Checking a neutron halo from elastic scattering

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We study the relative difference $E(q)$ of elastic scattering differential cross sections as function of the momentum tranfer q. Applied to neighboring nuclei incident on the same target, it yields qualitative information on the relative changes of matter distributions in an almost model independent way. Variations of the radius and of the surface thickness produce radically different pattern for $E(q)$. The method is well suited at medium and high energies. In the absence of data of sufficient quality, we consider ^{11}C and ^{11}Li elastic scattering on ^{12}C at about 60 MeV/nucleon incident energy for illustrative purpose.

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Nuclei close to the particle drip line offer a unique opportunity to study weakly bound systems subject to strong interactions. In this domain, much attention has been devoted to the spatial extension of the wave function. For instance, in the case of ^{11}Li, probing the neutron halo resulted from measurements of the total reaction cross sections and dissociation [1,2]. On the other hand, elastic scattering is known to provide information on the geometrical properties of the beam-target systems. Theoretical studies have investigated the influence of the ^{11}Li neutron halo on the differential cross sections at low and intermediate energies [3,4].

The purpose of the present work is to show that the relative difference of two differential cross sections is well suited to emphasize shape differences. Furthermore, in spite of the fact that interpretating the scattering of strong interacting particles requires a model, the relative difference underlines specific features merely connected to geometrical aspects. Consequently, such an analysis is practically model independent, at least at a qualitative level.

The situation is well illustrated by the model of Inopin and Berezhnoy [5,6]. Ignoring spin and isospin complications, the scattering amplitude is given in this case by

$$F(q) = \frac{ikR_0}{q} J_1(qR_0) e^{-\beta^2 q^2}, \tag{1}$$

where R_0 is the strong interaction radius of the system, β simulates its surface thickness, k is the incident momentum in the c.m., and $q = 2k\sin\theta/2$ is the momentum transfer.

The relative difference between two differential cross section is by definition

$$E(q) = 2\ \frac{\sigma_1 - \sigma_2}{\sigma_1 + \sigma_2}, \tag{2}$$

where $\sigma_i =\mid f(q) \mid^2$, for $R_0 = R_i$, $\beta = \beta_i$.

It is then immediate to check that, in this simple model, $E(q)$ displays radically different patterns according to whether the two systems differ by their radii ($R_1 \neq R_2$, β fixed) or by their surface thickness ($\beta_1 \neq \beta_2$, R fixed). Indeed, $E(q)$ undergoes strong oscillations with poles in the first case, and varies very smoothly in the second case. The same general behavior remains valid in more sophisticated calculations, although variations of R and β cannot be always disentangled.

As a second example, we have worked out another simple model, somewhat more realistic, to support the interpretation of $E(q)$. In a very crude way, the double convolution integral representing the beam-target interaction through an effective interaction

$$f(r) = \int \rho_a(\mathbf{r}')\rho_b(\mathbf{r}'')V_{\text{eff}}(\mathbf{r} - \mathbf{r}' + \mathbf{r}'')d\mathbf{r}'d\mathbf{r}''$$

has been parametrized by a two-parameter Fermi function

$$f(r) = \rho_0\ \gamma \frac{1}{1 + \exp(\frac{r-c}{\beta})}\ , \tag{3}$$

assuming spherical symmetry. Spin and isospin degrees of freedom are averaged, and the scattering amplitude is calculated by means of the asymptotic formula of Amado, Dedonder, and Lenz [7], which approximates the Glauber model with accuracy for $q > 1\ \text{fm}^{-1}$. The differential cross section is then given by

$$\sigma(q) = \frac{2}{3}\frac{k^2}{q^{8/3}} \mid 2\pi\rho_0\beta\gamma c^2 \mid^{2/3} e^{2\Phi}[\cosh(2\Sigma) + \cos(2\Psi)]. \tag{4}$$

Here we have

$$\Phi = -\pi\beta q - \tfrac{\sigma}{2}\text{Re}\tilde{t} + \tfrac{3}{2}\text{Re}(\alpha^{2/3})(qc)^{1/3}\cos\tfrac{\pi}{6}\ ,$$

$$\Sigma = -\tfrac{3}{2}\text{Im}(\alpha^{2/3})(qc)^{1/3}\sin\tfrac{\pi}{6} + \eta\text{Im}t_c - \tfrac{r\sigma}{2}\text{Im}\tilde{t}\ ,$$

$$\Psi = \tfrac{5\pi}{6} - \tfrac{\sigma}{2}\text{Im}\tilde{t} + qc + \tfrac{3}{2}\text{Re}(\alpha^{2/3})(qc)^{1/3}\sin\tfrac{\pi}{6}$$

$$+\tfrac{3}{2}\text{Im}(\alpha^{2/3})(qc)^{1/3}\cos\tfrac{\pi}{6}\ .$$

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Furthermore,

$$\mathrm{Re}\tilde{t} = 1.46\rho_0(2\pi\beta)^{1/2}(c^2+\pi^2\beta^2)^{1/4}\cos\tfrac{\pi\beta}{2c}\ ,$$

$$\mathrm{Im}\tilde{t} = 1.46\rho_0(2\pi\beta)^{1/2}(c^2+\pi^2\beta^2)^{1/4}\sin\tfrac{\pi\beta}{2c}\ ,$$

$$\alpha = \pi\beta\rho_0\sigma\sqrt{1+r^2}e^{i\phi_\alpha}\ ,$$

$$\phi_\alpha = -\arctan(r), \mathrm{Im}t_c \approx -2\arctan\left(\frac{\pi\beta}{c}\right).$$

The notation follows the one of Ref. [7], and we refer the reader to this original article for details. The interaction strength is parametrized by $\gamma = \frac{\sigma}{2}(1-ir)$, where σ is the effective nucleon-nucleon total cross section and r is the ratio of real to imaginary part. Actually, σ appears always multiplied by ρ_0, so that the relevant quantity is $\lambda = (\sigma\rho_0)^{-1}$, as can be checked easily. For the present purpose, λ will be considered as a free parameter, a point of minor importance since in first approximation $E(q)$ is independant of λ.

As usual, η is the Sommerfeld parameter. The quantity $\tilde{t}$ is related to the profile function given by $t(b) = \int_{-\infty}^{\infty} f(r)dz$. Similarly, t_c is the profile function of the Coulomb interaction folded with the density.

By taking the logarithmic derivative of Eq. (4) with respect to c and β, respectively, we can check how $E(q)$ is reflecting a change in radius or in surface thickness. In spite of the complexity of the full expression, it is easy to verify that $E(q)$ is dominated by two factors of $\sigma(q)$. The first one is the exponential falloff, $e^{-2\pi\beta q}$, leading to a linear variation of $E(q)$ under a change in β. The second one is the qc component in the $\cos(2\Psi)$ term, which induced the oscillatory behavior of $E(q)$ due to a change in radius. The typical patterns of $E(q)$ corresponding to the variation of either c or β are displayed in Figs. 1 and 2, respectively. They are less schematic than those produced by the Inopin- Bereznhoy [6] model, but the general trends are confirmed. Note that the large amplitude oscillation at low q ($q < 1$ fm^{-1}) is not relevant, as it lies outside the domain of validity of Eq. (4).

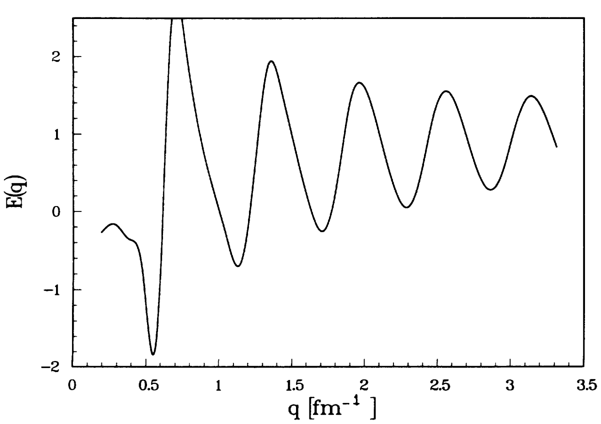

FIG. 1. Relative difference $E(q)$ corresponding to a variation of the radius c. The scattering amplitude is calculated by using the asymptotic formula of Amado, Dedonder, and Lenz [8]. The parameters are those fitting ^{11}C-^{12}C scattering at 60 MeV/nucleon.

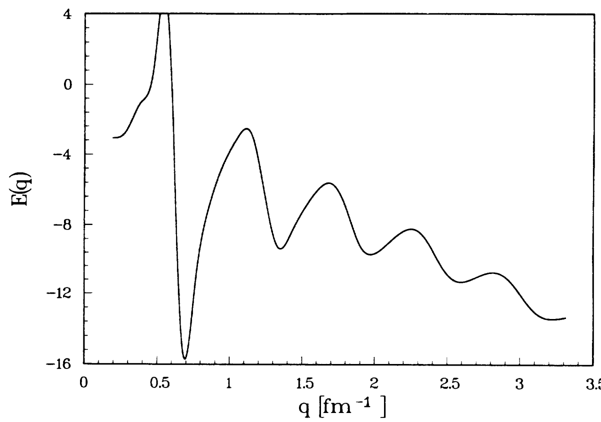

FIG. 2. Same as Fig. 1 for a variation of the surface thickness β.

Since a neutron halo is expected to develop a long tail in the density distribution, and thus to increase noticeably the surface thickness of the nucleus it belongs to, the function $E(q)$ appears a good tool to exhibit such an effect, provided the reference nucleus is well chosen, i.e., has approximately the same radius. The present work being devoted to ^{11}Li with a rms radius $\approx$ 3.2 fm, the reference nucleus should be taken among the $2s-1d$ shell nuclei. Such a choice, however, results in a large charge number difference, which could induce inconvenient effects arising from the Coulomb phase, especially at low energy.

Indeed, the most favorable data to be analyzed along with the relative difference method would be elastic differential cross sections on a proton target at 0.8–1.0 GeV/nucleon incident energy. In this range, the nucleon-nucleon interaction is rather well known, the Glauber model is reliable, and Coulomb effects are small except at very forward angles and at diffraction minima. No such data are yet available. For illustrative purposes, we shall discuss the case of ^{11}Li and ^{11}C on a ^{12}C target at 637 MeV and 620 MeV, respectively. These data have

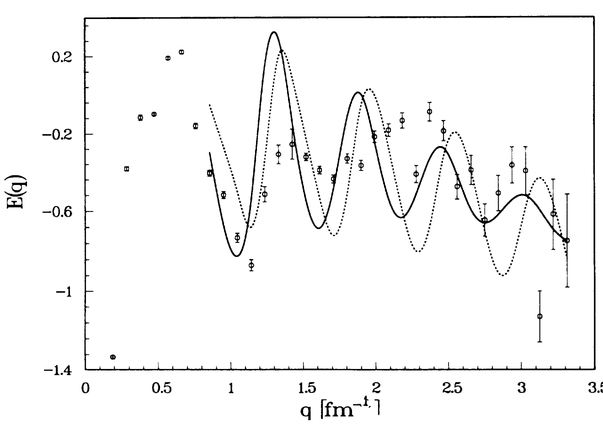

FIG. 3. Relative difference $E(q)$ of ^{11}Li and ^{11}C scattered by ^{12}C at 60 MeV/nucleon. Experimental data are taken from Ref. [8]. The solid curve corresponds to $E_1(q)$ and the dashed curve to $E_2(q)$, two model calculations as explained in the text.

been collected in the same experiment [8], and are the only ones at the moment providing us with an estimate of $E(q)$.

Comparing the two patterns of Figs. 1 and 2 to the experimental values of $E(q)$ displayed in Fig. 3, we conclude that the data indicate a superposition of changes in both radii and surface thickness. The large radius difference is, however, hiding somewhat the increase in surface thickness, which is essentially given by the slow average decrease of $E(q)$. In order to explore the situation in more details, we have proceeded as follows.

In a first step, calculations have been performed for ^{11}C-^{12}C elastic scattering at 620 MeV [8] by using Eq. (4). This is a four-parameter fit to the experimental data. We have only retained the best-fit values according to a χ^2 test to data at $q > 1$ fm^{-1}. For the two parameters fixing the interaction we obtained $\lambda = 1$ fm and $r = 1.72$; this last value is close to the one which can be inferred from the work of Satchler, McVoy, and Hussein [3]. The two geometrical parameters are found to be $c = 5.62$ fm and $\beta = 0.63$ fm. Keeping the interaction parameters fixed, we next have varied c and β so to fit the experimental data of ^{11}Li-^{12}C at 637 MeV [8]. Note that these data do not correspond to purely elastic scattering; some inelastic contributions could not be eliminated. They are used essentially to show how the method is working, and to give a preliminary feeling in the absence of data of higher quality. We obtain $c' = 6.33$ fm and $\beta' = 0.72$ fm. From these two set of parameters, the resulting $E_1(q)$ is displayed in Fig. 3, and compared to experimental values. Note that the actual c difference of 0.7 fm reflects the measured rms radius difference between ^{11}Li and ^{11}C, which is about this value.

As the shape of the experimental $E(q)$ clearly indicates a superposition of an increase in radius and surface thickness, we have calculated a second theoretical curve $E_2(q)$. It combines in a linear way the two logarithmic derivatives with respect to c and β calculated from the ^{11}C parameters with $\Delta c = 0.6$ fm and $\Delta\beta = 0.1$ fm. This second curve provides us with a fit comparable to the first one, which means that the Coulomb phase does not produce drastic effects on $E(q)$.

From the present attempt we conclude that $E(q)$ constitutes a good tool to exhibit matter distribution changes, at least at a qualitative level. Consequently, it should be successful in checking the presence of a neutron halo. In the absence of sufficiently accurate data at high energies, the low energy case we have analyzed shows the potentiality of the method, although the ^{11}Li data are contaminated by some inelastic contributions. These results are nevertheless encouraging, and the method is expected to yield reliable conclusions once applied at proper incident energies. Furthermore, with higher velocities, the influence of the Coulomb phase will diminish and varying the reference nucleus towards heavier elements may bring very interesting variations of $E(q)$ with A.

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