Photopion production in ^{3}H and ^{3}He

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The photopion production of $^3He(\gamma, \pi^+)^3H$ has been calculated by the distorted-wave impulse approximation in order to include the final state interaction (FSI) between π^+ and ^{3}H. The optical potentials used to describe the FSI are the Stricker, McManus, and Carr potential and the Kim potential. The inclusion of the FSI improves significantly the theoretical estimations compared with the results of plane-wave impulse approximation. In case of low momentum transfer the contribution of $E_{1^+}(3/2)$ amplitude is shown to play an additional role in obtaining an agreement with the experimental results.

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I. INTRODUCTION

The elementary photopion production amplitude has been well understood by the Chew-Goldberger-Low-Nambu (CGLN) method [1]. However, it has an ambiguity in the transformation of multipole amplitudes when applied to nuclear photopion production. CGLN's multipole amplitudes have to be transformed to the off-shell amplitudes of the π-A center of mass (c.m.) system from the on-shell amplitudes of the π-N c.m. system where CGLN multipole amplitudes have been defined.

On the contrary, the Blomqvist-Laget (BL) amplitudes [2,4] based on the Feymann diagrams of the Born terms with pseudovector coupling and the s-channel propagation of Δ resonance particle can be extended to off-shell external lines. The Born terms correspond to the nonresonant reaction and mainly contribute to the spin-flip reaction due to the predominance of the Kroll-Rudermann terms. The s-channel propagation of Δ particle corresponds to the resonant reaction and mainly contributes to the non-spin-flip reaction. Although the off-shell extension in the BL amplitudes may not be unique due to their nonrelativistic reduction form, they can be used efficiently as an elementary amplitude for the photopion production in nucleus because it is described in terms of an interacting nucleon momentum in nucleus.

It has been pointed out [19] that the BL I amplitude, due to the violation of the unitarity [20], is unsatisfactory in reproducing exactly the $E_{1^+}(3/2)$ and the $M_{1^+}(3/2)$ multipole amplitudes occurring from the Δ propagation. To satisfy the unitarity, BL I was modified into BL II amplitude [4] by considering the background resonance, which is the contribution of the nonresonance reaction to the $E_{1^+}(3/2)$ and the $M_{1^+}(3/2)$ multipole amplitudes. BL II amplitude obtained in this way satisfies the Watson theorem [3] derived from the unitarity and reproduces [4] the $E_{1^+}(3/2)$ and the $M_{1^+}(3/2)$ multiple amplitudes.

The $E_{1^+}(3/2)$ multipole amplitude has been usually neglected in nuclear photopion reaction, apart from the recent work of Kamalov *et al.* [37] on the polarization observables in $^3He(\gamma, \pi^+)^3H$ reaction, because it is small compared to the $M_{1^+}(3/2)$ multipole amplitude. But in case of low-angle region of outgoing pion the inclusion of the $E_{1^+}(3/2)$ amplitude improves [4] a theoretical estimation of the photopion production in nucleon level. It means that the contribution of $E_{1^+}(3/2)$ amplitude might also influence the nuclear photopion production in lower momentum transfer region.

In nuclear photopion production [5], by the selection rules from nuclear structure, there exist three types of reaction. One is the reaction where Δ propagation term is large compared with the Born terms as in the neutral photopion production. In this case the Δ-h approach [6] has been very successful. The other is the reaction where the Born terms are large compared to the Δ propagation contribution. The reactions of $^{10}B(\gamma, \pi^+)^{10}Be$ and $^6Li(\gamma, \pi^+)^6He$ are typical of this reaction because they are the spin-flip transitions of $\Delta J^\pi = 1^+$ and $\Delta J^\pi = 3^+$, respectively. Another type is the reaction of $^3He(\gamma, \pi^+)^3H$. This reaction has the spin-flip ($\Delta J^\pi = 1^+$) and the non-spin-flip ($\Delta J^\pi = 0^+$) transitions and as a result has the nearly equal contribution from both the Δ propagation term and the Born terms as shown later on.

It is possible, by studying the nuclear structure through the electron scattering, to choose artificially the momentum transfer region where the Δ resonance reaction is dominant. For instance one can select the momentum transfer region of $Q^2 \simeq 1.0\,\mathrm{fm}^{-2}$ in the reaction of $^{13}C(\gamma, \pi^-)^{13}N_{g.s.}$. In this momentum transfer the spin-flip reaction is suppressed, as seen in the $M1$ form factor of ^{13}C, and as a consequence the Δ-h approach works well for this reaction. But in case of $^3He(\gamma, \pi^+)^3H$ at the momentum transfer of $Q^2 = 0.48$–4.9 fm^{-2} (the region where there exists experimental results) initiated by

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unpolarized photon beam it is difficult to choose the reaction where the Δ propagation term is dominant.

The above situations occurring in the reaction $^3\mathrm{He}(\gamma,\pi^+)^3\mathrm{H}$ make it difficult to apply the Δ-h approach in this reaction, even if we can make use of the wave function including the Δ component. Moreover, in the Δ-h approach the nonresonant part has the uncertainty of transformation from the on-shell to off-shell amplitude as in the CGLN approach to nuclear photopion reaction. By these reasons we make a distorted wave impulse approximation (DWIA) calculation based on the BL amplitudes to describe this $^3\mathrm{He}(\gamma,\pi^+)^3\mathrm{H}$ reaction. The Δ medium effect in pion multiple scattering is included implicitly by the phenomenological optical potential. The DWIA calculation of the $^3\mathrm{He}(\gamma,\pi^+)^3\mathrm{H}$ reaction would give information about the Δ dynamics in the three-body nucleus.

Moreover, apart from the pion threshold region, the plane-wave impulse approximation (PWIA) calculations of $^3\mathrm{He}(\gamma,\pi^+)^3\mathrm{H}$ have shown a big disagreement with the experimental results [7–9] in the energy region above the Δ resonance peak. Many attempts to improve such disagreements have been done by the consideration of the nucleon Fermi motion [11], the first-order rescattering [10], or the phenomenological three-body wave functions [12]. Any calculations did not succeed in solving the disagreements up to 1990. In this year we showed [31,33] that the disagreement was improved by the inclusion of the FSI between π^+ and $^3\mathrm{H}$. To include the FSI one has to microscopically consider the complete pion multiple scattering with the inclusion of Δ formation in nucleus or macroscopically use the optical potential, which should reproduce the pion elastic scattering. We calculated the reaction by the DWIA where the FSI is taken into account in terms of an optical potential for π^+-$^3\mathrm{H}$.

The validity of optical potential approach to deal with the FSI in this reaction was also confirmed by S. S. Kamalov *et al.* in 1991 [34]. They used a first-order optical potential, but discarded a second-order potential and pion absorptions. Instead of neglecting these parts, they included single charge exchange (SCE) additionally to their calculations.

The optical potential approach would cause a double counting coming from pion recattering with the same nucleon. The emitted pion from a nucleon by gamma reaction would rescatter with the same nucleon (RSS) or with other nucleons (RSD) in the first order of multiple scattering and then collide with another nucleon successively (all of these amplitudes are included in the optical potential of π^+-$^3\mathrm{H}$ and cannot be included in the optical potential of π^+-$^2\mathrm{H}$ because of the higher-order multiple-scatteing amplitudes). But if one uses an effective operator for the elementary photopion production, the RSS contribution is already included in it. One must remedy a problem of this double counting of the RSS contribution in DWIA calculation. This RSS contribution would be important in low-energy region because the pion wavelength is large compared to the scatterer and propagates with a wide angle just as a spherical wave in nucleus. But in higher energy region, RSS probability is suppressed compared to the RSD probability because the pion wavelength is shorter and scatters with a preferred direction [25], and a pion emitted by gamma reaction has a suppressed forward propagation by the $(\mathbf{k}\times\mathbf{q})$ terms peculiar in photopion reaction [25].

Two methods could be suggested in principle. One way is to separate the RSS contribution from the pion optical potential. But it is not promising because the pion optical potential is based on the π-N amplitude, which needs to know the off-shell π-N amplitude to separate the RSS contribution. Its behavior is not known exactly, i.e., depends on the models [36].

The second way to separate the RSS contribution is to do with the γ-N amplitude. This is practically possible, and many authors [30] have done it especially for the π^0-N amplitude related with low energy theorem (LET). But this also has the off-shell amplitude of π-N and depends on the dynamics of the pion in off shell. However, the phenomenological separable π-N model by Nozawa *et al.* [30] shows the following facts.

In the case of charged photopion reaction at pion threshold the RSS contributions are small (about maximally 10% of the main contribution) compared with the usual Born terms and, moreover, they cancel each other. In the neutral case each RSS contribution becomes important because there is no seagull term, which is the main term in the charged reaction. It means that in the charged photopion reaction at low energy region RSS contributions are small enough to be neglected. Combined with the fact, as explained above, that in the higher energy region RSS contributions are small compared with the RSD contributions, and the double counting of RSS contributions would not cause any serious problems even if one uses the π-$^3\mathrm{H}$ optical potential in the DWIA calculation of the $^3\mathrm{He}(\gamma,\pi^+)^3\mathrm{H}$ reaction.

Moreover, the previous calculation $^3\mathrm{He}(\gamma,\pi^+)^3\mathrm{H}$ supports this fact. For example, the Goulard calculation [16] of $^3\mathrm{He}(\gamma,\pi^+)^3\mathrm{H}$ around pion threshold showed that the rescattering effect including RSS and RSD is small (see Table 4). But the results of Vergados [35] in $^4\mathrm{H}(\gamma,\pi^0)^4\mathrm{He}$ at pion threshold and Nozawa's in $N(\gamma,\pi^0)N$ reaction showed that these RSS and RSD contributions are important in neutral photopion reaction. It means that the double counting from RSS should be deducted in the neutral photonuclear reaction or the SCE mechanism including the (γ,π^0) reaction. However, in this article the SCE mechanism is considered implicitly through the absorption parts in the optical potential.

In this paper not only the effect from the FSI is taken into account, but the contributions from the $E_{1^+}(3/2)$ amplitude is also included. The $E_{1^+}(3/2)$ amplitude has been neglected in all previous calculations of nuclear photopion reaction. The further improved theoretical estimation is obtained in the low-momentum transfer region by means of the inclusion of $E_{1^+}(3/2)$ amplitude.

We exploit two kinds of optical potentials to investigate the dependece of optical potentials: One is the potential proposed by Stricker, McManus, and Carr (SMC) [13], which is known to be useful to describe the pion interaction with p-shell nuclei. The other is Kim's potential [21] that improves the SMC potential for pion elastic scattering of π-$^3\mathrm{He}$ in the region of $T_\pi^{\mathrm{lab}} = 180$–$240$ MeV.

In Sec. II the detailed formalism necessary for this reaction is shown with a brief introduction of the optical potentials used here. In Sec. III the results of PWIA and DWIA calculations are shown with discussions about the effects from the FSI and the $E_{1^+}(3/2)$ amplitude. Finally, a conclusion is presented in Sec. VI.

II. FORMALISM

A. Elementary photopion production amplitude

The BL I operator [2] used here as the elementary amplitude of $N(\gamma,\pi^{0,\pm})N'$ is expressed as

$$H_{\rm BL} = (L + L_\Delta) + i\boldsymbol{\sigma}\cdot(\mathbf{K}+\mathbf{K}_\Delta). \tag{1}$$

Here $\boldsymbol{\sigma}$ is a spin operator for nucleon. The nonresonance reaction part $L + i\boldsymbol{\sigma}\cdot\mathbf{K}$ is the contribution from the pseudovector Born terms (s and u channel of nucleon pole, the pion pole, and the $\boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}$ terms). The detail form of L and $\mathbf{K}$ is given by

$$L = \frac{ge}{2m}\left[\frac{\tau_c\mu_N}{2E_a(p_a^0-E_a)} + \frac{\mu_N\tau_c}{2E_b(p_b^0-E_b)}\right]\mathbf{q}\cdot(\mathbf{k}\times\boldsymbol{\epsilon}), \tag{2}$$

$$\begin{aligned}\mathbf{K} = {} & \frac{ge}{2m}\left[c\tau_c + \frac{mE_\pi\tau_cQ_N}{2E_a(p_a^0+E_a)} + \frac{mE_\pi Q_N\tau_c}{2E_b(p_b^0+E_b)}\right]\cdot\boldsymbol{\epsilon} \\ & + \left[\frac{\tau_c\mu_N}{2E_a(p_a^0-E_a)} - \frac{\mu_N\tau_c}{2E_b(p_b^0-E_b)}\right](\mathbf{k}\boldsymbol{\epsilon}\cdot\mathbf{q} - \boldsymbol{\epsilon}\mathbf{k}\cdot\mathbf{q}) \\ & + \left[-c\tau_c\frac{(\mathbf{k}-\mathbf{q})\boldsymbol{\epsilon}\cdot\mathbf{q}}{\mathbf{k}\cdot\mathbf{q}-k^0E_\pi} - \frac{\mathbf{q}\boldsymbol{\epsilon}\cdot\mathbf{p}_i\tau_cQ_N}{2E_a(p_a^0-E_a)}\right. \\ & \left. - \frac{\mathbf{q}\boldsymbol{\epsilon}\cdot\mathbf{p}_f Q_N\tau_c}{2E_b(p_b^0+E_b)}\right],\end{aligned} \tag{3}$$

where

$$\mu_N = \mu_p\left(\frac{1+\tau_3}{2}\right) + \mu_n\left(\frac{1-\tau_3}{2}\right)$$

with

$$\begin{aligned}&\mu_p = 1+\kappa_p,\quad \mu_n=\kappa_n,\quad \kappa_p=1.78,\quad \kappa_n=-1.91,\\ &Q_N = \frac{1+\tau_3}{2},\quad \tau_c=\tau_{\pm1,0}\\ &(\tau_+ \text{ for } \pi^-,\quad \tau_- \text{ for } \pi^+, \text{ and } \tau_0 \text{ for } \pi_0),\end{aligned} \tag{4}$$

with

$$\tau_0=\tau_3,\quad \tau_{\pm1}=\mp\frac{\tau_1\pm i\tau_2}{\sqrt{2}},$$
$$\frac{g^2}{4\pi}=14.5,\quad \frac{e^2}{4\pi}=\alpha=\frac{1}{137}.$$

By the predominance of the $\boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}$ term and the pion pole term in Eq. (3) this nonresonance reaction part is mainly spin-flip reaction. On the other hand, the resonance reaction part from the Δ particle propagation is mainly a non-spin-flip reaction because $\mathbf{K}_\Delta$ is smaller than L_Δ as follows:

$$L_\Delta = -\frac{2}{3}\frac{C_\pi C_\gamma G_1G_3}{p_\Delta^2-m_\Delta^2+i\Gamma_\Delta m_\Delta}\left[\frac{m_\Delta-m}{m}\mathbf{q}\cdot(\mathbf{p}_i\times\boldsymbol{\epsilon}) - \mathbf{q}\cdot(\mathbf{k}\times\boldsymbol{\epsilon})\right]\tau_c \tag{5}$$

$$\mathbf{K}_\Delta = -\frac{1}{3}\frac{C_\pi C_\gamma G_1G_3}{p_\Delta^2-m_\Delta^2+i\Gamma_\Delta m_\Delta}\left\{\left[(-1+\lambda)\mathbf{k}\cdot\mathbf{q} + \frac{m_\Delta-m}{m}\mathbf{q}\cdot\mathbf{p}_i\right]\boldsymbol{\epsilon} + (\mathbf{q}\cdot\boldsymbol{\epsilon})\left[\mathbf{k}(1+\lambda\sin\theta) - \frac{m_\Delta-m}{m}\mathbf{p}_i\right]\right\}\tau_c. \tag{6}$$

Here $\mathbf{k}$ is the initial photon momentum, and $\mathbf{q}$ is the outgoing pion momentum. The momenta of the initial and the final nucleon in nuclei are denoted, respectively, as $\mathbf{p}_i$ and $\mathbf{p}_f$ and

$$E_a = (\mathbf{p}_a^2+m^2)^{1/2} \neq p_a^0,$$

$$E_b = (\mathbf{p}_b^2+m^2)^{1/2} \neq p_b^0,$$

and

$$E_\pi = (\mathbf{q}^2+m_\pi^2)^{1/2}.$$

$\boldsymbol{\epsilon}$ is the polarization vector of incident photon and m is the nucleon mass and m_Δ is the mass of the Δ particle.

The contribution from the $E_{1^+}(3.2)$ multipole amplitude is included as the λ and $\lambda\sin\theta$ in Eq. (6) with the definition of $\lambda = 3G_Em_\Delta/m$ and the scattering angle θ.

Here

$$G_E = e^{i\phi E}\frac{2p_\Delta^0 m_\Delta \alpha}{(3m_\Delta + m)(m_\Delta + m)}$$

with the phenomenological phase factor ϕ_E and $\alpha = 0.8$ given by the BL II amplitude [4]. The Δ width Γ_Δ, coupling functions G_1 and G_3, which correspond to, respectively, $\gamma N\Delta$ and $\pi N\Delta$ couplings, are as follows:

$$G_1 = g_1\left(\frac{m_\Delta - m}{m_\pi}\right)\left(\frac{4\pi}{137}\right)^{1/2},$$
$$G_3 = \left(\frac{2.13}{m_\pi}\right)\left[\frac{1+(R|\mathbf{q}_\Delta|)^2}{1+(R|\mathbf{q}|)^2}\right]^{1/2} \text{MeV}^{-1}, \qquad (7)$$
$$\Gamma_\Delta = \Gamma(\Delta)\left(\frac{|\mathbf{q}|}{|\mathbf{q}_\Delta|}\right)^3 \frac{m_\Delta}{Q}\left[\frac{1+(R|\mathbf{q}_\Delta|)^2}{1+(R|\mathbf{q}|)^2}\right] \text{MeV},$$

where $g_1 = 0.282$, $\Gamma(\Delta) = 109$ MeV, $R = 0.0552$ MeV^{-1}. $C_\pi C_\gamma$ is the isospin coefficient of the system and is given by

$$C_\pi C_\gamma = \frac{1}{3}\tau_{+1}\left(=\frac{\sqrt{2}}{3}\tau_+\right) \quad \text{for } \gamma n \to p\pi^-$$
$$= -\frac{1}{3}\tau_{-1}\left(=\frac{\sqrt{2}}{3}\tau_-\right) \quad \text{for } \gamma p \to p\pi^+$$
$$= \frac{2}{3}\tau_0 \quad \text{for } \gamma N \to N\pi^0. \qquad (8)$$

Our formalism used here has the explicit isospin dependence by the isospin tensor operators $\tau_{\pm 1,0}$. They are very convenient for the calculation of the rescattering mechanism. The BL II model [4] has the same form as the previous BL I model, but uses the following parameter set in place of the previous parameter set:

$$m_\Delta = 1225 \text{ MeV}, \quad \Gamma(\Delta) = 110 \text{ MeV}, \quad g_1 = 0.34,$$
$$R = 0.007 \text{ MeV}^{-1}, \quad G_3 = \frac{2.18}{m_\pi}e^{i\Phi_{M,E}}, \qquad (9)$$
$$\Phi^M \simeq \frac{(1.07 + 0.0138|\mathbf{q}|)}{100|\mathbf{q}|}.$$

The numerical parameter Φ_E is shown in Ref. [4].

B. Photopion production in ^{3}He$(\gamma,\pi^+)^3$H reaction

Using the impulse approximation, we can derive the following formalism for the reaction of ^{3}He$(\gamma,\pi^+)^3$H:

$$\frac{d\sigma}{d\Omega}|\text{c.m.} = \frac{1}{(2\pi)^2}\frac{|\mathbf{q}^{(\text{c.m.})}|}{|\mathbf{k}^{(\text{c.m.})}|}\frac{k_0^{(\text{c.m.})}q_0^{(\text{c.m.})}E_i^{(\text{c.m.})}E_f^{(\text{c.m.})}}{W^{(\text{c.m.})^2}}$$
$$\times\frac{1}{4}\sum_{M_0,M_0',\epsilon}|M_{fi}|^2; \qquad (10)$$
$$M_{fi} = \frac{1}{2k^0 2q^0\frac{E_i E_f}{M^2}}T_{fi}.$$

The superscript (c.m.) in Eq. (10) denotes the c.m. system of γ-^{3}H or γ-^{3}He where ^{3}H and ^{3}He are assumed to be equivalent except for the isospin. The kinematical variables without (c.m.) means the γ-N c.m. system in nucleus. T_{fi} is defined in the following way:

$$T_{fi} = \langle\Psi_{^3\text{H}}\Phi_\pi^{(-)}|H_{\text{B.L.}}|\Psi_{^3\text{He}}\Phi_\gamma^{(-)}\rangle$$
$$= (2\pi)^3\int d\mathbf{k}_3\int d\mathbf{r}d\boldsymbol{\rho}\Psi_{^3\text{H}}(\text{r},\boldsymbol{\rho})\Phi_\pi^{(-)^*}(\text{q},\tfrac{2}{3}\boldsymbol{\rho})$$
$$\times H_{\text{B.L.}}(\mathbf{k}_3)\Phi_\gamma^{(+)}(\mathbf{k},\tfrac{2}{3}\boldsymbol{\rho})\Psi_{^3\text{He}}(\mathbf{r},\boldsymbol{\rho}), \qquad (11)$$

where the integration variable $\mathbf{k}_3$ is the momentum of an interacting nucleon. Both $\Phi_\gamma^{(+)}(\mathbf{k},\frac{2}{3}\boldsymbol{\rho})$ and $\Phi_\pi^{(+)}(-\mathbf{q},\frac{2}{3}\boldsymbol{\rho})$ were expanded by multipole amplitudes. The spatial part $\Phi_\pi(|\mathbf{q}|\frac{2}{3}\boldsymbol{\rho})$ of pion wave function is the distorted wave givenby the numerical solution of the Klein-Gordon equation with an optical potential. The pion nonlocalities are not included explicitly. But the pion momentum in the intermediate states depends on the active nucleon's momentum whose nonlocal Fermi motion is taken fully into account. In this sense we have considered implicitly the nonlocalities of the pion, more or less. Our DWIA calculation corresponds to the results between the nonlocal dWIA and local (in both nucleon and pion) DWIA calculation of Tiator and Wright's calculation [39]. Of course, the best calculation should be done in the momentum space. But our Sendai group's three-body wave functions are constructed in configuration spaces in order to exactly take the Coulomb effect into account. In the case of PWIA $\Phi_\pi(|\mathbf{q}|\frac{2}{3}\boldsymbol{\rho})$ is simply given by spherical Bessel function $j_l(|\mathbf{q}|\frac{2}{3}\boldsymbol{\rho})$.

The wave function of the three-body nucleus used here is as follows:

$$\Psi_3(\mathbf{r},\boldsymbol{\rho}) = \sum_\alpha|\alpha(12,3)\rangle\Psi_3(r,\rho) \qquad (12)$$
$$= \sum|(L,S)J,(l,s=\tfrac{1}{2})j,J_0=\tfrac{1}{2},M_0\rangle|(I,\tfrac{1}{2})T=\tfrac{1}{2},T_z\rangle\Psi_3(r,\rho),$$

where we use the Jacobi coordinates ($\mathbf{r}$ is the coordinate vector between 1 and 2 particle, and $\boldsymbol{\rho}$ corresponds to the distance vector from the center of mass of the interacting pair to the spectator particle 3). (L, S, J) represents angular momenta for the pair of particle 1 and 2, while $(l, s = \frac{1}{2}, j)$ corresponds to the spectator particle 3 which interacts with the incident photon. T stands for the isospin of the three-body nucleus. $T_z = -\frac{1}{2}$ for ^{3}He, and $T_z = \frac{1}{2}$ for ^{3}He.

The ^{3}He wave function $\Psi_3(\mathbf{r}, \boldsymbol{\rho})$ used here is the 34-channel angular-momentum configuration space solution of the Faddeev equation [14]. However, we perform all calculations with the wave function truncated at the fifth channel expect for the threshold region, where the whole wave function was used. The five channels are

$$^{2S+1}L_J(l_j) = {}^1S_0(s_{1/2}),\ {}^3S_1(s_{1/2}),\ {}^3D_1(s_{1/2}),\ {}^3S_1(d_{3/2}),$$

and $^3D_1(d_{3/2})$. These five channels can explain the properties of ^{3}He within 95% and correspond, respectively, to α = 2, 1, 7, 8, and 3 channels in the wave function of Ref. [34]. For the two-body interaction of nucleons the Reid soft core (RSC) is used in the Faddeev equation.

C. The optical potentials of pion in three-body nucleus

The FSI between pion and three-body nucleus is included in terms of an optical potential. It means that the pion wave should be distorted by this optical potential. The distorted pion wave function in nucleus is calculated [29] by the Klein-Gordon equation:

$$(\nabla^2 + k^2 - 2\bar{\omega}U_{\text{opt}})\Phi(x) = 0, \tag{13}$$

where k $[=k_L/(1+\epsilon_0/A)]$ is the pion momentum in the pion-nucleus center of mass (π-A c.m.), and $\bar{\omega} = \omega/(1+\epsilon/A)$ with the pion total energy ω in the π-A c.m. system. Also $\epsilon_0 = \omega_L/M$, $\epsilon = \omega/M$ where M =(nuclear mass)/A = 931 MeV and the subscript L for ω and k indicates the laboratory system. The pion optical potential has the form [20] of

$$\begin{aligned} 2\bar{\omega}U_{\text{opt}} = {} & -4\pi\left[b(r) + p_2 B_0 \rho^2(r) + \frac{p_1 - 1}{2}\nabla^2 c(r) + \frac{C_0(p_2-1)}{2p_2}\nabla^2\rho^2(r)\right] \\ & +4\pi\left(\boldsymbol{\nabla}\cdot L(r)c(r)\boldsymbol{\nabla} + \frac{C_0}{p_2}\boldsymbol{\nabla}\cdot\rho^2(r)\boldsymbol{\nabla}\right) + 2\bar{\omega}V_c(r), \end{aligned} \tag{14}$$

where

$$b(r) = p_1[b_0\rho(r) - \epsilon_\pi b_1 \delta\rho(r)],$$

$$c(r) = (1/p_1)[c_0\rho(r) - \epsilon_\pi c_1\delta\rho(r)],$$

$$\delta\rho(r) = \rho_n(r) - \rho_p(r), \quad \rho(r) = \rho_n(r) + \rho_p(r),$$

and

$$L(r) = [1 + (4\pi/3)[(A-1)/A]c(r)]^{-1}.$$

The kinematic factors are

$$p_1 = (1+\epsilon)/(1+\epsilon/A) \text{ and } p_2 = (1+\epsilon/2)/(1+\epsilon/A).$$

Here B_0 and C_0 describe the absorption of pion in the multiple scattering. The complex quantities b_0, b_1, c_0, c_1 are obtained from the pion-nucleus scattering amplitudes. And the p-wave term, is modified by the Ericson-Ericson factor, which has the effect of weakening the p-wave attraction. This factor arises from the inclusion of short-range correlations between nucleons in nuclei.

The Coulomb potential is

$$V_c = \epsilon_\pi e^2 \int \frac{\rho_p^{ch}(r')}{|\mathbf{r}-\mathbf{r}'|} d^3r', \tag{15}$$

where $\epsilon_\pi = \pm 1$ stands for the pion charge of π^+ or π^-. The proton and the neutron point densities $\rho_p(r)$ and $\rho_n(r)$ are assumed to be of the form

$$\rho_p(r) = N_P e^{-2r^2/3r_p^2}, \tag{16}$$

$$\rho_n(r) = N_n e^{-2r^2/3r_n^2}, \tag{17}$$

with the normalizations

$$\int \rho_p(r) d^3r = Z, \qquad \int \rho_n(r) d^3r = N, \tag{18}$$

where r_p and r_n are the radius of the proton and neutron. We use r_p = 1.65 fm and r_n = 1.65 fm. More detailed information can be found in Refs. [13] and [21].

Since the SMC [13] and the Kim [21] potential were fitted by $T_\pi^{\text{lab}} \leq 250$ MeV, to apply to the region consid-

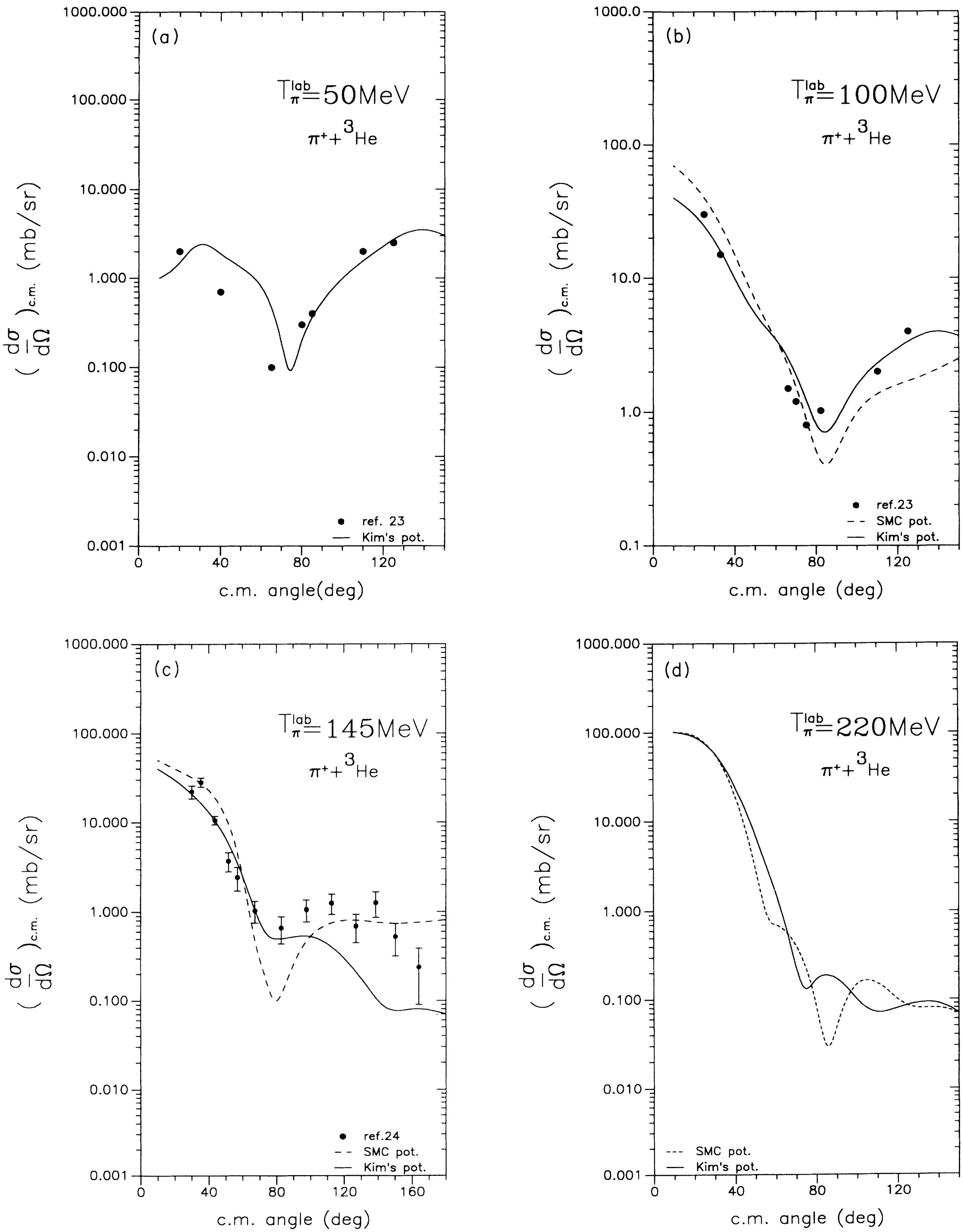

FIG. 1. The results of pion elastic scattering in three-body nucleus. The solid line is the results from the Kim potential. The dotted line is from the SMC potential. Experiments are from Refs. [22,24]. The result of $(\pi^+, {}^3H)$ at $T_\pi^{lab} = 180$ MeV is not shown because it is identical to the result of Ref. [21].

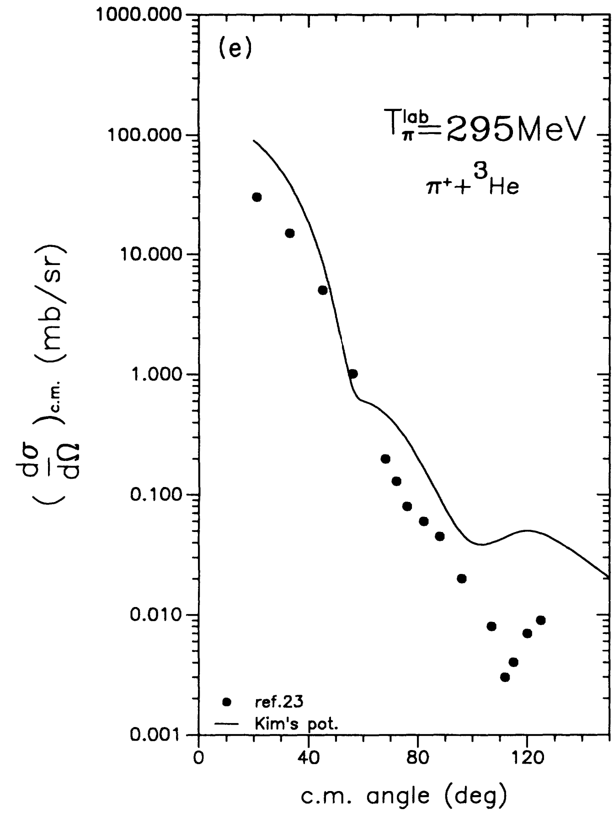

FIG. 1. (*Continued*).

ered here ($T_\pi^{\rm lab}$ = 60–310 MeV for $E_\gamma^{\rm lab}$ = 200–450 MeV), it is necessary to introduce a parameter set at $T_\pi^{\rm lab}$ = 295 Mev, where there exists the experiment of π-^{3}He elastic scattering. We determine the parameter to fit the pion elastic scattering and the pion total-cross-section experiments. All parameters used here are tabulated in Table I. The results of elastic differential, total absorption, total elastic, and total cross section [29] are shown in Figs. 1 and 2. For the differential cross section the Kim potential is better compared with the SMC potential, and for total cross section the SMC potential is more suitable to reproduce the Δ resonance shape. The absorption part represents the importance of inelastic channels including SCE and double-charge-exchange mechanism in this reaction.

In the $T_\pi^{\rm lab}$ region higher than around 200 MeV one would need the spin-flip part in the optical potential to reproduce the pion elastic scattering in the high angle region with more precision. But the spin-flip part mainly affects the high-angle region of the outgoing pion [21,27]. Since we are interested in a relatively small momentum transfer region, i.e., the small-angle region of pion, we do not include the spin-flip part in these optical potentials.

III. RESULTS AND DISCUSSIONS

A. PWIA results

In the charged pion threshold region the $\boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}$ term is a main contribution, due to the low-energy theorem. In

TABLE I. The pion optical potential parameters. The numbers in the bracket correspond to the imaginary part of optical potential. The columns in $T_\pi^{\rm lab}$ = 30, 40, and 50 MeV and lower part of $T_\pi^{\rm lab}$ = 180, 220 MeV are the parameters of the SMC potential. The upper part in the columns of $T_\pi^{\rm lab}$ = 180, 220 MeV and the column in $T_\pi^{\rm lab}$ = 140 MeV are the parameters of the Kim potential. The column in $T_\pi^{\rm lab}$ = 295 MeV is our fitting parameter for the experiment of Ref. [23].

$T_\pi^{\rm lab}$	30 MeV	40 MeV	50 MeV
$b_0(m_\pi^{-1})$	−0.027(0.002)	−0.027(0.003)	−0.027(0.004)
$b_1(m_\pi^{-1})$	−0.08(0.0)	−0.08(0.0)	−0.08(0.0)
$B_0(m_\pi^{-4})$	−0.05(0.05)	−0.05(09.05)	−0.05(0.05)
$c_0(m_\pi^{-3})$	0.24(0.005)	0.24(0.010)	0.24(0.020)
$c_1(m_\pi^{-3})$	0.22(0.003)	0.22(0.005)	0.22(0.01)
$C_0(m_\pi^{-6})$	−0.05(0.05)	−0.05(0.05)	−0.05(0.05)

$T_\pi^{\rm lab}$	140 MeV	180 Mev	220 MeV	295 MeV
$b_0(m_\pi^{-1})$	−0.045(0.068)	(−0.056(0.094) −0.084(0.002))	(−0.062(0.100) −0.091(0.029))	−0.106(0.054)
$b_1(m_\pi^{-1})$	−0.091(0.0)	(−0.093(0.0) −0.084(0.006))	(−0.097(0.0) −0.081(0.008))	−0.10(0.0)
$B_0(m_\pi^{-4})$	−0.111(0.128)	(−0.177(0.171) 0(0.182))	(−0.210(0.177) −0(0.182))	−0.022(0.18)
$c_0(m_\pi^{-3})$	−0.173(0.160)	(−0.021(0.250) 0.056(0.245))	(−0.145(0.264) −0.161(0.364))	−0.205(0.104)
$c_1(m_\pi^{-3})$	−0.159(0.103)	(−0.026(0.197) 0.056(0.245))	(−0.063(0.214) −0.077(0.182))	−0.002(0.142)
$C_0(m_\pi^{-6})$	−0.030(0.142)	(−0.039(0.204) 0(0.903))	(−0.038(0.217) 0(0.455))	−0.046(0.135)

Table II our values of spin-flip form factor at pion threshold are shown to be consistent with other theoretical calculations using the same RSC potential, and reproduce the experimental result within 92–94%. The remained difference can be improved by the more realistic wave functions.

In this calculation the Fermi motion is completely included by the integration of $\mathbf{k}_3$. The comparison of this complete integration [31] to the approximation of $\langle \mathbf{k}_3 \rangle = -\frac{1}{3}\mathbf{Q}$ in the π-^{3}He laboratory system, which corresponds to the optimal factorization [28], showed that this is a good approximation in this reaction. In the momentum space this approximation has also been shown to work by Tiator *et al.* [11,18].

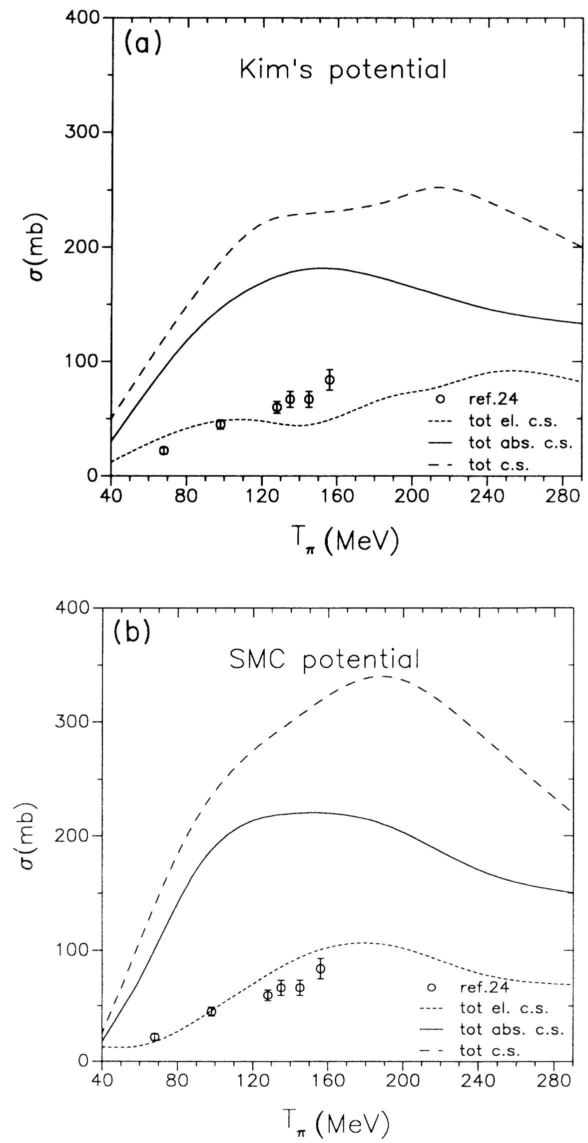

FIG. 2. The results of total cross section of pion scattering in the three-body nucleus. The solid line is for total absorption cross section. The long-dashed line is for the total cross section. The short-dashed line is for the total elastic cross section. Experiment is a total elastic cross section from Ref. [24].

TABLE II. The results of spin-flip form factor at pion threshold. The definition of spin-flip form factor and experiment are given by Ref. [15]. RSC is the Reid soft core potential. GPTT, SSCA, SSCB, and SSCD are the phenomenological nucleon-nucleon potential (see Ref. [12] for details). Finite size means that they corrected the nucleus charge radius, overestimated by the wave function, to the experimental charge radius.

Experiments [15]	0.52 ± 0.02
B. Goulard *et al.* [16]	0.474(Coulomb+RSC)
Los Alamos Group [17]	0.481(RSC)
	0.525(RSC+finite size)
J.L. Ballot *et al.* [12]	0.546(GPDT)
	0.547(SSCA)
	0.542(SSCB)
	0.541(SSCC)
Tiator [18]	0.495(RSC)
Our results	0.483(RSC by 5 channel)
	0.480(RSC by 34 channel)

Figure 3 shows our results of the PWIA calculation. The effect of spectator particle $d_{3/2}$ states could not be neglected, as shown by the difference of long-dashed (three channel) and solid lines (five channel). The importance of $d_{3/2}$ states, especially regarding the $3S_1(d_{3/2})$ component, was stressed on the polarized photon asymmetry in Ref. [37]. The Δ contribution (short-dashed line) shows that both contributions from the resonance and the nonresonance part work without any prevailing behavior of one part in this reaction. Moreover, the strong interference between the resonance and the nonresonance part causes the shift of peak from the Δ resonance position toward the experimental peak. (See the difference of peak position of the solid line and short-dashed line.) The pion rescattering usually has been said to be responsible for the shift of the resonance position [11]. But the first-order term in the forward elastic scattering approximation, the simplest of rescattering processes, did not show [31] the shift effect in the region of momentum transfer ≤ 6.0 fm^{-2}. We have to take into account the full rescattering series for the microscopic calculation. It is an almost hopeless task, due to the complexity of the process. In the case of neutral pion production in three-body nucleus, the rescattering of charge exchange should be considered because the charged amplitude is larger than the neutral amplitude.

The most important thing we have to notice in the results of PWIA is that above the Δ resonance theoretical results largely overestimate the experimental results. It is possible to guess that this overestimation is due to the neglect of the FSI.

B. DWIA results

The DWIA results for the reaction ^{3}He$(\gamma, \pi^+)^3$H in the momentum transfer region 0.48–4.9 fm^{-2} using the optical potentials are shown in Fig. 4. Our DWIA calculation showed [31] that the differences in the results from these two optical potential sets are within a few percent. It means that the distortion of the outgoing pion wave is not affected by the details of optical potentials, provided that the potentials can reproduce the experiment of pion elastic scattering in the low-angle region to some extent. But in the momentum transfer region higher than that considered here, one needs a more refined optical potential which could reproduce the experiments in the high-angle region.

The FSI influences strongly the PWIA calculation. It removes the overestimation of PWIA above the Δ energy region and reproduces remarkably the experiment of $Q^2 = 3.0$, 3.9, and 4.9 fm^{-2}. but in the low momentum transfer region($Q^2 = 0.48$ and $1.0\,\mathrm{fm}^{-2}$) a disagreement between theoretical and experimental results still remains. In Sec. III C we show it can be improved further by the consideration of the $E_{1^+}(3/2)$ amplitude.

The effect of the FSI is shown, respectively, for the cases of only Δ and only N in Fig. 5 by comparing the DWIA results with the PWIA results. Here "only Δ" means the switching off of the Born terms and "only N" the switching off of the Δ propagation contribution in elementary amplitude. In the case of only Δ, any remarkable effect apart from a small change in Δ width does not appear as expected because we are not using the Δ-h approach. If one would use the Δ-h approach, the FSI effect, i.e., the effect from pion multiple scatteing, would appear in only Δ. In the DWIA calculation the pion multiple scattering including the Δ medium effect

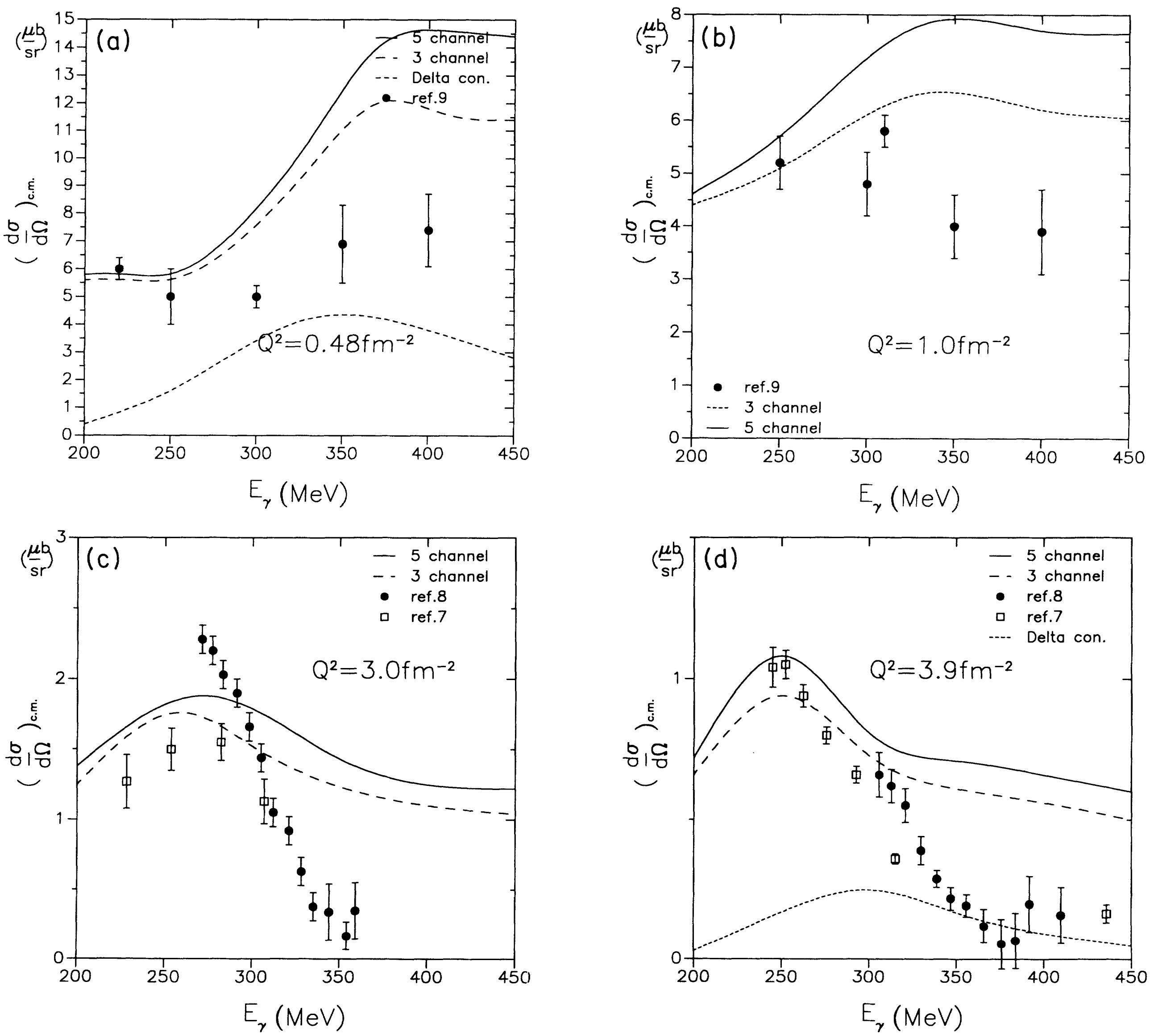

FIG. 3. The results of PWIA. The solid line is the calculation using five-channel wave function. The long-dashed line is for three-channel wave function. The short-dashed line is the contribution of Δ particle with five-channel wave function. Experiments are from Refs. [7–9].

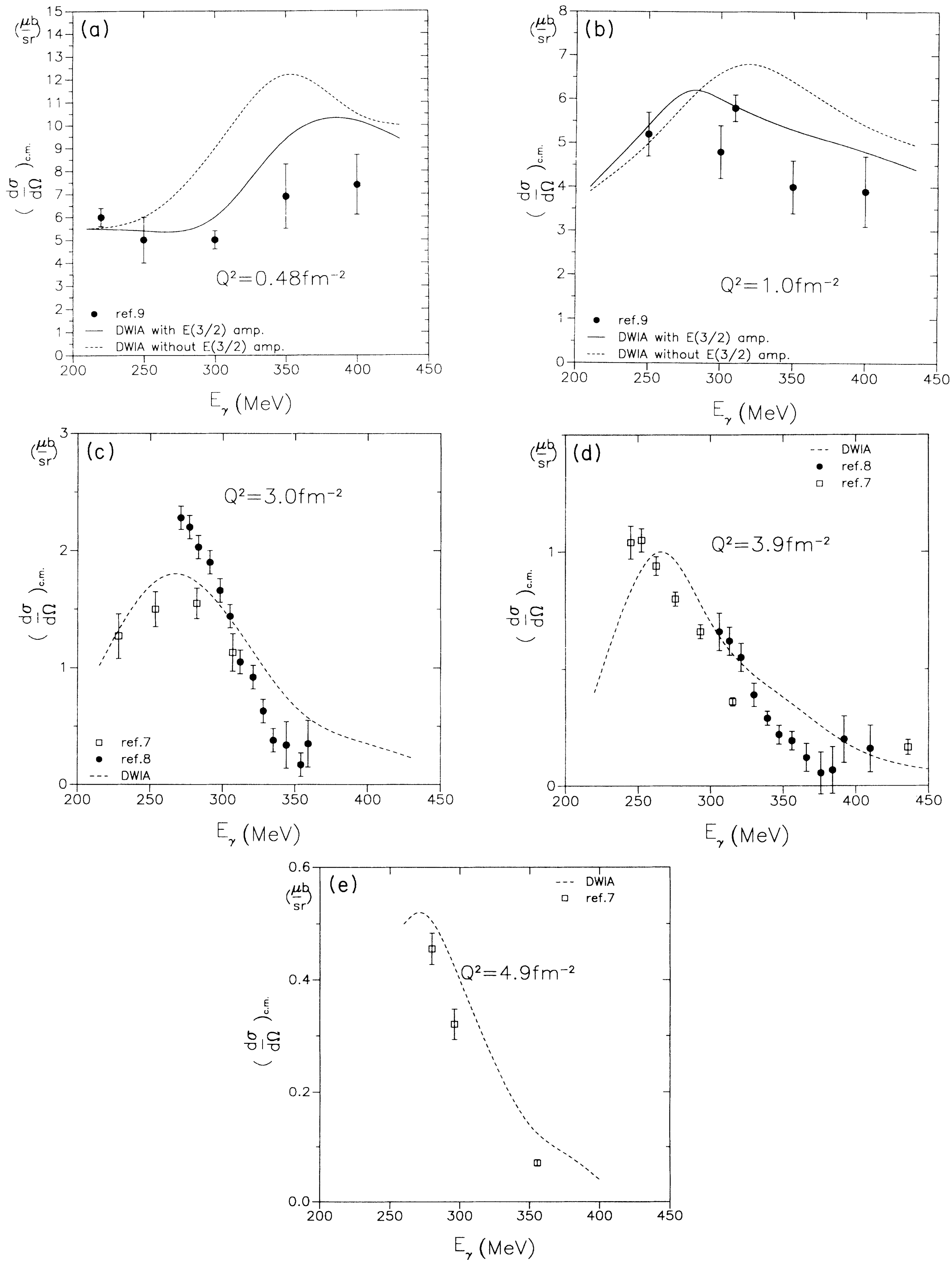

FIG. 4. The results of DWIA and the $E_{1^+}(3/2)$ amplitude contribution. The five-channel wave function has been used. The solid line is the result of the DWIA calculation with $E_{1^+}(3/2)$ contribution. The short-dashed line is the result of only the DWIA calculation. In the DWIA we used pion partial waves to $l \leq 5$. Experiments are from Refs. [7–9].

in nucleus is taken into account implicitly in an optical potential, as shown in Fig. 2 and affects mainly a case of only N.

In the case of the only N, the shape of resonance peak appears in place of nearly straight line in PWIA. The resonance shape might be due to the Δ contribution in pion multiple scattering. In other words the cross section is reduced by the FSI in both the low- and high-energy region because of the following reasons. In the low-energy region the s-wave contribution from $\boldsymbol{\sigma} \cdot \boldsymbol{\epsilon}$ term decreases by the pion s-wave absorption and the low-energy theorem [25] of pion in the nucleus and, as a consequence, the pion pole term (mainly p wave) is an only main term and responsible for the resonance shape. Going to higher energy region the absorption mechanism of pion $l \geq 1$ wave and attenuation of the pion by the optical potential decreases the differential cross section.

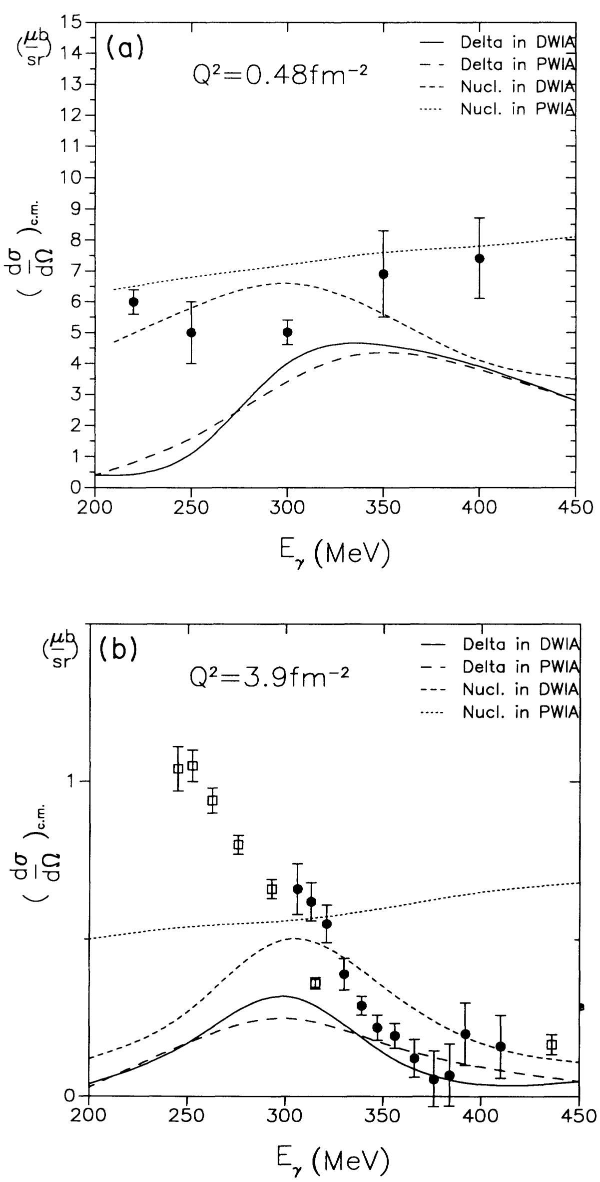

FIG. 5. The effect of FSI. The dotted line is for only N in PWIA. The short-dashed line is for only N in DWIA. The solid line is for only Δ in DWIA. The long-dashed line is for only Δ in PWIA.

C. The $E_{1^+}(3/2)$ multipole amplitude and the unitarity contribution

Since the $E_{1^+}(3/2)$ multipole amplitude that corresponds to two $\gamma N \Delta$ gauge couplings [38] is small compared with the $M_{1^+}(3/2)$ multipole amplitude, the $E_{1^+}(3/2)$ amplitude has usually been neglected. However, in the forward scattering angle region (0°–50°) the outgoing pion has a non-negligible contribution to the elementary charged photopion production [4]. The elementary reaction in this region corresponds to the reaction $^3\text{He}(\gamma,\pi^+)^3\text{H}$ in the low-momentum transfer region (0.48 fm^{-2}–1.0 fm^{-2}). It means that the contribution from the $E_{1^+}(3/2)$ cannot be neglected in the low momentum transfer region, and its effect might be further enhanced by the $d_{3/2}$ state component [37]. Really, the inclusion of the $E_{1^+}(3/2)$ yields remarkable improvements to the experimental results at $Q^2 = 0.48$ and 1.0 fm^{-2} than the only DWIA results as in Fig. 4, while in the case of $Q^2 = 3.0$, 3.9, and 4.9 fm^{-2} it did not give [31] any drastic change.

The BL I model is incomplete with respect to the unitarity [4,19]. The phenomenological inclusion of interference of the background resonance term and the Δ propagation term reproduced the $E_{1^+}(3/2)$ and the $M_{1^+}(3/2)$ multipole amplitude exactly in the BL II model. But in the charged photopion production the effect of the unitarization does not give the discernible effect. The unitarization influences mainly the neutral pion production [19] and $^{14}\text{N}(\gamma,\pi^+)^{14}\text{C}$ reaction, since the unitarization influences mainly the resonance reaction.

IV. CONCLUSION

It is necessary to treat the full rescattering processes microscopically in order to explicitly describe the FSI. But it is not only too troublesome due to the complexity of processes, but also the convergence of the rescattering series is not guaranteed. Moreover, in the Δ resonance region the validity of the perturbation expansion may be doubtful [26] because of the large coupling constant of Δ.

Therefore we calculated the reaction using two kinds of optical potentials for the FSI. If we use these optical potentials, the results for the elastic pion scattering are inferior to the results by other theoretical optical potentials [27] in the large-angle region. But for the reaction of $^3\text{He}(\gamma,\pi^+)^3\text{H}$ we are convinced from our results that these optical potentials are eligible for use in this reaction. The momentum transfer region, we calculated in this reaction does not need such a large angle for the outgoing pion.

Our results using the DWIA give significant improvements to the theoretical results compared with the PWIA. The FSI of the pion is one of the important ingredients necessary to understand the photopion reaction even in the three-body nucleus. It should include all possible elastic and inelastic multiple scatterings of pion. The single charge-exchange mechanism might be the most important process [34,37] in inelastic scattering mechanisms. But we have taken into account not only SCE but also other inelastic channels implicitly through the imaginary parts in the optical potential. It leads to the relatively simple calculation, but turns out to be a viable approach in this reaction. Also, our results show that we should not be free from the consideration of the $E_{1^+}(3/2)$ amplitude contributions as well as the FSI to describe the $^3\mathrm{He}(\gamma,\pi^+)^3\mathrm{H}$ reaction in low momentum transfer.

The Δ-h approach has been known to be very successful for the description of the photopion reaction of p-shell nuclei where the resonant reaction is dominant. But in the reaction where the nonresonant reaction becomes dominant, it has the uncertainty of the multipole amplitude's transformation from on-mass shell to off-mass shell, as in the CGLN method. The strong interference of the nonresonance and resonance part makes the application of the Δ-h concept unsatisfactory [6] in this reaction.

The hybrid model [32] that the nonresonance reaction in Δ-h approach is replaced by the PV Born terms may be an efficient method for this reaction. But the BL II model and the Δ-h model have a similar structure in the description of the $E_{1^+}(3/2)$ and $M_{1^+}(3/2)$ multipole amplitudes, in the respect that both approaches consider the contribution of the background resonance to the $E_{1^+}(3/2)$ and the $M_{1^+}(3/2)$ multipole amplitudes, and the Δ propagator in the BL II model is replaced by the nonrelativistic propagator, except various phenomenological terms. It suggests the possibility of incorporating systematical calculation of the Δ-h concept into the BL II model.

Finally, it should be noted that the polarized photon beam might be suitable for the separation of the nonresonance and the resonance reaction. The polarized target with polarized photon beam also may be an interesting method of the suppression of the $\boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}$ term. The suppression would be very useful in studying the secondary important interaction term [30]. It may be a useful key to further understanding of the electromagnetic structure of the three-body nucleus.

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