Simple parametrization of fragment reduced widths in heavy ion collisions

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A systematic analysis of the observed reduced widths obtained in relativistic heavy ion fragmentation reactions is used to develop a phenomenological parametrization of these data. The parametrization is simple, accurate, and completely general in applicability.

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Fragment momentum distributions measured in relativistic heavy ion collisions are typically observed to be Gaussian shaped [1–5]. Statistical models have been invoked [6–10], with varying degrees of success, to explain the observed distributions. Inconsistencies between statistical theories and experiments for heavier nuclear systems [2,5] and other measurements [11–13] were explained by including the dynamical contributions [14] from the collision, in addition to the internal Fermi motion of the colliding nuclei. Although progress toward understanding fragment momentum distributions is being made, a unified picture of the widths of these distributions is lacking. In this Brief Report, we present a simple, phenomenological parametrization based partly upon geometric considerations. The width predictions obtained using this parametrization are in excellent agreement for all experimental results published to date.

Within the framework of an independent particle model, Goldhaber [8] assumed zero net three-momentum in the nucleus and showed that a parabolic dependence of the momentum distribution variance (width) in each Cartesian coordinate $(i = x, y, z)$

$$\sigma_i^2 = \sigma_{0,i}^2 A_F(A_P - A_F)/(A_P - 1), \tag{1}$$

could be obtained. In this model σ_0 is related to the Fermi momentum of the fragmenting nucleus (p_F) by

$$\sigma_0^2 = p_F^2/5. \tag{2}$$

Over the past two decades, it has become customary to adopt the functional form of Eq. (1) to describe the measured widths as

$$\sigma_{\mathrm{expt}}^2 = \sigma_{0,\mathrm{expt}}^2 A_F(A_P - A_F)/(A_P - 1), \tag{3}$$

where the $\sigma_{0,\mathrm{expt}}$ deduced from the experimental distribution variances are compared with σ_0 obtained from Eq. (2).

Here, we take the functional form of Eq. (3) and consider some observed systematics for $\sigma_{0,\mathrm{expt}}$: (a) $\sigma_{0,\mathrm{expt}}$ is $\sim$ 70–80 MeV/c for carbon and oxygen fragments [1] and slightly larger ($\sim$ 95 MeV/c) for Ar [3,4] fragments, (b) much larger values of $\sigma_{0,\mathrm{expt}}$ are noted for La [2] and Au [5] fragments ($\sim$ 110 and $\sim$ 200 MeV/c, respectively), (c) there is essentially no target dependence for C and O fragment momentum distributions [1], (d) there is some target dependence observed for Au fragment momentum distributions [5], and (e) the momentum reduced widths decrease slightly with increasing energy for Au [5] and also for Ar [3,4] fragments.

From (a) and (b) above, the experimental observations suggest that the reduced width increases with the mass number of the fragmenting projectile nucleus. Consequently, a simple linear dependence on projectile nucleus mass appears to be a reasonable starting point. Therefore, we chose a form given by $a + bA_P$, where a and b are parameters and A_P is the mass number of the projectile nucleus. The systematics represented by (c) and (d) suggests a weak dependence on the mass number of the target nucleus where the fragment width increases slowly for heavier targets. This can be included in a natural way by incorporating the Coulomb interaction between the projectile and target nuclei. Finally, from (e) we note that there is a weak, inverse dependence of the width on the projectile kinetic energy. Combining the weak energy dependence with the target dependence noted previously, we assume a form given by $(1 + \alpha E_C/T_{\mathrm{lab}})$ where α is another parameter, T_{lab} is the beam energy in MeV/nucleon, and E_C is the Coulomb energy given by

$$E_C = \frac{1.44 Z_P Z_T}{r_P + r_T}, \tag{4}$$

In [4], Z_1 $(i = P, T)$ are the projectile and target charge numbers and r_i are the uniform distribution nuclear radii given by $(i = P, T)$

$$r_i = \sqrt{5/3}(r_i)_{\mathrm{rms}}, \tag{5}$$

where the nuclear rms radii are taken from electron scattering measurements [15].

The best fit to the available experimental widths (for $A_P \geq 12$) is given by

$$\sigma_{0,\mathrm{expt}} = (1 + E_c/4T_{\mathrm{lab}})(70 + 2A_P/3). \tag{6}$$

Figure 1 displays the predictions from Eq. (6), as a function of projectile mass number, compared with the experimental observations [1–5]. The quoted experimental

TABLE I. System and energy dependence of fragment reduced widths (σ_0).

System (Proj. & target)		Energy (A GeV)	σ_0 (MeV/c) Present work	Expt.	System (Proj. & target)		Energy (A GeV)	σ_0 (MeV/c) Present work	Expt.
^{12}C	+ Be	1.05	78.1				0.8	138.9	
	+ C		78.1	77.8±2.7			1.0	136.3	
	+ Al		78.3				1.2	134.6	
	+ Cu		78.6				2.1	130.9	
	+ Ag		78.8		^{93}Nb	+C	0.2	138.6	
	+ Pb		79.3				0.4	135.3	
^{12}C	+Be	2.10	78.0				0.6	134.2	
	+C		78.1	81.4±2.2			0.8	133.6	
	+Al		78.1				1.0	133.3	
	+Cu		78.3				1.2	133.1	
	+Ag		78.4				2.1	132.6	
	+Pb		78.6		^{139}La	+C	0.2	173.2	
^{16}O	+Al	0.0925	85.0				0.4	167.9	
	+Au		99.5				0.6	166.2	
^{16}O	+Al	0.1175	84.1				0.8	165.3	
	+Au		95.1				1.0	164.8	
^{16}O	+Be	2.10	80.7				1.2	164.4	169±12
	+C		80.8	82.7±1.3			2.1	163.7	
	+Al		80.9		^{197}Au	+C	0.2	218.2	
	+Cu		81.0				0.4	209.7	
	+Ag		81.2				0.6	206.9	
	+Pb		81.5				0.8	205.5	
^{40}Ar	+C	0.213	99.0	94±5			1.0	204.7	199±2[a]
		1.65	97.0	x: 90.8±7.9			1.2	204.1	
				y: 97.0±8.7			2.1	202.9	
				z: 103±8.1	^{197}Au	+Ag	0.2	305.8	
^{40}Ar	+KCl	1.65	97.4	x: 92.6±23			0.4	253.6	
				y: 76.5±19			0.6	236.2	
				z: 114.5±8.1			0.8	227.5	
^{84}Kr	+Au	0.2	117.7				1.0	222.2	219±3[a]
		0.4	151.8				1.2	218.7	
		0.6	143.2				2.1	211.3	

[a]Reference [5], these values are for multiplicity ≥ 3, fragment mass number ≤ 75, and $200 \leq E$ (A MeV)≤ 986.

uncertainties, which are comparable to the size of the plotted symbols, are not shown. The agreement between Eq. (6) and measurement is excellent. Additional insight into the various dependences on beam energy and collision system mass numbers is shown in Table I. Note that all the observed systematics summarized in (a)–(e) are nicely reproduced.

Several comments on the excellent agreement between the parametrization and the data are warranted. First, additional measurements for the mass range $50 < A_P < 140$ are needed to verify the assumed linearity in Eq. (6). Second, systematic verification of the assumed energy and target mass dependences are also needed. Finally, underlying questions such as why a simple, geometry-based parametrization yields a reasonable description of the fragment momentum width requires further study and consideration. Nevertheless, the present parametrization should serve as a useful tool for experimental simulations and analyses.

In conclusion, we have proposed and tested a reasonable phenomenological parametrization for fragment mo-

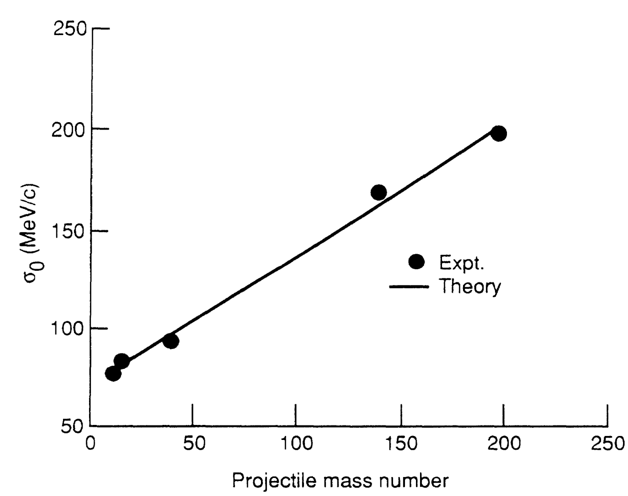

FIG. 1. Reduced width σ_0 in MeV/c as a function of projectile mass number A_P.

mentum reduced width in relativistic heavy ion collisions. The parametrization is simple, accurate, and nearly universal in its applicability.

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