Medium modifications to the ω-meson mass in the Walecka model

H.-C. Jean,[1,2] J. Piekarewicz,[2] and A. G. Williams,[1,2,3]
[1] *Department of Physics, Florida State University, Tallahassee, Florida 32306*
[2] *Supercomputer Computations Research Institute, Florida State University, Tallahassee, Florida 32306*
[3] *Department of Physics and Mathematical Physics, University of Adelaide, South Australia 5005, Australia*
(Received 4 November 1993)

We calculate the effective mass of the ω meson in nuclear matter in a relativistic random-phase approximation to the Walecka model. The dressing of the meson propagator is driven by its coupling to particle-hole pairs and nucleon-antinucleon ($N\bar{N}$) excitations. We report a reduction in the ω-meson mass of about 170 MeV at nuclear-matter saturation density. This reduction arises from a competition between the density-dependent (particle-hole) dressing of the propagator and vacuum polarization ($N\bar{N}$ pairs). While density-dependent effects lead to an increase in the mass proportional to the classical plasma frequency, vacuum polarization leads to an even larger reduction caused by the reduced effective nucleon mass in the medium.

PACS number(s): 21.65.+f, 24.10.Jv

I. INTRODUCTION

Understanding the role played by the nuclear medium in modifying hadronic properties is one of the most interesting and challenging problems facing nuclear physics today. For example, the spin-independent (or old) European Muon Collaboration (EMC) experiment revealed a medium-modified electromagnetic coupling of the nucleon relative to its free-space value. Trying to explain the origin of this modification, in terms of, either, conventional nuclear physics effects (e.g., binding energy, Fermi motion, correlations) or more exotic mechanisms (e.g., nucleon swelling), is still the source of considerable debate.

Also very interesting is the study of the modification of meson properties in the medium. Indeed, many interesting phenomena in finite nuclei have been attributed to an in-medium reduction of the mass of the rho meson [1]. These phenomena include the lack of an enhancement in the ratio of the spin-longitudinal to spin-transverse responses measured in quasielastic $(\vec{p},\vec{n})$ scattering [2], the enhancement of the K^+-nucleon interaction in the medium [3], and the behavior of certain spin observables measured in inelastic $(\vec{p},\vec{p}\,')$ transitions [4].

In the medium, a meson gets modified due to its coupling to nuclear excitations. This modification is contained in the meson self-energy whose imaginary part is a physical observable characterizing the linear response of the nuclear system to an external probe. To date, much work has been done (experimentally and theoretically) in understanding the response of the nuclear system in the spacelike region (i.e., $q_\mu^2 < 0$). All of the information gathered so far about the response of the nuclear system to a variety of probes (e.g., e^-, π, K^+, N), and for a variety of kinematical conditions (covering the inelastic, giant-resonance, quasielastic regions) can only reveal the nature of the nuclear response in the spacelike region. In these experiments the coupling of the probe to timelike excitations can only occur virtually. It therefore becomes very interesting to study the behavior of the meson self-energy in the timelike region. This could be done, for example, in colliding (e^+e^-) experiments and relativistic heavy-ion collisions. Alternatively, it can be studied by directly measuring the medium dependence of the decay of the meson into lepton pairs. Indeed, a proposal has been put forward to measure (at CEBAF) the nuclear-mass dependence of vector mesons by detecting lepton pairs [5].

The dependence of meson properties on the density of the nuclear medium is far from understood. In particular, predictions for the shift in the value of the ω meson at normal nuclear-matter density range anywhere from -100 to $+100$ MeV. These predictions are based on a variety of models that include quantum hadrodynamics (QHD) [6], Nambu-Jona-Lasinio models [7], and QCD sum rules (QSR's) [8,9].

In this paper we attempt a more detailed analysis and so elect to use the simplest version of QHD, namely, the Walecka model, to study medium modifications to the ω-meson propagator in the timelike region. The Walecka model is a strong-coupling renormalizable field theory of nucleons interacting via the exchange of (isoscalar) scalar (σ) and vector (ω) mesons [10,11]. The model has already been used extensively in calculations of nuclear matter and finite nuclei. The saturation of nuclear matter and the strong spin-orbit splitting observed in finite nuclei were among the first successes of the model [10–13]. More recently, the model has been used to calculate such diverse topics as collective modes in nuclear matter [14], isoscalar magnetic moments [15,16], and electroweak [17–19] and hadronic responses from finite nuclei [20] with considerable success.

In spite of these successes, the theory remains (practically) untested in the timelike region, which holds special relevance for a model that incorporates negative-energy degrees of freedom from the outset. In this calculation we will show that the in-medium shift in the value of the ω-meson mass arises from a sensitive cancellation between two competing effects. On the one hand, the (virtual)

0556-2813/94/49(4)/1981(8)/$06.00

coupling of the meson to particle-hole excitations leads to an increase in the value of the mass. This is consistent with the result reported by Chin using an approximation to QHD that ignored the coupling of the ω meson to $N\bar{N}$ excitations (vacuum polarization) [6]. By also including vacuum polarization in the present calculation, we believe that we have performed a more consistent field-theoretical calculation. We have found that $N\bar{N}$ excitations generate a shift in the value of the ω-meson mass proportional to the shift of the nucleon mass in the medium. Since in QHD the scalar field is responsible for a reduction of the nucleon mass, $N\bar{N}$ excitations (by themselves) give rise to a reduction of the ω-meson mass. For values of the model parameters consistent with the description of nuclear matter at saturation, we have found that vacuum polarization overwhelms the corresponding density-dependent contribution, and, ultimately, leads to a reduction of the ω-meson mass in the medium.

We have organized the paper as follows. In Sec. II we present the formalism needed to calculate the self-energy corrections to the propagation of the ω meson through the nuclear medium. In Sec. III we present results for the effective mass of the ω meson with special emphasis on the competition between the density-dependent dressing of the meson and vacuum polarization. Finally, in Sec. IV we offer our conclusions and outlook for future work.

II. FORMALISM

In this section we calculate the self-energy corrections to the ω-meson propagator in nuclear matter. The dressed meson propagator will be calculated to one-loop order by solving Dyson's equation in a nuclear-matter ground state obtained from using a relativistic mean-field approximation to the Walecka model.

The Walecka model (or QHD-I) is a renormalizable, relativistic quantum field theory of the nuclear system which involves an explicit description of the nucleon (N) and meson (σ, ω) degrees of freedoms [10,11]. The Lagrangian density for the Walecka model is

$$\begin{aligned}\mathcal{L} = &\bar{\psi}[\gamma_\mu(i\partial^\mu - g_v V^\mu) - (M - g_s\phi)]\psi \\ &+\frac{1}{2}(\partial_\mu\phi\partial^\mu\phi - m_s^2\phi^2) \\ &-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m_v^2 V_\mu V^\mu + \delta\mathcal{L} \,, \end{aligned} \tag{1}$$

where ψ is the baryon field with mass M, ϕ is the neutral scalar-meson (σ) field with mass m_s, V^μ is the neutral vector-meson (ω) field with mass m_v, and $F^{\mu\nu} \equiv \partial^\mu V^\nu - \partial^\nu V^\mu$. The term $\delta\mathcal{L}$ contains renormalization counterterms. In this model, nucleons interact via the exchange of isoscalar mesons with the coupling of the scalar field ϕ to the baryon scalar density $\bar{\psi}\psi$, and the vector field V^μ to the conserved baryon current $\bar{\psi}\gamma_\mu\psi$ introduced through minimal substitution.

Since the exact solutions to the field equations are unknown (and perhaps unattainable), we resort to a mean-field approximation in the usual way. In a mean-field approximation, one replaces the meson field operators by their (classical) ground-state expectation values:

$$\phi \to \langle\phi\rangle \equiv \phi_0 \,, \tag{2a}$$
$$V^\mu \to \langle V^\mu\rangle \equiv g^{\mu 0}V^0 \,. \tag{2b}$$

The mean-field equations can now be solved exactly with the solution becoming increasingly valid with increasing baryon density. Traditionally, the mean-field equations have been solved in two approximations. In mean-field theory (MFT) one calculates a baryon self-energy which is generated by the presence of all the nucleons in the occupied Fermi sea. The effect of the (infinite) Dirac sea is, however, neglected. In contrast, in the relativistic Hartree approximation (RHA) one includes the contribution to the baryon self-energy arising from the occupied Fermi sea as well as from the full Dirac sea. As a consequence, the baryon self-energy diverges in the RHA and must be renormalized.

In both approximations, the mean scalar-meson field ϕ_0 is responsible for a (downward) shift of the nucleon mass M^* in the nuclear medium relative to its free-space value M. In contrast to the ground-state expectation value of the vector field, which is fully determined by the conserved baryon density,

$$g_v V^0 = \frac{g_v^2}{m_v^2}\langle\bar{\psi}\gamma^0\psi\rangle = \frac{g_v^2}{m_v^2}\rho_B \,, \tag{3}$$

the expectation value of the scalar field (and consequently the effective mass) is a dynamical quantity that must be determined self-consistently from the equations of motion

$$M - M^* = g_s\phi_0 = \frac{g_s^2}{m_s^2}\langle\bar{\psi}\psi\rangle(M^*) = \frac{g_s^2}{m_s^2}\rho_S(M^*) \,. \tag{4}$$

There are five parameters to be determined in the model. The nucleon mass and the ω-meson mass are fixed at their physical values ($M = 939$ MeV and $m_v = 783$ MeV, respectively). The other three parameters must be fixed from physical observables. For example, the ratios of the coupling constant to the meson mass ($C_s^2 = g_s^2M^2/m_s^2$ and $C_v^2 = g_v^2M^2/m_v^2$) can be chosen to reproduce the bulk binding energy (15.75 MeV) and density (corresponding to a Fermi momentum of $k_F = 1.3\ \text{fm}^{-1}$) of nuclear matter at saturation. Finally, the mass of the scalar meson is adjusted to reproduce the root-mean-square radius of ^{40}Ca. These parameters along with the effective nucleon mass M^* at saturation density are listed in Table I.

In order to compute the meson propagators in the nuclear medium we solve Dyson's equation in the random-phase approximation (RPA) [21]. This approximation is characterized by an infinite summation of the lowest-order proper polarization. In a relativistic theory of nuclear structure the polarization insertion, or meson self-energy, describes the coupling of the meson to two kinds of excitations: the traditional particle-hole pairs and nucleon-antinucleon ($N\bar{N}$) excitations. In the nuclear medium real particle-hole excitations can be produced only if the four-momentum carried by the meson is spacelike ($q_\mu^2 < 0$). In contrast, real $N\bar{N}$ pairs can be excited only in the timelike region ($q_\mu^2 > 0$).

To date, most of the relativistic RPA studies of the

TABLE I. Mean-field parameters in the Walecka model. The nucleon mass and the ω-meson mass were fixed at their physical values (M = 939 MeV, m_v = 783 MeV). The effective nucleon mass M^* is the appropriate value at nuclear-matter saturation density ($k_F = 1.3\ \mathrm{fm}^{-1}$).

Model	g_s^2	g_v^2	m_s (MeV)	C_s^2	C_v^2	M^*/M
MFT	109.626	190.431	520	357.469	273.871	0.541
RHA	54.289	102.770	458	228.198	147.800	0.730

nuclear system have been carried out in a kinematical domain considerably different from the one relevant to the present analysis. These studies have investigated the response of the nuclear system to a variety of probes. In all of these cases the four-momentum transferred to the nucleus (and hence carried by the mesons) was constrained to the spacelike region. Hence, $N\bar{N}$ pairs could only be virtually excited. In the present work we need to study the ω-meson self-energy in a (timelike) region around the position of the ω-meson pole ($q_\mu^2 \simeq m_v^2$). This is a kinematical region where both particle-hole as well as $N\bar{N}$ pairs can only be virtually created (at least at low density) and where the Walecka model has been largely untested.

In the Walecka model one cannot decouple the σ meson from the analysis of the ω-meson propagator. In the nuclear medium the ω and σ propagators are inextricably linked because of scalar-vector mixing. Scalar-vector mixing occurs, for example, when a particle-hole pair becomes excited in the medium by means of a (longitudinal) vector meson which subsequently decays into a scalar meson. Scalar-vector mixing is a purely density-dependent effect that generates a coupling between the Dyson's equation for the scalar- and vector-meson propagators. The full scalar-vector meson propagator and the lowest-order (one-loop) proper polarization can be represented by a set of 5×5 matrices D^{ab} and Π_{ab}, with the indices a and b in the range $-1, 0, 1, 2, 3$ [14]. In this representation Dyson's equation becomes a (coupled) matrix equation given by

$$D = D_0 + D_0 \Pi D \ , \tag{5}$$

where D_0 represents the lowest-order meson propagator,

$$D_0 = \begin{pmatrix} \Delta_0 & 0 \\ 0 & D_0^{\mu\nu} \end{pmatrix} \ , \tag{6}$$

written in terms of the noninteracting scalar- and vector-meson propagators

$$\Delta_0(q) = \frac{1}{q_\mu^2 - m_s^2 + i\eta} \ , \tag{7}$$

$$D_0(q) = \frac{1}{q_\mu^2 - m_v^2 + i\eta} \ , \tag{8}$$

$$D_0^{\mu\nu} = \left(-g^{\mu\nu} + q^\mu q^\nu / m_v^2\right) D_0(q) \ , \tag{9}$$

and where $q_\mu^2 \equiv q_0^2 - \mathbf{q}^2$. Notice that since the ω meson always couples to a conserved baryon current the $q^\mu q^\nu$ term in $D_0^{\mu\nu}$ will not contribute to physical quantities.

The lowest-order polarization insertions will be expressed in terms of the self-consistent nucleon propagator. The nucleon propagator is written as a sum of Feynman $[G_F(k)]$ and density-dependent $[G_D(k)]$ contributions, i.e.,

$$G(k) \equiv G_F(k) + G_D(k) \ , \tag{10}$$

$$G_F(k) = (\gamma^\mu k_\mu^* + M^*) \frac{1}{k_\mu^{*2} - M^{*2} + i\eta} \ , \tag{11}$$

$$G_D(k) = (\gamma^\mu k_\mu^* + M^*) \times \frac{i\pi}{E^*(\mathbf{k})} \delta(k_0^* - E^*(\mathbf{k})) \theta(k_F - |\mathbf{k}|) \ , \tag{12}$$

where the momentum $k^{*\mu}$ and energy $E^*(\mathbf{k})$ are defined, respectively, by $k^{*\mu} \equiv (k^0 - g_v V^0, \mathbf{k})$ and $E^*(\mathbf{k}) \equiv \sqrt{\mathbf{k}^2 + M^{*2}}$. The Feynman part of the propagator has the same analytic structure as the free propagator. The density-dependent part, on the other hand, corrects G_F for the presence of occupied states below the Fermi surface and vanishes at zero baryon density. In terms of the nucleon propagator the lowest-order scalar-scalar, vector-vector, and scalar-vector (mixed) polarizations are given, respectively, by

$$\Pi^s(q) = -ig_s^2 \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}[G(k)G(k+q)] \ , \tag{13a}$$

$$\Pi^v_{\mu\nu}(q) = -ig_v^2 \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}[\gamma_\mu G(k) \gamma_\nu G(k+q)] \ , \tag{13b}$$

$$\Pi^m_\mu(q) = ig_s g_v \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}[\gamma_\mu G(k) G(k+q)] \ . \tag{13c}$$

In the above expressions the traces include isospin and we have adopted the conventions of Ref. [22]. As in the case of the nucleon propagator we can write the above polarization insertions as a sum of two contributions,

$$\Pi^{ab}(q) \equiv \Pi_F^{ab}(q) + \Pi_D^{ab}(q) \ . \tag{14}$$

The Feynman contribution to the polarization, or vacuum polarization (Π_F^{ab}), is a bilinear function of G_F and describes the self-energy corrections to the meson propagators due to their coupling to $N\bar{N}$ excitations. The Feynman contribution to the polarization insertion diverges and must be renormalized. The density-dependent part of the polarization (Π_D^{ab}), on the other hand, is finite and contains at least one power of G_D. The density-dependent part of the polarization insertion describes the coupling of the meson to particle-hole excitations. In addition, it contains a term which has no nonrelativistic counterpart. This term arises from the negative-energy components in the Feynman propagator and describes the Pauli blocking of $N\bar{N}$ excitations. This term enforces the Pauli principle by preventing the nucleon from the $N\bar{N}$ pair (in Π_F^{ab}) to make a transition to an occupied state below the Fermi surface [18,23].

For a mean-field ground state obtained in the MFT (no vacuum loops) approximation, it has been shown that the consistent linear response of the system, and hence the consistent meson self-energy, is obtained by neglecting the Feynman part of the polarization insertion. This consistency is reflected, for example, in the proper treatment of (spurious) excitations associated with an overall translation of the center of mass of the system. Notice, however, that in the MFT approximation one retains the Pauli blocking of $N\bar{N}$ excitations even though one is neglecting the Feynman contribution to the polarization. Retaining the Pauli blocking of $N\bar{N}$ excitations has been proven essential to satisfy current conservation in calculations of the electromagnetic response of the nuclear system. Nevertheless, one should question an approximation that retains the Pauli blocking of an excitation that has not been put in from the outset. If the effect from Pauli blocking of $N\bar{N}$ excitations represents a small contribution to the overall size of the observables, then one might be justified in making this approximation. If, however, the observables are seen to depend heavily on this assumption, then one would be forced to neglect this approximation in favor of the RHA where vacuum loops are included in, both, the description of the ground state as well as in the linear response of the system. We now present results for the effective mass of the ω meson in both (MFT and RHA) approximations.

III. RESULTS

The effective mass of the ω meson (m_v^*) in nuclear matter is obtained by finding the value of the four-momentum q_μ^2 for which the imaginary part of the propagator attains its maximum. Alternatively, since the ω meson has a very narrow width in the region of interest, we can find the effective mass by searching for zeros in the inverse propagator. In the particular case of the transverse component of the polarization (unaffected by scalar-vector mixing) we obtain

$$\begin{aligned} D_T^{-1}(q) &\equiv D_{22}^{-1}(q) = D_{33}^{-1}(q) \\ &= q_\mu^2 - m_v^2 - \Pi_{22}(q;k_F) = 0 \ , \end{aligned} \tag{15}$$

where we have defined the x axis along the direction of three-momentum $\mathbf{q}$. The (transverse) effective ω-meson mass has been plotted in Fig. 1 as a function of the Fermi momentum (relative to its value at saturation, $k_F^0 = 1.30\ \mathrm{fm}^{-1}$) for two values of the three-momentum transfer. These MFT results are in agreement with those published in Ref. [14] and have been included here for completeness.

One can gain some insight into the physics driving these modifications to the ω-meson mass by examining the low-density limit of these results. By performing a low-density expansion of the transverse meson propagator one can show that the effective mass of the ω meson is given by

$$m_v^{*2} = m_v^2 + \Omega^2 + \mathcal{O}(m_v^2/4M^2) \ , \tag{16}$$

where we have introduced the classical plasma frequency

$$\Omega^2 = \frac{g_v^2 \rho_B}{M} \ . \tag{17}$$

The above set of equations indicate that the density-dependent dressing of the ω-meson propagator leads to an increase in the mass of the ω meson in the medium which is proportional to the classical plasma frequency. This is the (old) result obtained by Chin in 1977; e.g., see Ref. [6].

Two features of this plot are particularly noteworthy. First, the area between the dotted lines represents the region where the imaginary part of the transverse polarization is nonzero. Inside this region the ω meson can "decay" into $N\bar{N}$ pairs. This region is bounded from below by the curve

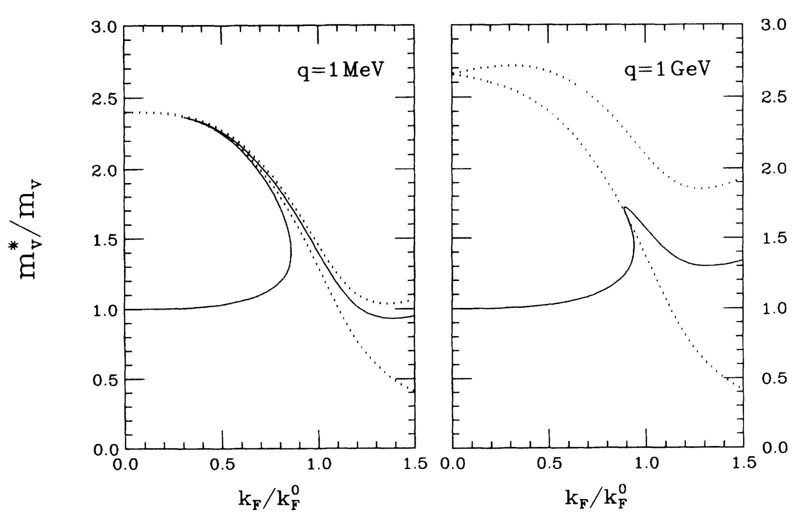

FIG. 1. Effective transverse ω-meson mass as a function of the Fermi momentum for $|\mathbf{q}| = 1$ MeV and $|\mathbf{q}| = 1$ GeV in MFT. The dotted lines enclose the (q^0) region where the imaginary part of the transverse polarization is nonzero and damping of the modes is possible.

$$[q^0]_{\rm min} = \begin{cases} \sqrt{\mathbf{q}^2 + 4M^{*2}} & \text{if } |\mathbf{q}| \leq 2k_F\ , \\ \sqrt{k_F^2 + M^{*2}} + \sqrt{(|\mathbf{q}| - k_F)^2 + M^{*2}} & \text{if } |\mathbf{q}| \geq 2k_F\ , \end{cases} \tag{18}$$

and from above by the curve

$$[q^0]_{\rm max} = \sqrt{k_F^2 + M^{*2}} + \sqrt{(|\mathbf{q}| + k_F)^2 + M^{*2}}\ . \tag{19}$$

This region is defined by imposing energy-momentum conservation for the (on-shell) production of a $N\bar{N}$ pair with the nucleon three-momentum constrained to be below the Fermi momentum. Note, however, that by itself this nonzero imaginary part contributes to an unphysical (i.e., negative) decay width for the ω meson. This can be seen, for example, by considering the case of $N\bar{N}$ pair production at $\mathbf{q} = 0$ ($N\bar{N}$ pair produced with equal magnitude but opposite direction of the three-momentum). In the absence of Pauli blocking the threshold for pair production would start at $q^0 = 2M^*$ (nucleon and antinucleon created at rest). In the medium, however, all nucleon states with momentum $0 \leq |\mathbf{k}| \leq k_F$ should be Pauli blocked and, hence, should not contribute to the width. The threshold for pair production in the medium should, therefore, move (for $\mathbf{q} = 0$) from $q^0 = 2M^*$ to $q^0 = 2\sqrt{k_F^2 + M^{*2}}$. This is precisely the region

$$[q^0]_{\rm min} = 2M^* \leq q^0 \leq 2\sqrt{k_F^2 + M^{*2}} = [q^0]_{\rm max}\ , \tag{20}$$

where the density-dependent contribution to the polarization (Π_D^{22}) develops an imaginary part which would exactly cancel the contribution from vacuum polarization to the decay width of the ω meson. Therefore, vacuum polarization must be included in the study of the damping of the meson (collective) modes. The other feature that one should stress is the many meson branches that are developed for $0.3 \sim k_F/k_F^0 \sim 0.9$. In Ref. [14] it has been suggested that this structure is also related to the Pauli blocking of $N\bar{N}$ excitations. Indeed, by removing the negative-energy (antiparticle) components from the Feynman propagator it was shown that the multibranch structure disappears. Hence, vacuum polarization might also play an important role in the modifications of the real part of the propagator and should be included in the study of the effective ω-meson mass.

We now present results for the effective ω-meson mass in the relativistic Hartree approximation (RHA). In the RHA one must include vacuum contributions to the nucleon self-energy in the calculation of the mean-field ground state as well as the vacuum dressing of the meson propagator. Details of the renormalization procedure can be found in Ref. [24] (note that in the present work the renormalization point is taken at $q_\mu^2 = m_v^2$ and not $q_\mu^2 = 0$). In Fig. 2 we present results for the effective ω-meson mass obtained by finding the zeros of the inverse transverse propagator in analogy to the MFT case [see Eq. (15)]. In the present RHA case, however, the ω-meson self-energy includes density-dependent corrections and vacuum polarization. We observe, in particular, that the multibranch structure present in the MFT calculation has now completely disappeared. Hence the effective mass in the medium (driven by virtual excitations) and the modified width (driven by the decay into $N\bar{N}$ pairs) are dramatically affected by vacuum polarization. Note, however, that the combined effect of the density-dependent dressing plus vacuum polarization generates, in contrast to the MFT case, a reduction in the value of the effective ω-meson mass in the medium. One can shed some light on this result by studying the low-density limit of vacuum polarization. In particular, if only vacuum polarization is included in the dressing of the propagator one obtains an effective meson mass given by

$$\begin{aligned} m_v^{*2} &= m_v^2 \left[1 + \frac{g_v^2}{3\pi^2}\frac{(M^* - M)}{M}\right] + \mathcal{O}(m_v^2/4M^2) \\ &= m_v^2 - \Omega^2 \frac{g_v^2}{3\pi^2}\frac{C_s^2}{C_v^2} + \mathcal{O}(m_v^2/4M^2)\ . \end{aligned} \tag{21}$$

This result indicates that the shift in the mass of the ω meson is proportional to the shift of the nucleon mass in the medium, and thus negative in the Walecka model. If we now add both effects, namely, the density-dependent dressing plus vacuum polarization, into the calculation of the fully dressed propagator, we obtain (the low-density limit of) the in-medium ω-meson mass in the RHA:

$$m_v^{*2} \simeq m_v^2 + \Omega^2 \left[1 - \frac{g_v^2}{3\pi^2}\frac{C_s^2}{C_v^2}\right]\ . \tag{22}$$

The above equation embodies the central result from the present work. It indicates that the shift in the value of the

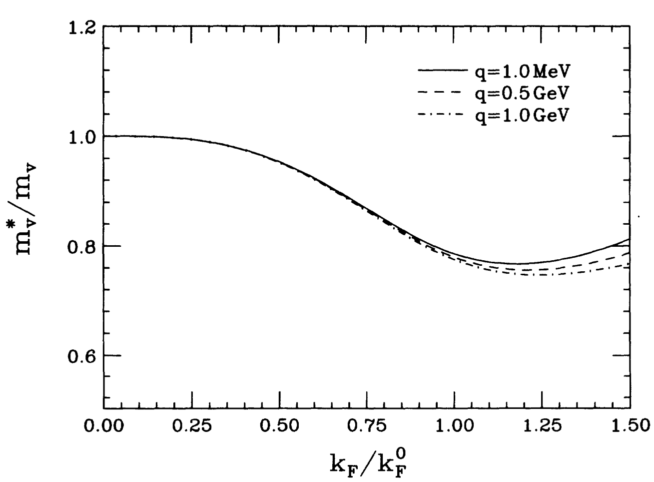

FIG. 2. Effective transverse ω-meson mass as a function of the Fermi momentum for $|\mathbf{q}| = 1$, 500, and 1000 MeV in the RHA. The damping of the modes occurs for values of q^0 outside the range shown in the figure.

ω-meson mass arises from a delicate competition between two effects. On the one hand, the density-dependent dressing of the ω-meson propagator leads to an increase in the mass proportional to the classical plasma frequency [see Eq. (16)]. On the other hand, the vacuum dressing of the propagator is proportional to $M^* - M$ and gives rise, in the present model, to the opposite effect. Because in the Walecka model the scalar field is responsible for a downward shift in the value of the nucleon mass in the medium, the ω meson "drags" along lighter $N\bar{N}$ pairs (relative to free space) that are, ultimately, responsible for reducing the ω-meson mass. The final outcome of this competition depends on the particular values of the coupling constants adopted in the model [see Eq. (22)]. In the present model $(g_v^2/3\pi^2)(C_s^2/C_v^2) \sim 5.36 > 1$ and the vacuum polarization dominates over the corresponding density-dependent dressing leading to a reduction of the ω-meson mass in the medium. In the Walecka model, the values of C_s^2 and C_v^2 were selected in order to reproduce, at the mean-field level, the binding energy and density of nuclear matter at saturation. By further associating the value of the vector-meson mass to the physical value of ω-meson mass, one obtains the $NN\omega$ coupling constant given in Table I. One should mention that this large value for the coupling constant is consistent with other estimates based on fits to empirical two-nucleon data [25]. Thus we believe that the parameters adopted in the present calculation are realistic, and that the shift in the value of the ω meson should be largely insensitive to a fine tuning of parameters. Other models, where the parameters have been constrained by different physical observables, might lead (and actually have led) to different predictions in the magnitude, and even in the direction, of the shift of the ω-meson mass [7–9]. Therefore, the value of the ω-meson mass in the nuclear medium is model dependent and should be determined experimentally [5].

We now close with a brief discussion of the longitudinal meson propagator. In studying the longitudinal effective mass of the ω meson in the medium one must find the zeros of the longitudinal propagator. Because of scalar-vector mixing, however, Dyson's equation for the longitudinal propagator becomes a 3×3 matrix equation (actually a 2×2 matrix equation because of current conservation). However, our calculations, as well as those of Ref. [14], suggest that the scalar-vector mixing is quite small; the shift in the value of the ω-meson mass is at most 1.5 MeV for all densities below the nuclear-matter saturation density. We have also shown that this result is fairly insensitive to the particular choices of width and renormalization point for the scalar propagator. In Fig. 3, the longitudinal effective mass is plotted as a function of the Fermi momentum for $|\mathbf{q}| = 1$ MeV (dashed line) and 1 GeV (dot-dashed line) along with the transverse mass (solid line). For small values of $|\mathbf{q}|$, the transverse and longitudinal effective masses are practically identical (in fact, they are not resolved in the figure). This can be traced to the low-density behavior of the longitudinal mass (ignoring scalar-vector mixing)

$$m_v^{*2} \simeq m_v^2 + \Omega^2 \left[\frac{1}{1+\mathbf{q}^2/m_v^2} - \frac{g_v^2}{3\pi^2} \frac{C_s^2}{C_v^2} \right] , \tag{23}$$

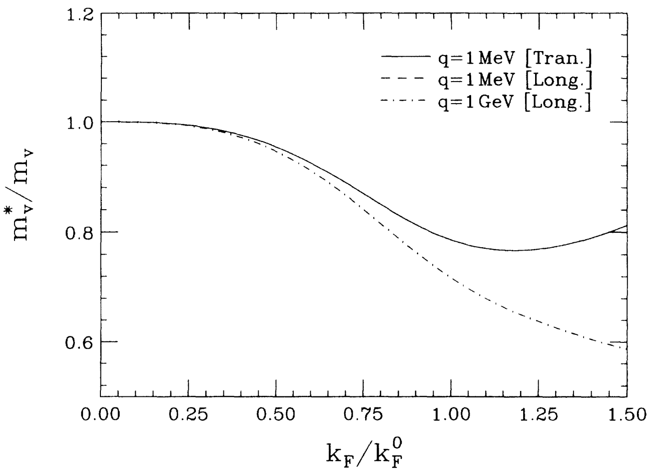

FIG. 3. Effective transverse ω-meson mass as a function of the Fermi momentum for $|\mathbf{q}| = 1$ MeV (solid line) in the RHA. Also shown is the longitudinal mass at $|\mathbf{q}| = 1$ MeV (dashed line) and $|\mathbf{q}| = 1$ GeV (dot-dashed line). The damping of the modes occurs for values of q^0 outside the range shown in the figure. Note that the transverse and longitudinal modes at 1 MeV are indistinguishable in this figure.

which should be compared to Eq. (22). For large values of the momentum, however, the longitudinal effective mass is more sensitive to the value of $|\mathbf{q}|$ and displays an even stronger reduction (see Table II).

IV. CONCLUSION AND FUTURE WORK

We have calculated the effective mass of the ω meson in nuclear matter in the Walecka model. The ground state of nuclear matter was obtained by solving the field equations in a mean-field approximation. The effective mass of the ω meson was subsequently obtained by solving Dyson's equation for the propagator in a relativistic random-phase approximation. We have calculated the dressing of the meson propagator in two approximations. In MFT one neglects vacuum polarization and computes the self-energy corrections to the propagator by including the coupling of the meson to particle-hole pairs and the Pauli blocking of $N\bar{N}$ excitations. In the RHA, on the other hand, one computes the full (one-loop) dressing of the propagator by also including vacuum polarization.

TABLE II. Effective ω-meson mass as a function of momentum transfer at nuclear-matter saturation density ($k_F = 1.3$ fm^{-1}) in the RHA. Meson masses and the momentum transfer are measured in MeV.

Mode	$\lvert\mathbf{q}\rvert$	m_v^*/m_v	m_v^*	$m_v - m_v^*$
Transverse	1	0.786	615.7	167
Transverse	500	0.780	611.0	172
Transverse	1000	0.776	607.5	176
Longitudinal	1	0.786	615.7	167
Longitudinal	1000	0.718	562.1	221

Both approximations have been used extensively in the past (with considerable success) in the study of the linear response of the nuclear system to a variety of probes. In all these cases the momentum transfer to the nucleus, and hence the momentum carried by the meson, was constrained to the spacelike region. In contrast to these findings, we have shown that in the timelike region probed in the present work, a MFT description is inappropriate. Because in MFT one includes the Pauli blocking of $N\bar{N}$ excitations, but not vacuum polarization, one obtains an unphysical (i.e., negative) contribution to the decay width of the ω meson. Furthermore, one generates a dispersion curve for the collective meson modes having a complicated multibranch structure that arises from the Pauli-blocked $N\bar{N}$ excitations.

These two obvious deficiencies of the MFT approach were corrected by calculating the meson propagator in the RHA. In contrast to the MFT results, the ω-meson mass displayed a very smooth behavior as a function of nuclear density. In addition, the damping of the meson modes was now caused by the decay of the ω meson into $N\bar{N}$ pairs. The Pauli blocking of $N\bar{N}$ excitations (already present in the MFT description) simply reduced the decay width of the ω meson by suppressing those transitions to nucleon states below the Fermi surface.

In the RHA the effective ω-meson mass was reduced relative to its free-space value. This reduction arose from two competing effects. On the one hand, the density-dependent dressing of the meson propagator (with no vacuum polarization) caused an increase in the ω-meson mass proportional to the classical plasma frequency. Vacuum polarization, on the other hand, led to a reduction in the mass. This reduction was, ultimately, traced to the corresponding reduction of the nucleon mass in the medium. For the particular values adopted in the model, vacuum polarization effects dominated over the density-dependent dressing and led to a reduction of about 170 MeV in the value of the ω-meson mass at nuclear-matter saturation density (see Table II). We argue that because the parameters of the model were constrained by bulk properties of nuclear matter at saturation, our finding should be largely insensitive to a fine tuning of parameters. However, since other theoretical models, constrained by a different set of observables, can apparently lead to different predictions, it is important to make an experimental determination of the effective ω-meson mass.

In the future we plan to examine the effects of a medium-modified ω mass to the photoproduction of e^+e^- pairs in a kinematical region around the ω-meson mass pole. Many dynamical components must be integrated in such a calculation. For example, in addition to the calculation of an effective ω-meson propagator in the nuclear medium, one needs a model for the $\gamma N \to \omega N$ transition amplitude that can be extrapolated off shell. Ideally, one would like to have a single underlying model for the calculation of both the dressing of the propagator and the photoproduction amplitude. One should also further study the effect of the three-momentum transfer on the photoproduction cross section. In free space the ω-meson propagator is a function of only one kinematical variable, namely, the four-momentum squared (q_μ^2). In the nuclear medium, the self-energy (and thus the propagator) depends, in addition, on the magnitude of the three-momentum transfer ($\mathbf{q}$). For small values of the three-momentum transfer the effective longitudinal and transverse masses of the ω meson are practically identical. At larger values of the three-momentum transfer ($|\mathbf{q}| \sim 1$ GeV), however, the two modes get well separated with the longitudinal mass reduced, near the nuclear-matter saturation density, by an additional 50 MeV relative to the transverse value. This suggests that at large enough values of the three-momentum transfer the spectrum for the production of e^+e^- pairs might show two well-separated (ω-meson) peaks. This statement obviously requires that the signal be extracted from the wide "backgrounds" associated with Bethe-Heitler (nonresonant) pairs and the formation and decay of the (isovector) ρ meson. This may be done by a careful study of the interference of these amplitudes, for example [5]. The study of a medium-modified ρ-meson propagator is, unfortunately, much more complicated in the context of a renormalizable quantum field theory and will not be addressed here; see Ref. [11]. We believe that detailed measurements of the photoproduction of lepton pairs from nuclei are essential and should provide invaluable insights into the formation, propagation, and decay of vector mesons inside the nuclear medium.

ACKNOWLEDGMENTS

This research was supported by the Florida State University Supercomputer Computations Research Institute and U.S. Department of Energy Contracts Nos. DE-FC05-85ER250000, DE-FG05-92ER40750, and DE-FG05-86ER40273. The research of A.G.W. was supported in part by the Australian Research Council.

[1] G.E. Brown and M. Rho, Phys. Lett. B **237**, 3 (1990); Phys. Rev. Lett. **66**, 2720 (1991).
[2] G.E. Brown and J. Wambach, State University of New York Report No. SUNY-NTG-93-6, LANL bulletin board nucl-th-9302009, 1993.
[3] G.E. Brown, C.B. Dover, P.B. Siegel, and W. Weise, Phys. Rev. Lett. **60**, 2723 (1988).
[4] Edward J. Stephenson and Jeffrey A. Tostevin, in *Spin and Isospin in Nuclear Interactions*, edited by S.W. Wissink, C.D. Goodman, and G.E. Walker (Plenum, New York, 1991).
[5] CEBAF Report PR 89-001, spokespersons Barry M. Preedom and Gary S. Blanpied.
[6] S.A. Chin, Ann. Phys. (N.Y.) **108**, 301 (1977).
[7] A. Hosaka, Phys. Lett. B **244**, 363 (1990).
[8] Tetsuo Hatsuda and Su Houng Lee, Phys. Rev. C **46**,

R34 (1992).
[9] Yuji Koike, Michigan State University Report No. MSUCL-898; LANL bulletin board hep-ph-9307302, 1993.
[10] J.D. Walecka, Ann. Phys. (N.Y.) **83**, 491 (1974).
[11] B.D. Serot and J.D. Walecka, in *Advances in Nuclear Physics*, edited by J.W. Negele and E. Vogt (Plenum, New York, 1986), Vol. 16.
[12] C.J. Horowitz and B.D. Serot, Nucl. Phys. **A368**, 503 (1981).
[13] C.J. Horowitz and B.D. Serot, Phys. Lett. **140B**, 181 (1984).
[14] K. Lim and C.J. Horowitz, Nucl. Phys. **A501**, 729 (1988); K. Lim, Ph.D. thesis, Indiana University, 1988.
[15] J.A. McNeil, R.D. Amado, C.J. Horowitz, M. Oka, J.R. Shepard, and D.A. Sparrow, Phys. Rev. C **34**, 746 (1986).
[16] R.J. Furnstahl and B.D. Serot, Nucl. Phys. **A468**, 539 (1987).
[17] K. Wehrberger and F. Beck, Phys. Rev. C **37**, 1148 (1988).
[18] C.J. Horowitz and J. Piekarewicz, Phys. Rev. Lett. **62**, 391 (1989); Nucl. Phys. **A511**, 461 (1990).
[19] C.J. Horowitz and J. Piekarewicz, Phys. Rev. C **47**, 2924 (1993).
[20] C.M. Kormanyos, R.J. Peterson, J.R. Shepard, J.E. Wise, S. Bart, R.E. Chrien, L. Lee, B.L. Clausen, J. Piekarewicz, M.B. Barakat, R.A. Michael, and T. Kishimoto, Phys. Rev. Lett. **71**, 2571 (1993).
[21] A.L. Fetter and J.D. Walecka, *Quantum Theory of Many-Particle Systems* (McGraw-Hill, New York, 1971).
[22] J.D. Bjorken and S.D. Drell, *Relativistic Quantum Fields* (McGraw-Hill, New York, 1965).
[23] J. Piekarewicz, Nucl. Phys. **A511**, 487 (1990).
[24] R.J. Furnstahl and C.J. Horowitz, Nucl. Phys. **A485**, 632 (1988).
[25] R. Machleidt, K. Holinde, and Ch. Elster, Phys. Rep. **149**, 1 (1987); R. Machleidt, in *Proceedings of the Relativistic Dynamics and Quark-Nuclear Physics*, edited by M.B. Johnson and A. Picklesimer (Wiley, New York, 1986).