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Global optical potentials for emitted alpha particles

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A set of global optical potentials for alpha particles with energies above 80 MeV has been extended to lower energies and proved appropriate to describe (n,α) reactions. It is found that the transmission coefficients for alpha-particle emission are rather strongly related to the fusion cross sections and elastic scattering at energies above 80 MeV, but not to elastic scattering at lower energy where specific features are comprised within complex optical potential parametrizations.

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The α-nucleus interaction plays an important role in studies of nuclear structure and nuclear reactions. The concept of the α-particle mean field is now widely used to unify the bound and scattering alpha-particle states, just as the nuclear mean field is used to calculate the properties of bound single-particle states and also the scattering of unbound nucleons by nuclei [1]. At positive energies the alpha-particle mean field is simply the familiar alpha-particle optical model potential, which describes very well the differential cross sections for the elastic scattering of alpha particles by nuclei. Unlike the nucleon case, there are however no global optical potentials for alpha particles that fit to good accuracy the scattering from many nuclei over a wide range of energies. Some optical model analyses of low-energy data [2,3] favored a squared Woods-Saxon form factor, i.e., a shape different from that used for the nucleon-nucleus potential. However, a global optical potential has been obtained by Nolte *et al.* [4] for alpha particles with energies above 80 MeV, these data being similarly fitted by using either a Woods-Saxon form factor or a squared Woods-Saxon form factor.

In addition to this potential ambiguity, it has been found in heavy ion induced reactions that there is a significant reduction in the barrier for the evaporation of alpha particles from nuclei excited to about 100 MeV and having large angular momenta [5,6]. Furthermore, the calculated alpha-particle emission probabilities from some fast-neutron induced reactions which give cross sections that fall short of the experimental data, particularly at lower emission energies, have been reported by Bychkov *et al.* [7]. In this paper we extend the global optical potential of Nolte *et al.* to low energies and use it to understand the different features of alpha-particle emission.

The main aspects of the standard optical potential parameter set of Nolte *et al.* [4] with Woods-Saxon type form factors and volume imaginary part (Table I), that are important for the present purpose, are as follows: First, it uses the result of Singh and Schwandt [8] and Put and Paans [9] who found in extensive analyses of differential cross sections for the elastic scattering from ^{24}Mg and ^{90}Zr, respectively, that the potential geometry becomes rather constant for alpha-particle energies above 80 MeV. In addition, the quality of the fits is worse for lower energies than for higher ones if the geometry of the real and imaginary wells is fixed to that obtained from a best fit to data above 100 MeV, so that optimum fitting requires a striking energy dependence for the real potential form factor.

Second, the energy dependence of the well depth parameters obtained by Put and Paans from elastic scattering on ^{90}Zr, with a fixed set of geometrical parameters, has also been adopted. In spite of the bad fits they obtained at low energy, a linear energy dependence of the real depth has been found over the whole energy range up to 150 MeV, while for the imaginary depth a near linear increase with energy up to about 80 MeV is followed by an approximately constant value. A similar energy dependence has been found for ^{24}Mg [8] (Table I).

On this basis Nolte *et al.* derived the mass dependence of both the real and imaginary potential depths as well as the diffuseness parameters for 140 MeV alpha-particle scattering on six target nuclei from ^{12}C to ^{208}Pb. Moreover, this potential was also found to reproduce the total reaction cross sections from small energies (<10 meV/nucleon) up to 200 MeV/nucleon for light and medium nuclei, whereas for heavy nuclei the agreement is achieved only up to 80 MeV/nucleon.

The most important information used for the extension of this global potential to lower energies has been the energy dependence of the imaginary well depth derived by Put and Paans from the ^{90}Zr data, with real and imaginary geometry fixed to the best-fit values around 100 MeV. Using the potential depth expression adopted by Nolte *et al.* [4]

$$W(A,E_\alpha)=b_0+b_1A^{1/3}+b_2E_\alpha \qquad (1)$$

and keeping the value of b_1 unchanged, the parameter b_2 value corresponding to these data (Table 4 and Fig. 5

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of Ref. [9]) is 0.15 ± 0.06 MeV^{-1}. Furthermore, a good overlap with the similar energy dependeence of the ^{24}Mg data [8] has been obtained by adopting the value $b_2 = 0.20$ MeV^{-1}. In these circumstances, use of the same mass dependence as was found above 80 MeV allowed the extension of the imaginary potential of Nolte *et al.* to lower energies (Table I), with the respective parameters within the limits of those derived by Schwandt and Singh in the case of the ^{24}Mg target nucleus.

The low-energy dependence of the imaginary well depth can also be found from the analysis of some (n,α) reaction cross sections. However, contrary to Bychkov *et al.* [7], we consider the alpha-particle emission spectra at an incident neutron energy of 15 MeV to be too sensitive to level density effects and pre-equilibrium emission, to be a useful probe of the alpha-particle optical potential. To avoid these doubtful features of the cross section calculations [10] we have analyzed (n,α) reaction data for only the first 3–5 MeV above the threshold. Quite useful in this respect are related measurements on isotopes around $A = 50$ used in fast and fusion reactors, which are of great interest for activation and radiation damage studies. To test the potential, statistical model calculations using the present optical potential are compared with experimental cross sections for the (n,α) reaction on the target nuclei ^{48,50}Ti, ^{51}V, ^{52}Cr, ^{54}Fe, and ^{59}Co up to an incident energy of about 10 MeV (Fig. 1). The generally good agreement shows that the global optical potential [4] can be extended to low energy and used to calculate low-energy alpha-particle evaporation. Some difficulties of the calculation are discussed in the next section.

The main assumptions underlying the Hauser-Feshbach statistical model calculations, which were carried out by means of the code STAPRE-H [11], and the other model parameters are given elsewhere [10]. These give consistent calculated cross sections (excitation functions and energy spectra) in good agreement with known experimental data for competing reaction channels. Actually, the alpha-particle transmission coefficients have determined all the calculated results within the energy range considered, since there is no pre-equilibrium emission. Among the various features of excited nuclei affecting the alpha-particle emission, these reactions depend only on the low-energy and low angular momentum region in the E^*-J plane [12] (of so-called type IV), so this is the least affected by nuclear shape and level density effects. In Fig. 2 we show the (n,α) reaction population cross sections for the ground state as well as for all the low-lying states explicitly taken into account in the Hauser-Feshbach calculation. This shows that the level density parameters have very little influence on this calculation.

The comparison in Fig. 1 of the results obtained using various optical potential parameter sets shows the close similarity between the present ones and those given by the Huizenga and Igo [13] potential. Quite different, low by about a factor of two, are the cross sections determined from the widely used average four-parameter set of McFadden and Satchler [3] (Fig. 1). Following Bychkov *et al.* [7] who showed that an increase of 20–25 % in the

TABLE I. Global optical potentials for alpha particles. Standard six-parameter Woods-Saxon form with volume imaginary component and symbols of Ref. [4] are used.

V (MeV)	r_v (fm)	a_v (fm)	W (MeV)	r_w (fm)	a_w (fm)	Ref.
$133-0.29E^{\text{c.m.}}$	1.30	0.74	$\begin{cases} 10+0.25E^{\text{c.m.}} (E<70 \text{ MeV}) \\ 34-0.09E^{\text{c.m.}} (E>70 \text{ MeV}) \end{cases}$	1.60	0.71	[8]
$101.1+6.051 \cdot Z/A^{1/3}-0.248 \cdot E$	1.245	$0.817-0.0085 \cdot A^{1/3}$	$26.82-1.706 \cdot A^{1/3}+0.006 \cdot E$	1.57	$0.692-0.02 \cdot A^{1/3}$	[4]
$101.1+6.051 \cdot Z/A^{1/3}-0.248 \cdot E$	1.245	$0.817-0.0085 \cdot A^{1/3}$	$\begin{cases} 12.64-1.706 \cdot A^{1/3}+0.20 \cdot E (E<73 \text{ MeV}) \\ 26.82-1.706 \cdot A^{1/3}+0.006 \cdot E (E>73 \text{ MeV}) \end{cases}$	1.57	$0.692-0.02 \cdot A^{1/3}$	this work

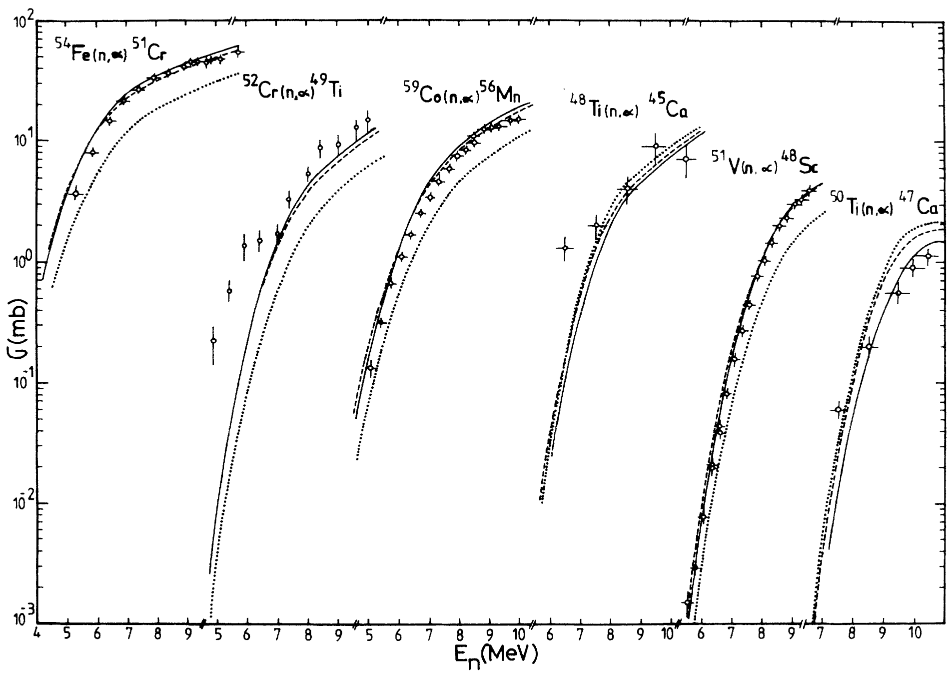

FIG. 1. Comparison of the calculated (n,α) reaction cross sections with experimental data for the target nuclei ^{48,50}Ti [25], ^{51}V [26], ^{52}Cr [27], ^{54}Fe [28], and ^{59}Co [29], shown according to the increasing asymmetry parameter $(N-Z)/A$ of the system. The statistical model results are obtained by using the optical potentials for alpha particles from present work (full curves), Ref. [13] (dash-dotted curves), and Ref. [3] (the parameter set Ca9 for the residual nuclei ^{45,47}Ca, and the average four-parameter set for the rest of the cases, dotted curves).

diffuseness parameter of the real part of this potential is sufficient to give acceptable agreement with experimental emission spectra, and replacing the value $a = 0.52$ fm of the average four-parameter set by 0.62 fm (Fig. 2), it is found that extension to lower energies of the Nolte *et al.* global potential gives similar results to the empirical method [7,10]. The reactions within these figures have been arranged according to the "isotope effect" [14], i.e., absolute (n,α) reaction cross sections decreasing with the asymmetry parameter $(N-Z)/A$ of the target nucleus. A good description has been obtained even over three orders of magnitude and down to the smallest cross sections.

The (n,α) reaction excitation functions for the ^{48,50}Ti and ^{52}Cr target nuclei require further comment. The large disagreement between the experimental and calculated data in the last case for incident energies below 7 MeV corresponds to a change of the experimental slope around this energy. However, since up to still higher energies only the ground state is populated (Fig. 2), this behavior seems unphysical. The case of the (n,α) reactions on titanium isotopes is very complex, starting with the difference between the predictions of McFadden and Satchler's average four-parameter set and their best-fit optical potential parameters (set Ca9) for 24.7 MeV alpha-particle scattering from ^{40}Ca [3]. Since the average four-parameter set was proved able to describe well total α-reaction and (α,n)-reaction cross sections around the Coulomb barrier in the same mass range [17], total α-particle reaction cross sections given by this set and the best-fit parameters for the nuclei around $A = 50$ (^{40}Ca, ^{55}Mn, and ^{59}Co) [3] have been compared too (Fig. 3) by

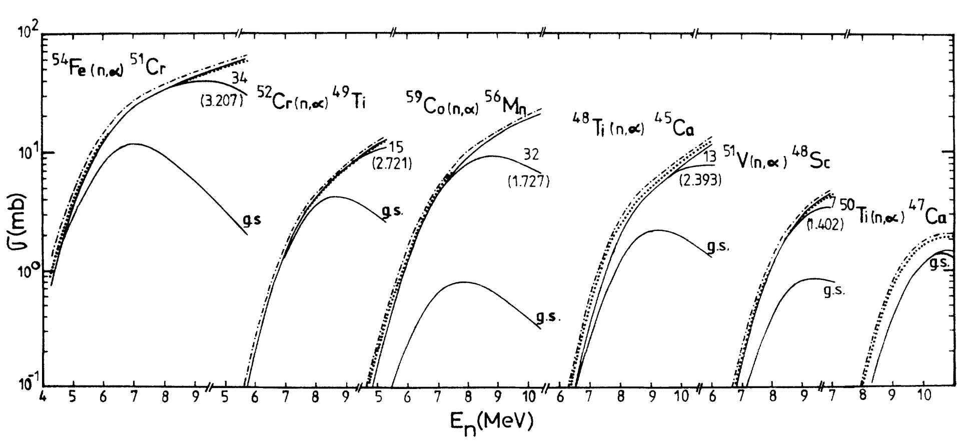

FIG. 2. Calculated cross sections for (n,α) reactions as in Fig. 1, by using the optical potential for alpha particles from present work (thick full curves), Ref. [4] (dash-dotted curves), and the average four-parameter set of Ref. [3] modified by increasing the diffuseness parameter to $a = 0.62$ fm [7,10] (dotted curves). There are shown also the population cross sections for the ground state (g.s.) and the given number of the residual nucleus low-lying levels up to given excitation energy (within brackets, in MeV) involved in the Hauser-Feshbach calculations (thin full curves).

using the optical model code SCAT2 [18]. It is found that the average four-parameter set is very close to the optimum six-parameter sets for the last two nuclei, but not for ^{40}Ca. In the last case the best-fit parameter set agrees with our potential while both overestimate significantly the cross sections of the (n,α) reactions on the ^{48,50}Ti isotopes. An explanation of this behavior could be the special place held by the calcium isotopes in the understanding of nuclear structure, shown by the accurately measured charge rms radii which are highly anomalous relative to the normal increase with $A^{1/3}$ [15]. By reducing the present optical potential diffuseness parameter for the ^{45,47}Ca residual nuclei, according to the decreasing values found by Andl *et al.* [16] for the diffuseness of the Fermi functions describing the charge distributions of the Ca isotopes, we obtain (n,α) reaction cross sections in rather good agreement with experimental data except at the lowest energy (full curve in Fig. 1).

The alpha-particle optical potential analyses have been lately focused on two main questions, encountered in any search for a global parameter set (e.g., [4,8,9,19]): (i) The optical model parameter sets obtained from elastic scattering at high energies do not describe the low-energy elastic scattering or complete fusion data. (ii) The statistical model cross sections for alpha-particle emission from hot nuclei are much underestimated by the optical potentials that account for elastic scattering on the cold ground-state nuclei.

The first question concerns the data at energies above 80 MeV which are described by constant shape parameters while energy-dependent form factors for both the real and imaginary potentials have to be used at lower energy. However, Singh and Schwandt [8] found that no abrupt and large energy dependence is needed in the nucleon-nucleus folding model with "Woods-Saxon equivalent" parameter values which are consistent, within uncertainties, with those obtained with the corresponding optical model analysis (Table I). They considered the folded form factor to be more realistic, and the anomalous energy dependence as a phenomenological model attempt to reproduce the realistic potential at low energies by using relatively larger radii and smaller diffuseness. The diffuseness is changed because the elastic scattering is sensitive only to the tail region of the potential at lower energies but becomes progressively more sensitive to the nuclear interior at higher energies. This has supported the use of the constant geometry in the present global potential extension to lower energy, with a considerably larger diffuseness. This trend was proved also by the empirical adjustment method [7,10].

The three-turning point WKB approximation for scattering by a complex local optical potential, due to Brink and Takagawa, is quite helpful for understanding both these questions [20]. In this approximation, when the absorption is moderate in the interior region of the potential, the scattering amplitude can be written as a sum of a term describing a wave reflected at the barrier (external turning point), and a second one corresponding to a wave which penetrates into the interior region of the interaction potential and is reflected on the internal classical turning point. Moreover, Delbar *et al.* [21] showed for alpha-particle scattering from ^{40,44}Ca that the barrier term dominates at small scattering angles and becomes more important as energy increases, while the internal contribution dominates at large angles and low energy. Since the scattering amplitude has two components which are not only sensitive to the potential in different parts of space, but also vary differently with energy, the usual Woods-Saxon parametrization may be inadequate. Particularly in the low incident energy region the internal contribution to the scattering amplitude is greater for

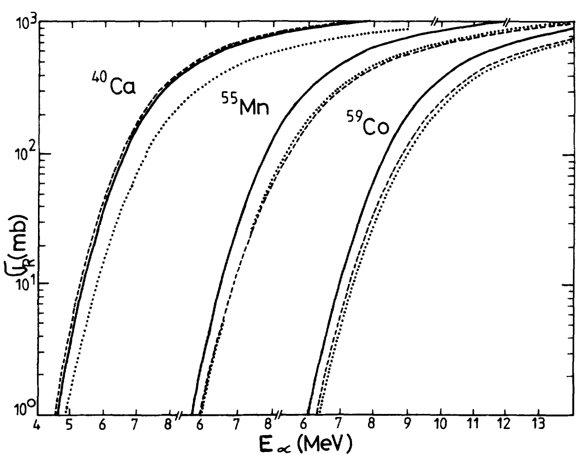

FIG. 3. Calculated alpha-particle total reaction cross sections for the target nuclei ^{40}Ca, ^{55}Mn, and ^{59}Co, by using optical potentials from present work (full curves), the best-fit parameter sets Ca9, Mn3, and Co2, respectively (dashed curves), and average four-parameter set (dotted curves) of Ref. [3].

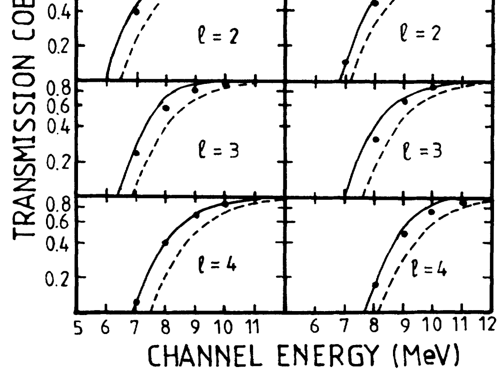

FIG. 4. Transmission coefficients versus channel energy for alpha particles on ^{48}Ti and ^{56}Mn, and partial waves l=(0–4)$\hbar$, calculated by using the optical model potentials from present work (full curves, Ref. [3] (dashed curves), and Ref. [13] (dots).

^{40}Ca than for ^{44}Ca, and is responsible for the anomalous large angle scattering (ALAS) which depends smoothly on the incident energy and disappears above 55 MeV [21].

The different inherent sensitivities of the scattering and reaction cross sections to the potential at various radii explain significant differences between the optical potentials obtained from comparisons with elastic scattering data and fusion cross sections [6]. This is the reason for the very different reaction cross sections given by the parameter sets of McFadden and Satchler [3] and Huizenga and Igo [13], apart from the special case of ^{40}Ca (Fig. 3). Recalling that fusion reactions are the inverse of evaporative decay, McMahan and Alexander [6] concluded that the transmission coefficients for evaporation calculations are more appropriately related to fusion cross sections than to elastic scattering. In this respect we emphasize the correspondence between predictions of the present extended global optical potential and that of Huizenga and Igo, which is based on both elastic scattering and reaction excitation function data [22]. Nevertheless, the reason for the better agreement with the experimental (α, n) reaction on ^{48}Ti and ^{51}V provided only by McFadden-Satchler potential [17] is less clear. Analysis of additional cases could show other facts, e.g., smaller imaginary potential depth expected for nuclei near closed shells.

The complementary case of emitted particles from excited nuclei has recently been highlighted by Alexander *et al.* [23], the major problem being that neither elastic scattering nor the total of all reactions is the time-reversed reaction for particle evaporation. They show that only the penetrability for the escape of a particle, initially inside the hot nucleus, is needed by the statistical evaporation model while the conventional complex optical potential model transmission coefficients account for important aspects of elastic scattering such as transparency, peripheral absorption, and shape resonances. The optical model transparency, i.e., penetration both into and again out of the well, has proved especially significant for nucleons but essentially absent for the more strongly absorbed alpha particles described by the parameter set of Huizenga and Igo [13]. However, this effect can be related to the internal contribution to the semiclassical scattering amplitude within the three-turning point WKB approximation [20], vanishing at higher energies (e.g., above 55 MeV for ALAS on ^{40}Ca [21]).

We conclude that the present extension of the global optical potential obtained by fitting elastic scattering data above 80 MeV, and partially the Huizenga-Igo parameter set, do not already include the optical model transparency effect. The increased sensitivity to the nuclear interior relative to the nuclear tail region, in alpha-particle scattering at higher energy [8], also makes less probable the interactions in peripheral and distant collisions that do not penetrate the real barrier. In this way the correct transmission coefficients provided by these optical potentials for emitted particles can be explained. On the other hand, by taking into account parameter sets from elastic scattering at 24.7 MeV [3] and comparing the respective transmission coefficients with the extended global optical potential predictions, one can also observe the transparency effect for alpha particles. Figure 4 shows these coefficients for $l=(0\text{–}4)\hbar$ (similarly to the nucleon case [23]) as the low partial waves are particularly important for evaporation at low energies.

It can be concluded that the transmission coefficients for alpha-particle evaporation are rather strongly related to the fusion cross sections and elastic scattering at energies above 80 MeV, but not to elastic scattering at lower energy where specific features are comprised within complex optical potential parametrizations. This also explains why the global optical potential of Nolte *et al.* fails to reproduce elastic scattering data for small energies, whereas good agreement resulted from comparison with systematics of the volume integrals [4] (similar in some respect with the total reaction cross section test). Additional analyses of total and (α, N) reaction cross sections, of high interest for nuclear astrophysics, would give an alpha-particle mean field representation over a large energy range. The present extension of the imaginary potential depth to lower energy could be also checked by using the dispersion relations connecting the real and imaginary parts of the potential [1], previous experience being encouraging [24,30].

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