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Multiphonon vibrational states in deformed nuclei

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The ^{154}Gd nucleus was studied by γ-ray spectroscopy following the $(\alpha, 2n)$ reaction at an energy of 26 MeV. We identified the double-phonon $K = 4^+$ $\gamma\gamma$ vibrational band in this nucleus at 1645.8 keV. Also, we present the systematics of possible $K = 4^+$ double-phonon vibrational bands in the deformed rare-earth region of nuclei.

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I. INTRODUCTION

Vibrational degrees of freedom in both spherical and deformed nuclei are described by phonon excitations resulting from the oscillations of a liquid drop. In spherical nuclei, harmonic vibrational motion results in an excitation spectrum consisting of equally spaced degenerate phonon multiplets. Although exact harmonic phonon excitations have never been observed, there are numerous examples of nuclei exhibiting near-harmonic or anharmonic vibrational motion. In fact, one- and two-phonon excitations have been observed in tens of nuclei, as well as isolated cases of three-phonon excitations [1,2]. In the low-lying excited states of deformed nuclei, there are two types of quadrupole vibrations superimposed on the rotational states: β and γ. The β vibration has its angular momentum aligned along the symmetry axis, whereas the γ vibration breaks axial symmetry and has a projection of $K = 2$ on the symmetry axis. There are many deformed nuclei with known single β and γ bands. However, the existence of collective two-phonon vibrations of the type $\beta\beta$, $\beta\gamma$, and $\gamma\gamma$ has been an open question [3–5] in nuclear structure physics for over three decades and therefore the focus of a considerable controversy. The essence of the controversy was the nonobservation of any two-phonon vibrational states at the expected energy ranges near twice the excitation energies of the single β and γ phonons.

A sampling of the various theoretical approaches concerned with the existence of collective multiphonon excitations includes the generalized coordinate method (GCM) [6], the self-consistent collective coordinate method (SCCM-1) [7], the multiphonon method (MPM) [8], the dynamic deformation model (DDM) [9], the quasiparticle-phonon nuclear method (QPNM) [10], and, more recently, the self-consistent cranking model (SCCM-2) [11], amongst others. All of these models can be viewed through a unified perspective of attempts to solve the many-body nuclear problem through the inclusive treatment of collective and single-particle degrees of freedom to varying degrees. For example, the GCM neglects single-particle effects and determines the properties of the nucleus by its shape. The DDM constructs a basis of multiparticle-hole (multiquasiparticle) states corresponding to different deformations. The MPM and the QPNM models both include collective phonons coupled to quasiparticles, but differ significantly in the number of phonons included. The QPNM is limited to two phonons, whereas the MPM may include up to eight phonons. The SCCM approach treats quadrupole collective modes such as the γ vibration as large-amplitude, nonlinear nuclear phenomena. The predictions of these models vary in the degree of anharmonicity and collectivity for the two-phonon vibrational excitations.

The questions posed early in the history of nuclear structure regarding the vibrational states of deformed nuclei have remained unanswered. The characterization of the single β and γ phonons and the systematics of the two-phonon excitations hold the key to our understanding of one of the principle elementary excitation modes of the nucleus.

Nuclei in the $A = 150$ region of the chart of nuclides represent the most extensively studied region of nuclei. Yet, there was no definitive information on the existence of any two-phonon vibrational states until a recent lifetime measurement [12] in the ^{168}Er nucleus. This measurement showed that the lowest lying $K = 4^+$ band is in fact a two-phonon $\gamma\gamma$ band. That is, the members of this band are strongly connected by $E2$ transitions to the single-phonon γ band with transition probabilities that are equivalent in collectivity to transitions from the single-phonon states to the ground state. Intriguingly, the energy ratio of the two-phonon to one-phonon vibrational excitations was found to be 2.5. Although this single observation allowed the exclusion of the QPNM

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approach, no single model could be chosen amongst the others, even though they involve drastically different assumptions and predictions.

The primary motivation for this work was to carry out a systematic search for two-phonon vibrational excitations in the deformed rare-earth region of nuclei. We studied several nuclei in this region, including ^{154,156}Gd and ^{162}Dy. The scope of this paper is limited to presenting the new data in ^{154}Gd, and the newly emerging systematics of double-phonon ($K = 4^+$) $\gamma\gamma$ vibrational bands in the deformed rare-earth region from our compilation of 16 possible candidates.

II. EXPERIMENT

The ^{154}Gd nucleus had previously been studied through various reactions [13–20]. We have used the $(\alpha, 2n)$ reaction to study the transitions and levels in the ^{154}Gd nucleus by γ-ray spectroscopy. The experiment was carried out at the University of Notre Dame FN tandem accelerator using the University of Pittsburgh multidetector array [21]. The α beam energy was 26 MeV. The target was a 700 μg/cm^2 thick self-supporting foil of enriched (98.3%) ^{152}Sm. The detector array consisted of five HP-Ge detectors (30% efficiency) with BGO anti-Compton shields. Energy and efficiency calibrations were done by using a ^{152}Eu source at the target position. The measurements included γ-ray singles, γ-γ coincidences, directional correlations (DCO ratios) [22], and angular distributions. Approximately 20 million coincidence events were recorded.

A. Results

A partial level scheme for the ^{154}Gd nucleus is shown in Fig. 1. A minimum of 10 bands were populated up to a maximum spin of 16 in the ground-state band and the β band. The β band was mainly populated from levels above 3.6 MeV. Some weak feeding of the β band was observed from the 2^+ and 3^+ members of a $K = 2^+$ band at 1531 keV. No inband transitions were observed for this $K = 2^+$ band and no transition intensities could be extracted. The most recent work [19] in this nucleus utilizing the (^{9}Be,5n) reaction proposed a total of 68 new γ-ray transitions and 49 new levels up to a maximum spin of 26. Our reaction allowed us to confirm 13 of their lower spin states and 23 of their new transitions. In addition, we have observed several new transitions and placed a number of new levels at lower spins. In spite of the wealth of information in Ref. [19], the γ band was not observed. We set no hardware multiplicity constraints and we were able to observe the strong population of the γ band through a $K = 4^+$ band at 1645.8 keV. A partial level diagram focusing on the depopulation of the $K = 2^+$ and the $K = 4^+$ bands is shown in Fig. 2.

1. $K = 2$ γ band at 996 keV

The γ band was previously known [20] up to J^π of 7^+. A total of 21 transitions were known to depopulate the six levels in the γ band. The energy levels and transitions within the two bands of interest are tabulated in Table I. Within the γ band, a new 304.75 keV transition (a doublet) was placed between the $5^+_\gamma \rightarrow 3^+_\gamma$ members of the band. This assignment was warranted and justified by multiple coincidence gates in spite of the existence of two other nearly identical transitions of 303.22 keV ($7^+_4 \rightarrow 5^+_4$) and 304.75 keV ($6^+_4 \rightarrow 6^+_\gamma$). The spectrum obtained by gating on the 479.1-keV transition clearly shows the presence of a 304.75-keV transition. Similarly, by gating on 411.9-keV transition which feeds into the 6^+_4 level at 1911 keV, we were able to place a new 343.0-keV transition between the 6^+ and 4^+ members of the γ band. Gating on the 892-keV transition, it was possible to separate the 649.7-keV transition of $4^+_4 \rightarrow 2^+_\gamma$ from the newly observed 647.57-keV transition. New placements

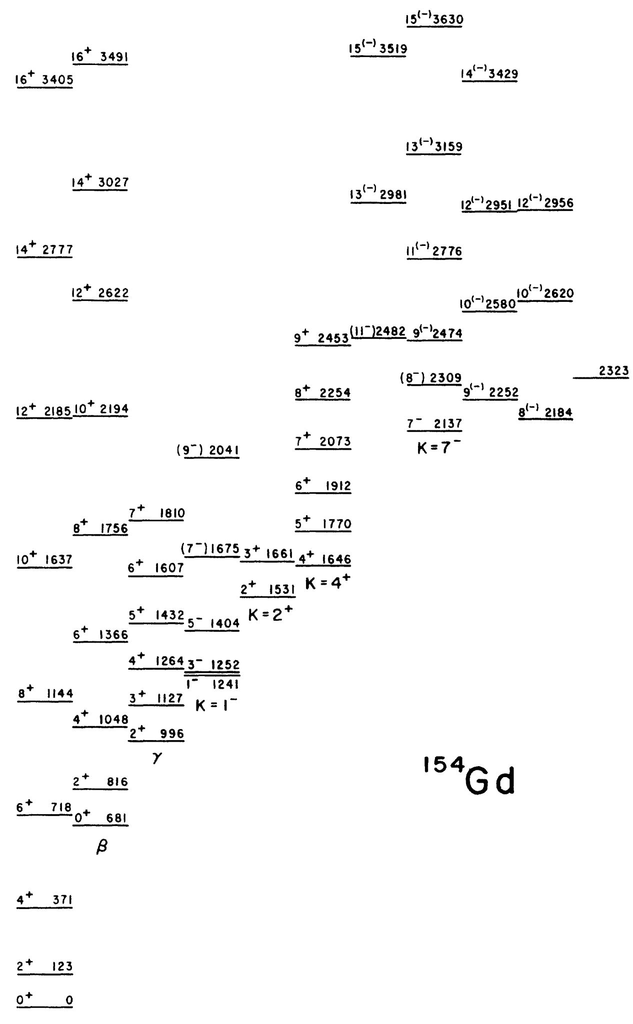

FIG. 1. The level scheme of the g.s. and excited bands populated in this study of ^{154}Gd.

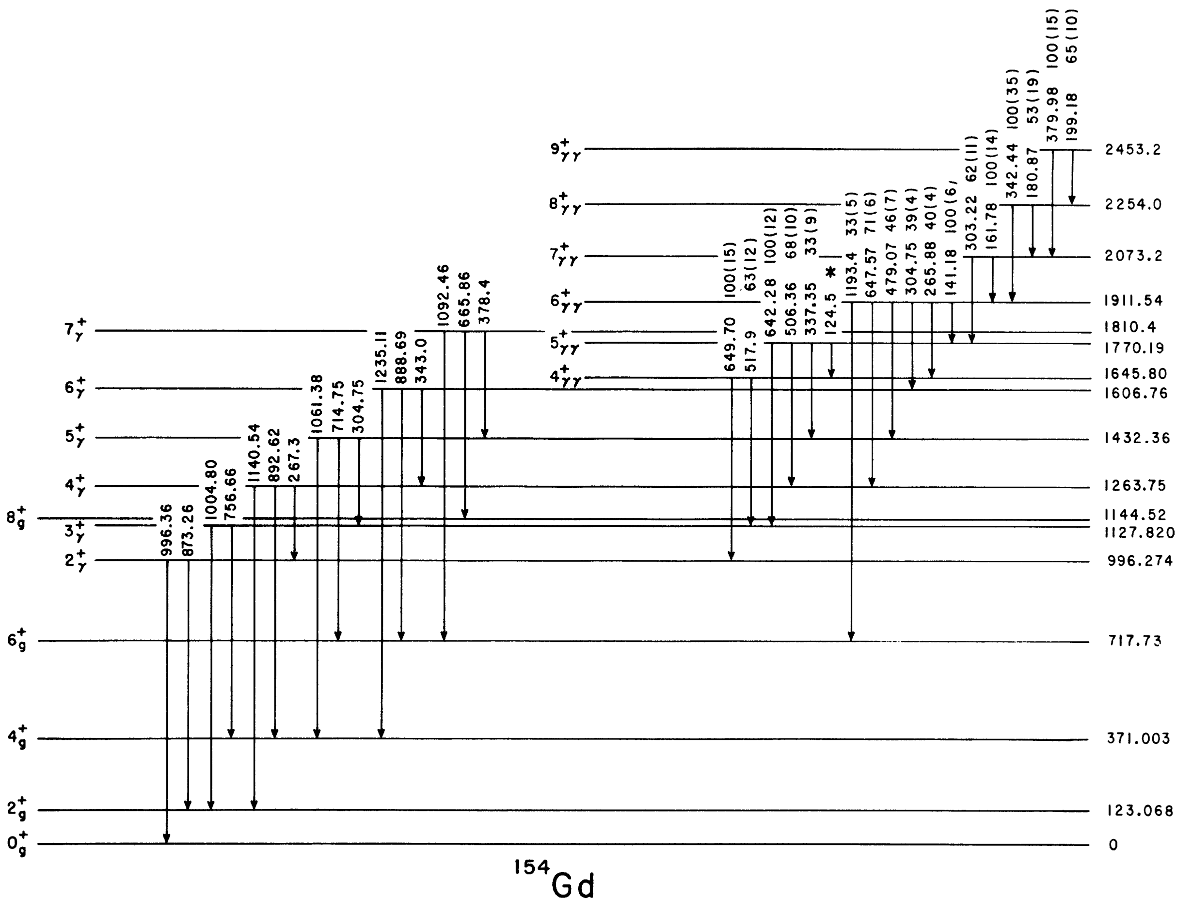

FIG. 2. A partial level diagram showing the observed depopulation of the $K = 2^+$ γ band at 996.27 keV and the $K = 4^+$ band at 1645.8 keV. Energies of the γ rays and their intensities are shown above the transition arrows. Uncertainties in the energies are given in Table I.

include the 441.0-keV and 512.0-keV transitions on top of the 7^+_γ level becuase they are in coincidence with all three transitions 378.4 keV, 665.8 keV, and 1092.5 keV that depopulate the 7^+_γ level. Figures 3(a)–3(c) show the spectra gated by the 888 keV, 1092 keV, and the 1004 keV transitions, respectively. Previously, the authors of Ref. [18] had placed a 441.3-keV transition between a 11^- state at 2482.3 keV and a $9^{(-)}$ state at 2040.5 keV. This is not confirmed in our data. We propose that this 441.0-keV transition belongs to a level at 2251.9 keV.

2. *K = 4 band at 1645.8 keV*

The $K = 4^+$ band at 1645.8 keV was previously known up to the 6^+ state with tentative (7^+) and (8^+) level assignments. A total of 27 transitions were known to depopulate the five levels in the band. We have extended the $K = 4^+$ band to a J^π of (9^+) and added a number of new transitions.

The $5^+_4 \to 4^+_4$ transition in this band has the same energy as the $2^+_g \to 0^+_g$ transition. The $6^+_4 \to 5^+_4$ transition is 141.18 keV. We placed a new 342.44-keV transition between the 8^+_4 and 6^+_4 members of the $K = 4^+$ band. This transition could be separated from the 343.0-keV transition within the γ band by gating on γ rays of 647.57 keV and 479.07 keV. We confirm the (7^+_4) and (8^+_4) states proposed by Ref. [18] and, in addition, we propose a new level at 2453.2 keV. Previously, Ref. [18] had placed a 199.3-keV transition between levels at 2272.8 keV and at 2073.7 keV. Our data clearly disagrees with this possibility due to not only the coincidence of the 199.18-keV transition with the 161.78-keV or the 141.18-keV transitions, but also the coincidence with the 180.87-keV transition. Two transitions, the 199.2(3) keV and the 380.4(7) keV, are also placed in the neighboring isotopes ^{156}Gd and ^{155}Gd. We have considered the possibilities of populating levels in these nuclei. The ^{156}Gd nucleus would have to be produced via the $(\alpha, 2n)$ reaction on the 0.69% ^{154}Sm component of the target, while the ^{155}Gd nucleus would result from the (α, n) reaction on ^{152}Sm. In ^{156}Gd, the 199.2 keV and 380.4 keV γ rays correspond to the $4^+_g \to 2^+_g$ and $8^+_g \to 6^+_g$ transitions, respectively, and should therefore be seen in coincidence. In ^{154}Gd, these two transitions are not in coincidence, while in ^{155}Gd they are placed in a sequence very similar to the ^{154}Gd case with an anticoincidence relationship between

TABLE I. A listing of all the known energy levels of the $K = 2^+$ γ band and the $K = 4^+$ $\gamma\gamma$ band along with their populating and depopulating transitions. Transitions populating and depopulating a given level are shown with their levels of origin.

Level (keV)	I^π, K	Depopulating E_γ (keV)	Final level (keV)	Populating E_γ (keV)	Initial level (keV)
996.274	2^+,2	873.26(6)	123.068	267.3(1)	1263.75
		996.36(17)	0	649.70(6)	1645.80
1127.820	3^+,2	756.66(8)	371.003	304.75(15)	1432.36
		1004.80(8)	123.068	517.9(1)	1645.80
				642.28(7)	1770.19
1263.75	4^+,2	267.3(1)	996.274	343.0(2)	1606.76
		892.62(12)	371.003	506.36(6)	1770.19
		1140.54(12)	123.068	647.57(23)	1911.54
1432.36	5^+,2	304.75(15)	1127.820	337.35(9)	1770.19
		714.75(13)	717.73	378.4(2)	1810.4
		1061.38(6)	371.003	479.07(18)	1911.54
1606.76	6^+,2	343.0(2)	1263.75	304.75(15)	1911.54
		888.69(13)	717.73		
		1235.11(19)	371.003		
1810.4	7^+,2	378.4(2)	1432.36	440.96(9)	2251.4
		665.86(14)	1144.52	512.0(2)	2323.3
		1092.46(6)	717.73		
1645.80	4^+,4	517.9(1)	1127.820	124.5(2)	1770.19
1770.19	5^+,4	124.5(2)	1645.80	141.18(6)	1911.54
		337.35(9)	1432.36	303.22(9)	2073.2
		506.36(6)	1263.75		
		642.28(7)	1127.820		
1911.54	6^+,4	141.18(6)	1770.19	161.78(4)	2073.2
		265.88(7)	1645.80	226.04(4)	2137.49
		304.75(15)	1606.76	342.44(7)	2254.0
		479.07(18)	1432.36	411.9(2)	2323.3
		647.57(23)	1263.75		
		1193.4(4)	717.73		

the 141.18-keV and the 303.22-keV transitions. We have ruled out any significant contributions from both isotopes (^{155,156}Gd) due to the observed coincidence relationships of the 199.18-keV and the 379.98-keV transitions with those of the 141.18 keV and the 303.22 keV γ rays.

A new level is proposed at 2453.2 keV as a member of the $K = 4^+$ band based on the observed coincidences and the relative intensities of the 199.18-keV and 379.98-keV transitions. This level, along with the previously tentative two levels, are fitted smoothly by a simple rotational energy formula [23]. Figure 4 shows the fit in a plot of $(E/2J)$ vs $2J^2$. The experimental DCO ratios are shown in Fig. 5 for the 141.18-keV, 161.78-keV, 180.87-keV, and 199.18-keV transitions, as well as several of the known stretched $E2$ and $E1$ transitions in ^{154}Gd. These ratios [24] were obtained by gating on the forward angle detectors (35°) and getting the ratios of intensities for the backward detectors at 135 and 90 degrees. The best fit of the ratios for the known $E2$ transitions is 1.7, while the value for the $E1$ transitions is 1.1 for this system. The ratios for the four transitions of interest are approximately 1.5, which suggests neither stretched $E2$ nor $E1$. The fact that 161.78-keV, 180.87-keV, and 199.18-keV transitions all have the same DCO ratios as the 141.18 keV suggests $E2/M1$ admixed transitions or $\Delta J = 1$ $E2$ transitions. The angular distribution of the 141.18-keV transition was previously measured [18] to have a quadrupole-to-dipole ratio given by $\delta = 6.1^{+5.3}_{-2.2}$. The extreme lower δ value would be 3.9, implying a lower limit of 93.8% for the $E2$ component of the transition.

III. DISCUSSION

The aim of our study was the search for double-phonon vibrations of the type $\beta\beta$, $\gamma\gamma$, and $\beta\gamma$. The signatures of the first two would be predominant $E2$ decay via collective transitions to the single-phonon vibration of its type, whereas the $\beta\gamma$ type would decay to both types of vibrations. Due to the weak side-feeding of the β band, we were unable to identify bands that show a strong connection to it in this nucleus. The situation was quite different for the γ band at 996 keV, since the data clearly show a strong connection from the $K = 4^+$ band at 1645.8 keV. We therefore propose the 1645.8 keV 4^+ level as the bandhead of the two-phonon $\gamma\gamma$ vibration in ^{154}Gd. The supporting evidence consists of the rotational band structure of the levels built on this state, the strong preference of these states to decay to the $K = 2^+$ γ band via $E2$ transitions, and an approximate "$B(E2)$ test" [25] to show that the transitions interconnecting these two bands are indeed collective.

Relative $B(E2)$ values are given in Table II for the de-

TABLE II. A listing of the initial and final spins, energies, and relative intensities of transitions depopulating the 6^+ member of the $K = 4^+$ band at 1645.80 keV. The observed intensity of the 141.18-keV transition is deduced from the percentage of $E2$ implied by $\delta = 6.1$.

J_i^π	J_f^π	$E\gamma$ (keV)	Relative $B(E2)$ values	Multipolarity	Relative intensities This work	Ref. [20]
6_4^+	5_4^+	141.18	1.3×10^5	$\delta = 6.1$[18]	100(6)	100(10)
	4_4^+	265.88	2.2×10^3	$E2$	40(4)	54(5)
	6_γ^+	304.75	1.1×10^3	$E2$[15]	39(4)	20(5)
	5_γ^+	479.07	137	$E2$[15]	45(7)	45(5)
	4_γ^+	647.57	47	$E2$[15]	71(6)	
	6_g^+	1193.39	1	if pure $E2$	33(5)	41(6)

population of the 6^+ member of the $K = 4^+$ band. The values in this table are extracted from a spectrum gated by the 161.78-keV γ-ray transition that directly feeds this level. This method should yield intensities similar to the singles γ-ray measurements, unless there is another transition of the same energy in coincidence. The difference in the intensity of the $6_4^+ \rightarrow 6_\gamma^+$ 304.75-keV transition observed in this study in comparison of Ref. [20] is most probably due to this fact. The multipolarities for four of the six transitions depopulating this 1911.54-keV level were previously known [15] to be pure $E2$. We have assumed that the 1193.4-keV transition between $6_4^+ \rightarrow 6_g^+$ is pure $E2$, thereby giving an upper limit of its $B(E2)$ strength. Although the ratio of the $B(E2)$ values of the

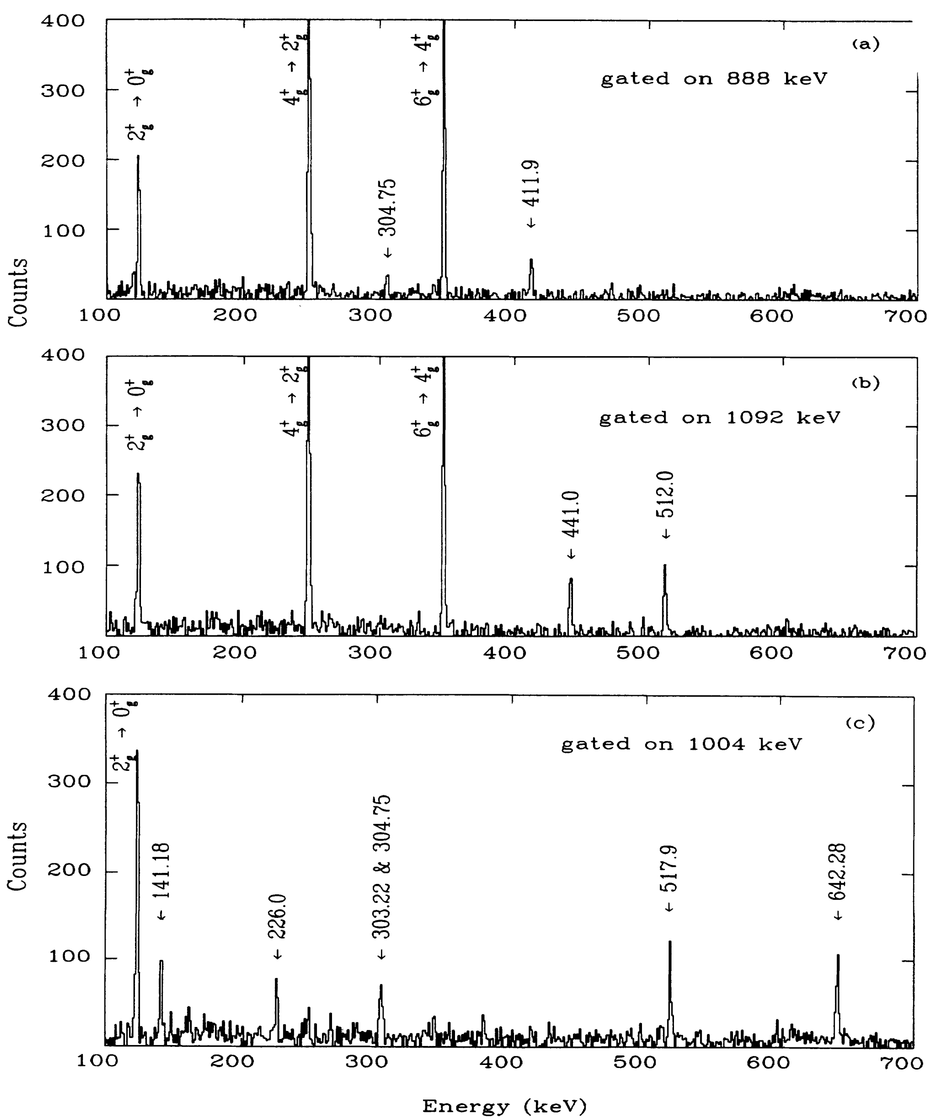

FIG. 3. (a)–(c) Portions of select γ-ray spectra gated by the 888-keV, the 1092-keV, and the 1004-keV transitions in ^{154}Gd, respectively.

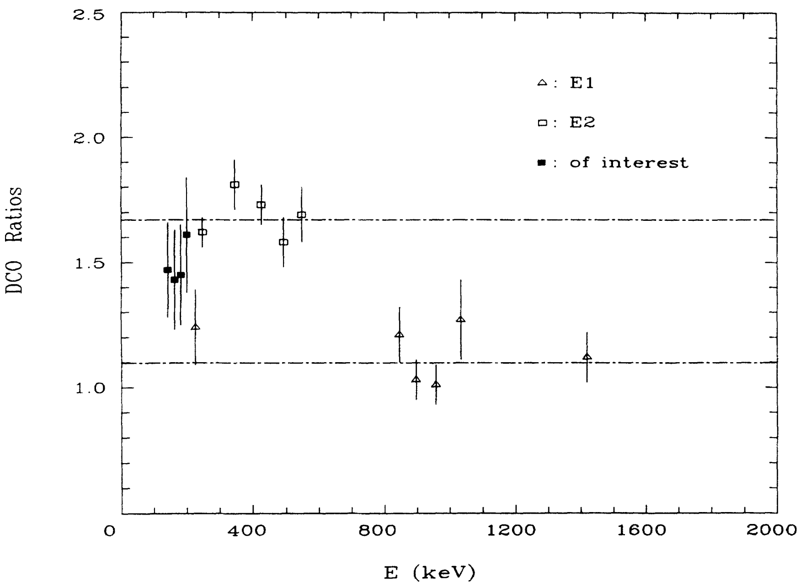

FIG. 4. A plot of the deduced DCO ratios vs the energies of the respective transitions. The open rectangles and triangles represent the known $E2$ and $E1$ transitions, respectively. The filled-in rectangles are the transitions of unknown multipolarity.

$6_4^+ \to 5_4^+$ and the $6_4^+ \to 4_4^+$ transitions do not show the expected Alaga ratio of 5.6 if one uses the measured multipolarity of the $6_4^+ \to 5_4^+$ (the multipolarity of this transition is presently being remeasured), the energy spacings of the levels in the band are clearly rotational. Also, there is a strong preference in decay to the single-phonon γ band rather than the ground-state band, as shown in Table II, of relative $B(E2)$ values. This is true throughout all the levels of this band. The gated spectrum in Fig. 3(c) shows the prominent feeding of the γ band by transitions in the $K = 4^+$ band.

Rotational bands built on vibrational excitations of a deformed nucleus should depopulate within the rotational band by approximately the same $E2$ transition matrix elements as the ground-state rotational band. The quadrupole moments of single-phonon vibrational bands have been shown [26] to be equal to those of the g.s. rotational bands. In the absence of measured level lifetimes, one can use the relative $B(E2)$ values to evaluate in single particle units the collectivity of the connecting transitions between the $K = 4^+$ and the $K = 2^+$ γ bands. The lifetimes and absolute $B(E2)$ values are known for the first 2^+ states in this region and the $6_4^+ \to 4_4^+$ transition is pure $E2$. It is therefore possible to normalize one inband $B(E2)$ to the other. For example, the $B(E2 : 2_g^+ \to 0_g^+)$ in ^{154}Gd is 0.764 e^2b^2 (156 W.u.). The $B(E2 : 6_4^+ \to 4_4^+)$ would be 0.240 e^2b^2 (49 W.u.). Using the relative branching ratios from Table II along with this normalization yields a $B(E2 : 6_4^+ \to 5_\gamma^+) = 0.015$ e^2b^2 (3.1 W.u.). In comparison, the $B(E2 : 2_\gamma^+ \to 0^+)$ value is 6 W.u. If the $K = 4^+$ band at 1645.8 keV was a quasiparticle excitation, the transitions depopulating the band should be a fraction of a Weisskopf unit.

The previous characterization of this band at 1645.8 keV was that of a two-proton quasiparticle configuration: $\frac{3}{2}^+[411\uparrow] + \frac{5}{2}^+[413\downarrow]$ [16,18] based on (d,d') scattering measurements. Inelastic scattering experiments are traditionally viewed as being ideal for locating collective states. Measurements of the ^{154}Gd nucleus had not shown the population of a 4^+ level at excitation energies of 2 MeV (approximately twice the energy of the γ vibration). However, a careful examination of Fig. 2 in Ref. [14] shows a big bump at the excitation energy of the band (approximately 1645 keV) that could easily mask effects in the order of a few μb/sr. The same study [14] shows that in the neighboring isotope of ^{156}Gd, the 4^+ state at 1511 keV has a cross section of 5 μb/sr (at 90°), which is reasonable given the small probability of multiple excitation. In fact, the lifetime of the $K = 4^+$ state at 1511 keV in the ^{156}Gd nucleus was previously

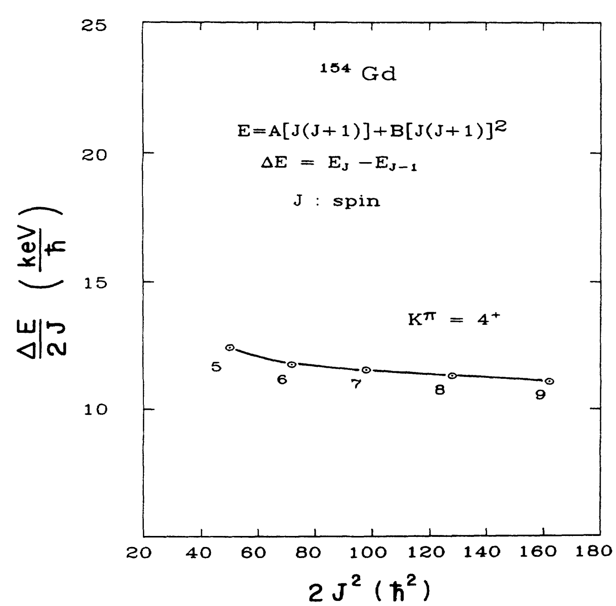

FIG. 5. A fit of the new and previously known levels of the $K = 4^+$ band at 1645.80 keV to the rotational energy formula.

measured [27] and the ratios of the absolute $B(E2)$ values for the transitions depopulating the 4^+ state to the $K = 2^+$ γ band are the same order of magnitude in collectivity as the decay transitions from the γ band to the ground-state band. That is, the values of $B(E2 : 4_4^+ \rightarrow 4_\gamma^+) : B(E2 : 4_4^+ \rightarrow 3_\gamma^+) : B(E2 : 4_4^+ \rightarrow 2_\gamma^2)$ are 2.0:3.6:1.8 W.u. in comparison with a $B(E2 : 2_\gamma^+ \rightarrow 0_g^+)$ of 4.7 W.u. This is similar to the case in ^{168}Er.

One of the most surprising features of these studies in the Gd isotopes in comparison to the ^{168}Er was the ratio of excitation energies of the two-phonon to one-phonon bandhead levels. This ratio is 1.65 in ^{154}Gd and 1.31 in ^{156}Gd compared to 2.5 in ^{168}Er. In all cases, the bands were previously known. We therefore expanded our search to the entire deformed rare-earth region by reexamining the existing data [28]. Our aim was to locate all low-lying 4^+ bands, to calculate the relative $B(E2)$ values, and to pick out the 4^+ bands that have the signatures of strong preference in decay to the 2^+ γ band via $E2$ transitions.

The compiled systematics of the excitation energy ratios of the two-phonon to one-phonon bandhead levels (the energy ratio R) is shown in Fig. 6. The chosen 4^+ bands are not necessarily the lowest-lying $K = 4^+$ bands. In some cases, the first $K = 4^+$ band does not decay to the $K = 2^+$ γ band. In ^{158}Gd, for example, it is the second 4^+ state ($R = 1.62$) that has the signature of strong decay to the γ band. Also, some of these 4^+ bands decay to lower-lying negative parity bands in addition to the γ band with significant intensities, similar to the case of ^{168}Er. The $B(E1)$ values in ^{168}Er were shown [29] to be 10^{-7} W.u. Many of these 4^+ bands have only one or two known levels and their only known decay route is to the 2^+ γ bands. We have found a total of 16 $K = 4^+$ bands that decay to the γ bands in the region.

An outstanding feature of the systematics is the wide range of energy ratios of the excitation energies of the $K = 4^+$ and the $K = 2^+$ bandheads (R). The only regularities that we have observed are the dependence of R on the energy separation of the two single-phonon β and γ vibrations, and on their relative ordering in excitation energy. For example, the largest R values in Fig. 6 correspond to the very large differences in β and γ excitation energies (^{168}Er and ^{164}Dy). The energy differences are shown below the R values in Fig. 6. The sign of the energy difference, $\Delta E_{\beta\gamma} \equiv E_\beta - E_\gamma$, roughly divides R values into two groups above and below a median value of 1.64. For $\Delta E_{\beta\gamma} > 0$, the R values are in general larger than this median value, and for $\Delta E_{\beta\gamma} < 0$, the R values are smaller than the median value. The only significant exception is ^{178}Hf.

We believe that previous characterizations of these states as collective double-phonon vibrations have not been forthcoming due to the unexpectedly wide range of observed anharmonicities as demonstrated by the ratio R. Of course, the most definitive proof would be to obtain the *lifetimes* of all the candidate bands shown here. However, the existing lifetime measurements, and the relative $B(E2)$ values where lifetimes are not yet available, all suggest that the 4^+ states shown in Fig. 6 may be collective double-phonon $\gamma\gamma$ vibrational states and not two quasiparticle excitations.

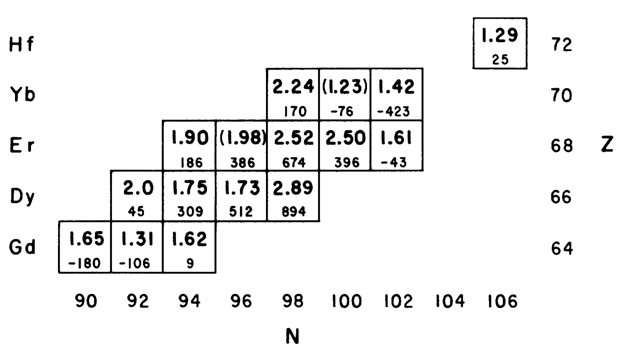

FIG. 6. The larger bolder numbers in this figure are the R values (energy ratios of the excitation energies of the $K = 4^+$ and the $K = 2^+$ γ bandheads) deformed nuclei in the $A = 150$ region. The numbers are given in parentheses if the spin of the $K = 4$ (4^+) state is placed in parentheses in the Nuclear Data Sheets by the evaluator. The smaller figures below the R values are the energy differences of the single phonon β and γ bands.

IV. CONCLUSIONS

In conclusion, the deformed nucleus ^{154}Gd has been studied via the $(\alpha, 2n)$ reaction. Several new transitions and levels have been placed. We have presented evidence for the characterization of the $K = 4^+$ band at 1645.8 keV in ^{154}Gd as a double-phonon $\gamma\gamma$ vibrational band. The anomalous excitation energy ratio of $R = 1.65$ for ^{154}Gd was shown to be part of the systematic trends in this region of nuclei. We present, in total, 16 candidates for double-phonon $K = 4^+$ $\gamma\gamma$ vibrational bands. The ratios of R do not simply depend on the energies of the single-phonon γ bands, but seem to be correlated with the energies of both the β and γ vibrational bands. Further experimental work is in progress to determine the $E2/M1$ admixtures of transitions in ^{154}Gd with better precision as well as further studies to find the higher-lying members of these $K = 4^+$ bands. Theoretical work is much needed in order to understand the effects responsible for the wide range of observed anharmonicities.

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