Effects of ^{10}Li virtual states on the structure of ^{11}Li

I. J. Thompson
Department of Physics, University of Surrey, Guildford GU2 5XH, United Kingdom

M. V. Zhukov
Department of Physics, Chalmers University of Technology and Göteborg University, S-41296 Göteborg, Sweden
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The presence of a low-lying $1s_{1/2}$ virtual state in the $n+^9$Li system, when included in Faddeev three-body calculations, has a pronounced effect on the structure of the ^{11}Li halo. The resulting ^{9}Li momentum distributions, calculated in the Serber model, have a narrow width comparable with the experimental data. The structure of ^{11}Li would then have nearly 50% of s-wave motion between the neutrons and the core. The three-body system may also approximately fulfill the conditions for the appearance of Efimov states, and there may exist low-lying excited states near the breakup threshold.

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Many experiments with neutron-rich radioactive nuclear beams have confirmed that ^{11}Li has a large neutron halo, or dilute neutron skin, which extends to large radii. Such experiments have measured the total reaction cross sections [1] for the radioactive projectiles, and also, following the breakup of ^{11}Li, the inclusive momentum distributions of the ^{9}Li [2,3] and neutron [4] fragments. These distributions are consistently narrow over a wide range of target nuclei and incident energies. Their narrow width implies, by the uncertainty principle, that the halo neutrons are widely dispersed in space.

Considered as three-body systems of core + two valence neutrons, ^{11}Li (and similar nuclei such as ^{6}He and ^{14}Be) possess a number of unusual properties [5]. None of the two-body subsystems are bound: the nuclei are only bound because of *all* the attractions between the three fragments. The dineutron system has a barely unbound virtual state (scattering length –17 fm), and the $n+^9$Li system has no bound states. The ^{11}Li nucleus is more weakly bound ($E \approx -0.32$ MeV [6,7]) than ^{6}He or ^{14}Be, so the valence neutrons are more likely to be outside the (^{9}Li) core. In this exterior region, the ground-state wave function must follow three-body asymptotic laws [8], which depend on the internal two-body distances only in specific combinations. All these features can be taken into account by solving the Faddeev or other three-body equations with corresponding strict three-body asymptotics.

It appears that ^{11}Li has further features which make it a particularly important nucleus. In the usual models [9,10], the $0s$ and $0p_{3/2}$ states in the ^{9}Li core are occupied, and it was thought that the halo neutrons would occupy the next vacant $0p_{1/2}$ states to fill the p shell. We review evidence below, however, which indicates that in ^{10}Li there are low-energy opposite-parity intruder states from the next sd shell [11]. This is known to occur for the ground state of ^{11}Be [12], and for states near 1.7 MeV in ^{12}B. What is significant here is that an intruder state now appears to come very close to the $n+^9$Li threshold, and that this is in the s-wave channel. We show that these features have a profound effect on the long-range correlations in the ^{11}Li three-body system, as there are now barely unbound virtual states in each of the two-body subsystems. Thus, ^{11}Li approaches the Efimov case [13] of zero-energy states in each subsystem, in which limit there would be a large number of three-body bound states.

In the previous models of ^{11}Li [9,10] the halo neutrons were largely in a $(0p_{1/2})^2$ configuration. Experiments have seen a low-lying $n+^9$Li resonance [14,15], and this was expected to be a $0p_{1/2}$ single-particle state. The resulting three-body Faddeev wave functions [10] yielded suitable binding energies ≈ -0.32 MeV [6,7], and gave rms matter radii close to the experimental value ~ 3.15 fm, provided that the $0p_{1/2}$ single-particle resonance was at around +0.20 MeV [10]. A state at such an energy was seen by Amelin *et al.* [16], but other results [14,15,17] suggest the resonance should be at +0.4 MeV or higher. The quantum numbers of the resonances, however, have not been obtained in the experiments.

Recently, evidence has accumulated [7] which points to the existence of a barely unbound ground state in ^{10}Li. There are peaks in neutron-^{9}Li concidence spectra near zero relative velocity [18,19] which could correspond to s-wave states. From the ^{11}B(^{7}Li,^{8}B)^{10}Li reaction there is evidence [17] for not only a broad $0p$ peak at +0.50 MeV, but also a narrow peak near zero energy with width below 0.1 MeV. In an s-wave channel, this peak should be interpreted as the effect of a virtual state near the single-particle threshold.

In this note we consider the effect of a near-threshold s-wave virtual state in the $n+^9$Li interaction on the structure of the ^{11}Li neutron halo. The position of s virtual states is characterized by a scattering length a_0, and a virtual state would explain the ^{10}Li data of [17] if it had a scattering length of modulus at least 10–50 fm, although the evidence is not yet conclusive. Such unnatural-parity intruder states could arise [12] from the coupling of core excitation with neutrons in the *sd* shell. In the present work we examine the effects on the ^{11}Li structure of increasing the depth of the *s*-wave $n-^{9}$Li potential to give various $1s_{1/2}$ virtual-state scattering lengths, while keeping the *p*-wave potential to give a $0p_{1/2}$ resonance between +0.15 and +0.50 MeV.

Since there remains ambiguity in the shape of the n-^{9}Li potential, we consider the Woods-Saxon geometry of [10], and vary the *s*- and *p*-wave strengths V_s and V_p and spin-orbit strength V_{so} to fit specified 1*s* scattering lengths and $0p_{1/2}$ resonance energies. We keep the $0p_{3/2}$ bound state at −4.1 MeV, the observed ^{9}Li neutron separation energy, and orthogonalize the halo neutron wave functions to this state as well as to the deep $0s_{1/2}$ occupied state. We use the realistic super-soft-core *nn* potential (SSC) [20]. The Faddeev three-body wave functions now contain a superposition of $(0p_{1/2})^2$ and $(1s_{1/2})^2$ configurations, with (two-dimensional) radial wave functions decaying asymptotically according to the three-body separation energy E_{11}. These energies are shown in Fig. 1, and Table I selects those parameter combinations which give binding energies nearest to −0.32 MeV [6,7]. These ground state binding energies depend only weakly on the potential geometry once its *s*- and *p*-wave pole positions are specified, so we show only the results for a set of Woods-Saxon potentials *P*0–*P*4 with various depths, and $r_0 = 1.27$ fm ($R = 9^{1/3}r_0$) and $a = 0.67$ fm for both central and spin-orbit components.

Comparing the *P*0 and *P*3 calculations in Table I, the inclusion of a 1*s* virtual state has increased the weight of the $s_{1/2}$ state of *n*-core motion from 3% to 45%, and the $^{1}S_0$ state of *nn* relative motion from 38% to 60%. This gives a greater pairing contribution to the binding energy. The rms matter radius is increased (from 3.05 fm to 3.64 fm) because *s* waves lead to a weaker "centrifugal barrier" which governs the asymptotic decay rate of the three-body wave function in the hyper-radius ρ (defined by $\rho^2 = \frac{20}{11}R^2_{(nn)c} + \frac{10}{18}r^2_{nn}$).

The Faddeev wave functions for ^{11}Li can also be used in the Serber model of spectator/participant breakup to yield the momentum distributions of ^{9}Li fragments [2,3].

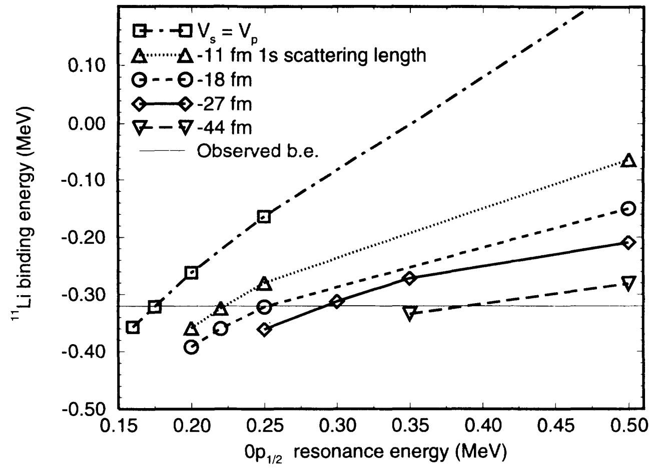

FIG. 1. Variation of the ^{11}Li three-body binding energy with the $0p_{1/2}$ resonance energy, for various positions of the $1s_{1/2}$ virtual-state pole. The cases nearest the observed −0.32 MeV [6,7] are listed in Table I.

This model works well for ^{6}He fragmentation [21], and for ^{11}Be [22]. It was a puzzle in Ref. [10] that the Fourier distributions of the $(0p_{1/2})^2$-dominated wave functions were twice as wide as the data of [2], as can seen again for the data of [3] with the upper curves in Fig. 2. The narrow widths are not expected to be caused by the reaction mechanism, because Ref. [23] shows for ^{6}He fragmentation that the distribution of the "core" fragment is only weakly affected by final-state interactions. For neutrons the discrepancy between the calculated Serber and empirical momentum distributions [4] is greater than for the core, but, by [23], final state interactions must be more important here.

Including ^{10}Li virtual states helps to resolve this puzzle. The Serber width for the ^{9}Li fragment decreases from 73 MeV/c for *P*0 to 31 MeV/c for *P*3. The distributions are shown in Fig. 2(a), and the distributions limited for the aperture of [3] in Fig. 2(b). The large decrease in width arises because, in momentum as well as in coordinate space, *s* waves are finite at the origin while *p* waves have a node there. The *P*4 combination (with the largest

TABLE I. Effect of low-lying 1*s* intruder states: Binding energies E_{11}, rms radii, weights of selected channels, and Serber widths without and with the experimental acceptances of [3]. The SSC *nn* potentials were used. All $0p_{3/2}$ eigenstates are at $E = -4.1$ MeV. The matter rms radius of ^{9}Li is taken as 2.32 fm. The last line shows the effect of a 0*s*-state scattering length $a_0 = -13$ fm [25]. Note that weights, momentum distributions and rms radii only use the wave functions out to hyper-radius $\rho = 50$ fm.

	1*s* (0*s*) a_0 (fm)	$0p_{1/2}$ reson. (MeV)	E_{11} g.s. (MeV)	R_m rms (fm)	$(s_{1/2})^2$ weight (%)	$(p_{1/2})^2$ wt. (%)	$^{1}S_0$ (*nn*) wt. (%)	$^{3}P_1$ (*nn*) wt. (%)	$p_\perp$ (^{9}Li) hwhm (MeV/c)	p(^{9}Li) [3] hwhm (MeV/c)
*P*0	0.7	0.175	−0.33	3.05	3	94	38	59	73	40
*P*1	−11	0.22	−0.32	3.28	23	72	52	44	38	28
*P*2	−18	0.25	−0.32	3.39	31	64	53	37	35	27
*P*3	−27	0.30	−0.33	3.64	45	51	60	29	31	26
*P*4	−44	0.35	−0.31	3.73	64	30	67	16	28	25
*G*1	(−13)		−0.31	3.40	97	1	96	4	35	31

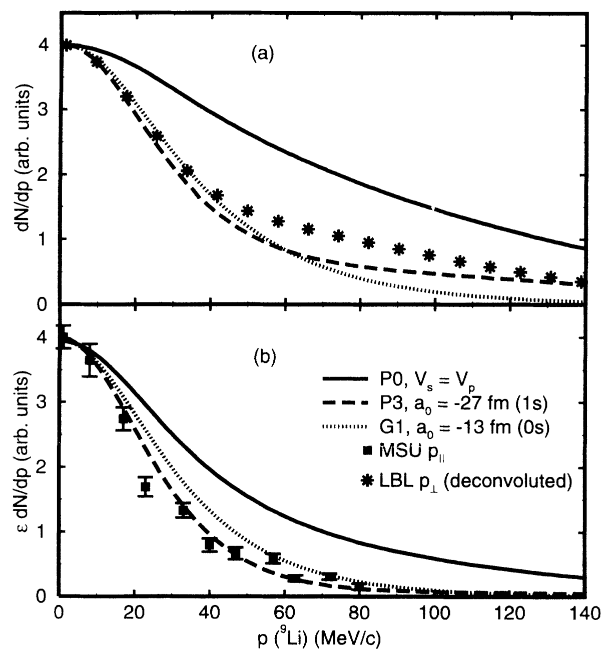

FIG. 2. Serber distributions for the rectilinear momentum of the ^{9}Li core following ^{11}Li fragmentation on light targets. The squares show $p_{||}$ data from [3], and the stars are the $p_{\perp}$ data of [2] corrected by the authors of [2] for multiple scattering in the target. Part (b) includes the effect of the finite angular acceptances in the experiment of [3].

$1s$ weight) has the narrowest width, but the $P3$ case is nearest to the Michigan $p_{||}$ data [3]. The shapes for neutron cross sections (not shown) also become narrower. The ratio of core to neutron widths increases from less than unity to ≈ 1.3 for the virtual-state models, moving toward the value $\sqrt{2}$ indicating no strong angular correlations between the neutrons. This is in correspondence with the results collected in [24] from experimental data at different energies.

A ^{10}Li virtual state near zero energy is the most important cause of the narrow momentum widths. This is demonstrated by using the early [25] shallow ($V = -7.8$ MeV) potential of Gaussian shape ($R = 2.55$ fm), which supports a $0s$ virtual state with a scattering length of -13 fm, but no p-wave resonance. This $G1$ case (with a $0s$ virtual state) has almost as narrow a Serber width as the $P2$ calculation, although it has a greater rate of decay at large momenta.

The larger rms matter radius of the $P1$–$P4$ ground states appears to be a natural consequence of increasing s-wave admixtures in the n-core wave function. Scattering lengths of less than -27 fm (cases $P3$–$P4$) lead to ^{11}Li being significantly larger than that determined from Glauber fits to the total-reaction cross section data [1]. However, there is no model-independent connection between the measured cross sections and the rms radii, and we need to reexamine the size determination in terms of a more microscopic reaction theory than used in [1]. Ogawa *et al.* [26] show, for example, that the optical limit of the Glauber model (as used in [1]) overestimates the reaction cross sections by 10–20%. The four-body Glauber theory of [27] provides a more accurate analysis, but requires optical potentials for ^{9}Li scattering.

The additional binding energy obtained in s-wave virtual state models depends on certain properties of the nn potential. The p-wave configuration that enters into the ^{11}Li g.s. is $^{3}P_1$, for which the potential is repulsive. If this repulsion is neglected (as in [9]), then the binding energy is increased by 0.30 for the $P0$ case, and by 0.15 MeV for $P3$. The structure of halo nuclei is very sensitive to such changes, and hence to aspects of the nn potential not always considered important.

We conclude that these calculations have shown that the presence of a low-lying $1s_{1/2}$ virtual state in the ^{10}Li system has pronounced effects on the structure of the ^{11}Li halo. The Serber-model widths of the ^{9}Li fragments become as narrow as the experimental data. Future calculations will use the wave functions obtained here to calculate the Coulomb dipole strength function $dB(\mathcal{E}1, E)/dE$ (as in [28]) to fit the data of [29], and also the nuclear breakup cross sections (as in [27]). These results will be compared with measurements of total reaction cross sections [1], and will confirm or deny our picture of ^{11}Li as having nearly 50% of s-wave motion between the neutrons and the core, and with larger rms radius than was previously believed.

We again note that ^{11}Li may then approximately fulfil Efimov's conditions [13], being a three-body system with a virtual state near zero energy in all pair-wise interactions. If the virtual states were at zero energy, then an infinite number of bound states may be expected; otherwise, only low-lying excited state(s) near the breakup threshold might be found. Either prospect is intriguing.

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