Identical energy bands of uranium isotopes in the interacting boson model

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The energy spectra and $B(E2)$ values of the uranium isotopes $^{232-238}$U are calculated in the interacting-boson-plus-fermion-pair model. It was found that in order to reproduce the identical energy bands, weak boson pairing, a weak angular momentum interaction and a strong quadrupole interaction are needed.

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Recently superdeformed rotational bands with almost identical energies have been discovered in the mass 150 and 190 regions [1–4]. The occurrences of identical energy bands which suggest a constant moment of inertia are not expected from general arguments. The $A^{5/3}$ mass dependence, the changes of deformation with mass, the orbital alignment effects, and changes in pairing all seem to be able to produce some changes in moments of inertia. The suppression of these effects has been discussed by Stephens *et al.* [5,6]. It was pointed out that the phenomena of identical rotational bands are not restricted to the superdeformed nuclei [7]. It was shown that in the actinide region many nuclides show identical rotational bands. Particularly, the ground-state rotational bands of ^{236}U and ^{238}U are almost identical up to $I = 24$. It was also pointed out that the variation of moment of inertia in the rare-earth region is much less than the rigid-body $A^{5/3}$ dependence [8,9]. Therefore, the occurrence of identical bands seems to be more common for both superdeformed and normal deformed nuclei than generally expected. It was suggested in Ref. [7] that for the identical bands in normal deformation nuclei the single-particle states are well characterized and provide a better chance to understand the mechanism in producing the identical energy bands.

In this work we perform a phenomenological calculation on the uranium isotopes in the scheme of the interacting boson approximation (IBA) boson-plus-fermion-pair model. It is hoped to get some clue from the characteristics of the interaction parameters and the wave functions.

In the calculation, two kinds of basis states are included in the model space, i.e., the pure boson states and states with bosons and a pair of fermions. The fermions are assumed to occupy the neutron $1j_{15/2}$ and proton $1i_{13/2}$ orbitals. This seems to be a natural choice since these two orbitals are quite near to the Fermi surface of the actinides.

For the Hamiltonian, we adopted the standard one of the boson-plus-fermion model [10]. That is,

$$H = H_B + H_F + H_{BF},$$

$$H_B = \epsilon_d n_d + a_1 P^\dagger \cdot P + a_2 L \cdot L + a_3 Q \cdot Q,$$

which is the Hamiltonian of IBA-1 in multipole expansion form [11]. H_F is the fermion pair Hamiltonian, which includes the one-body and two-body terms and takes the form

$$H_F = \epsilon_j (2j+1)^{1/2} (a_j^\dagger \times \tilde{a}_j)^{(0)} + \frac{1}{2} \sum_J V^J (2J+1)^{1/2} [(a_j^\dagger \times a_j^\dagger)^J \times (\tilde{a}_j \times a_j)^J]^{(0)}.$$

The single nucleon j can take the values of 13/2 and 15/2, respectively:

$$H_{BF} = \alpha \hat{Q} \cdot (a_j^\dagger \times \tilde{a}_j)^{(2)} + \beta Q \cdot [(a_j^\dagger \times a_j^\dagger)^{(4)} \times \tilde{d} - d^\dagger \times (\tilde{a}_j \times \tilde{a}_j)^{(4)}]^{(2)}.$$

Here

$$\hat{Q} = (d^\dagger \times \tilde{s} + s^\dagger \times \tilde{d})^{(2)} - \frac{\sqrt{7}}{2} (d^\dagger \times \tilde{d})^{(2)},$$

which is the SU(3) generator quadrupole operator. The fermion-fermion interaction strength V^J's are calculated from the Yukawa potential with the Rosenfeld mixture. The harmonic oscillator wave functions are used. The oscillation constant ν is chosen as $\nu = 0.96 A^{-1/3} \times 10^{22}$ s^{-1} with $A = 230$.

The whole Hamiltonian is then diagonalized in the selected model space. The energy eigenvalues are fitted with the energy levels of $^{232-238}$U, and the interaction parameters, the coupling parameters α, β, and fermion single-particle energies are determined by the least-squares fittings. The calculated and experimental energy levels of $^{232-238}$U are shown in Figs. 1–4. In general, the ground-state bands can be reproduced quite well. This means that the similarities between the moments of inertia of the different uranium isotopes can be reproduced in general. However, since the values of the moments of inertia depend on the energy gaps quite sensitively, we can only say that the moments of inertia (for example, for ^{236}U and ^{238}U) deduced from the calculated energy levels are similar but not identical. Note that the energy spectra of uranium isotopes were calculated in the interacting boson model without fermion pairs [12,13]. In those calculations the calculated energy levels for high spin states were usually much higher than the experimental data. The discrepancies increased quickly as the spin became higher. These discrepancies can be avoided

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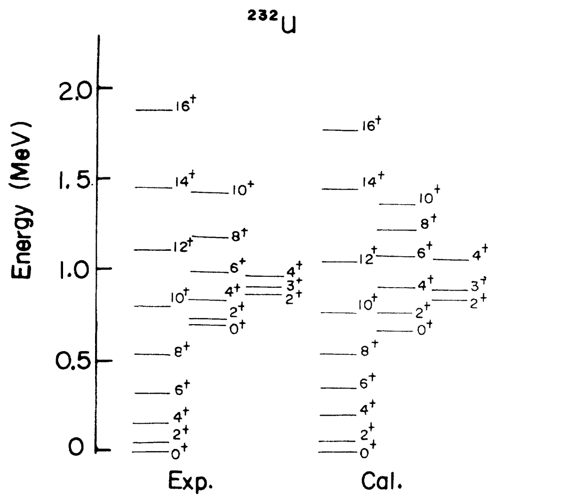

FIG. 1. The calculated and experimental energy levels for ^{232}U. The experimental data are adopted from Refs. [15,16].

by introducing the fermion-pair degrees of freedom. As can be seen from Figs. 3 and 4, up to $I = 30$ the energy spectra can be reproduced quite well. The β and γ bands can be also reproduced. However, the theory-experiment agreements are not as good as for the ground-state bands.

The best fitted interaction parameters are shown in Table I. There are several interesting points which are worthy of mentioning. First, except for the single-d boson energy, all other boson-boson interaction and boson-fermion coupling parameters can be unified for all of the four uranium isotopes. This suggests that the boson cores are quite stable with respect to the change of mass. In other words, the characteristics of the boson core do not

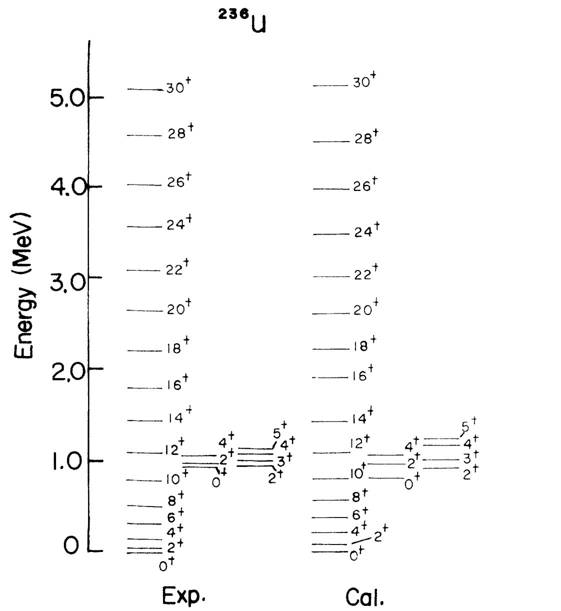

FIG. 3. The calculated and experimental energy levels for ^{236}U. The experimental data are adopted from Ref. [15].

depend on the number of bosons sensitively. Furthermore, the unification of α and β values indicates that the boson core and fermion-pair interactions are similar for the isotope string, and very likely the effect of fermion pair alignments on the core is small. Second, in the same table, parameters adopted in our previous similar calculations are also displayed for comparison. In order to get a general idea we show the range of the parameters obtained in the previous calculations. The single-d boson

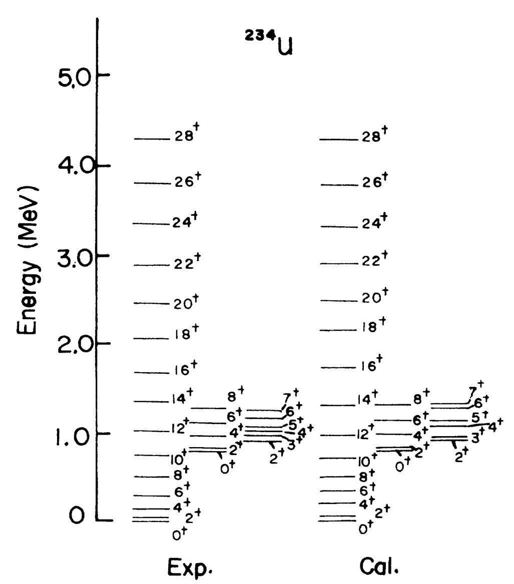

FIG. 2. The calculated and experimental energy levels for ^{243}U. The experimental data are adopted from Refs. [14,17].

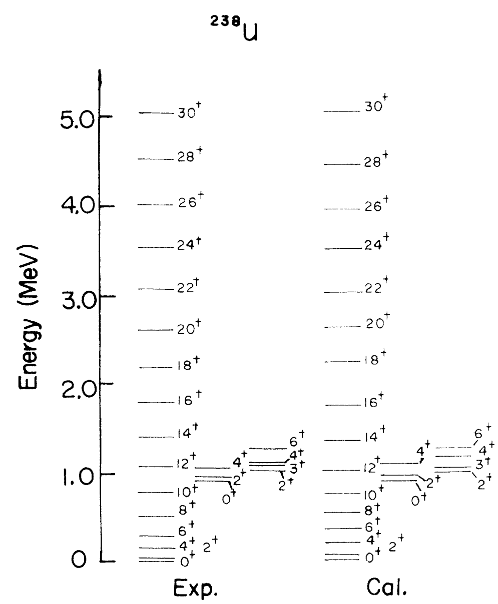

FIG. 4. The calculated and experimental energy levels for ^{238}U. The experimental data are adopted from Ref. [18].

TABLE I. The best fitted boson-boson interaction (in MeV) and boson-fermion coupling parameters of the uranium isotopes. Parameters of the previous calculations on the rare-earth and Pt-Os regions are also displayed for comparison.

	ϵ_d	$P^\dagger \cdot P$	$L \cdot L$	$Q \cdot Q$	α	β
^{232}U	0.393	0.007	0.001	−0.01	0.07	0.04
^{234}U	0.393	0.007	0.001	−0.01	0.07	0.04
^{236}U	0.465	0.007	0.001	−0.01	0.07	0.04
^{238}U	0.471	0.007	0.001	−0.01	0.07	0.04
Rare earth region	0.3–0.7	0.02–0.13	0.003−−0.01	0.004−−0.01	0.02–0.11	0.004–0.05
Pt-Os region	0.4–0.6	0.027–0.25	0.008–0.01	0.008−−0.007	0.037–0.09	0.02

energy and the boson-fermion coupling parameters α, β of the uranium isotopes are similar to those of the rare-earth and Pt-Os regions. However, the pairing interaction and angular momentum interaction are much smaller than the corresponding values in the rare-earth and Pt-Os regions. The $Q \cdot Q$ interaction, on the other hand, is larger than the corresponding values in the rare-earth and Pt-Os regions. In order to explain the inertness of the moments of inertia of the superdeformed bands it was speculated that some compensations happen between the change of deformation and orbital alignments. In our calculation, the low value of $L \cdot L$ interaction suggests weak orbital alignment. The unification of boson interaction parameters in the whole isotope string also suggests that the change of deformation versus mass number is very small. Therefore, our calculation supports the reduction of these effects of orbital alignment and change of deformation on the variation of moment of inertia in the sense that they are separately small. Finally, the large and unified $Q \cdot Q$ interaction confirms that the isotope string has a large and stable deformation. The adopted fermion single-particle energies are $\epsilon_{15/2} = 0.605$ MeV (for the whole isotope string) and $\epsilon_{13/2} = 0.450$ MeV (for ^{232}U and ^{234}U), 0.553 MeV (for ^{236}U) and 0.656 MeV (for ^{238}U). These single-fermion energies are considerably smaller than those in the rare-earth and Pt-Os regions which correspond to other single-fermion orbitals.

The validity of the model calculation can be further tested by computing the $B(E2)$ values. The $E2$ operator in the interacting-boson-plus-fermion model is given by

$$T^{(2)} = e_B Q_B + \alpha e_F (a_j^\dagger \times \tilde{a}_j)^{(2)} + \beta e_B [(a_j^\dagger \times a_j^\dagger)^{(4)} \times \tilde{d} - d^\dagger \times (\tilde{a}_j \times \tilde{a}_j)^{(4)}]^{(2)}$$

where

$$Q_B = (s^\dagger \times \tilde{d}) + (d^\dagger \times \tilde{s}) + \chi (d^\dagger \times \tilde{d})^{(2)}.$$

In this work j can take the values 13/2 and 15/2. The values of α and β are adopted from the Hamiltonian and the effective charges e_B and e_F are chosen to be 0.2 and 0.5, respectively. For ground-state band transitions the $B(E2)$ values change quite smoothly with the value of χ. The choice of the value of χ is correlated with the choice of e_B. In this work the value of $-\sqrt{7}/2$ is chosen, which matches the value of $e_B = 0.2$, and Q_B agrees with the Q that appears in the Hamiltonian. The calculated and experimental $B(E2)$ values of ^{236}U and ^{238}U for the ground-state bands are shown in Fig. 5. In the low-spin region the theory-experiment agreements are in general quite good. However, around $I = 14$ or 16 the calculated $B(E2)$ values drop quite sharply as compared with the experimental values. This is because the fermion-pair-plus-boson configurations come into play and this seems to be a common discrepancy that happens in all boson-plus-fermion-pair type calculations. However, for the case of the uranium isotopes it was found that both $(i_{13/2})^2$ and $(j_{15/2})^2$ configurations mix with the boson configurations mildly in the low and intermediate spin regions. Only in the very-high-spin region does the $(j_{15/2})^2$ alignment become quite strong and the $(j_{15/2})^2$-plus-boson configurations become dominant. Recall that in the previous calculations with the same model for the

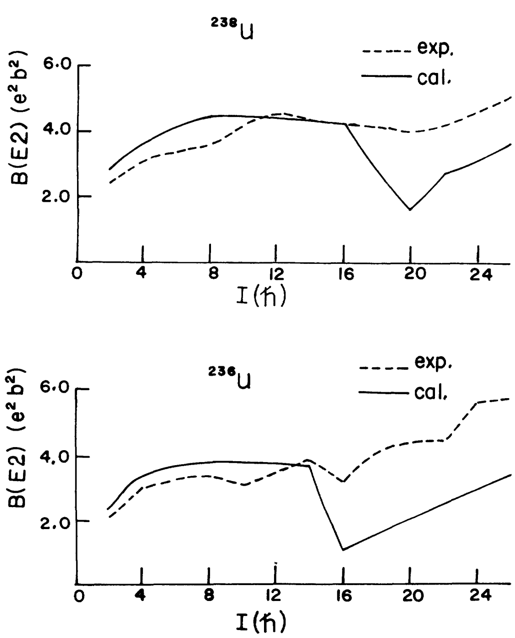

FIG. 5. The calculated and experimental $B(E2)$ values for the ground-state bands of ^{236}U and ^{238}U. The experimental data are adopted from Refs. [15,18].

TABLE II. Wave function intensity distribution of ^{238}U.

State	Pure boson configurations intensity	Boson plus $(1j_{13/2})^2$ configurations intensity	Boson plus $(1j_{15/2})$ configurations intensity
0_1^+	0.863	0.065	0.072
0_2^+	0.969	0.015	0.016
2_1^+	0.864	0.064	0.072
2_2^+	0.867	0.062	0.071
2_3^+	0.908	0.042	0.050
3_1^+	0.833	0.078	0.089
4_1^+	0.857	0.067	0.075
4_2^+	0.844	0.071	0.085
4_3^+	0.833	0.076	0.091
6_1^+	0.821	0.084	0.095
6_2^+	0.775	0.099	0.125
8_1^+	0.769	0.108	0.123
10_1^+	0.711	0.134	0.154
12_1^+	0.657	0.159	0.184
14_1^+	0.610	0.179	0.210
16_1^+	0.570	0.197	0.233
18_1^+	0.364	0.144	0.491
20_1^+	0.011	0.006	0.984
22_1^+	0.000	0.000	1.000
24_1^+	0.000	0.000	1.000
26_1^+	0.000	0.000	1.000
28_1^+	0.000	0.000	1.000
30_1^+	0.000	0.000	1.000

rare-earth and Pt-Os regions the $B(E2)$ values drop almost to zero at the onset of the fermion-pair configuration. Here the $B(E2)$ values drop to a certain value and then rise again versus the increase of spin in accordance with the experimental data. However, the calculated $B(E2)$ values in the high-spin region are usually smaller than the experimental data by a factor of 2. This discrepancy seems to suggest that the calculated wave functions are too pure, and it is speculated that the situation can be improved if more single-particle orbitals are included in the calculation. In Table II we present the wave function intensity distributions of ^{238}U as an illustration. It seems that small fermion-pair admixtures come into play in the region of rather low spins. In the region of intermediate spins both proton $(1i_{13/2})^2$ and neutron $(1j_{15/2})^2$ configurations have considerable contributions. Only in the very-high-spin region do the neutron $(1j_{15/2})^2$-plus-boson configurations become dominant. In Table III we also present some calculated and experimental $B(E2)$ values for interband low-spin state transitions. From the table, we can see that the calculated results agree with the experimental data qualitatively.

In summary, a phenomenological boson-plus-fermion-pair calculation is performed on the even uranium isotopes. The identical ground-state energy bands of ^{236}U and ^{238}U can be reproduced. The optimization of the interaction parameters indicates weak boson-boson pairing, a weak angular momentum-angular momentum interaction and a strong quadrupole-quadrupole interaction. The strong quadrupole-quadrupole interaction and the stability of the interaction parameters with respect to the change of mass numbers indicate that the deformations of the uranium isotopes are large and stable. Since our calculation shows very small boson orbital angular momentum interaction, this seems to indicate that the two factors, the change of deformation and the orbital alignments, are both small for the uranium isotopes.

TABLE III. The calculated and experimental $B(E2)$ values for some interband low-spin state transitions. The $2_1^+ \rightarrow 0_1^+$ $B(E2)$ values of ^{232}U and ^{234}U are also displayed for comparison. The experimental data are adopted from Refs. [14,15].

	J_i	J_f	Expt.	Calc.
^{232}U	2_1	0_1	2.0	1.9
^{234}U	2_1	0_1	2.0	2.2
	0_2	2_1	$>6.1\times10^{-4}$	0.011
	2_2	4_1	<0.010	0.006
	2_2	2_1	<0.015	0.047
	2_2	0_1	<0.019	0.010
	2_3	2_1	0.042	0.012
	2_3	0_1	0.025	0.035
^{238}U	2_2	4_1	0.043	0.018
	2_2	0_1	0.003	0.011
	2_3	4_1	0.003	0.011
	2_3	0_1	0.011	0.050
	2_4	4_1	0.003	0.010
	2_4	2_1	0.046	0.010

Note that in order to explain the reduction of the effects of the two factors to the change of the moment of inertia it is conjectured that the two factors mutually compensate each other. It was pointed out in Ref. [7] that the properties of identical ground-state bands occur in other actinides. An examination of the ground-state bands of ^{238}Pu and ^{240}Pu, ^{246}Cm and ^{248}Cm, and ^{250}Cf does show identical band structure for the experimentally available low-spin states [7]. Therefore, it is conjectured that the characteristics of the interaction parameters found in this work may be also true for other actinides such as Pu, Cm, and Cf. Apparently, more experimental and theoretical work is needed for a better understanding of the occurrence of identical bands in actinides and superdeformed nuclei.

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