function F_2^D can be presented in the form of a sum of convolution integrals such as (3), where the moments of effective distributions are the matrix elements of the twist-2 operators on the deuteron state and in the framework of our approach the meson and nucleon structure functions are

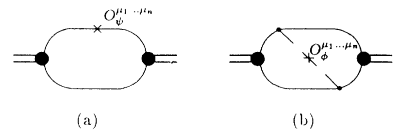

FIG. 1. Diagrams for the matrix elements (9) of the twist-2 contribution to the moments of the deuteron structure function in the ladder approximation.

considered as given.

Therefore, in an effective meson-nucleon theory and operator product expansion method with the twist-2 approximation, the deuteron structure function is presented in the form of a convolution of the meson and nucleon structure functions with their effective distributions. This result is valid in the exact deep inelastic limit $Q^2 \gg m^2$, which is studied in the present paper. However, the convolution approximation will break down beyond the leading twist approximation and one can anticipate nontrivial observable phenomena at moderate Q^2, Q^2 approximately few GeV^2.

C. Relativistic description within Bethe-Salpeter formalism

The matrix elements of the twist-2 operators (12) and (13) on the deuteron state in Bethe-Salpeter formalism are computed explicitly using the Mandelstam method [43].

The corresponding moments of the deuteron structure function in the ladder approximation (Fig. 1) have the explicit form Eq. (14) with

$$\mu_n^{N/D} = \frac{1}{2M_D^n}\int \frac{d^4p}{(2\pi)^4}\bar{\Phi}_D(p)\{\hat{S}^{-1}(p_2)(\gamma_0^{(1)}+\gamma_3^{(1)})(p_{10}+p_{13})^{n-1} + \hat{S}^{-1}(p_1)(\gamma_0^{(2)}+\gamma_3^{(2)})(p_{20}+p_{23})^{n-1}\}\Phi_D(p)\ , \tag{15}$$

$$\mu_n^{\sigma/D} = \frac{g_\sigma^2}{M_D^n}\int \frac{d^4p\,d^4p'}{(2\pi)^8}\bar{\Phi}_D(p)\frac{(k_0+k_3)^n}{(k^2-\mu_\sigma^2)^2}\Phi_D(p')\ , \tag{16}$$

where the kinematical variables in the rest frame are defined by

$$p = (p_0,\mathbf{p}),\quad p' = (p_0',\mathbf{p}'),\quad k = p - p',\quad P = (M_D,\mathbf{0}),\quad p_1 = \frac{P}{2}+p,\quad p_2 = \frac{P}{2}-p\ , \tag{17}$$

where p_{ik} is kth ($k = 0,\ldots,3$) component of four-vector p_i ($i = 1,2$) and we use deep inelastic kinematics: $pq \approx q_0(p_0+p_3)$ and the OZ-axis direction is chosen such that $q = (q_0, \mathbf{0}_\perp, -q_3)$. The vertex functions $\bar{\Phi}_D(p)$ and $\Phi_D(p)$ are the Bethe-Salpeter amplitudes in momentum space:

$$\Phi(p) = \int d^4x\langle 0|T(\psi_2\,(Y-x/2)\,\psi_1(Y+x/2))|D\rangle e^{iP_DY}e^{ipx}\ , \tag{18}$$

$$\bar{\Phi}(p) = \int d^4x\langle D|T(\bar{\psi}_1(Y+x/2)\bar{\psi}_2(Y-x/2))|0\rangle e^{-iP_DY}e^{-ipx}\ , \tag{19}$$

and we denote the Dirac propagator and the inverse Dirac propagator as

$$\hat{S}(p_i) = \frac{\hat{p}_i + m}{p_i^2 - m^2} , \tag{20}$$

$$\hat{S}^{-1}(p_i) = \hat{p}_i - m . \tag{21}$$

D. Convolution form of the deuteron structure function

Using the results of the two previous sections, we find an explicit expression for the deuteron structure function by the inverse Mellin transform of the moments (14), (15), and (16). In view of the inverse powerlike behavior of the ladder Bethe-Salpeter amplitude in the momentum space, the contributions (15) and (16) to the deuteron structure function moments, (14) appear as divergent integrals at high enough values of n. A formal procedure is adopted applying regularization in the integrals (15) and (16) in the form of a simple cutoff at some $p_{\max}$. The regularization is then removed by taking the limit $p_{\max} \to \infty$ when the inverse Mellin transform has been calculated. In the present paper, to estimate nuclear effects in the deuteron structure function at moderate x, we calculate moments only at lowest n, where the regularization procedure is not needed.

The inverse Mellin transform of (10) is defined as

$$F(x) = \frac{1}{2\pi i} \int_{\gamma - i\infty}^{\gamma + i\infty} dn\, x^{-n} \mathsf{M}_n(F) . \tag{22}$$

Using the explicit form of the moments (14)–(16), we "restore" the deuteron structure function to a convolution form:

$$F_2^D(x) = \int_0^1 f^{N/D}(\xi) F_2^N(x/\xi) d\xi + \int_0^1 f^{\sigma/D}(\xi) F_2^\sigma(x/\xi) d\xi , \tag{23}$$

where

$$f^{N/D}(\xi) = \frac{1}{M_D} \int \frac{d^4p}{(2\pi)^4} \bar{\Phi}_D(p) \hat{S}^{-1}(p_2)(\gamma_0^{(1)} + \gamma_3^{(1)}) \Phi_D(p) \times \left\{ \theta(p_{10} + p_{13}) \delta\left(\xi - \frac{p_{10} + p_{13}}{M_D}\right) + \theta(-p_{10} - p_{13}) \delta\left(\xi + \frac{p_{10} + p_{13}}{M_D}\right) \right\} , \tag{24}$$

$$f^{\sigma/D}(\xi) = \frac{g_\sigma^2}{M_D} \int \frac{d^4p\, d^4p'}{(2\pi)^8} \bar{\Phi}_D(p) \frac{(k_0 + k_3)}{(k^2 - \mu_\sigma^2)^2} \Phi_D(p') \times \left\{ \theta(k_0 + k_3) \delta\left(\xi - \frac{k_0 + k_3}{M_D}\right) + \theta(-k_0 - k_3) \delta\left(\xi + \frac{k_0 + k_3}{M_D}\right) \right\} . \tag{25}$$

In (24) we have taken into account the symmetry of the deuteron with respect to the two nucleons. Note that a natural variable in the operator product expansion of the deuteron amplitude is $x = -q^2/2M_D q_0 \equiv x_D = (m/M_D)x_N$, which is different from x_N in (3). The variable x_N is utilized usually in discussions of the structure functions, including the nuclear ones, and the experimental data are usually presented in this variable. The transition from x_D to x_N is trivial. However, the dependence of the deuteron structure functions on the variable x_D shows that, in principle, the deuteron structure function (23) is defined in the region of $x_N \in (0, M_D/m)$ (M_D/m is the exact threshold of the reaction), i.e., including the region beyond the single nucleon kinematics $x_N = 1$. This "cumulative" region in nuclear case is due to the Fermi motion of nucleons. For the analysis of the structure functions at large x_N, the high-n behavior of moments of the structure functions is essential [14]. Therefore, when the region $x_N > 1$ is discussed, the behavior of the moments at large n must be studied more carefully. In particular, the validity of using approximations, such as the ladder approximation and leading twist approximation, should be checked in the high-n limit.

It is to be stressed that the convolution formula (23) is not assumed here, but it is a result of the calculation within the framework formulated in the Sec. II B. This formula has a clear and obvious physical interpretation. The first term on the right-hand side (RHS) represents the contribution of the nucleons to the full deuteron structure function F_2^D, the *relativistic impulse approximation.* Here both nucleons are manifestly off mass shell with regard to the conservation of the four-momenta in all vertexes of the Feynman diagrams for the process. The second term is the contribution of the *meson exchange current.* In the model with several mesons, there will be a sum over all mesons on the RHS of (23).

To have a normal physical interpretation, the functions $f^{N/D}$ and $f^{\sigma/D}$ must satisfy the normalization conditions [44]. Since the baryonic charge in the present model is connected only with nucleonic fields, we should have

$$\int_0^1 f^{N/D}(\xi) d\xi = 2 , \tag{26}$$

where 2 is number of baryons in the deuteron. Then the integral of the structure function F_2 of any hadron target is proportional to the momentum carried by its constituents. For the deuteron structure function (23) we should have

$$\langle \xi \rangle_D \equiv \int_0^1 f^{N/D}(\xi)\xi \, d\xi + \int_0^1 f^{\sigma/D}(\xi)\xi \, d\xi = 1 \; , \quad (27)$$

which means that all the momentum of the deuteron is carried by its constituent, the nucleons and mesons. The following section is devoted to the derivation of the sum rules (26) and (27) in the Bethe-Salpeter formalism.

E. Baryon number and energy-momentum sum rules

The Bethe-Salpeter amplitudes $\bar{\Phi}_D(p)$ and $\Phi_D(p)$ satisfy the normalization condition [3,43,45], which in ladder approximation has the form

$$\int \frac{d^4p}{(2\pi)^4} \bar{\Phi}_D(p) S^{-1}(p_1;p_2) \left[\frac{\partial S(p_1;p_2)}{\partial P_\mu} \right]_{P^2=M_D^2} S^{-1}(p_1;p_2)\Phi_D(p) = -2iP_\mu \; , \quad (28)$$

where

$$S(p_1;p_2) = \hat{S}(p_1)\hat{S}(p_2) \; . \quad (29)$$

The physical meaning of the normalization (28) is clarified by an explicit calculation of $\partial S(p_1,p_2)/\partial P_\mu$:

$$\frac{\partial S(p_1;p_2)}{\partial P_\mu} = -S(p_1;p_2) \left[\frac{\partial S^{-1}(p_1;p_2)}{\partial P_\mu} \right] S(p_1;p_2) = -\frac{1}{2}\{\hat{S}(p_1)\gamma_\mu^{(1)} + \hat{S}(p_2)\gamma_\mu^{(2)}\} S(p_1;p_2) \; . \quad (30)$$

Substituting (30) into Eq. (28) one finds that (28) is nothing but the expression for the conserved vector current:

$$\langle D | \{ e_1 \bar{\psi}_1 \gamma_\mu^{(1)} \psi_1 + e_2 \bar{\psi}_2 \gamma_\mu^{(2)} \psi_2 \} | D \rangle = 2P_\mu (e_1 + e_2) \; . \quad (31)$$

Thus the integral (26) is evaluated explicitly:

$$\int_0^1 f^{N/D}(\xi) d\xi = \frac{1}{M_D} \int \frac{d^4p}{(2\pi)^4} \bar{\Phi}_D(p) \hat{S}^{-1}(p_2)(\gamma_0^{(1)} + \gamma_3^{(1)})\Phi_D(p) = 2 \; , \quad (32)$$

as it should be.

It is easily shown that the twist-2, $n = 2$ operators (12) and (13) are related to the energy-momentum tensor of the theory under consideration. Thus we begin the derivation of the integral (27) from a consideration of the trace of the energy-momentum tensor $\Theta_{\mu\nu}$.

For the theory with the Lagrangian (5), (6), and (7), the trace of the energy-momentum tensor is [46]

$$\Theta_\mu^\mu(x) = m\bar{\psi}(x)\psi(x) + \mu_\sigma^2 \phi^2(x) \quad (33)$$

$$= \mathcal{H}(x) + \frac{i}{2}\bar{\psi}(x)(\boldsymbol{\gamma}\overleftrightarrow{\boldsymbol{\partial}})\psi(x) - \boldsymbol{\partial}\phi(x)\boldsymbol{\partial}\phi(x) - \frac{3}{2}g_\sigma \phi(x)\bar{\psi}(x)\psi(x) \; , \quad (34)$$

where $\mathcal{H}$ is the total Hamiltonian density. In the rest frame of the deuteron, one should have simultaneously

$$\langle D | \Theta_\mu^\mu(0) | D \rangle = 2M_D^2 \quad (35)$$

and

$$\int d^3x \langle D | \mathcal{H}(x) | D \rangle = M_D \langle D | D \rangle = 2M_D^2 V \; ; \quad (36)$$

i.e., in spite of the difference of the explicit expressions of $\Theta_\mu^\mu(x)$ and $\mathcal{H}(x)$, they should have the same matrix element for the deuteron state. It can be shown that the difference between them is equal to zero in view of the virial theorem (B6), (B7), and (B10) (see Appendix B).

Coming back to the energy-momentum sum rule (27) for the deuteron, we see that $\langle \xi \rangle_D$ is just a sum of the moments $\mu_n^{N/D}$ and $\mu_n^{\sigma/D}$ at $n = 2$. Taking into account the explicit form of the moments (15) and (16) and averaging for the unpolarized deuteron, the following relations hold:

$$\langle D | \Theta^{03}(0) | D \rangle = \langle D | \Theta^{30}(0) | D \rangle = 0 \; , \quad (37)$$

$$\langle D|\partial_3\phi(0)\partial_3\phi(0)|D\rangle = \tfrac{1}{3}\langle D|\boldsymbol{\partial}\phi(0)\boldsymbol{\partial}\phi(0)|D\rangle \ , \tag{38}$$

$$\langle D|\bar{\psi}(0)(\gamma_3\overleftrightarrow{\partial})\psi(0)|D\rangle = \tfrac{1}{3}\langle D|\bar{\psi}(0)(\boldsymbol{\gamma}\overleftrightarrow{\boldsymbol{\partial}})\psi(0)|D\rangle \ , \tag{39}$$

and we get

$$\begin{aligned}\langle\xi\rangle &= \frac{1}{2M_D^2}\langle D|\{\Theta^{00}(0)+\Theta^{33}(0)\}|D\rangle \\ &= \frac{1}{2M_D^2}\langle D|\left\{\Theta^{\mu}_{\mu}(0)+\frac{4}{3}\left(-\frac{i}{2}\bar{\psi}(0)(\boldsymbol{\gamma}\overleftrightarrow{\boldsymbol{\partial}})\psi(0)+\boldsymbol{\partial}\phi(0)\boldsymbol{\partial}\phi(0)+\frac{3}{2}g_\sigma\phi(0)\bar{\psi}(0)\psi(0)\right)\right\}|D\rangle \ . \end{aligned}\tag{40}$$

Therefore the sum rule (27) is satisfied in view of (35) and the virial theorem (see Appendix B).

F. Nonrelativistic limit of the theory

There is a popular opinion that theoretical considerations of nuclear processes at high momentum transfers automatically require a relativistic description of the nuclear structure. It is very often, but not always, true. The choice of kinematical conditions is important. For instance, the structure functions $F_2^A(x_N, Q^2)$ of nuclei in deep inelastic scattering ($Q^2 \gg 1$ GeV^2) for a wide region of the Bjorken variable x_N ($x_N < 1$) can be considered as scattering on nucleons, which move nonrelativistically in a nuclear potential, the x–rescaling model [31–34]. Only beyond the single nucleon kinematics ($x_N > 1$) do the high momentum components of the nuclear wave function become important. However, a fully relativistic analysis of the reactions is of course preferable, especially in the problematic cases and in crucial regions of x_N and Q^2. Furthermore, there are some doubts about the equivalence of relativistic and nonrelativistic descriptions of deep inelastic reactions [27,29] and comparison of consistent relativistic and nonrelativistic results would help to clarify the issue.

The consistent nonrelativistic reduction of the Bethe-Salpeter equation and/or Bethe-Salpeter amplitude is unknown. Moreover a statement of the problem to reduce this equation or amplitude may be incorrect if it is to be considered independently of the dynamics of the initial field theory.

An unambiguous way to find the nonrelativistic limit of the theory is to reduce the initial field-theoretical formalism and then consider the dynamical problem within a nonrelativistic theory. This can be done by a consistent method suggested in [47] and is adopted to deep inelastic scattering on deuterons in Refs. [26,13]. This method is very similar to the famous Foldy-Wouthuysen transform, and it is briefly outlined below (here we schematically follow Refs. [26,13] and all details can be found therein).

The classical equation of motion for the theory with Lagrangian (5), (6), and (7) has the form

$$(i\hat{\partial}-m)N(x) = g_\sigma\phi(x)N(x) \ , \tag{41}$$

$$(\Box+\mu_\sigma^2)\phi(x) = -g_\sigma\bar{N}(x)N(x) \ , \tag{42}$$

where the Dirac bispinor field $N(x)$ could be determined in terms of the large and small components $f(x)$ and $\chi(x)$, respectively.

Antinucleon degrees of freedom are eliminated by a nonrelativistic reduction of the matrix Eq. (41). To achieve this objective we employ Eq. (41) to express the small component $\chi(x)$ of the spinor field $N(x)$ in terms of a large component $f(x)$ in leading order of the $1/m$ expansion:

$$\chi(x) = -\frac{i}{2m}\boldsymbol{\sigma}\boldsymbol{\partial}f(x) \ . \tag{43}$$

Note that the usual nonrelativistic limit is nothing but the retention of only the leading terms in the $1/m$ expansion. In interacting meson-nucleon theory, this approximation restricts the calculations to the lowest order in the coupling constant g^2, which leads to the Schrödinger equation with the static one-boson-exchange potential.

In principle, the expression (43) allows one to calculate any covariant object in terms of the nonrelativistic spinor field $f(x)$. However, it has been noted [48,47] that the field $f(x)$ does not obey conditions of normalization of the probability density and charge and consequently cannot serve as a true second-quantized field. A new field $\psi(x) = (\hat{I}+\hat{F})f(x)$ is introduced. The operator $\hat{I}$ is a unit operator, and $\hat{F}$ is defined so that the field $\psi(x)$ obeys all the conditions for a canonical second-quantized field. For the case under consideration, we have

$$\psi(x) = \left(\hat{I}-\frac{\Delta}{8m^2}\right)f(x) \ . \tag{44}$$

The case of the pseudoscalar exchanges is presented in Refs. [47,26].

To compute any observable, the corresponding fully relativistic expression should be rewritten using Eqs. (43) and (44). For instance, the Hamiltonian is found from the Lagrangian (5)–(7) and has the form

$$H_0^\sigma = \frac{1}{2}\int d^3x\{[\nabla\phi(x)]^2+\dot{\phi}(x)\dot{\phi}(x)+\mu_\sigma^2\phi(x)\phi(x)\} \ , \tag{45}$$

$$H_0^N + H_{\rm int} = \int d^3x \left\{ \frac{1}{2m} \nabla\psi^\dagger(x)\nabla\psi(x) + m\psi^\dagger(x)\psi(x) \right\}$$
$$+ g_\sigma \int d^3x \{\psi^\dagger(x)\phi(x)\psi(x)\} \ .$$

The same reduction should be applied to the interaction operator in (2), i.e., in the leading twist approximation to the operators (12) and (13). To compute the amplitude (2), we need to solve the bound state problem, which in the consistent nonrelativistic limit must lead to the Schrödinger equation. The bound state can be found from the following equation

$$H|D\rangle = M_D|D\rangle \ . \tag{46}$$

Since the meson number operator commutes with the Hamiltonian (45), the physical state can be represented in the Tamm-Dancoff method as a superposition of states with bar nucleons and a varying number of virtual mesons. For the deuteron one gets

$$|D\rangle = \sqrt{1-Z_D}\varphi_0^D|NN\rangle + \varphi_1^D|NN\sigma\rangle + \varphi_2^D|NN\sigma\sigma\rangle + \cdots \ , \tag{47}$$

where Z_D is the constant of renormalization of the wave function determined by the condition $\langle D|D\rangle = 1$. In the rest system of the deuteron, it is convenient to redefine the wave function φ_0^D as

$$\varphi_0^D(\mathbf{p}_1,\mathbf{p}_2) = (2\pi)^3\delta^{(3)}(\mathbf{p}_1+\mathbf{p}_2)\varphi_0^D(\mathbf{p}_1) \ , \tag{48}$$

and then it obeys the usual Schrödinger equation with the one-boson-exchange potential,

$$2\frac{\mathbf{p}^2}{2m}\varphi_0^D(\mathbf{p}) - \int \frac{d^3\mathbf{k}}{(2\pi)^3}\frac{g_\sigma^2}{\omega^2(\mathbf{k})}\varphi_0^D(\mathbf{p}+\mathbf{k}) = \varepsilon_D\varphi_0^D(\mathbf{p}) \ , \tag{49}$$

where $\varepsilon_D = M_D - 2m$ and $\omega(\mathbf{k}) = \sqrt{\mathbf{k}^2+\mu^2}$. In what follows we will use for the deuteron wave function φ_0^D another notation Ψ_D bearing in mind that when other meons $(\pi,\omega,\rho,\ldots)$ are included, Ψ_D will represent the well-known nonrelativistic deuteron function computed, for instance, by the Bonn or Paris groups [20,49]. The wave functions $\varphi_{1,2}^D$ are expressed in terms of the wave function Ψ_D, and its explicit form can be found from Eq. (46).

The baryon charge in the present nonrelativistic theory is by construction conserved. To check the consistency of the approach, let us compute the matrix element of the energy-momentum tensor, Eq. (33), with the deuteron state:

$$\langle D|\theta_\mu^\mu(0)|D\rangle = 2\int \frac{d^3\mathbf{p}}{(2\pi)^3}\left(m - \frac{\mathbf{p}^2}{2m}\right)\Psi_D^\dagger(\mathbf{p})\Psi_D(\mathbf{p}) + 2\mu_\sigma^2\int \frac{d^3\mathbf{p}_1 d^3\mathbf{p}_2}{(2\pi)^6}\frac{g_\sigma^2}{\omega^4(\mathbf{k})}\Psi_D^\dagger(\mathbf{p}_1)\Psi_D(\mathbf{p}_2) \ , \tag{50}$$

where $\mathbf{k} = \mathbf{p}_1+\mathbf{p}_2$. Now we rewrite (50) in the coordinate representation

$$\langle D|\theta_\mu^\mu(0)|D\rangle = 2m + \int d^3\mathbf{r}\,\Psi_D^\dagger(\mathbf{r})\left(-\frac{\Delta}{m} + r\frac{d}{dr}V(r) + V(r)\right)\Psi_D(\mathbf{r}) \ , \tag{51}$$

where V is the nucleon-nucleon potential. Using the virial theorem (see Appendix B) we have

$$\langle rdV/dr\rangle = 2\langle T\rangle \ , \tag{52}$$

where $\langle T\rangle \equiv \langle p^2/m\rangle$ is the mean kinetic energy of nucleons in the deuteron and $\langle V\rangle = \varepsilon_D - \langle T\rangle$. Then we get

$$\langle D|\theta_\mu^\mu(0)|D\rangle = 2m + \varepsilon_D \equiv M_D \ . \tag{53}$$

Thus we obtain a consistent nonrelativistic approach, which allows a description of the bound state, the deuteron, and the reactions with this data, starting from a fully covariant theory. Note that this solution does not utilize the Bethe-Salpeter equation and appears independent of the Bethe-Salpeter formalism. However, it was shown by Schweber [50] that the Schrödinger equation is equivalent to the nonrelativistic Bethe-Salpeter equation. Therefore, at least in principle, there is the possibility of a consistent nonrelativistic reduction of the Bethe-Salpter equation.

Calculating the reduced matrix elements (9) by the nonrelativistic reduction of the twist-2 operators (12) and (13), a consistent nonrelativistic limit of the moments is obtained [13,14]. The effective distribution function of the nucleons in the nonrelativistic limit is

$$f^{N/D}(y) = \int \frac{d^3\mathbf{p}}{(2\pi)^3}|\Psi_D(\mathbf{p})|^2\left(1+\frac{p_z}{m}\right)\delta\left(y - \left[1 + \frac{\mathbf{p}^2}{2m^2} + \frac{p_z}{m}\right]\right) + \int \frac{d^3\mathbf{p}\,d^3\mathbf{k}}{(2\pi)^6}\Psi_D^\dagger(\mathbf{p})V(\mathbf{k})\Psi_D(\mathbf{p}+\mathbf{k})\frac{1}{k_z}\left\{\delta\left(y - \left|1+\frac{k_z}{2m}\right|\right) - \delta\left(y - \left|1-\frac{k_z}{2m}\right|\right)\right\} \ . \tag{54}$$

This distribution, like distribution (4), is normalized to one nucleon. In the g^2 approximation, moments of this function coincide with moments of the effective distribution of the nucleons of the x-rescaling model (4). The explicit form of the nucleonic contribution to the moments of the deuteron structure function F_2^D [13,14] is

$$\mathsf{M}_n^D(F_2^D) \simeq \mathsf{M}_n^N(F_2^N)\left(1 + \frac{1}{6}n^2\frac{\langle T\rangle}{m} + \frac{2}{3}n\frac{\langle T\rangle}{m} + n\frac{\langle V\rangle}{m}\right) + \mathcal{O}(g^3) \ . \tag{55}$$

This will be used in discussion of the results for the Bethe-Salpeter formalism.

III. BETHE SALPETER EQUATION FOR THE DEUTERON

A. Spinor-spinor Bethe-Salpeter equation

Here we use the technique, which is often utilized in meson physics [4,5] and "canonical" studies of the properties of the Bethe-Salpeter equation [3], but has not been applied to the description of the NN system in the framework of the Bethe-Salpeter formalism with both nucleons off mass shell [7–11,51]. A similar parametrization of the relativistic nucleon-nucleon vertex of the deuteron was utilized in the simpler case of a formalism with one nucleon on mass shell [52]. In some details our technique is similar to the one given by Gourdin [8], but our formalism has an explicit covariant form and is easily applicable to deep inelastic lepton scattering. The spinor-spinor Bethe-Salpeter equation in the ladder approximation has the form

$$\Phi(p) = iS(p_1,p_2)\sum_B \int \frac{d^4p'}{(2\pi)^4}\frac{g_B^2\Gamma_B^{(1)}\Gamma_B^{(2)}}{(p-p')^2-\mu_B^2}\Phi(p') , \tag{56}$$

where μ_B is the mass of meson B; Γ_B is the meson-nucleon vertex, corresponding to the meson B. For convenience, we introduce the "charge-conjugated" amplitude

$$\Phi = \Psi\gamma^c \quad \text{or} \quad \Psi = -\Phi\gamma^c , \tag{57}$$

where γ^c is the conjugation matrix (A4).

In the general case, the amplitudes Ψ and Φ appear as 4×4 matrices in the space of indices of the spinor fields. Let the row index correspond to the indices of the first particle and the column index to the second particle. Thus Eq. (56) for amplitude Ψ reads

$$\Psi(p) = iD(p_0,\mathbf{p}^2)\sum_B \int \frac{d^4p'}{(2\pi)^4}\frac{g_B^2}{(p-p')^2-\mu_B^2} \times\hat{\Lambda}(p_1)\Gamma_B\Psi(p')\Gamma_B\hat{\Lambda}(p_2) , \tag{58}$$

where $\hat{\Lambda}(p) = (\hat{p}+m)$ and D is the "two-particle scalar propagator,"

$$D^{-1}(p_0,\mathbf{p}^2) \equiv [\omega^2(\mathbf{p}) - p_0^2 - \tfrac{1}{4}M_D^2]^2 - p_0^2M_D^2 . \tag{59}$$

We parametrize the Bethe-Salpeter amplitude using the decomposition in terms of a complete set of the Dirac matrices, their bilinear combinations (A5), and the 4×4-identity matrix, $\hat{\mathbb{1}}$, as

$$\Psi(p) = \hat{\mathbb{1}}\psi_s(p) + \gamma_5\psi_p(p) + \gamma_\mu\psi_v^\mu(p) + \gamma_5\gamma_\mu\psi_a^\mu(p) +\sigma_{\mu\nu}\psi_t^{\mu\nu}(p) . \tag{60}$$

In the rest frame we use a specific notation for the three-vector components:

$$\psi_v^\mu(p) \equiv (\psi_v^0(p), \boldsymbol{\psi}_v(p)) , \tag{61}$$

$$\psi_a^\mu(p) \equiv (\psi_a^0(p), \boldsymbol{\psi}_a(p)) . \tag{62}$$

Using the antisymmetric property of the tensor component $\psi_t^{\mu\nu}$, we introduce three-vector notation in the rest frame:

$$\psi_t^{0i}(p) \equiv \boldsymbol{\psi}_t^0(p) , \tag{63}$$

$$\psi_t^{ij}(p) \equiv \varepsilon^{ijk}\psi_t^k(p) \quad [\psi_t^k(p) \equiv \boldsymbol{\psi}_t(p)] , \tag{64}$$

where $i,j,k = 1,2,3$ and other tensor components are equal to zero.

So we have four scalar functions ψ,

$$\psi_s(p), \psi_p(p), \psi_v^0(p), \psi_a^0(p) ; \tag{65}$$

and four three-vector functions $\boldsymbol{\psi}$,

$$\boldsymbol{\psi}_v(p), \boldsymbol{\psi}_a(p), \boldsymbol{\psi}_t^0(p), \boldsymbol{\psi}_t(p) . \tag{66}$$

Inserting the parameterization (60) into (58) and using the orthogonality of our basis (A6), we obtain a set of equations for the components (65) and (66). Calculations for exchanges of any kind are straightforward, but cumbersome. For example, for the scalar meson exchange, we get

$$\psi_s(p) = \hat{\mathsf{K}}\{[m^2 - p^2 + \tfrac{1}{4}M_D^2]\psi_s + mM_D\psi_v^0 + 2iM_D(\mathbf{p}\cdot\boldsymbol{\psi}_t^0)\} , \tag{67}$$

$$\psi_p(p) = \hat{\mathsf{K}}\{[m^2 + p^2 - \tfrac{1}{4}M_D^2]\psi_p - 2mp_0\psi_a^0 + 2m(\mathbf{p}\cdot\boldsymbol{\psi}_a) + 2M_D(\mathbf{p}\cdot\boldsymbol{\psi}_t)\} , \tag{68}$$

$$\psi_v^0(p) = \hat{\mathsf{K}}\{mM_D\psi_s + [m^2 + p^2 - 2p_0^2 + \tfrac{1}{4}M_D^2]\psi_v^0 + 2p_0(\mathbf{p}\cdot\boldsymbol{\psi}_v) + 4im(\mathbf{p}\cdot\boldsymbol{\psi}_t^0)\} , \tag{69}$$

$$\psi_a^0(p) = \hat{\mathsf{K}}\{-2mp_0\psi_p + [m^2 - p^2 + 2p_0^2 - \tfrac{1}{4}M_D^2]\psi_a^0 - 2p_0(\mathbf{p}\cdot\boldsymbol{\psi}_a)\} , \tag{70}$$

$$\boldsymbol{\psi}_v(p) = \hat{\mathsf{K}}\{-2p_0\mathbf{p}\psi_v^0 + 2\mathbf{p}(\mathbf{p}\cdot\boldsymbol{\psi}_v) + [m^2 + p^2 - \tfrac{1}{4}M_D^2]\boldsymbol{\psi}_v + iM_D[\mathbf{p}\times\boldsymbol{\psi}_a] + 4imp_0\boldsymbol{\psi}_t^0 + 4im[\mathbf{p}\times\boldsymbol{\psi}_t]\}\ , \quad (71)$$

$$\boldsymbol{\psi}_a(p) = \hat{\mathsf{K}}\{-2m\mathbf{p}\psi_p + 2p_0\mathbf{p}\psi_a^0 - iM_D[\mathbf{p}\times\boldsymbol{\psi}_v] - 2\mathbf{p}(\mathbf{p}\cdot\boldsymbol{\psi}_a) + [m^2 - p^2 + \tfrac{1}{4}M_D^2]\boldsymbol{\psi}_a + 2mM_D\boldsymbol{\psi}_t\}\ , \quad (72)$$

$$\boldsymbol{\psi}_t^0(p) = \hat{\mathsf{K}}\{\tfrac{1}{2}iM_D\mathbf{p}\psi_s + imp\psi_v^0 - imp_0\boldsymbol{\psi}_v + [m^2 - p^2 + 2p_0^2 - \tfrac{1}{4}M_D^2]\boldsymbol{\psi}_t^0 - 2\mathbf{p}(\mathbf{p}\cdot\boldsymbol{\psi}_t^0) + 2p_0[\mathbf{p}\times\boldsymbol{\psi}_t]\}\ , \quad (73)$$

$$\boldsymbol{\psi}_t(p) = \hat{\mathsf{K}}\{-\tfrac{1}{2}M_D\mathbf{p}\psi_p - im[\mathbf{p}\times\boldsymbol{\psi}_v] + \tfrac{1}{2}mM_D\boldsymbol{\psi}_a + [m^2 + p^2 - 2p_0^2 + \tfrac{1}{4}M_D^2]\boldsymbol{\psi}_t + 2p_0[\mathbf{p}\times\boldsymbol{\psi}_t^0] + 2\mathbf{p}(\mathbf{p}\cdot\boldsymbol{\psi}_t)\}\ , \quad (74)$$

where the operator $\hat{\mathsf{K}}$ is defined by

$$\hat{\mathsf{K}}\psi = iD(p_0,\mathbf{p}^2)\int \frac{d^4p'}{(2\pi)^4}\frac{g_\sigma^2}{(p-p')^2-\mu^2}\psi(p')\ . \quad (75)$$

B. Partial wave decomposition and the deuteron state

To solve the set of Eqs. (67)–(74) we first perform the integration on the angular variables. To do this we expand the scalar functions ψ in spherical harmonics and the vector functions $\boldsymbol{\psi}$ in the vector spherical harmonics [53,54]:

$$\psi(p) = \frac{1}{|\mathbf{p}|}\sum_{J,M}\psi(p_0,|\mathbf{p}|;JM)Y_{JM}(\Omega_p)\ , \quad (76)$$

$$\boldsymbol{\psi}(p) = \frac{1}{|\mathbf{p}|}\sum_{J,M}\Big\{\psi(p_0,|\mathbf{p}|;J-1JM)\mathbb{Y}_{JM}^{J-1}(\Omega_p) + \psi(p_0,|\mathbf{p}|;JJM)\mathbb{Y}_{JM}^{J}(\Omega_p) + \psi(p_0,|\mathbf{p}|;J+1JM)\mathbb{Y}_{JM}^{J+1}(\Omega_p)\Big\}\ . \quad (77)$$

Then we expand the boson propagator in (75) as

$$\frac{1}{(p-p')^2-\mu^2} = -\frac{2\pi}{|\mathbf{p}|\cdot|\mathbf{p}'|}\sum_{l,\lambda}Y_{l\lambda}^*(\Omega_p')Y_{l\lambda}(\Omega_p)\mathsf{Q}_l(y_\mu)\ , \quad (78)$$

where $\mathsf{Q}_l(y_\mu)$ is the Legendre function of the second kind (see Appendix C) and

$$y_\mu = \frac{|\mathbf{p}|^2 + |\mathbf{p}'|^2 + \mu^2 - (p_0 - p_0')^2}{2|\mathbf{p}|\cdot|\mathbf{p}'|}\ . \quad (79)$$

Using the orthogonality properties of the harmonics Y_{Lm} and $\mathbb{Y}_{JM}^L$ and relations (C4)–(C10) from Appendix C, we get a set of equations for the partial wave amplitudes ψ. It is remarkable that the full system of equations splits into two independent subsystems for the amplitudes:

$$\psi_s(JM),\ \ \psi_v^0(JM),\ \ \psi_v(J-1JM),\ \ \psi_v(J+1JM),\ \ \psi_a(JJM),\ \ \psi_t^0(J-1JM),\ \ \psi_t^0(J+1JM),\ \ \psi_t(JJM) \quad (80)$$

and

$$\psi_p(JM),\ \ \psi_a^0(JM),\ \ \psi_a(J-1JM),\ \ \psi_a(J+1JM)\ ,\ \ \psi_v(JJM),\ \ \psi_t^0(JJM),\ \ \psi_t(J-1JM),\ \ \psi_t(J+1JM). \quad (81)$$

This splitting is the result of the parity conservation in NN interaction, and the two sets of amplitudes (80) and (81) correspond to states with different parity. Indeed, the parity transformation operator for amplitude (60) reads

$$\hat{\mathcal{P}}\Psi(p_0,\mathbf{p}) = \gamma_0\Psi(p_0,-\mathbf{p})\gamma_0\ . \quad (82)$$

Applying the $\hat{\mathcal{P}}$ to (60) and taking into account the parity of the spherical harmonics (76) and (77), we find that the amplitudes (80) belong to the parity $(-1)^J$ and (81) belong to the parity $(-1)^{J+1}$.

We are now in a position to define the system of equations for the deuteron state by requiring definite quantum numbers for the angular momentum ($J = 1$) and parity ($\mathcal{P} = +1$). Taking into account that J and M are good quantum numbers and that only $J = 1$ is interpreted as the deuteron state, we omit these quantum numbers in formulas for the amplitudes. For the states with different $L = J$, $J \pm 1$, we introduce new indices (for the state without quantum number L we understand $L = J$). The final notations are

$$\psi(p_0,|\mathbf{p}|;1M) = \psi_1(p_0,|\mathbf{p}|)\ , \quad (83)$$

for components $\psi_p(JM)$ and $\psi_a^0(JM)$, and

$$\psi(p_0,|\mathbf{p}|;01M) = \psi_0(p_0,|\mathbf{p}|)\ , \quad (84)$$

$$\psi(p_0,|\mathbf{p}|;11M) = \psi_1(p_0,|\mathbf{p}|)\ , \quad (85)$$

$$\psi(p_0,|\mathbf{p}|;21M) = \psi_2(p_0,|\mathbf{p}|)\ , \quad (86)$$

for all remaining components. The states with $L = 0,1,2$ are states with S, P, D configurations in three-dimensional momentum space. We also introduce the notation $\Psi_M^D(p_0,\mathbf{p})$ for the deuteron Bethe-Salpeter amplitude. This amplitude has components listed in (81) with $J = 1$, and all other components are zeros.

For the scalar meson exchange, we get

$$\psi_{p1}(p_0,|\mathbf{p}|)=\hat{\mathsf{K}}_1\Bigg\{[m^2+p^2-\tfrac{1}{4}M_D^2]\psi_{p1}-2mp_0\psi_{a1}^0 +2m|\mathbf{p}|\left[\frac{1}{\sqrt{3}}\psi_{a0}-\left(\frac{2}{3}\right)^{1/2}\psi_{a2}\right]+2M_D|\mathbf{p}|\left[\frac{1}{\sqrt{3}}\psi_{t0}-\left(\frac{2}{3}\right)^{1/2}\psi_{t2}\right]\Bigg\}, \tag{87}$$

$$\psi_{a1}^0(p_0,|\mathbf{p}|)=\hat{\mathsf{K}}_1\left\{-2mp_0\psi_{p1}+\left[m^2-p^2+2p_0^2-\frac{1}{4}M_D^2\right]\psi_{a1}^0-2p_0|\mathbf{p}|\left[\frac{1}{\sqrt{3}}\psi_{a0}-\left(\frac{2}{3}\right)^{1/2}\psi_{a2}\right]\right\}, \tag{88}$$

$$\psi_{v1}(p_0,|\mathbf{p}|)=\hat{\mathsf{K}}_1\Bigg\{\left[m^2+p^2-\frac{1}{4}M_D^2\right]\psi_{v1}-M_D|\mathbf{p}|\left[\left(\frac{2}{3}\right)^{1/2}\psi_{a0}+\frac{1}{\sqrt{3}}\psi_{a2}\right] +4imp_0\psi_{t1}^0-4m|\mathbf{p}|\left[\left(\frac{2}{3}\right)^{1/2}\psi_{t0}+\frac{1}{\sqrt{3}}\psi_{t2}\right]\Bigg\}, \tag{89}$$

$$\psi_{a0}(p_0,|\mathbf{p}|)=\hat{\mathsf{K}}_1\Bigg\{-\frac{2}{\sqrt{3}}m|\mathbf{p}|\psi_{p1}+\frac{2}{\sqrt{3}}p_0|\mathbf{p}|\psi_{a1}^0+\left(\frac{2}{3}\right)^{1/2}M_D|\mathbf{p}|\psi_{v1} -\frac{2}{\sqrt{3}}|\mathbf{p}|^2\left[\frac{1}{\sqrt{3}}\psi_{a0}-\left(\frac{2}{3}\right)^{1/2}\psi_{a2}\right]+\left[m^2-p^2+\frac{1}{4}M_D^2\right]\psi_{a0}+2mM_D\psi_{t0}\Bigg\}, \tag{90}$$

$$\psi_{a2}(p_0,|\mathbf{p}|)=\hat{\mathsf{K}}_1\Bigg\{\frac{2\sqrt{2}}{\sqrt{3}}m|\mathbf{p}|\psi_{p1}-\frac{2\sqrt{2}}{\sqrt{3}}p_0|\mathbf{p}|\psi_{a1}^0+\frac{1}{\sqrt{3}}M_D|\mathbf{p}|\psi_{v1} +\frac{2\sqrt{2}}{\sqrt{3}}|\mathbf{p}|\left[\frac{1}{\sqrt{3}}\psi_{a0}-\left(\frac{2}{3}\right)^{1/2}\psi_{a2}\right]+\left[m^2-p^2+\frac{1}{4}M_D^2\right]\psi_{a2}+2mM_D\psi_{t2}\Bigg\}, \tag{91}$$

$$\psi_{t1}^0(p_0,|\mathbf{p}|)=\hat{\mathsf{K}}_1\left\{-imp_0\psi_{v1}+\left[m^2-p^2+2p_0^2-\frac{1}{4}M_D^2\right]\psi_{t1}^0+2ip_0|\mathbf{p}|\left[\left(\frac{2}{3}\right)^{1/2}\psi_{t0}+\frac{1}{\sqrt{3}}\psi_{t2}\right]\right\}, \tag{92}$$

$$\psi_{t0}(p_0,|\mathbf{p}|)=\hat{\mathsf{K}}_1\Bigg\{-\frac{1}{2\sqrt{3}}M_D|\mathbf{p}|\psi_{p1}+\left(\frac{2}{3}\right)^{1/2}m|\mathbf{p}|\psi_{v1}+\frac{1}{2}mM_D\psi_{a0} +\left[m^2+p^2-2p_0^2+\frac{1}{4}M_D^2\right]\psi_{t0}+\frac{2\sqrt{2}}{\sqrt{3}}ip_0|\mathbf{p}|\psi_{t1}^0 +2\frac{1}{\sqrt{3}}|\mathbf{p}|^2\left[\frac{1}{\sqrt{3}}\psi_{t0}-\left(\frac{2}{3}\right)^{1/2}\psi_{t2}\right]\Bigg\}, \tag{93}$$

$$\psi_{t2}(p_0,|\mathbf{p}|)=\hat{\mathsf{K}}_1\Bigg\{\frac{\sqrt{2}}{2\sqrt{3}}M_D|\mathbf{p}|\psi_{p1}+\frac{1}{\sqrt{3}}m|\mathbf{p}|\psi_{v1}+\frac{1}{2}mM_D\psi_{a2} +\left[m^2+p^2-2p_0^2+\frac{1}{4}M_D^2\right]\psi_{t2}+\frac{2}{\sqrt{3}}ip_0|\mathbf{p}|\psi_{t1}^0 -\frac{2\sqrt{2}}{\sqrt{3}}|\mathbf{p}|^2\left[\frac{1}{\sqrt{3}}\psi_{t0}-\left(\frac{2}{3}\right)^{1/2}\psi_{t2}\right]\Bigg\}, \tag{94}$$

where the operator $\hat{\mathsf{K}}_1$ for the scalar theory is defined by

$$\hat{\mathsf{K}}_1\psi_L = -ig_\sigma^2 D(p_0,\mathbf{p}^2)\int\frac{dp_0' d|\mathbf{p}'|}{(2\pi)^3}Q_L(y_\mu)\psi_L(p_0',|\mathbf{p}'|). \tag{95}$$

Note that Eq. (58) is invariant under the transformation

$$p_0, p_0' \to -p_0, -p_0' \ , \tag{96}$$

which is a consequence of the symmetry under the exchange of the particles and properties under the time-reversal transformation of a state. Because of that, all components $\psi(p_0,|\mathbf{p}|)$ are even or odd under $p_0 \to -p_0$. From the explicit form of the set of equations (87)–(94), we find that the components ψ_{t1}^0 and ψ_{a1}^0 are odd and other components are even.

C. Wick rotation

To solve numerically the system of singular equations (87)–(94), we use the well-known Wick rotation [55–57], presented via the substitutions

$$p_0 \to ip_0, \quad p_0' \to ip_0' \ , \tag{97}$$

where we keep the "old" notations for the "new," rotated, momenta. It is also instructive to change the phase of one of the amplitudes:

$$\psi_{a1}^0(p_0,|\mathbf{p}|) \to i\psi_{a1}^0(p_0,|\mathbf{p}|) \ . \tag{98}$$

After these transformations the rotated system (87)–(94) is real.

The transition (97) to the Euclidean space removes all singularities from Eq. (58). The propagator (20) now reads

$$D^{-1}(p_0,\mathbf{p}^2) = [\omega^2(\mathbf{p}) + p_0^2 - \tfrac{1}{4}M_D^2]^2 + p_0^2 M_D^2 \ . \tag{99}$$

Singularities from the exchange boson propagator are removed by the new definition of y_μ in (78):

$$y_\mu = \frac{|\mathbf{p}|^2 + |\mathbf{p}'|^2 + \mu^2 + (p_0 - p_0')^2}{2|\mathbf{p}|\cdot|\mathbf{p}'|} \ . \tag{100}$$

Applying the transforms (97) and (98) to the system (87)–(94), we get a system of rotated Bethe-Salpeter amplitudes for the deuteron.

D. Normalization condition and observables

The Wick-rotated deuteron amplitude $\Psi_M^D(p_0,\mathbf{p})$ satisfies the normalization condition, which is derived from (28):

$$P_\mu = \frac{1}{3}\sum_M\int\frac{d^4p}{(2\pi)^4}\mathrm{Tr}\{\gamma_0\Psi_M^{D\dagger}(p_0,\mathbf{p})\gamma_0\gamma_\mu\Psi_M^D(p_0,\mathbf{p})(\hat{p}_2 - m)\} \ . \tag{101}$$

In the rest frame of the deuteron $[P_D = (M_D,\mathbf{0})]$, this leads to a normalization condition of the form

$$\begin{aligned} M_D = 2\int\frac{dp_0 d|\mathbf{p}|d\Omega_p}{(2\pi)^4}\{&4m[(\boldsymbol{\psi}_t^\dagger\cdot\boldsymbol{\psi}_a) + (\boldsymbol{\psi}_a^\dagger\cdot\boldsymbol{\psi}_t)] \\ &-M_D[\psi_p^\dagger\psi_p + \psi_a^{0\dagger}\psi_a^0 + (\boldsymbol{\psi}_v^\dagger\cdot\boldsymbol{\psi}_v) + (\boldsymbol{\psi}_a^\dagger\cdot\boldsymbol{\psi}_a) + 4(\boldsymbol{\psi}_t^{0\dagger}\cdot\boldsymbol{\psi}_t^0) + 4(\boldsymbol{\psi}_t^\dagger\cdot\boldsymbol{\psi}_t)] \\ &+4[\psi_p^\dagger(\mathbf{p}\cdot\boldsymbol{\psi}_t) + (\mathbf{p}\cdot\boldsymbol{\psi}_t^\dagger)\psi_p] + 2i((\boldsymbol{\psi}_v^\dagger\cdot[\mathbf{p}\times\boldsymbol{\psi}_a]) + ([\mathbf{p}\times\boldsymbol{\psi}_a^\dagger]\cdot\boldsymbol{\psi}_v))\} \ , \end{aligned} \tag{102}$$

where the components of all ψ and $\boldsymbol{\psi}$ are the real solutions of the Wick-rotated set of equations (87)–(94).

The normalization condition is the first and simplest example of the calculation of observables in the Bethe-Salpeter formalism. Other observables are the moments of the structure functions (14), which also can be calculated in terms of the rotated amplitude Ψ_M^D. It can be done, since there are no extra singularities in the integrals (15) and (16). At the same time, to calculate the distribution functions (24) one needs to exercise care in handling of the singular δ functions. The explicit expression for the nucleon contribution to the moments of deuteron structure function reads

$$\mathsf{M}_n(F_2^D) = \mathsf{M}_n(F_2^N)\frac{1}{3}\sum_M\int\frac{d^4p}{(2\pi)^4}\mathrm{Tr}\{\gamma_0\Psi_M^{D\dagger}(p_0,\mathbf{p})\gamma_0[(\gamma_0+\gamma_3)(p_{10}+p_{13})^{n-1}]\Psi_M^D(p_0,\mathbf{p})(\hat{p}_2 - m)\} \ . \tag{103}$$

To present the moments (103) in terms of the real components, the definition of Ψ_M^D and straightforward (but tedious) algebra are used.

IV. NUMERICS, RESULTS, AND DISCUSSION

A. Numerical estimation of the nuclear effects

To calculate the Bethe-Salpeter amplitude for the deuteron and to perform a full analysis of the structure functions, we should solve the Bethe-Salpeter equation with a realistic NN potential [1,11,20]. However, for unpolarized deep inelastic scattering it is possible to make a first estimate of the nuclear effects on the structure function via a much simpler model. In fact, the nonrelativistic estimate (55) prompts us that the deviation of the moments of the deuteron structure function F_2 from those of the physical nucleon is defined by the mean values of the kinetic energy $\langle T\rangle$ and the potential energy $\langle V\rangle$. It is obvious that one can fit these two parameters even in the simplest deuteron model with scalar exchange; as such, a model incorporates two parameters, the coupling constant and the mass of the exchange meson. Therefore, to perform the first quantitative estimate of the nuclear effects in deuteron structure functions, we consider this simplified model of the deuteron. Solving exactly the dynamical problem for the deuteron in this model, we obtain *realistic* estimates for off-mass-shell effects (also referred to as "binding effects") and, for effects of the motion of the nucleons in the deuteron, the Fermi motion. Since the Bethe-Salpeter amplitude is quite a complex object, we, unfortunately, are not able to present the analytical estimates in a manner similar to nonrelativistic ones (55). However, we obtain numerically the Bethe-Salpeter amplitude, including explicit dependence on the off-mass-shell energy of nucleons, and the observables of deep inelastic scattering on the deuteron.

We are considering a system of two spinor fields, nucleons with mass m, interacting with the exchange of a light scalar field ("meson") with mass μ. For the model the following are given:

$$m = 0.939 \text{ GeV}, \quad M_D = 2m + \varepsilon_D, \quad \varepsilon_D = -2.2246 \text{ MeV}, \tag{104}$$

and there is only one independent parameter μ. We choose the parameter $\mu \sim 200$–400 MeV, so as to have the mean value of the momentum of nucleons in the deuteron (or radius of the deuteron). The dependent parameter g must be defined such that the system has only one bound state with total momentum $J = 1$ at fixed energy ε_D.

The eigenvalue problem of the spinor-spinor Bethe-Salpeter equation for the deuteron with scalar exchange, (87)–(94), is solved numerically by using a standard procedure for two-dimensional integration with a Gaussian mesh. The amplitude is then normalized in accordance with Eq. (101). Note that even in the simplified model the deuteron has all eight components in the Bethe-Salpeter amplitude Ψ_M^D, though the D-wave components are strongly suppressed in comparison to the case of a realistic interaction.

To estimate the deuteron structure function F_2^D in the medium x_N region , $0.2 \le x_N \le 0.8$, it is sufficient to calculate the first few moments of the structure function. Indeed, let us consider the nucleonic contribution to the deuteron structure functions, given by Eq. (3). Since $f^{N/D}(y)$ has a very sharp peak near $\langle y\rangle \sim 1$, the integrand in (3) can be expanded at this point and we get

$$F_2^{N/D}(x_N) \approx F_2^N(x_N/\langle y\rangle) + \frac{1}{2}(\langle y^2\rangle - \langle y\rangle^2)\frac{\partial^2 F_2^N(x_N/y)}{\partial y^2}\Big|_{y=\langle y\rangle} + \cdots . \tag{105}$$

The values $\langle y\rangle$ and $\langle y^2\rangle$ are moments of $f^{N/D}(y)$ at $n = 2,3$. The respective moments (15), obtained in other variables, are transformed by

$$\langle y^n\rangle = \frac{1}{2}\left(\frac{M_D}{m}\right)^n \mu_{n+1}^{N/D} . \tag{106}$$

The nucleonic contribution to the deuteron momentum [see also Eq. (27)] can be written in the form

$$\langle y\rangle = 1 - \delta_N, \quad \delta_N \ll 1 , \tag{107}$$

where δ_N is the part of the deuteron momentum carried by the mesonic component. The value of δ_N controls the magnitude of binding effects in the deuteron. For example, the nonrelativistic estimate (55) gives $\delta_N \approx 5.0 \times 10^{-3}$, where we take $\langle T\rangle \approx 15$ MeV. The value of δ_N in the present model is dependent on the mass parameter μ. Our calculations for the deuteron with the scalar exchange give $\delta_N \approx 3.9 \times 10^{-3}$ at mass $\mu = 0.28m$, i.e., the same order of magnitude as the nonrelativistic estimate. The results of the calculation of the moments in the Bethe-Salpeter formalism are presented in Table I for the various masses of the exchange meson.

Using the calculated values of $\langle y\rangle$ and $\langle y^2\rangle$ and the expansion (105), the behavior of the deuteron structure function in the Bethe-Salpeter formalism is estimated. The model dependence of the parameter μ is shown in Fig. 2. The model dependence of the results is weak, and our estimates are in reasonable agreement with the nonrelativistic calculations

To compare explicitly the results of the different approaches to the deuteron structure functions, we present the results of the Bethe-Salpeter formalism, the nonrelativistic and the light-cone calculations in Fig. 3. The binding effects in the model of the scalar deuteron are of the same order as in the nonrelativistic calculation. A more plausible evaluation of the deuteron structure function will be obtained in calculations with realistic meson-nucleon models. However, it is clear that binding effects in the relativistic description are not negligible, and they are of the same magnitude as those in the nonrelativistic approach. At the same time, results, in the Bethe-Salpeter formalism differ from the light-cone calculations, which give $\delta_N \equiv 0$ [17,27,29,58].

TABLE I. First moments of the effective distribution function of nucleons in the deuteron calculated in different approaches.

	Nonrelativistic calculations	Bethe-Salpeter formalism $\mu = 0.2$	$\mu = 0.28$	$\mu = 0.425$	Light-cone calculations
$\langle y\rangle$	0.9950	0.9964	0.9961	0.9956	1.0000
$\langle y^2\rangle$	0.9953	0.9976	0.9980	0.9987	1.0086

B. Open questions and further investigations

Further calculations of the deuteron structure functions should be based on realistic models of the meson-nucleon theory. It is possible that observables calculated in different realistic models will be in a reasonable agreement. Special attention should be paid to the dependence of the strong meson-nucleon form factors on the cutoff mass parameter Λ [62,63]. To this end a detailed numerical analysis within a realistic model is now being undertaken.

The nuclear effects in the spin-independent structure function of the deuteron, F_2^D, clearly will be of the same magnitude as in our simple estimates. The only exception is that we will be able to discuss the full range of the Bjorken variable x_N, including $x_N > 1$. The most interesting case is to calculate the spin-dependent structure function g_1^D and its first moment. The present formalism is very convenient for a covariant calculation of the observables of polarized deep inelastic scattering on the deuteron. Indeed, to include the polarized case we should simply extend the operator product expansion basis to the axial operators [15,18,59]. The axial operators $O_A^{\mu_1\cdots\mu_n}$ for a system of nucleons and scalar mesons are

$$\hat{O}_A^{\mu_1\cdots\mu_n}(0) = \left(\frac{i}{2}\right)^n \{\bar{\psi}(0)\gamma_5\gamma^{\mu_1}\overleftrightarrow{\partial}^{\mu_2}\cdots\overleftrightarrow{\partial}^{\mu_n}\psi(0)\}\ , \tag{108}$$

and expressions for moments of the structure function $\mathsf{M}_n(g_1^D)$ can be easily written down and are similar to Eqs. (15) and (16). Then the structure function g_1^D can be recovered by the inverse Mellin transform. We would like to remind the reader of some technical difficulties appearing in the numerical calculation of both the unpolarized and polarized structure functions via the Wick-rotated amplitude. In fact, to compute the integrals of the form (24) and (25) with the nonzero imaginary part of the variable p, we should continue the Dirac δ function to the imaginary plane. After we do this we will get explicit formulas for the structure functions F_2^D, g_1^D, etc., in a convolution form.

The convolution form of the structure functions is appropriate for an analysis of the experimental data with the aim to extract the neutron structure functions from the combination of the structure functions of the proton and deuteron. In the case of the spin structure function g_1, the unfolding of the convolution is, generally speaking, an unresolvable task using the traditional methods, because of the nodes of this function and special methods must be utilized [60].

As to a second convolution term in Eq. (23), the meson exchange current contribution to the deuteron structure function, we anticipate its contribution to the spin-independent structure function F_2^D at $x \sim 0.0$–0.3 to be small. Such a conclusion is based on the numerical

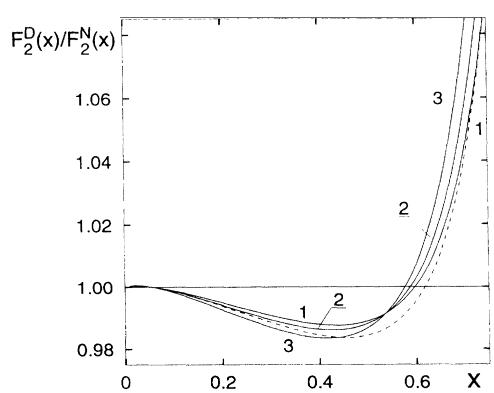

FIG. 2. Ratio of the deuteron and nucleon structure functions $F_2^D(x)/F_2^N(x)$ calculated in the Bethe-Salpeter formalism. Solid curves present the results of the calculation of the present work (see text) with the different mass of exchange meson: $1, \mu = 0.2m$; $2, \mu = 0.28m$; $3, \mu = 0.425m$. The dashed line presents the result of the nonrelativistic calculations. The structure function F_2^N is taken from Ref [14].

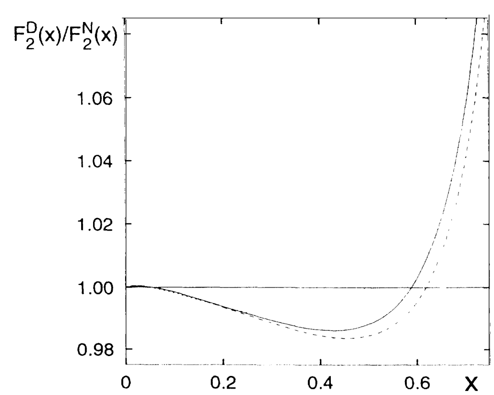

FIG. 3. Ratio of the deuteron and nucleon structure functions $F_2^D(x)/F_2^N(x)$ calculated in different theoretical approaches. Curves (see also the text): solid, the results of calculations within the Bethe-Salpeter formalism with the mass of the exchange meson $\mu = 0.28m$; dashed, nonrelativistic estimate; dotted, light-cone calculation. All curves are calculated by the formula (105) using the $\langle y\rangle$ and $\langle y^2\rangle$ corresponding to the respective approach (see Table I).

estimate of δ_N and previous nonrelativistic calculations [26,62]. To complete the phenomenological applicability of the present approach at small x, we must develop a formalism to incorporate the nuclear shadowing corrections to the reaction. This phenomenon gives the most significant contribution to the deuteron structure function at small x_N, say $x_N < 0.05$ [29,61,62], which is the same region where the mesonic exchange currents contribute to the deuteron structure function. This leads to a partial cancellation of the mesonic and shadowing corrections to the deuteron structure functions in the nonrelativistic case [62,60].

V. CONCLUSIONS

We have considered a relativistic description of deep inelastic scattering on the deuteron within the Bethe-Salpeter formalism and the Wilson operator product expansion method, where we make the usual well-defined approximations: (1) The *ladder approximation* for the Bethe-Salpeter amplitude; this approximation is relevant for the weakly bound and weakly relativistic system, such as the deuteron. (2) The *twist-2 approximation* in the operator product expansion method; this approximation corresponds to neglecting corrections of order $\sim x^2m^2/Q^2$. As a result it is possible to provide a fully relativistic description of the reaction.

One of the main advantages of the method is the possibility to consider the deuteron structure functions in the whole range of the Bjorken variable $x \in (0, M_D/m)$ and to give a rigorous covariant description of the spin structure of the deuteron.

In the standard operator product expansion method we obtain an explicit form of the nucleon contribution and mesonic exchange corrections to moments of the deuteron structure function F_2^D. The structure function of the deuteron is recovered by the inverse Mellin transform of the moments and is presented as the sum of two convolution terms, nucleon (relativistic impulse approximation) and meson (contribution of the meson exchange currents). The sum rules for the baryon number and energy momentum of the deuteron are derived using the normalization condition of the Bethe-Salpeter amplitude and a virial theorem of the field theory. It is found that a fully relativistic approach to deep inelastic scattering of leptons on the deuteron provides us with a self-consistent description.

We have presented numerical estimates of the nuclear effects in the deuteron structure function evaluated within the Bethe-Salpeter formalism. The calculations are carried out in a model of the deuteron as a system of two spinor nucleons interacting by the scalar meson exchange. This model is relevant to estimate the magnitude of nuclear corrections to the unpolarized deuteron structure function F_2^D. It is shown that the effects of the binding of the nucleons are significant and relativistic calculations within the Bethe-Salpeter formalism are in agreement with previous nonrelativistic estimates. At the same time, the results obtained in the Bethe-Salpeter formalism differ from light-cone calculations, since light-cone calculations do not include the dynamics properly.

The reasonable quantitative agreement of the presented calculations of the deuteron structure function at $x \sim 0.0$–0.8 in the nonrelativistic and relativistic approaches confirms our expectation that these approaches have to give similar results within the boundaries of validity of the nonrelativistic approximation. However, it does not imply that the relativistic effects in the deuteron structure function are small or negligible in general. It only shows that in a slightly relativistic system such as the deuteron (or any other atomic nucleus) we should find *special* kinematic conditions of the experiment to display the relativistic effects. Polarized deep inelastic scattering of leptons on deuteron provides a possibility to search for relativistic effects in the deuteron. These investigations are topical today in view of the numerous new and anticipated experimental data with polarized deuterons, which are expected to clarify the "spin crisis" [64]. Also, we can expect nontrivial relativistic phenomena at high x, where an accurate account of the relativistic nucleon motion will allow one to search for other possible degrees of freedom in nuclei, such as the Δ isobar [65] or multiquarks [17]. The precise evaluation of the structure functions in this region is important for QCD analysis of the experimental data, since a significant fraction of the full momentum can be carried by the "superfast" quarks [16,14]. This may lead to errors in the determination of the QCD parameters, such as α_s or $\Lambda_{\rm QCD}$, from the nuclear data, when the behavior of the nuclear structure function $\sim (1 - x_N)^\gamma$ as $x_N \to 1$ is assumed.

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APPENDIX A: METRIC, GAMMA MATRICES, AND VECTORS

We use the covariant normalization of the states

$$\langle P|P'\rangle = 2P_0(2\pi)^3\delta(\mathbf{P} - \mathbf{P}') \ , \tag{A1}$$

with $(2\pi)^3\delta(\mathbf{P} - \mathbf{P}') \to V$ when $\mathbf{P}' \to \mathbf{P}$ and the metric

$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} . \tag{A2}$$

The γ matrices are chosen in the explicit form

$$\gamma_0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \gamma_i = \begin{pmatrix} 0 & \boldsymbol{\sigma} \\ -\boldsymbol{\sigma} & 0 \end{pmatrix} ,$$
$$\gamma_5 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} . \tag{A3}$$

To transform the BS amplitude to convenient form we define the conjugation matrix:

$$\gamma^c = \gamma_3\gamma_1 \text{ with property } \gamma^c\gamma_\mu^T = \gamma_\mu\gamma^c . \tag{A4}$$

The 15 matrices G_i $(i = 1, \ldots, 15)$,

$$\gamma_\mu, \quad \gamma_5, \quad \gamma_5\gamma_\mu, \quad \sigma_{\mu\nu} = \frac{i}{2}(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu) , \tag{A5}$$

and $G_{16} = \mathbb{1}$ present the full basis in space of 4×4 matrices, satisfying the Clifford algebra with orthogonality:

$$\mathrm{Tr}(G_iG_j) = \begin{cases} \pm 4, & i = k , \\ 0, & i \neq j \end{cases} . \tag{A6}$$

The Levi-Civita tensor

$$\varepsilon^{\alpha\beta\gamma\delta} = -\varepsilon_{\alpha\beta\gamma\delta} = \begin{cases} 1 \text{ if } \alpha\beta\gamma\delta \text{ is even permutation of } 0,1,2,3 , \\ -1 \text{ if } \alpha\beta\gamma\delta \text{ is odd permutation of } 0,1,2,3 , \\ 0 \text{ in all other cases} \end{cases} . \tag{A7}$$

Components of the four-vectors,

$$p = (p_0, \mathbf{p}) = (p_0, \{p^i\}) = (p_0, -\{p_i\}) . \tag{A8}$$

For the three-dimensional (3D) vectors **a** and **b** we denote the scalar and vector products in the form

$$(\mathbf{a} \cdot \mathbf{b}) \text{ and } [\mathbf{a} \times \mathbf{b}] , \tag{A9}$$

respectively.

The caret notation,

$$\hat{p} = \gamma_\mu p^\mu = \gamma_0 p_0 - (\boldsymbol{\gamma} \cdot \mathbf{p}) . \tag{A10}$$

APPENDIX B: VIRIAL THEOREMS OF FIELD THEORY

The virial theorem is a well-known theorem of classical mechanics, statistical physics, and quantum mechanics. Very often the proof of this theorem is based on specific properties of the Hamiltonian of the system. The more general method based on spatial (3D) dilatations [66,67] is not so widely known. This type of transformations of the *field* has been used in nonlinear field theories to derive the virial-like theorems for these fields (see, e.g., Ref. [68]) and in the effective relativistic theory of nuclei ("Dirac phenomenology") [24]. We consider here a proof for the specific cases of field-theoretical virial theorems.

We start from the general variational principle of quantum mechanics for the ground state of the system:

$$\delta\langle g|H|g\rangle = 0 , \tag{B1}$$

where H is the total Hamiltonian of the theory and $|g\rangle$ is the ground state and eigenvector of H. The Hamiltonian density is a polynomial form of the set $\{\phi_i\}$ of fields and their differentials, with possible explicit dependence on x:

$$H = \int d^3x \mathcal{H}(x, \phi_1(x) \cdots \phi_i(x) \cdots) . \tag{B2}$$

Let us require that (B1) imply a special class of variations. This special class is an infinitesimal 3D dilatation of the fields of the theory. Explicitly,

$$\phi_i(\mathbf{x}, t) \to \tilde{\phi}_i(\mathbf{x}, t) \equiv \sqrt{\lambda^3}\phi_i(\lambda\mathbf{x}, t) ,$$
$$\lambda \approx 1 + \epsilon . \tag{B3}$$

Variation of (B1) under (B3) is then given as

$$\delta\langle g|H|g\rangle = \langle g| \int d^3x \mathcal{H}(x, \tilde{\phi}_1(x) \cdots \tilde{\phi}_i(x) \cdots)|g\rangle$$
$$-\langle g|H|g\rangle \tag{B4}$$
$$\equiv \langle g| \int d^3x \delta\mathcal{H}(x)|g\rangle + O(\epsilon^2) . \tag{B5}$$

Calculating the explicit form of $\delta\mathcal{H}(x)$, one gets the virial theorem for the field theory:

$$\langle g| \int d^3x \delta\mathcal{H}(x)|g\rangle = 0 . \tag{B6}$$

Moreover, since the variation (B3) and the integration in (B6) are independent we have also a stronger relation

$$\langle g|\delta\mathcal{H}(x)|g\rangle = 0 . \tag{B7}$$

To generalize these considerations to the case of any eigenstate $|n\rangle$ of the Hamiltonian H one should follow the general variational principle.

Examples. For nonrelativistic quantum mechanics with the Hamiltonian of the form

$$H = \int d^3x \Bigg\{ m\psi^\dagger(x)\psi(x) + \frac{\boldsymbol{\partial}\psi^\dagger(x)\boldsymbol{\partial}\psi(x)}{2m}$$
$$+\psi^\dagger(x)V(x)\psi(x) \Bigg\} , \tag{B8}$$

we get the well-known result

$$\langle n|\int d^3x\left\{\frac{\boldsymbol{\partial}\psi^\dagger(x)\boldsymbol{\partial}\psi(x)}{m}\right.$$
$$\left.-\psi^\dagger(x)[\mathbf{x}\boldsymbol{\partial}V(x)]\psi(x)\right\}|n\rangle=0\ . \quad \text{(B9)}$$

The virial theorem for the theory with the Lagrangian defined by Eqs. (5), (6), and (7) has the form

$$\langle n|\int d^3x\left\{-\frac{i}{2}\bar{\psi}(x)(\boldsymbol{\gamma}\overleftrightarrow{\boldsymbol{\partial}})\psi(x)+\boldsymbol{\partial}\phi(x)\boldsymbol{\partial}\phi(x)\right.$$
$$\left.+\frac{3}{2}g_\sigma\phi(x)\bar{\psi}(x)\psi(x)\right\}|n\rangle=0\ . \quad \text{(B10)}$$

APPENDIX C: SOME FORMULAS CONCERNING SPHERICAL HARMONICS AND VECTOR SPHERICAL HARMONICS

The explicit form of the Legendre function of the second kind $Q_l(y)$ at particular l,

$$Q_0(y)=\frac{1}{2}\ln\left(\frac{y+1}{y-1}\right)\ , \quad \text{(C1)}$$

$$Q_1(y)=\frac{y}{2}\ln\left(\frac{y+1}{y-1}\right)-1\ , \quad \text{(C2)}$$

$$Q_2(y)=\frac{1}{4}(3y^2-1)\ln\left(\frac{y+1}{y-1}\right)-\frac{3}{2}y\ . \quad \text{(C3)}$$

Some useful relations, including spherical harmonics and vector spherical harmonics [54],

$$\mathbf{n}Y_{JM}(\Omega_p)=\left\{\left(\frac{J}{2J+1}\right)^{1/2}\mathbb{Y}^{J-1}_{J-M}(\Omega_p)\right.$$
$$\left.-\left(\frac{J+1}{2j+1}\right)^{1/2}\mathbb{Y}^{J+1}_{JM}(\Omega_p)\right\}\ , \quad \text{(C4)}$$

$$\mathbf{n}\cdot\mathbb{Y}^{J+1}_{JM}(\Omega_p)=-\left(\frac{J+1}{2J+1}\right)^{1/2}Y_{JM}(\Omega_p)\ , \quad \text{(C5)}$$

$$n\cdot\mathbb{Y}^{J}_{JM}(\Omega_p)=0\ , \quad \text{(C6)}$$

$$\mathbf{n}\cdot\mathbb{Y}^{J-1}_{JM}(\Omega_p)=\left(\frac{J}{2J+1}\right)^{1/2}Y_{JM}(\Omega_p)\ , \quad \text{(C7)}$$

$$\mathbf{n}\times\mathbb{Y}^{J+1}_{JM}(\Omega_p)=i\left(\frac{J}{2J+1}\right)^{1/2}\mathbb{Y}^{J}_{JM}(\Omega_p)\ , \quad \text{(C8)}$$

$$\mathbf{n}\times\mathbb{Y}^{J}_{JM}=i\left\{\left(\frac{J+1}{2J+1}\right)^{1/2}\mathbb{Y}^{J-1}_{JM}(\Omega_p)\right.$$
$$\left.+\left(\frac{J}{2J+1}\right)^{1/2}\mathbb{Y}^{J+1}_{JM}(\Omega_p)\right\}\ , \quad \text{(C9)}$$

$$\mathbf{n}\times\mathbb{Y}^{J-1}_{JM}(\Omega_p)=i\left(\frac{J+1}{2J+1}\right)^{1/2}\mathbb{Y}^{J}_{JM}(\Omega_p)\ , \quad \text{(C10)}$$

where $\mathbf{n}=\mathbf{p}/|\mathbf{p}|$ is the unit vector specified by the polar angles $\Omega_p=(\vartheta_p,\phi_p)$ of momentum $\mathbf{p}$.

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