Energy-shell contributions of the three-particle–three-hole excitations

A. Mariano and F. Krmpotić
Departamento de Física, Facultad de Ciencias Exactas, Universidad Nacional de La Plata, C.C. 67, 1900 La Plata, Argentina

A. F. R. de Toledo Piza
Instituto de Física, Universidade de São Paulo, Brazil
(Received 27 April 1993)

The response functions for the extended second and third random phase approximation are compared. A second-order perturbation calculation shows that the first-order amplitude for the direct 3p3h (three-particle–three-hole) excitation from the ground state cancels with those that are engendered by the 1p1h-3p3h coupling. As a consequence nonvanishing 3p3h effects to the 1p1h response involve off energy shell renormalization only. On shell 3p3h processes are absent.

PACS number(s): 21.60.Jz, 21.60.Cs

Many efforts have been devoted during the last few years to developing generalized random phase approximations (RPA), which go beyond the standard one-particle–one-hole (1p1h) approach [1]. This has been accomplished by including additional correlation effects in both the ground state and the excited states [2–16]. The reasons for that were mainly (i) the problem of the missing strength in the Gamow-Teller (GT) resonances, induced by (p,n) reactions [17,18], and (ii) the issue of the missing charge and missing dip strength in quasielastic electron scattering [19]. In particular, the extended second RPA (ESRPA), which explicitly includes the 2p2h ground state correlations (GSC), was extensively used to describe the above-mentioned nuclear excitations [2,4–6,10–12,13,15]. Yet, it is self-evident that when the 2p2h admixtures are present in the ground state, the external excitation field can lead, not only to the 1p1h and 2p2h states in the final nucleus, but also to the 3p3h states. However, as the ESRPA does not involve the 3p3h propagator these excitations cannot appear within the response function as real on the energy-shell processes. Recently the 3p3h degrees of freedom were explicitly included within a Tamm-Dancoff approach (TDA), and their effects on the non-energy-weighted GT sum rule were discussed [14]. Also an extended third RPA (ETRPA) [20], which possesses as the TDA limit the formalism developed in Ref. [14], has been used to study the effects of 3p3h excitations on the static strength function for quasielastic electron scattering [16].

The purpose of this paper is to present some results for on the energy-shell 3p3h effects in the response function. This is done in the context of the full ETRPA approach which is therefore reviewed below. The nature of the resulting response function is then confronted to what one obtains using the ESRPA by performing a perturbative expansion of the responses in each case. The possibility of having a three nucleon ejection process is finally analyzed in this framework.

Let us start with the linear response to an external field $\hat{F}$ defined as

$$R(E) = -i \int_{-\infty}^{\infty} \langle\tilde{0}|T[\hat{F}^{H\dagger}(t)\hat{F}^{H}(0)]|\tilde{0}\rangle e^{iEt} dt \ , \qquad (1)$$

where $\hat{F}^H(t) \equiv e^{i\hat{H}t}\hat{F}e^{-i\hat{H}t}$, $\hat{H} = \hat{H}_0 + \hat{V}$, with $\hat{H}_0$ and $\hat{V}$ being, respectively, the Hartree-Fock (HF) mean field and the residual interaction. The spectral representation of the response function, in terms of a set $\{|\nu\rangle\}$ of eigenstates of the Hamiltonian $\hat{H}$, reads

$$R(E) = \sum_{\nu} \left[\frac{\langle\tilde{0}|\hat{F}|\nu\rangle\langle\nu|\hat{F}^{\dagger}|\tilde{0}\rangle}{E - E_\nu + i\eta} - \frac{\langle\tilde{0}|\hat{F}^{\dagger}|\nu\rangle\langle\nu|\hat{F}|\tilde{0}\rangle}{E + E_\nu - i\eta} \right] \ , \qquad (2)$$

where η is an infinitesimal positive number.

Within the equation of motion method [1], the set $\{|\nu\rangle\}$ is generated as

$$|\nu\rangle = \Omega^{\dagger}_{\nu}|\tilde{0}\rangle; \Omega^{\dagger}_{\nu} = \sum_i X^{\nu}_i C^{\dagger}_i - \sum_j Y^{\nu}_j C_j \ , \qquad (3)$$

and

$$\Omega_\nu|\tilde{0}\rangle = 0, \quad \text{for all } \nu \ . \qquad (4)$$

The operators $C^{\dagger}_i$ and C_i (with $C^{\dagger}_i \equiv a^{\dagger}_{p1} \cdots a^{\dagger}_{pi} a_{h1} \cdots a_{hi}$) create and annihilate i particle-hole pairs on the HF vacuum $|-\rangle \equiv |0\text{p}0\text{h}\rangle$, respectively.

The equation of motion for $\Omega^{\dagger}_{\nu}$

$$\langle\tilde{0}|[\Omega_\nu, [H, \Omega^{\dagger}_{\mu}]]|\tilde{0}\rangle = E_\nu \langle\tilde{0}|[\Omega_\nu, \Omega^{\dagger}_{\mu}]|\tilde{0}\rangle \delta_{\nu,\mu}, \qquad (5)$$

where E_ν stands for the excitation energy of the state $|\nu\rangle$, leads to the RPA-like eigenvalue problem

$$\mathcal{A}\mathcal{X}^{\nu} = E_\nu \mathcal{N}\mathcal{X}^{\nu} \ , \qquad (6)$$

with

$$\mathcal{A} = \begin{pmatrix} A & B \\ B^* & A^* \end{pmatrix} \ , \quad \mathcal{X}^{\nu} = \begin{pmatrix} X^{\nu} \\ Y^{\nu} \end{pmatrix} \ ,$$
$$\mathcal{N} = \begin{pmatrix} N & 0 \\ 0 & -N^* \end{pmatrix} \ . \qquad (7)$$

The submatrices A, B, and N given by

$$A_{i,j} = \langle\tilde{0}|[C_i,[H,C_j^\dagger]]|\tilde{0}\rangle,\quad B_{i,j} = \langle\tilde{0}|[C_i,[H,C_j]]|\tilde{0}\rangle,$$
$$N_{i,j} = \langle\tilde{0}|[C_i,C_j^\dagger]|\tilde{0}\rangle\ , \tag{8}$$

and using Eqs. (2)–(6) it is possible to write the response in representation independent form as

$$R(E) = \mathcal{F}^\dagger(E\mathcal{N} - \mathcal{A} + i\eta\mathcal{I})^{-1}\mathcal{F}\ , \tag{9}$$

where $\mathcal{F}$ is defined as

$$\mathcal{F} \equiv \begin{pmatrix} F^A \\ F^B \end{pmatrix}\ ,\quad \text{with}\ \begin{matrix} F_i^A = \langle\tilde{0}|[C_i,\hat{F}]|\tilde{0}\rangle\ , \\ F_i^B = F_i^{A*}(\hat{F}\to\hat{F}^\dagger)\ . \end{matrix} \tag{10}$$

After splitting the Hilbert space of $ipih$ states into a P space that includes only the 1p1h states and the Q space that spans on the rest of the states, the response function can be written as

$$R(E) = \hat{\mathcal{F}}_P^\dagger(E)\mathcal{G}_P(E)\tilde{\mathcal{F}}_P(E) + \mathcal{F}_Q^\dagger(E)\mathcal{F}_Q\ , \tag{11}$$

where

$$\mathcal{G}_P(E) = [E\mathcal{N}_P + i\eta\mathcal{I}_P - \mathcal{A}_P - (\mathcal{A}_{PQ} - \mathcal{N}_{PQ}E)\mathcal{G}_Q(E)(\mathcal{A}_{QP} - \mathcal{N}_{QP}E)]^{-1}\ , \tag{12}$$

with

$$\mathcal{G}_Q(E) = [E\mathcal{N}_Q + i\eta\mathcal{I}_Q - \mathcal{A}_Q]^{-1}\ , \tag{13}$$

and

$$\tilde{\mathcal{F}}_P(E) = \mathcal{F}_P - \mathcal{N}_{PQ}\mathcal{F}_Q + \mathcal{A}_{PQ}\mathcal{G}_Q(E)\mathcal{F}_Q\ . \tag{14}$$

In standard RPA the state $|\tilde{0}\rangle$ is approximated by the HF ground state and the Q space is absent, while the so-called extended RPA incorporates perturbative ground state 2p2h admixtures and a perturbatively suggested truncation of the dynamical matrices and excitation operator. It is obtained by the following.

(i) Evaluating the matrix elements (8) and (10) for [11]

$$|\tilde{0}\rangle = c_0|0\rangle + \sum_{2_0} c_{2_0}|2_0\rangle\ , \tag{15}$$

where

$$c_0 \cong 1 - \frac{1}{2}\sum_{2_0}|c_{2_0}|^2, c_{2_0} \cong -\frac{V_{2_00}}{E_{2_0}}\ , \tag{16}$$

$2_0 \equiv (p_1p_2h_1h_2)_0$ represents the 2p2h ground state admixtures, E_{2_0} the corresponding unperturbed energy, and $V_{2_00} \equiv \langle 2_0|V|0\rangle$.

(ii) Keeping terms up to second order in $\hat{V}$ for the forward sector within the P space, terms linear in $\hat{V}$ for the backward sector within the P space and for the coupling between the P and Q spaces, and only terms of zeroth order within the Q space. Under these conditions the norm matrix elements read [5]

$$N_{ij} = \delta_{ij} + \Delta N_{ij} \tag{17}$$

where $i \equiv ipih$ and the nonzero ΔN_{ij} are

$$\Delta N_{11'} = \sum_{2_0,2_0'} c_{2_0}^* c_{2_0'}\langle 2_0|\hat{D}_{11'}|2_0'\rangle, \Delta N_{13} = \sum_{2_0} c_{2_0}^*\langle 1;2_0|3\rangle, \tag{18}$$

where $\hat{D}_{11'} = [\hat{C}_1,\hat{C}_{1'}^\dagger] - \delta_{11'}$ and $\langle 1;2_0|3\rangle$ is the overlap between the 1p1h⊗(2p2h)$_0$ and 3p3h final state configurations. (Note that within the quasiboson approximation $\hat{D}_{11'} \equiv 0$.) The explicit result for the matrix element $\langle 2_0|\hat{D}_{11'}|2_0'\rangle$ is

$$\langle(p_1p_2h_1h_2)_0|\hat{D}_{ph,p'h'}|(p_1'p_2'h_1'h_2')_0\rangle = -[1 + P(h_1,h_2)P(h_1',h_2')] \times[\delta_{p,p'}\delta_{h_1,h'}P^-(h,h_2)P^-(p_1,p_2)\delta_{h_1',h}\delta_{h_2,h_2'}\delta_{p_2,p_2'}\delta_{p_1,p_1'}] + p \leftrightarrow h\ , \tag{19}$$

where $P^-(i,j) \equiv [1 - P(i,j)]$, while the operator $P(i,j)$ exchanges the arguments i and j.

The forward going energy matrix elements are evaluated in the same way and one gets

$$A_{ij} = \delta_{ij}E_j + V_{ij} + \Delta A_{ij}\ , \tag{20}$$

where $V_{ij} \equiv \langle i|\hat{V}|j\rangle$ and the nonzero matrix elements ΔA_{ij} are

$$\Delta A_{11'} = \sum_{2_0,2_0'}(E_1 - E_{2_0})c_{2_0}^* c_{2_0'}\langle 2_0|\hat{D}_{11'}|2_0'\rangle,$$
$$\Delta A_{13} = \Delta N_{13}E_3\ . \tag{21}$$

The one-body matrix elements are

$$F_i^A \begin{cases} f_1 + \sum_{1'}\Delta N_{11'}f_{1'} & \text{for } i = 1, \\ \sum_{2_0} c_{2_0}f_{i2_0} & \text{for } i > 1\ , \end{cases} \tag{22}$$

where

$$f_1 \equiv \langle 1|\hat{F}|0\rangle \text{ and } f_{i2_0} \equiv \langle i|\hat{F}|2_0\rangle\ . \tag{23}$$

Before proceeding it is convenient to introduce the unperturbed Green's function:

$$\mathcal{G}^0(E) \equiv \begin{pmatrix} G^0(E) & 0 \\ 0 & G^{0*}(-E) \end{pmatrix}\ , \tag{24}$$

where $G^0(E) \equiv [E^+ - A(\hat{H} = \hat{H}_0)]^{-1}$ (with $E^+ \equiv E + i\eta$) and rewrite the perturbed Green's function within the space P in the form

$$\mathcal{G}_P(E) = [(\mathcal{G}_P^0(E))^{-1} - \mathcal{K}_P(E)]^{-1} , \tag{25}$$

where

$$\mathcal{K}_P(E) \equiv \mathcal{K}_{11'}(E) = \begin{pmatrix} V_{11'} + \Sigma_{11'}(E) & B_{11'} \\ B^*_{11'} & V^*_{11'} + \Sigma^*_{11'}(-E) \end{pmatrix} , \tag{26}$$

with

$$\Sigma_{11'}(E) = \Delta\Sigma^{(2)}_{11'}(E) + \Delta\Sigma^{(3)}_{11'}(E) + \sum_{i=2,3} V_{1i} G^0_{ii}(E) V_{i1'} , \tag{27}$$

and

$$\Delta\Sigma^{(2)}_{11'}(E) = \Delta A_{11'} - \Delta N_{11'} E ,$$
$$\Delta\Sigma^{(3)}_{11'}(E) = -[2V_{13} - \Delta N_{13}(G^0_{33}(E))^{-1}]\Delta N_{31'} . \tag{28}$$

In the above equations $V_{ij'}$ stands for the matrix representation of the residual interaction within the $ipih \otimes jpjh$ subspace.

The response function now reads

$$R(E) = \hat{\mathcal{F}}_{1'}(E) \mathcal{G}_{11'}(E) \tilde{\mathcal{F}}_{1'}(E) + \sum_{i=2,3} \mathcal{F}^\dagger_i \mathcal{G}^0_{ii}(E) \mathcal{F}_i , \tag{29}$$

where

$$\tilde{\mathcal{F}}_1(E) \equiv \begin{pmatrix} \tilde{F}^A_1(E) \\ \tilde{F}^B_1(E) \end{pmatrix} , \quad \begin{array}{c} \tilde{F}^A_1(E) = f_1 + \Delta\tilde{F}_1(E), \\ \text{with } \Delta\tilde{F}_1(E) = \Delta F^{(2)}_1 + \Delta F^{(3)}_1 + \sum_{i=2,3} V_{1i} G^0_{ii}(E) F_i, \\ \Delta F^{(2)}_1 = \Delta N_{11'} f_{1'} , \Delta F^{(3)}_1 = -\Delta N_{13} F_3 . \end{array} \tag{30}$$

From the expressions for $\Delta A_{11'}$ and $\Delta N_{11'}$, given by Eqs. (18) and (21), respectively, the matrix elements $\Delta\Sigma^{(2)}_{11'}(E)$ and $\Delta\tilde{F}^{(2)}_1$ can be expressed as

$$\Delta\Sigma^{(2)}_{11'}(E) = -\sum_{2_0,2'_0} c^*_{2_0} c_{2'_0} \langle 2_0 | \hat{D}_{11'} | 2'_0 \rangle (E - E_1 + E_{2'_0}) , \tag{31}$$

$$\Delta\tilde{F}^{(2)}_1 = \sum_{2_0,2'_0} c^*_{2_0} c_{2'_0} \langle 2_0 | \hat{D}_{11'} | 2'_0 \rangle f_{1'} . \tag{32}$$

Moreover, from the relationships

$$V_{13} = -\sum_{2_0} c^*_{2_0} E_{2_0} \langle 1; 2_0 | 3 \rangle , f_{3,2_0} = \sum_1 \langle 3 | 1; 2_0 \rangle f_1 , \tag{33}$$

one obtains

$$\Delta\Sigma^{(3)}_{11'} = \sum_{2_0,2'_0} c^*_{2_0} c_{2'_0} \langle 1; 2_0 | 1'; 2'_0 \rangle (E - E_1 + E_{2'_0}) , \tag{34}$$

$$\Delta\tilde{F}^{(3)}_1 = -\sum_{2_0,2'_0} c^*_{2_0} c_{2'_0} \langle 1; 2_0 | 1'; 2'_0 \rangle f_{1'} . \tag{35}$$

We can note here that

$$\langle 1; 2_0 | 1'; 2'_0 \rangle = \langle 2_0 | (\hat{D}_{11'} + \hat{d}_{11'}) | 2'_0 \rangle,$$

$$\text{with } \hat{d}_{11'} \equiv \delta_{11'} + C^\dagger_{1'} C_1 , \tag{36}$$

and thus in summary we get the following.

(i) In the ESRPA (where the Q space includes only the 2p2h excitations)

$$\Sigma_{11'}(E) = -\sum_{2_0,2'_0} c^*_{2_0} c_{2'_0} \langle 2_0 | \hat{D}_{11'} | 2'_0 \rangle (E - E_1 + E_{2'_0}) + \sum_2 \frac{V_{12} V_{21'}}{E^+ - E_2} , \tag{37}$$

$$\tilde{\mathcal{F}}_1(E) = f_1 + \sum_{2_0,2'_0;1'} c^*_{2_0} c_{2'_0} \langle 2_0 | \hat{D}_{11'} | 2'_0 \rangle f_{1'} + \sum_{2,2_0} \frac{V_{12} f_{22_0} c_{2_0}}{E^+ - E_2} ; \tag{38}$$

(ii) in the ETRPA (where the Q space includes both the 2p2h and 3p3h excitations)

$$\Sigma_{11'}(E) = \sum_{2_0,2'_0} c^*_{2_0} c_{2'_0} \langle 2_0 | \hat{d}_{11'} | 2'_0 \rangle (E - E_1 + E_{2'_0}) + \sum_{i=2,3} \frac{V_{1i} V_{i1'}}{E^+ - E_i} , \tag{39}$$

$$\tilde{\mathcal{F}}_1(E) = f_1 - \sum_{2_0,2'_0} c^*_{2_0} c_{2'_0} \langle 2_0 | \hat{d}_{11'} | 2'_0 \rangle f_{1'} + \sum_{i=2,3;2_0} \frac{V_{1i} f_{i2_0} c_{2_0}}{E^+ - E_i} . \tag{40}$$

The results (37) and (38) are in essence those obtained previously by Arima and collaborators [5,11] and by the Jülich group [6,12]. On the other hand, when terms containing the matrix elements $\langle 2_0 | \hat{d}_{11'} | 2'_0 \rangle$ are neglected in Eqs. (39) and (40), one finds the results derived in our previous works [16].[1]

In order to elucidate some of the content of these equations we turn next to a perturbative expansion of the response function and examine the leading corrections to the unperturbed 1p1h response $R^0(E) = \sum_1 |f_1|^2/(E^+ - E_1)$. To achieve maximum simplicity we first omit the residual interaction within the 1p1h sector and backward contributions, so that to second order the Bethe-Salpeter

[1]These terms give rise to disconnected graphs, which are nonphysical, as well as to double connected graphs represented in Figs. 1(d) and 1(e), respectively. As seen from relations (43) and (47) below, they do not contribute to the response function.

equation Eq. (25) reads

$$G_{11'}(E) \cong G^0_{11}(E) + G^0_{11}(E)\Sigma_{11'}(E)G^0_{1'1'}(E) \ , \quad (41)$$

which substituted into Eq. (11) leads to the desired approximation for the response function. Within the ESRPA one gets

$$R(E) \cong R^0(E) + \sum_{2,2_0,2'_0} c^*_{2_0}\frac{f^*_{22_0}f_{22'_0}}{E^+ - E_2}c_{2'_0} + 2\sum_{1,2,2_0}\frac{\mathcal{R}(f^*_1 f_{22_0}c_{2_0})}{E^+ - E_1}\frac{V_{12}}{E^+ - E_2}$$
$$+\sum_{1,1'}\frac{f^*_1}{E^+ - E_1}\left[\sum_{2_0,2'_0} c^*_{2_0}c_{2'_0}\langle 2_0|\hat{D}_{11'}|2'_0\rangle(E - E_1 - E_{2_0}) + \sum_2\frac{V_{12}V_{21'}}{E^+ - E_2}\right]\frac{f_{1'}}{E^+ - E_{1'}} \ , \quad (42)$$

and in the ETRPA

$$R(E) \cong R^0(E) + \sum_{i=2,3;2_0,2'_0} c^*_{2_0}\frac{f^*_{i2_0}f_{i2'_0}}{E^+ - E_i}c_{2'_0} + 2\sum_{i=2,3;1,2_0}\frac{\mathcal{R}(f^*_1 f_{i2_0}c_{2_0})}{E^+ - E_1}\frac{V_{1i}}{E^+ - E_i}$$
$$-\sum_{1,1'}\frac{f^*_1}{E^+ - E_1}\left[\sum_{2_0,2'_0} c^*_{2_0}c_{2'_0}\langle 2_0|\hat{d}_{11'}|2'_0\rangle(E - E_1 - E_{2_0}) - \sum_{i=2,3}\frac{V_{13}V_{31'}}{E^+ - E_i}\right]\frac{f_{i'}}{E^+ - E_{1'}} \ . \quad (43)$$

Now the two expressions (42) and (43) can be shown to be equivalent. This results in fact from explicitly performing the sums over 3p3h states in Eq. (43). To do that one first rewrites these sums making use of relations (33) and (36) as

$$\sum_{3,2_0,2'_0} c^*_{2_0}\frac{f^*_{32_0}f_{32'_0}}{E^+ - E_3}c_{2'_0} = \sum_{3,2_0,2'_0} c^*_{2_0}f^*_1\frac{\langle 2_0|(\hat{D}_{11'} + \hat{d}_{11'})|2'_0\rangle}{E^+ - E_1 - E_{2_0}}f_{1'}c_{2'_0} \ , \quad (44)$$

$$2\sum_{1,3,2_0}\frac{\mathcal{R}(f^*_1 f_{32_0}c_{2_0})}{E^+ - E_1}\frac{V_{13}}{E^+ - E_3} = -2\sum_{1,2_0,2'_0} c^*_{2_0}f^*_1\frac{E_{2_0}\langle 2_0|(\hat{D}_{11'} + \hat{d}_{11'})|2'_0\rangle}{(E^+ - E_1)(E^+ - E_1 - E_{2_0})}f_{1'}c_{2'_0} \ , \quad (45)$$

and

$$\sum_{1,1',3}\frac{f^*_1}{E^+ - E_1}\frac{V_{13}V_{31'}}{E^+ - E_3}\frac{f_{1'}}{E^+ - E_{1'}} = \sum_{1,1',2_0,2'_0}\frac{f^*_1 c^*_{2_0}E_{2_0}\langle 2_0|(\hat{D}_{11'} + \hat{d}_{11'})|2'_0\rangle E_{2'_0}f_{1'}c_{2'_0}}{(E^+ - E_1)(E^+ - E_1 - E_{2_0})(E^+ - E_{1'})} \ . \quad (46)$$

The result of performing the sum is

$$\sum_{1,1',2_0,2'_0}\frac{f^*_1}{(E^+ - E_1)}c^*_{2_0}c_{2'_0}\langle 2_0|(\hat{d}_{11'} + \hat{D}_{11'})|2'_0\rangle(E - E_1 - E_{2_0})\frac{f_{1'}}{(E^+ - E_{1'})} \ , \quad (47)$$

which substituted into Eq. (43) gives the expression (42) also for the ETRPA response.

The cancellation among the 3p3h on the energy-shell contributions can be exhibited also making use of the Rayleigh-Schrödinger perturbation expansion, i.e.,

$$|\tilde{i}\rangle = |i\rangle + |i\rangle^{(1)} + \cdots \quad \text{and } \tilde{E}_i = E_i + E_i^{(1)} + \cdots ,$$
$$i = ipih \ , \quad (48)$$

where the perturbed wave functions and energies are indicated by the symbol $\sim$ and the superscript points the order of the correction introduced by the residual interaction $\hat{V}$ on the unperturbed quantities $|i\rangle$ and E_i. The amplitude for the $\hat{F}$ excitation from the correlated ground state to the perturbed 3p3h states reads

$$\langle\tilde{3}|\hat{F}|\tilde{0}\rangle = \frac{\langle\tilde{3}|[\hat{H},\hat{F}]|\tilde{0}\rangle}{\tilde{E}_3 - \tilde{E}_0} = \frac{\langle 3|[\hat{H},\hat{F}]|0\rangle}{E_3 - E_0} + \mathcal{O}(\hat{V}^2) \ , \quad (49)$$

with

$$\langle 3|[\hat{H},\hat{F}]|0\rangle = \sum_1\langle 3|\hat{V}|1\rangle\langle 1|\hat{F}|0\rangle - \sum_{2_0}\langle 3|\hat{F}|2_0\rangle\langle 2_0|\hat{V}|0\rangle$$
$$\equiv 0 \ , \quad (50)$$

where the last equivalence is a direct consequence of the relations (33), i.e.,[2]

$$\sum_{1} V_{31} f_1 = -\sum_{1,2_0} c_{2_0} E_{2_0} \langle 1; 2_0 | 3 \rangle f_1 = \sum_{2_0} f_{3,2_0} V_{2_0 0} \ . \tag{51}$$

Thus we see once more that, up to the second order in $\hat{V}$, the 3p3h final states do not contribute to the response function and that $|\langle \tilde{3} | \hat{F} | 0 \rangle|^2 \cong \mathcal{O}(\hat{V}^4)$. The Goldstone diagrams for the fourth order 3p3h on the mass-shell contributions to the response function are shown in Figs. 1(e) and 1(f).

At first glance it might look as if the connected Goldstone diagrams associated with the terms (44), (45), and (46) of the ETRPA response [illustrated in Figs. 1(a), 1(b), and 1(c), respectively] should give rise to on the mass-shell 3p3h contributions, through the imaginary part of the propagator $(E^+ - E_3)^{-1}$. However, Eq. (47) shows that these contributions in fact cancel out so that the 3p3h sector only affects the 1p1h excitations by coupling them with the virtual intermediate states $|1; 2_0\rangle$. Thus in spite of including the 3p3h propagator in the Green's function, three nucleon ejection does not occur in the leading order processes. The above-mentioned diagrams also explain the physical meaning of the fourth term in the expression (42). The cancellation of on shell 3p3h contributions results from the destructive interference between amplitudes involving creation of the 3p3h state from a ground state correlation and from V_{31} coupling, respectively. A similar calculation in which the backward part of Eq. (25) and/or the residual interaction within 1p1h space are kept up to the relevant order leads again to the same result. It is worth stressing that this does not depend on the form of the two-body force used as residual interaction or on the size of single particle space.

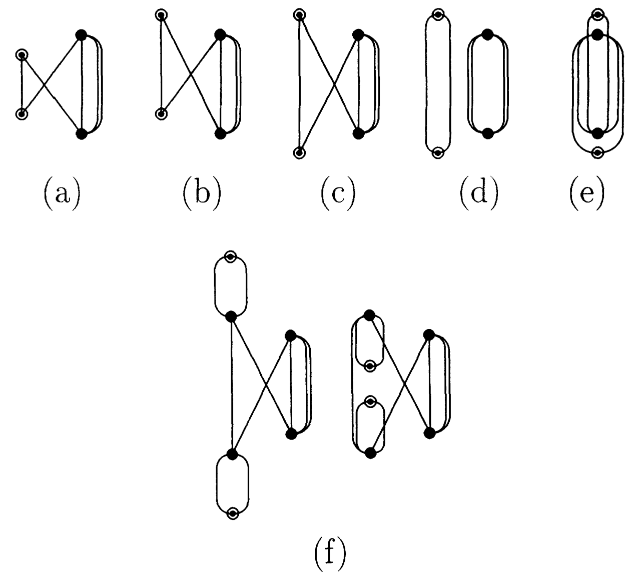

FIG. 1. Graphical representation of the second- and fourth-order contribution to the response function. The dotted circles (⊙) denote the one-body vertices and the filled ones (•) indicate the two-body matrix elements. The diagrams (a), (b), and (c) correspond, respectively, to the terms given by Eqs. (44), (45), and (46). Second-order unlinked and double-linked graphs analogous to the diagram (c) are shown in (d) and (e), respectively. The last ones, although contained in Eqs. (39) and (40), do not contribute to the response function. Finally, (f) illustrates the fourth order on the energy-shell 3p3h processes.

F.K. acknowledges the financial support of the CONICET from Argentina.

[2]Note that $\hat{H}_0$ does not contribute since $\langle 3 | [\hat{H}_0, \hat{F}] | 0 \rangle = 0$.

[1] D. J. Rowe, *Nuclear Collective Motion* (Methuen, London, 1970).
[2] G. F. Bertsch and I. Hamamoto, Phys. Rev. C **26**, 1323 (1982).
[3] C. Yannouleas, M. D. Worzecka, and J. J. Griffin, Nucl. Phys. **A397**, 239 (1983).
[4] W. M. Alberico, M. Ericson, and A. Molinari, Ann. Phys. (N.Y.) **154**, 356 (1984).
[5] K. Takayanagi, K. Shimizu, and A. Arima, Nucl. Phys. **A444**, 436 (1985).
[6] S. Drożdż, V. Klemt, J. Speth, and J. Wambach, Phys. Lett. **166B**, 253 (1986).
[7] M. H. Macfarlance, Phys. Lett. B **182**, 265 (1986).
[8] J. Hirsch, A. Mariano, M. Faig, and F. Krmpotić, Phys. Lett. B **210**, 55 (1988).
[9] S. Adachi and E. Lipparini, Nucl. Phys. **A489**, 445 (1988).
[10] S. Nishizaki, S. Drożdż, J. Wambach, and J. Speth, Phys. Lett. B **215**, 231 (1988).
[11] K. Takayanagi, K. Shimizu, and A. Arima, Nucl. Phys. **A477**, 205 (1988); **A481**, 313 (1988).
[12] S. Drożdż, S. Nishizaki, J. Speth, and J. Wambach, Phys. Rep. **197**, 1 (1990).
[13] W. M. Alberico, A. De Pace, A. Drago, and A. Molinari, Universitá di Torino Report No. DFTT 7/90, 1990.
[14] A. Mariano, J. Hirsch, and F. Krmpotić, Nucl. Phys. **A518**, 523 (1990).
[15] K. Takayanagi, Phys. Lett. B **230**, 11 (1989); Nucl. Phys. **A510**, 162 (1990); **A516**, 276 (1990); **A522**, 494 (1991).
[16] A. Mariano, E. Bauer, F. Krmpotić, and A. F. R. de Toledo Piza, Phys. Lett. B **268**, 332 (1991).
[17] C. Gardee, J. Rapaport, T. N. Taddeucci, C. D. Goodman, C. Foster, D. E. Bainum, C. A. Goulding, M. G. Greenfield, D. J. Horen, and E. Sugarbaker, Nucl. Phys. **A369**, 258 (1981).
[18] J. Rapaport, in *The Interaction Between Medium Energy Nucleons in Nuclei*, Proceedings, edited by H. O. Meyer, American Institute of Physics, AIP Conf. Proc. No. 97 (AIP, New York, 1982), p. 365.
[19] P. Barreau, M. Bernheim, J. Duclos, J. M. Finn, Z.

Meziani, J. Morgenstern, J. Mougey, D. Royer, B. Saghai, D. Tarnowski, S. Turck-Chieze, M. Brussel, G. P. Capitani, E. De Sanctis, S. Frullani, F. Garibaldi, D. B. Isabelle, E. Jaus, I. Sick, and P. D. Zimmerman, Nucl. Phys. **A402**, 515 (1983).

[20] A. Mariano, *Proceedings of the XV Workshop on Nuclear Physics, Buenos Aires, Argentina* (World Scientific, Singapore, 1993), p. 216.