Two-neutrino double beta decay of ^{48}Ca

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(Received 17 June 1993)

We have performed quasiparticle random phase approximation (QRPA) calculations for the $2\nu\beta\beta$ decay matrix element $M^{2\nu}_{\rm GT}$ of ^{48}Ca using realistic effective interactions derived from the Paris, Bonn, and Reid potentials. Unlike earlier QRPA calculations where the BCS self-energies were suppressed, we have retained these self-energies together with the use of the single-particle levels provided by a ^{40}Ca core. Different choices for the single-particle levels are employed, and their effects on $M^{2\nu}_{\rm GT}$ are compared. A majority of our calculated $M^{2\nu}_{\rm GT}$ matrix elements are in good agreement with the experimental bounds of $|M^{2\nu}_{\rm GT}| < 0.05$ MeV^{-1}, and exhibit good stability with respect to the variation of the particle-particle interaction strength g_{pp}.

PACS number(s): 23.40.Hc, 21.30.+y, 27.40.+z

There are about ten [1–4] nuclei whose two-neutrino double beta ($\nu\nu\beta\beta$) decays have been observed, and among them ^{48}Ca is of particular theoretical interest. Most $\beta\beta$ nuclei are far from the closed shells, and for them exact shell model calculations are prohibitively difficult. To treat these nuclei one is compelled to employ the commonly used quasiparticle random phase approximation (QRPA) (see, for example, Ref. [5]).

^{48}Ca is a double closed-shell nuclei, and it is the lightest known nucleus whose $\nu\nu\beta\beta$ decay has been observed. Comparing with other $\beta\beta$-decay nuclei such as ^{76}Ge, ^{48}Ca is much more suitable for being treated by the nuclear shell model. A relatively small model space consisting of the four $0f1p$ orbits outside a closed ^{40}Ca core may be adequate. Hence ^{48}Ca has been a favored nucleus for theoretical $\beta\beta$ studies as it can be investigated using either QRPA or exact shell model methods [6–12]. And it is probably the only nucleus for which a high accuracy $\beta\beta$ calculation can be made [10].

In this Brief Report we wish to perform a $2\nu\beta\beta$ calculation for ^{48}Ca using a modified QRPA method recently suggested by Stout and Kuo [13]. This method differs from earlier QRPA methods mainly in its treatment of the single-particle (s.p.) energies. In earlier QRPA calculations of $\beta\beta$ decays, the s.p. self-energy terms given by the BCS gap equations were generally suppressed. Stout and Kuo have found that the retention of those self-energies was important in stabilizing the QRPA results with respect to the variation of the particle-particle interaction strength and in reproducing the experimental $M^{2\nu}_{\rm GT}$ matrix elements of ^{76}Ge, ^{82}Se, and ^{100}Mo. Thus it may be of interest to further examine their method by applying it to a $M^{2\nu}_{\rm GT}$ calculation for ^{48}Ca.

In the following we shall first briefly review the QRPA formalism in order to discuss the BCS self-energies. Two major inputs to $\beta\beta$-decay calculations are the s.p. energies and the effective nucleon-nucleon (NN) interactions. We shall carry out several calculations using different choices for the s.p. energies, to investigate the dependence of our calculated $M^{2\nu}_{\rm GT}$ on these choices. For the effective NN interactions we shall use the G-matrix interactions [14,13] derived from the Paris, Bonn-A, and Reids NN potentials. (From now on the Bonn-A potential shall be referred to simply as the Bonn potential.) There have been various schemes in assigning the energy denominators used in QRPA calculations of $M^{2\nu}_{\rm GT}$. To our knowledge, studies concerning the choices of such energy denominators have hardly been reported in the literature. We shall also perform a series of calculations using different choices for the energy denominator and discuss our results.

As is well known [14], in QRPA $M^{2\nu}_{\rm GT}$ calculations a first step is to solve the following BCS gap equations separately for the active protons and for the active neutrons:

$$\Delta_a = \frac{1}{\sqrt{2j_a+1}} \sum_b \sqrt{2j_a+1}\, u_a v_b (-1)^{l_b} G(aabb0), \quad (1)$$

$$G(abcdJ) \equiv -\frac{1}{2}(1+\delta_{ab})^{1/2}(1+\delta_{cd})^{1/2}\langle abJ|V_{\rm eff}|cdJ\rangle, \quad (2)$$

$$\mu_a = \frac{2}{\sqrt{2j_a+1}} \sum_{bJ} (2J+1) v_b^2 G(ababJ), \quad (3)$$

$$v_a^2 = \frac{1}{2}\left\{1 - \frac{\varepsilon_a - \mu_a - \lambda}{\varepsilon_a^{\rm QP}}\right\}, \quad u_a^2 + v_a^2 = 1, \quad (4)$$

$$\varepsilon_a^{\rm QP} = [(\varepsilon_a - \mu_a - \lambda)^2 + \Delta_a^2]^{1/2}, \quad (5)$$

$$A = \sum_a (2j_a+1) v_a^2. \quad (6)$$

In the above, Δ is the pairing gap, μ is the s.p. self-energy, and $V_{\rm eff}$ is the NN effective interaction to be derived from the Paris, Bonn, and Reid NN potentials. ε is the unperturbed s.p. energy and $\varepsilon^{\rm QP}$ denotes the quasiparticle energy. v^2 and u^2 represent respectively the occupation and emptiness of the s.p. orbit, and λ is the chemical potential determined by the condition that the total number of active nucleons is A, which, for example, represents the number of active neutrons when

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solving the neutron gap equations. Recall that the above gap equations are solved separately, once for the active protons and again for the active neutrons. When solving the neutron gap equations, clearly μ_a of Eq. (3) represents the average interaction of neutron a with all the other active neutrons. μ_a does not involve the interaction of neutron a with any other protons. Similarly μ_a of the proton gap equations also does not involve any proton-neutron interactions.

Following Stout and Kuo [13] we take the s.p. energies ε as those given by a ^{40}Ca core, and retain the self-energy correction μ given by the gap equations. There are different ways of determining the s.p. energies with respect to a ^{40}Ca core, and it is not clear which set of the s.p. energies one should use for $\beta\beta$ calculations. As presented in Table I we have considered three such sets of s.p. energies. The column under OKW [15] corresponds to the experimental s.p. energies for a valence nucleon (hole) outside a ^{40}Ca core. These s.p. energies are often used for particle-hole calculations of ^{40}Ca. The column under WS denotes the Woods-Saxon s.p. energies corresponding to a ^{40}Ca potential well [16]. The s.p. levels of ^{40}Ca have been successfully described by Hartree-Fock calculations using the Skyrme-type effective interactions. We have also considered one set of such s.p. energies, denoted SKII [17] in Table I. Note that for OKW and WS the neutron and proton s.p. energies are taken to be the same, while for SKII they are different, mainly by a nearly constant upward shift of the proton s.p. levels.

Within the general framework of proton-neutron (pn) QRPA (see, e.g., Ref. [18] and references quoted therein), the two-neutrino Gamow-Teller matrix element is written as

$$M_{\rm GT}^{\nu\nu} = \sum_{a,b} \langle 0_f^+ || t_- \sigma || 1_b^+ \rangle \langle 1_b^+ | 1_a^+ \rangle \langle 1_a^+ || t_- \sigma || 0_i^+ \rangle D_1(a), \tag{7}$$

$$D_1(a) = [E_a + Q_{\beta\beta}/2 + m_e - E_i]^{-1}. \tag{8}$$

Here $Q_{\beta\beta}$ is the $\beta\beta$-decay Q value, which is 4.271 MeV for ^{48}Ca. m_e is the electron rest mass. The Gamow-Teller reduced matrix elements are given as

$$\langle 1_a^+ || t_- \sigma || 0_i^+ \rangle = \sum_{p,n} \langle p || \sigma || n \rangle [X_a^{pn} u_p v_n - Y_a^{pn} v_p u_n],$$

$$\langle 0_f^+ || t_- \sigma || 1_b^+ \rangle = \sum_{p,n} \langle p || \sigma || n \rangle [\bar{X}_b^{pn} \bar{v}_p \bar{u}_n - \bar{Y}_b^{pn} \bar{u}_p \bar{v}_n]. \tag{9}$$

The overlap integral

$$\langle 1_b^+ | 1_a^+ \rangle = \sum_{p,n} (\bar{X}_b^{pn} X_a^{pn} - \bar{Y}_b^{pn} Y_a^{pn}) \tag{10}$$

between two J^π states belonging to two different sets is introduced. The quantities without (with) a bar are defined with respect to the parent 0_i^+ (daughter 0_f^+) state. The u and v coefficients are obtained from solving the gap equations. The RPA amplitudes (X and Y) and ($\bar{X}$ and $\bar{Y}$) are obtained from the pnQRPA equations [18] for ^{48}Ca and ^{48}Ti, respectively. As usual, a strength parameter g_{pp} [18] is introduced in the pnQRPA equations.

In Figs. 1 to 4 we present some of our results for the

TABLE I. Different sets of ^{40}Ca single particle energies, in MeV, and the ^{48}Ca $\beta\beta$ decay matrix element $M_{\rm GT}^{2\nu}$ at $g_{pp} = 1.0$. The experimental bounds [20] for this matrix element is $|M_{\rm GT}^{2\nu}| < 0.05$ MeV^{-1}. The various effective interactions used in the calculation are explained in the text.

			OKW	WS	SKII	
	nlj		ε	ε	ε_p	E_n
	$0d_{5/2}$		−13.5	−17.1	−15.68	−22.75
	$0d_{3/2}$		−7.3	−13.4	−10.17	−17.13
	$1s_{1/2}$		−9.8	−13.0	−9.6	−16.67
	$0f_{7/2}$		0.0	−5.3	−1.07	−7.73
	$0f_{5/2}$		6.5	1.0	5.39	−0.79
	$1p_{3/2}$		2.1	−1.5	3.11	−2.69
	$1p_{1/2}$		4.1	0.5	4.39	−1.30
$M_{\rm GT}^{2\nu}$	Paris	G	−0.063	−0.050	−0.023	
		LESU	−0.031	−0.051	−0.014	
	Bonn	G	−0.080	−0.063	−0.027	
		LESU	*	−0.155	−0.035	
	Reid	G	−0.036	−0.031	−0.013	
		LESU	−0.071	−0.012	0.003	

$M_{\rm GT}^{2\nu}$ matrix element of ^{48}Ca. They have been obtained with three choices [13,18] for the effective interactions, each using both the Paris and the Bonn NN potentials. (1) G : Here we consider the effective interaction as given by the bare G matrix only, calculated with a Pauli exclusion operator specified by $(n_1, n_2, n_3) = (6, 15, 28)$ and harmonic oscillator parameter $\hbar\omega = 10.0$ MeV. (2) G-2nd: Here the effective interaction is given by the sum of the nonfolded diagrams first- and second-order in G. In this scheme a large pairing contribution is given by the well-known core polarization diagram G_{3p1h}. (3) LESU: The effective interaction in this choice is given by the Q-box folded-diagram series, calculated with the Lee-Suzuki iteration method, with the Q box given by irreducible diagrams first- and second-order in G.

Figure 1 displays our results using the WS s.p. spectrum of Table I and the effective interactions derived from the Paris NN potential. At $g_{pp} = 1$, the $M_{\rm GT}^{2\nu}$ matrix element given by effective interactions G and LESU are quite close to each other, and both are within the experimental limit of $|M_{\rm GT}^{2\nu}| < 0.05$ MeV^{-1}. The calculation with G-2nd becomes unstable at $g_{pp} \approx 0.9$.

The above are obtained with the ^{40}Ca s.p. energies and with the self energies μ retained. A main difference of our present calculation with earlier QRPA calculations is on the treatment of the self energies μ. Thus it may be of interest to repeat our calculation according to the earlier treatment of the s.p. energies, namely using the ^{48}Ca Woods-Saxon s.p. energies together with the suppression of μ. The results are generally not as good. For example, the curve denoted by circles in Fig. 1 is obtained with the above ^{48}Ca procedure and using the bare reaction matrix G as the effective interaction. It is seen that $M_{\rm GT}^{2\nu}$ calculated in this way becomes unstable before $g_{pp} = 1$ and tends to exceed the experimental bounds.

A few words about some numerical aspects of our calculations may be added. All the seven orbits of Table

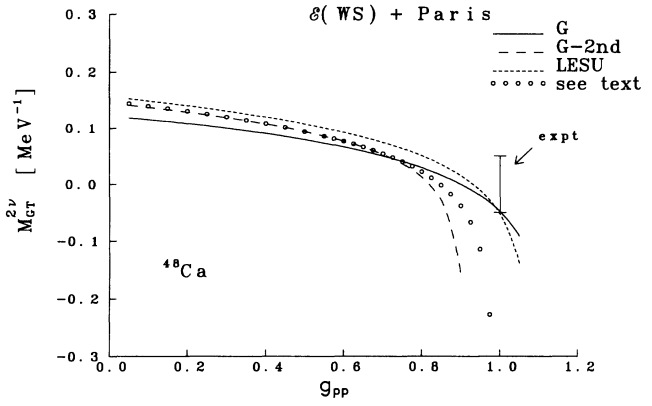

FIG. 1. $M^{2\nu}_{GT}$ matrix elements calculated with the WS s.p. spectrum of Table I and the Paris NN interaction.

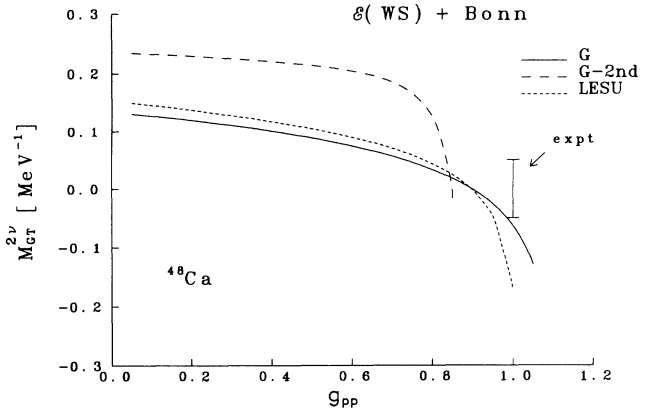

FIG. 2. Same as Fig. 1 except for the use of the Bonn NN interaction.

I are included in our gap-equation calculation. For the neutron gap equations, the *sd* orbits are nearly all full and this had given us difficulty in obtaining gap-equation solutions with high accuracy. However, this difficulty can be overcome by using a carefully designed iteration and interpolation procedure and we have done so.

Let us now return to present more results obtained with our present treatment about the s.p. energies. Our $M^{2\nu}_{GT}$ matrix elements calculated with the Bonn potential and the ^{40}Ca WS spectrum are shown in Fig. 2. Generally speaking, the Bonn potential seems to give stronger particle-particle interaction. Comparing with Fig. 1 where the Paris potential is employed, the results of Fig. 2 seem to have the tendency of becoming unstable at smaller g_{pp} values. At $g_{pp} = 1$, the calculated $M^{2\nu}_{GT}$ values of Fig. 2 are outside the experimental bounds.

In Fig. 3 we present our results using the SKII s.p. spectrum of Table I and the Paris NN potential. The results presented in Fig. 4 are the same except for the use of the Bonn NN potential. In both figures, the results for G and for LESU are rather close to each other, and are within the experimental bounds at $g_{pp} = 1$. The effective interaction G-2nd seems to be too strong, mainly due to the strong pairing force provided by the G_{3p1h} core polarization diagram, and consequently the calculated $M^{2\nu}_{GT}$ using G-2nd becomes unstable way before $g_{pp} = 1$.

The use of the OKW s.p. spectrum of Table I has given results generally worse than those shown in Figs. 1 to 4, the calculated $M^{2\nu}_{GT}$ often exhibiting a stronger dependence on g_{pp}. For example, as indicated by "*" of Table I, there is no solution for $M^{2\nu}_{GT}$ at $g_{pp} = 1$ when using the OKW spectrum together with the Bonn LESU effective interaction. At this point, the QRPA has complex eigenvalues and consequently the RPA amplitudes X and Y become undefined [13].

The numerical values of our calculated $M^{2\nu}_{GT}$ matrix elements at $g_{pp} = 1$, using the Paris, Bonn, and Reid NN potentials are summarized in Table I. Generally speaking, the results obtained from the ^{40}Ca WS and SKII s.p. spectra are better than those given by the OKW spectrum. For the Paris and the Reid cases, the results given by G and by LESU are essentially equivalent to each other. Zucker *et al.* [10] have performed an essentially exact shell model calculation for the two-neutrino double beta decay for ^{48}Ca, obtaining a value of 0.04 MeV^{-1} for the $M^{2\nu}_{GT}$ matrix element. It is encouraging that their value and our $M^{2\nu}_{GT}$ values are rather close to each other. But it must be borne in mind that our calculation and theirs have a number of differences. They used a modified Kuo-Brown effective interaction which is similar, but not exactly the same, to our Paris-G and -LESU interactions. Also, ours is an *sd-fp* shell approximate (*pn*QRPA) calculation while theirs is an "exact" *fp*-shell calculation. Rigorous comparisons between *pn*QRPA and feasible exact shell model results will be very useful in assessing the reliability of the former. Some studies in this direction have been carried out by Brown and Zhao [12]; they have compared the ^{48}Ti(n,p) and ^{48}Ca(p,n) Gamow-Teller transitions obtained both by exact shell model calculation and by a modified QRPA method.

The results presented so far have all been calculated with the denominator $D_1(a)$ of Eq. (8), which has been adopted by several authors (see Ref. [18] and references quoted therein). Because of the two different sets of intermediate states involved in Eq. (8), there is actually some ambiguity about the adoption of this energy denominator. For example, it would seem to be also reasonable to use an average energy denominator [19], namely, replac-

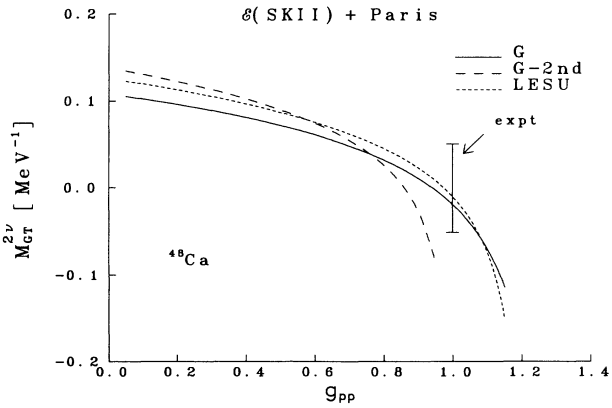

FIG. 3. $M^{2\nu}_{GT}$ matrix elements calculated with the SKII s.p. spectrum of Table I and the Paris NN interaction.

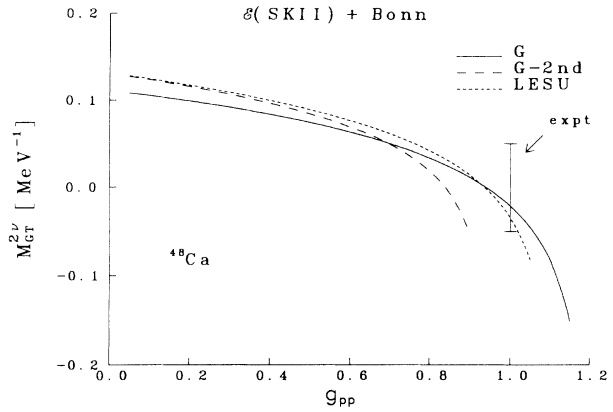

FIG. 4. Same as Fig. 3 except for the use of the Bonn NN interaction.

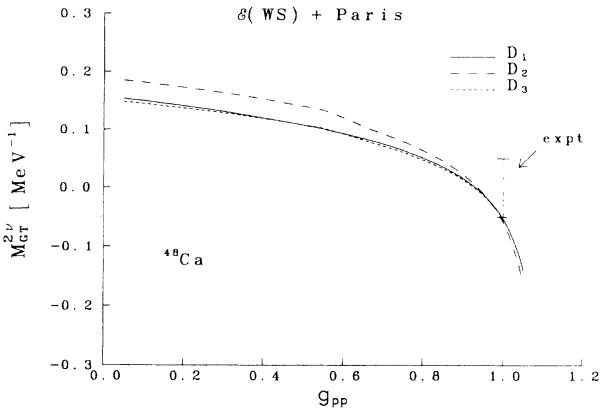

FIG. 5. Comparison of $M^{2\nu}_{GT}$ matrix elements obtained with three different choices for the energy denominator.

ing $D_1(a)$ by

$$D_2(ab) \equiv \left[\frac{E_a + E_b}{2} + \frac{Q_{\beta\beta}}{2} + m_e - E_i\right]^{-1}, \qquad (11)$$

where $(E_b - E_i)$ is the excitation energy corresponding to the daughter nucleus (^{48}Ti). We have performed calculations using $D_2(ab)$. In addition, the choice of

$$D_3(ab) = \tfrac{1}{2}[D_1(a) + D_1(b)] \qquad (12)$$

has also been studied.

In Fig. 5 we present some of our results for comparing the above three denominators. Using the above three denominators, the $M^{2\nu}_{GT}$ matrix elements shown are all calculated with the same Paris LESU interaction and the same ^{40}Ca WS s.p. spectrum. At small g_{pp} values, the matrix elements exhibit some noticeable dependence on the choice of the energy denominators. But it has been a surprise to us that the $M^{2\nu}_{GT}$ values at $g_{pp} \approx 1$ are remarkably close to each other. This is a desirable and welcoming result, and would be of interest for further investigation.

In summary, we have performed QRPA calculations of $M^{2\nu}_{GT}$ for ^{48}Ca using realistic effective interactions derived from the Paris, Bonn-A, and Reid NN potentials. We have considered two treatments for the s.p. energies; one using the s.p. energies provided by a ^{40}Ca core and with the BCS self energies μ retained and the other using ^{48}Ca s.p. energies together with μ suppressed. Within the context of our present calculations, the former seems to give more desirable results; in providing a better stability with respect to the variation of the particle-particle interaction strength g_{pp} and in attaining reasonably good agreement with the experimental bounds. As found before [14,18], the results for $M^{2\nu}_{GT}$ given by the effective interactions G and LESU are rather similar to each other, while the effective interaction G-2nd seems to give a too strong pairing interaction. We have also performed calculations using various choices for the energy denominators, and for our present calculations the $M^{2\nu}_{GT}$ matrix elements given by the three commonly used forms for the energy denominators are practically identical to each other.

The work of one of us (T.T.S.K.) was supported in part by the National Science Council (Taiwan, Republic of China), and by the U.S. DOE Grant DE-FG02-88ER40388.

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