Measurable decay modes of barium isotopes via exotic cluster emissions

Satish Kumar and Raj K. Gupta
Physics Department, Panjab University, Chandigarh-160014, India
(Received 8 November 1993)

Cluster decays of $^{112-120}$Ba nuclei are calculated within a preformed cluster model. The α-nuclei ^{4}He, ^{8}Be, ^{12}C, ^{16}O, and ^{20}Ne are predicted as the possible decay modes, lying within the limits of present experimental methods. Other than the α particle, ^{12}C decay of ^{112}Ba is shown to be the most probable one with half-life time $T_{1/2} \sim 10^4$ s, stressing the role of doubly magic $^{100}_{50}$Sn daughter nucleus in cluster radioactivity. The use of different Q values for $T_{1/2}$ estimates and presence of nuclear structure effects in Geiger-Nuttall plots are also discussed.

PACS number(s): 23.60.+e, 21.60.Gx, 23.90.+w, 27.60.+j

Recently, one of us and collaborators [1] have pointed out some new instabilities against exotic cluster decays for some "stable" nuclei in the region $50 < Z < 82$. In particular, ^{12}C and ^{16}O decays of ^{120}Ba are predicted to have half-life times $\sim 10^{22}$ and 10^{26} s, respectively, which are well below the presently measured upper limit for heavy cluster decays of radioactive nuclei. However, ^{120}Ba is a short lived nucleus with the measured [2] half-life of only 24 s and hence the predicted cluster-decay probabilities are too small for the experimental possibilities. Apparently it is of interest to search other possibilities with larger cluster decay constants (shorter decay half-life times). Knowing that all the so-far-observed daughter nuclei in radioactive cluster-decay studies are the magic or nearly magic spherical nuclei, the isotopes of barium, lighter than ^{120}Ba, suggest an exciting new possibility of even a doubly magic $^{100}_{50}$Sn$_{50}$ as one of the daughter product.

Experiments are now being planned [3] at GSI, Darmstadt (Germany), for producing ^{114}Ba whose theoretically estimated half-life is still lower $\sim$ 0.1–1 s. We understand that similar efforts are going on in Dubna (Russia) for producing ^{116}Ba. The aim of these experiments is not only to produce new, lighter isotopes of barium, but also to measure the possible emissions of ^{12}C and α particles from these "stable" nuclei. The only theoretical estimate available on cluster decays of these nuclei is the simple barrier penetration calculation of Poenaru *et al.* [4]. These authors use a unified fission model (UFM) whose parameters are fitted to a large volume of data on α decay and the ^{14}C decay of ^{223}Ra. We choose to work here with the preformed cluster model (PCM) of Malik and Gupta [5] which has been used quite extensively now [6–10] and is perhaps the only theoretical prescription known for calculating the cluster preformation probability based on collective model picture of the nucleus [11]. The calculations are made for $^{112-120}_{56}$Ba nuclei.

In the PCM of Malik and Gupta [5], the decay half-life time $T_{1/2}$ or the decay constant λ is defined as

$$\lambda = P_0 \nu_0 P \quad (T_{1/2} = \ln 2/\lambda). \tag{1}$$

Considering a coupled motion in dynamical collective coordinates of mass asymmetry $\eta = (A_1 - A_2)/A$ and relative separation R, Malik and Gupta solved the stationary Schrödinger equation [5,9]

$$H(\eta, R)\psi(\eta, R) = E\psi(\eta, R), \tag{2}$$

with the Hamiltonian constructed as

$$H(\eta, R) = V(\eta) + V(R) + V(\eta, R) + \tfrac{1}{2}B_{\eta\eta}\dot{\eta}^2 + \tfrac{1}{2}B_{RR}\dot{\mathbf{R}}^2 + \mathbf{B}_{R\eta}\dot{\mathbf{R}}\dot{\eta}. \tag{3}$$

For collective potentials calculated in the Strutinsky method and B_{ij} as cranking masses, both the coupling potential and coupling mass are small [12–15]. This reduces the problem to one of decoupled motions, with $P_0 \propto |\psi(\eta)|^2$ and $P \propto |\psi(R)|^2$.

The stationary Schrödinger equation for η motion is

$$\left(-\frac{\hbar^2}{2\sqrt{B_{\eta\eta}}}\frac{\partial}{\partial\eta}\frac{1}{\sqrt{B_{\eta\eta}}}\frac{\partial}{\partial\eta} + V(R)\right)\psi^\nu(\eta) = E_\eta^\nu\psi^\nu(\eta), \tag{4}$$

whose numerical solution (for fixed R) gives the ground-state ($\nu = 0$) cluster preformation probability

$$P_0(A_2) = |\psi^0(\eta)|^2\sqrt{B_{\eta\eta}(\eta)}\frac{2}{A}. \tag{5}$$

Choosing the inner turning point $R_a = R_1 + R_2$ $(= R_t)$ or $= C_1 + C_2$ $(= C_t)$, C_1 being the Süssmann central radii, the potential $V(\eta)$ in two spheres approximation is defined as the sum of nuclear binding energies, the Coulomb and proximity potentials. The mass parameters are the classical hydrodynamical masses [16].

For R motion, instead of solving the corresponding radial Schrödinger equation, the WKB method is used. Then, for the penetration path shown in Fig. 1 of Ref. [5],

$$P = P_i W_i P_b, \tag{6}$$

where, for simplicity, the internal deexcitation probability W_i is taken as unity, and the WKB penetrabilities are

$$P_i = \exp\left[-\frac{2}{\hbar}\int_{R_t}^{R_i}\{2\mu[V(R) - V(R_i)]\}^{1/2}dR\right], \tag{7}$$

$$P_b = \exp\left[-\frac{2}{\hbar}\int_{R_i}^{R_b}\{2\mu[V(R)-Q]\}^{1/2}dR\right]. \quad (8)$$

Both these integrals are solved analytically by parametrizing $V(R)$ suitably (for further details, see Refs. [5,9].

The assault frequency ν_0, for both the cluster and daughter nuclei formed in the ground state, is defined simply as

$$\nu_0 = \frac{v}{R_0} = \frac{\sqrt{(2E_2/\mu)}}{R_0}, \quad (9)$$

with kinetic energy of cluster $E_2 = (A_1/A)Q$ and $Q = E_1 + E_2$, the Q value. R_0 is the equivalent spherical parent nucleus radius.

The model of Malik and Gupta was extended recently [17,18] to include deformations of both the cluster and daughter nuclei. The barrier gets lowered considerably and the inner turning point R_a lies at Q value, with $R_a > R_0$. In other words, the simplifying assumptions of choosing $R_a = R_t$ and introducing the idea of internal deexcitation in the model of Malik and Gupta is found to assimilate completely the deformation effects of both the cluster and daughter nuclei.

Figure 1 shows the calculated fragmentation potential V as a function of cluster mass A_2 for various Ba isotopes. We notice that for the $N = Z$ ^{112}Ba nucleus, the minima in $V(A_2)$ occur only at $A_2 = 4n$ α nuclei. As the ratio $N{:}Z$ increases, the minima start appearing also at $A_2 = 4n + 2$ clusters. This result was first observed in Ref. [6]. The preformation probabilities P_0 are also large for the clusters referring to potential-energy minima [19].

Figures 2 and 3 give the Geiger-Nuttall (GN) plots between the calculated $T_{1/2}$ and Q values or penetrabilities P. We notice in Fig. 2 that the $\log_{10}T^{\rm cal}_{1/2}$(s) vs $Q^{-1/2}$ plots for each cluster are nearly straight lines but with different slopes and intercepts. Thus, the equation of the straight line (the GN law) for each cluster is different [18]. This difference of slope and intercept for each cluster is associated [11,18] to their having a different preformation factor P_0. Similarly, $\log_{10}T^{\rm cal}_{1/2}$(s) vs $-\ln P$ plots in Fig. 3

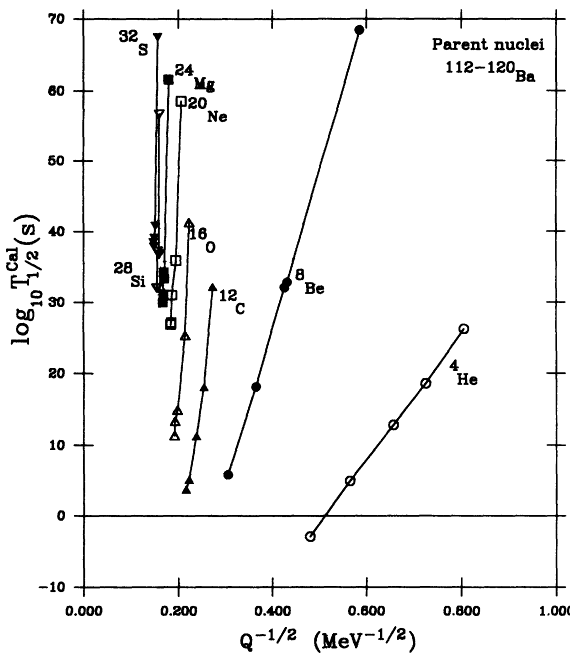

FIG. 2. Geiger-Nuttall plots of $\log_{10}T^{\rm cal}_{1/2}$(s) vs $Q^{-1/2}$ for various clusters emitted from $^{112-120}$Ba.

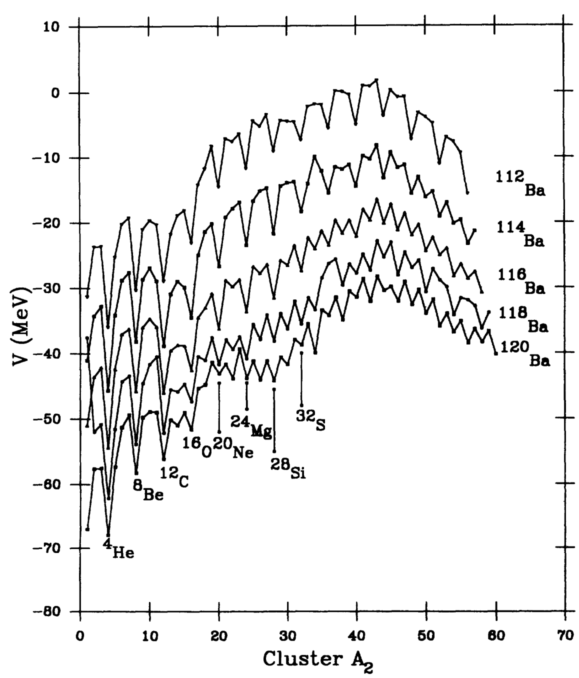

FIG. 1. Fragmentation potentials for $^{112-120}$Ba, at $R_a = C_1 + C_2$.

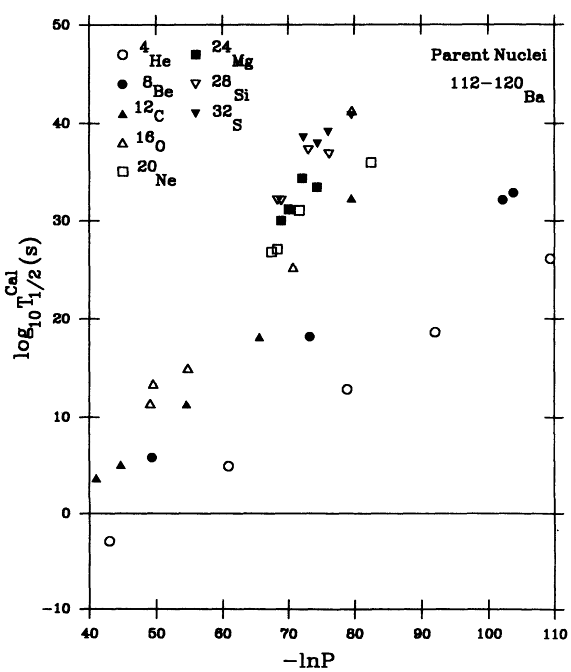

FIG. 3. Same as in Fig. 2, but for $\log_{10}T^{\rm cal}_{1/2}$(s) vs $-\ln P$.

are nearly straight lines for each cluster. The differences in slopes and small deviations from straight lines may arise due to the presence of nuclear (proximity) potential in $V(R)$ (in GN law, P is the pure Coulomb barrier penetrability). Thus, the nuclear structure effects are evident in Fig. 3. The fact that here the P_0 are different for different clusters and decrease with increase in their size is presented in Fig. 4. The variation of $-\log_{10}P_0$ with A_2 is reasonably well represented by a straight line, at least for $A_2 \leq 30$. For $A_2 > 30$, the values of P_0 increase rather than decrease, in agreement with earlier observations [8,20].

TABLE I. Predicted half-life times $T_{1/2}$ and other characteristics for certain cluster decays of Ba isotopes, using the preformed cluster model of Malik and Gupta [5]. The first turning point $R_a = C_1 + C_2$. For Q values the masses are of Möller and Nix [21] for $A \geq 16$ and Wapstra *et al.* [21] for $A < 16$ (see also footnote a).

Parent nucleus	Emitted cluster	Daughter nucleus	Q value (MeV)	Preformation probability P_0	Decay constant λ (s^{-1})	$\log_{10}T_{1/2}$ (s)
^{112}Ba	^{4}He	^{108}Xe	4.33	9.98×10^{-1}	5.68×10^{2}	−2.91
	^{8}Be	^{104}Te	10.72	9.13×10^{-7}	1.00×10^{-6}	5.84
	^{12}C	^{100}Sn	21.46	2.44×10^{-8}	1.24×10^{-4}	3.75
	^{16}O	^{96}Cd	26.94	3.00×10^{-14}	3.00×10^{-14}	13.36
	^{20}Ne	^{92}Pd	28.77	2.98×10^{-22}	6.50×10^{-32}	31.03
	^{24}Mg	^{88}Ru	35.19	2.44×10^{-25}	3.54×10^{-35}	34.29
	^{28}Si	^{84}Mo	40.89	7.48×10^{-28}	4.10×10^{-38}	37.23
	^{32}S	^{80}Zr	46.28	1.81×10^{-29}	2.09×10^{-39}	38.52
^{114}Ba	^{4}He	^{110}Xe	3.13	9.98×10^{-1}	7.64×10^{-6}	4.96
	^{8}Be	^{106}Te	7.52	1.39×10^{-8}	5.02×10^{-19}	18.14
	^{12}C	^{102}Sn	20.20	4.08×10^{-8}	5.27×10^{-6}	5.12
			18.34[a]	8.96×10^{-10}	1.47×10^{-10}	9.67
	^{16}O	^{98}Cd	27.17	1.94×10^{-12}	3.06×10^{-12}	11.36
	^{20}Ne	^{94}Pd	29.60	7.03×10^{-20}	1.06×10^{-27}	26.82
	^{24}Mg	^{90}Ru	35.62	4.48×10^{-23}	5.07×10^{-32}	31.14
	^{28}Si	^{86}Mo	41.99	9.22×10^{-25}	5.72×10^{-33}	32.08
	^{32}S	^{82}Zr	45.63	6.26×10^{-28}	8.45×10^{-39}	37.91
^{116}Ba	^{4}He	^{112}Xe	2.32	9.98×10^{-1}	1.03×10^{-13}	12.83
	^{8}Be	^{108}Te	5.50	5.97×10^{-10}	5.39×10^{-33}	32.11
	^{12}C	^{104}Sn	17.63	5.53×10^{-10}	3.44×10^{-12}	11.31
	^{16}O	^{100}Cd	25.44	1.69×10^{-13}	8.33×10^{-16}	14.92
	^{20}Ne	^{96}Pd	29.24	8.72×10^{-20}	5.50×10^{-28}	27.10
	^{24}Mg	^{92}Ru	35.79	2.08×10^{-22}	7.22×10^{-31}	29.98
	^{28}Si	^{88}Mo	41.73	1.83×10^{-24}	6.15×10^{-33}	32.05
	^{32}S	^{84}Zr	44.82	2.00×10^{-28}	5.50×10^{-40}	39.10
^{118}Ba	^{4}He	^{114}Xe	1.90	9.98×10^{-1}	1.80×10^{-19}	18.59
	^{8}Be	^{110}Te	5.35	5.87×10^{-10}	1.02×10^{-33}	32.83
	^{12}C	^{106}Sn	15.44	5.86×10^{-12}	5.19×10^{-19}	18.13
	^{16}O	^{102}Cd	21.76	6.80×10^{-17}	3.73×10^{-26}	25.27
	^{20}Ne	^{98}Pd	26.34	1.87×10^{-22}	8.24×10^{-37}	35.93
	^{24}Mg	^{94}Ru	34.49	1.93×10^{-23}	2.86×10^{-34}	33.39
	^{28}Si	^{90}Mo	39.65	4.24×10^{-26}	9.97×10^{-38}	36.84
	^{32}S	^{86}Zr	43.94	1.36×10^{-28}	1.06×10^{-41}	40.81
^{120}Ba	^{4}He	^{116}Xe	1.54	9.71×10^{-1}	4.73×10^{-27}	26.17
	^{8}Be	^{112}Te	2.92	1.71×10^{-10}	2.44×10^{-69}	68.45
	^{12}C	^{108}Sn	13.40	5.06×10^{-20}	4.08×10^{-33}	32.23
	^{16}O	^{104}Cd	20.10	5.12×10^{-29}	3.55×10^{-42}	41.29
	^{20}Ne	^{100}Pd	23.60	3.00×10^{-38}	1.70×10^{-59}	58.61
	^{24}Mg	^{96}Ru	31.40	9.05×10^{-46}	1.75×10^{-62}	61.60
	^{28}Si	^{92}Mo	39.80	1.90×10^{-46}	1.17×10^{-57}	56.77
	^{32}S	^{88}Zr	41.00	3.70×10^{-50}	2.45×10^{-68}	67.45

[a]Masses are from Comey *et al.* [21] and Wapstra *et al.* [21].

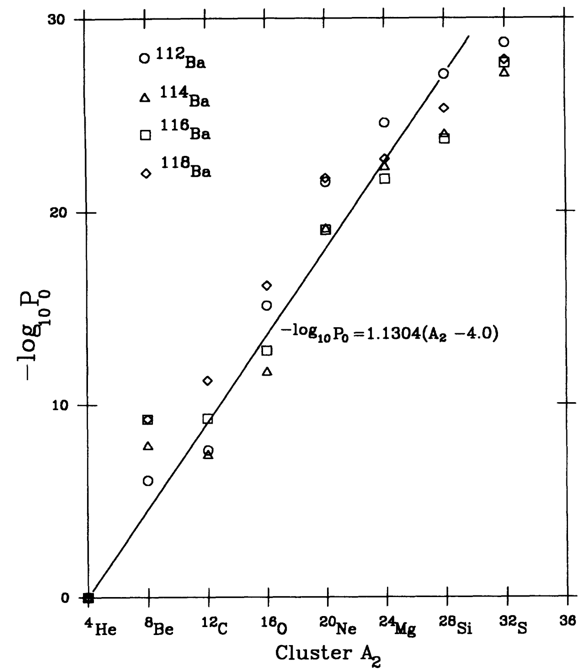

FIG. 4. Logarithm of cluster preformation probability P_0 vs mass A_2 of clusters emitted from $^{112-120}$Ba. The straight line gives the average behavior for $4 \leq A_2 \leq 30$.

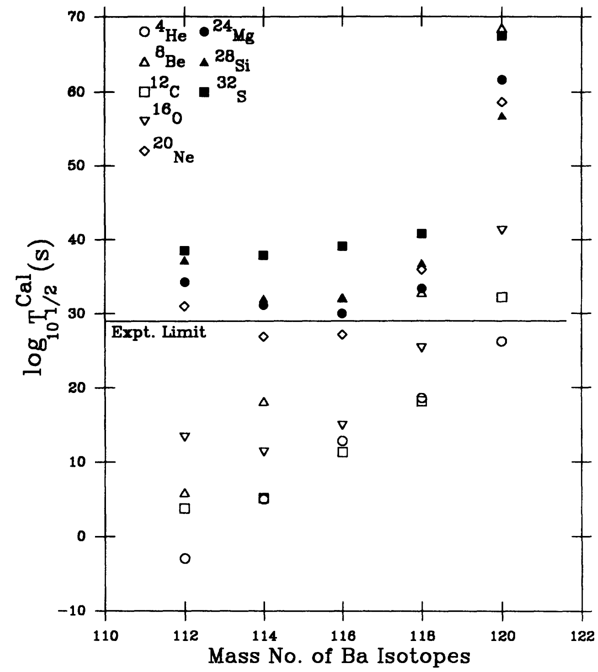

FIG. 5. Logarithm of calculated half-life times vs mass of Ba parents for various cluster decays. The limit of present experiments is also shown.

Table I and Fig. 5 present our calculations for the decay half-life times. First of all we notice that, other than α decay, ^{12}C decay of ^{112}Ba is the most probable (largest λ or shortest $T_{1/2}$). This means a clear preference for the doubly magic $Z = N = 50$ ^{100}Sn daughter nucleus in heavy cluster decays of Ba nuclei. Also, the α and ^{12}C decays of ^{114}Ba are predicted to be equally strongly probable. Notice that not only the Q values for the most probable decays of ^{112,114}Ba are larger but also the P_0 for these clusters to be preformed in ^{112,114}Ba are much larger than for other heavier parents. This is also true of many other decays. Figure 5 shows that many decays up to ^{20}Ne cluster lie below the limit of present experimental methods. Also, Table I illustrates for ^{12}C decay of ^{114}Ba that both P_0 and P change significantly if different Q-value estimates are used.

Summarizing, we have shown that the preformed cluster model of Malik and Gupta predict ^{4}He, ^{8}Be, ^{12}C, ^{16}O, and ^{20}Ne as the possible measurable decay modes of barium isotopes and, of these, ^{4}He and ^{12}C decays of ^{112}Ba are the most probable. This last result refers to a doubly magic daughter nucleus $^{100}_{50}$Sn$_{50}$. The decays with their daughters in the neighborhood of $^{100}_{50}$Sn can be labeled as the Sn radioactivity, since the so-far-observed cluster radioactivity occurs with $^{208}_{82}$Pb$_{126}$ or its neighboring nuclei as the daughters and is called the Pb radioactivity. The fact that in Sn radioactivity only $A_2 = 4n$ α nuclei are predicted to be the most probable, stems from $N/Z \approx 1$ for the best decaying parents.

The GN plots for various cluster decays of Ba isotopes are similar to the ones for the observed cluster decays of radioactive nuclei, except that here the Q values involved are much smaller. The penetrabilities P are of similar orders in the two cases (compare our Fig. 3 with Fig. 3 of Ref. [22]), though the potential used here contains the nuclear term and the GN law is for pure Coulomb potential. This difference in potentials is reflected in GN plots by their deviating slightly from straight lines and having different slopes. Thus, the importance of nuclear structure effects, in terms of cluster preformation factor P_0, are indicated in the GN plots. The P_0 for clusters are small, compared to one for α decay, and, for $A_2 \leq 30$, decrease as the size A_2 of cluster increases.

This work is supported in part by the University Grants Commission, Delhi.

[1] R. K. Gupta, S. Singh, R. K. Puri, and W. Scheid, Phys. Rev. C **47**, 561 (1993).

[2] X. Shu-wei *et al.*, Phys. Rev. C **46**, 510 (1992).

[3] A. Guglielmetti and R. Bonetti, private communication.

[4] D. N. Poenaru, D. Schnabel, W. Greiner, D. Mazilu, and R. Gherghescu, At. Nucl. Data Tables **48**, 231 (1991).

[5] S. S. Malik and R. K. Gupta, Phys. Rev. C **39**, 1992 (1989).

[6] S. S. Malik, S. Singh, R. K. Puri, S. Kumar, and R. K. Gupta, Pramana J. Phys. **32**, 419 (1989).

[7] R. K. Gupta, W. Scheid, and W. Greiner, J. Phys. G **17**, 1731 (1991).

[8] S. Singh, R. K. Gupta, W. Scheid, and W. Greiner, J. Phys. G **18**, 1243 (1992).
[9] R. K. Gupta, S. Singh, R. K. Puri, A. Săndulescu, W. Greiner, and W. Scheid, J. Phys. G **18**, 1533 (1992).
[10] S. Kumar and R. K. Gupta, Int. J. Mod. Phys. E (to be published).
[11] R. K. Gupta and W. Greiner, Int. J. Mod. Phys. E (to be published).
[12] J. Maruhn and W. Greiner, Phys. Rev. Lett. **32**, 548 (1974).
[13] R. K. Gupta, W. Scheid, and W. Greiner, Phys. Rev. Lett. **35**, 353 (1975).
[14] D. R. Saroha and R. K. Gupta, J. Phys. G **12**, 1265 (1986).
[15] S. S. Malik, N. Malhotra, D. R. Saroha, and R. K. Gupta, Report No. IC/86/128.
[16] H. Kröger and W. Scheid, J. Phys. G **6**, L85 (1980).
[17] S. Kumar, Ph.D. thesis, Panjab University, 1992.
[18] R. K. Gupta, *Frontier Topics in Nuclear Physics*, NATO Advanced Study Institute, edited by W. Scheid and A. Săndulescu (Plenum, New York, 1993).
[19] R. K. Gupta, Fiz. Elem. Chastits At. Yadra **8**, 717 (1977) [Sov. J. Part. Nucl. **8**, 289 (1977)].
[20] A. Săndulescu, R. K. Gupta, W. Greiner, F. Carstóiu, and M. Horoi, Int. J. Mod. Phys. E **1**, 379 (1992).
[21] P. E. Haustein, At. Nucl. Data Tables **39**, 185 (1988).
[22] A. A. Ogloblin *et al.*, Phys. Lett. B **235**, 35 (1990).