Nucleon-nucleon potentials and their test with bremsstrahlung

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(Received 16 August 1993)

A complete potential model for nucleon-nucleon bremsstrahlung is used to test t matrices of recent NN potentials. $pp\gamma$ and $np\gamma$ cross sections and spin observables are calculated using the Paris, Bonn-B, and inversion potentials to the most recent phase shift analyses and Bonn-B phase shifts. We confront our results with older noncoplanar $pp\gamma$ data from Harvard and most recent but inclusive data from Saclay and the Indiana University Cyclotron Facility. We obtain a good agreement between experiment and theory for all potentials. Nonlocality effects of potentials, supposedly discerned by bremsstrahlung, are small in comparison to data errors and differences between the potentials themselves.

PACS number(s): 13.75.Cs, 25.20.Dc

I. INTRODUCTION

The study of nucleon-nucleon bremsstrahlung in the framework of a potential model has traditionally been considered as a means for accessing the off-shell domain of the nuclear force [1,2]. Due to a general interest in off-shell t matrix studies and planning of improved new bremsstrahlung experiments in Bloomington, Uppsala, and Jülich we update and extend the theoretical predictions with several of today's most successful NN potentials and a complete bremsstrahlung program.

These experimental programs have for the theory and formulation of NN potentials several benefits. Among them are improvements of the available database below and above meson production threshold and interference studies of mesonic and electromagnetic processes. For the more traditional fields of NN potentials we expect some quantitative improvements and in particular a confinement within the numerous potential models.

Several boson exchange models have been refined over the years and their fit of data is such that χ^2 criteria cannot favor any of the set [3,4]. Nevertheless, there exist differences between the potentials and among them is the prediction of nonlocality from boson exchange models which is in contrast to the success of phenomenological local potentials [5,6]. The amount of nonlocality is also very different between boson exchange potentials and the very many different versions of Bonn potentials are a good example for this circumstance [4]. The possibility of constructing phase equivalent local potentials to nonlocal potentials with inversion or apply unitary transformations to the t matrices is indicative of the mathematically ill-posed problem which enters this physics [7].

Quantum inversion is a rigorous mathematical method to determine off-shell from on-shell t matrices with modest assumptions about the class of underlying potentials [8–10]. Thus, inversion is an appealing method to access potentials from experiment with the benefit of gaining insight into the ill-posedness of the problem. Contrary to inversion, boson exchange models use a fundamental ansatz for an equation of motion and fit parameters to data. A natural consequence of these different approaches is that models have more degrees of freedom to handle and their set of parameters is not unique. Inversion generally has a tendency to underestimate the degrees of freedom due to lack of sufficient or accurate data. In view of this background it is obvious that these numerical studies are also intended to show and compare the overlapping region of model and inversion approaches.

At present we assume that all inversion potentials are local energy independent potentials for partial wave radial Schrödinger equations with the restriction to a potential class [8]

$$\int_a^\infty r|V(r)|\,dr < \infty \quad \text{for } a \geq 0. \tag{1}$$

This fixes the continuation of the t matrix from on-shell into the off-shell domain uniquely and thus off-shell differences between various inversion potentials reflect the differences in the on-shell input [11]. By comparing various inversion t matrices in their on-shell and off-shell domain, one can see that, at least in the off-shell region tested by bremsstrahlung, the splitting between the forces is not perceptibly enlarged. However, this simple link between on-shell and off-shell similarity does not hold necessarily for potentials from other classes. The question is therefore, to what extent predicted off-shell effects in the NN force, i.e., effects stemming from model properties like nonlocalities are manifest in a physical application like bremsstrahlung. Since formal considerations cannot resolve the dispute about nonlocality, we performed calculations with the purpose to compare local and nonlocal potentials in quantities which can be measured with bremsstrahlung.

From the theoretical potentials we select Paris [3], as a representative r-space potential with momentum depen-

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dence, and Bonn-B, a q-space potential [4] with large nonlocality. Both potentials reproduce the two nucleon observables well and have also been successfully used in few- and many-nucleon calculations. Discrepancies of these potentials in breakup Faddeev calculations and few-body and nuclear-matter binding energies systematically show that something is missing in these potentials [12–14]. It is generally agreed upon that local potentials always underbind triton by 800 ± 150 keV [10,12] whereas Bonn-B misses the experimental binding energy of 8.45 MeV only by 250 keV [4]. This is a significant result in favor of a significant nonlocality in nucleon-nucleon potentials. Unfortunately, this improvement with Bonn-B is not unique since triton binding energy calculations with inclusion of some three-body forces give also agreement with experiment and very similar are the findings for nuclear matter calculations [13]. Some theoretical calculations claim small three-body effects for the triton binding energy [14]. At present, there seems to exist no better mechanism than bremsstrahlung which could decide between a genuine three body force and a two body nonlocality. It is our understanding and opinion that this qualitative features of an ambiguity between nonlocality and three-body force cannot be resolved by better fits to two nucleon scattering and bound state data. Certainly, it is favorable to have the fewest discrepancies between data and potential model results and measure this difference with χ^2. In view of QCD, as a possible fundamental theory for strong interactions, there is no doubt that the NN potentials used here are altogether phenomenologically motivated and optimally tuned operators with a limited predictive power outside the fitting range. We expect that an unquestionable advantage of these potentials will remain their effectiveness as interpolating and extrapolating means when we want to describe nuclear structure and low energy nuclear reactions with information taken from two nucleon data.

II. BREMSSTRAHLUNG POTENTIAL MODEL AND RESULTS

The potential model formalism, which we developed and use in this analysis, is an extension of [15–21]. In addition to pole terms of the Pauli operator with electromagnetic interactions, the transition amplitude contains rescattering terms of external currents and exchange contributions to order $\mathcal{O}(k^0)$. Relativistic spin corrections, as obtained from a Foldy-Wouthuysen reduction of the single-particle Dirac equation with an external electromagnetic field, are included to $\mathcal{O}(1/m^3)$ in the external single-scattering and to $\mathcal{O}(1/m^2)$ in the rescattering terms. For $pp\gamma$, the Coulomb potential corrections are included with the usual two potential t-matrix formalism. All other details of model and numerical calculations are given elsewhere [20,22].

Our potential selection comprises inversion potentials based on phase-shift analyses by Arndt-FA91 [23] and Nijmegen-II [6,10] and phase shifts from the Bonn-B potential [4]. The Nijmegen phases have a $\chi^2 \sim 1$, the lowest claimed so far in a phase-shift analysis. For comparison, we consider the boson exchange models from Paris in its configuration space parametrization [3] and the Bonn-B potential in momentum space [4].

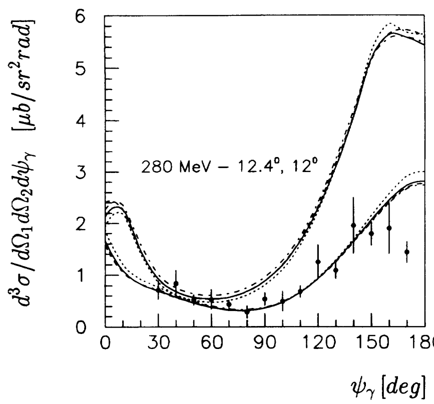

FIG. 1. Theoretical $pp\gamma$ cross sections based on Nijmegen-II inversion (solid), Paris (dash-dotted), Bonn-B (dashed), and Bonn-B phase shifts inversion (dotted) potentials. The lower curves with data [26] are exact and the upper curves are on-shell approximated results.

The photon energy k measures the off-shell characteristic of the process $N_1 + N_2 \to N_1 + N_2 + \gamma$. In the center-of-mass system (cms) of two nucleons before photon emission (initial cms), the energy conservation law requires

$$2\epsilon(p) = k + \epsilon(|\mathbf{p}' + \mathbf{k}/2|) + \epsilon(|\mathbf{p}' - \mathbf{k}/2|) \tag{2}$$

with

$$\epsilon(x) := \sqrt{x^2 + m^2}. \tag{3}$$

p and p' denote incident and outgoing relative momenta of two nucleon systems respectively and m is the nucleon mass. The left side is determined by the kinetic energy of the projectile and is a constant. Thus, large photon energies are attained at high energies and small momenta of the outgoing nucleons. It is well known [24] that the latter situation corresponds to small nucleon angles, i.e., proton emission near the axis. This is the region where off-shell effects in bremsstrahlung amplitudes are expected to be enhanced. At the same time, however, the lower partial waves in the NN t matrix, especially the S states, become more important. Thus, a simple consideration of kinematics by Eq. (2) establishes the link between high off-shell signature and S-wave dominance of the bremsstrahlung process [25].

As a first test we compare our potential set in a kinematic situation where off-shell effects are expected to be large. This is shown in Figs. 1 and 2 for the smallest proton angles available in the TRIUMF experiment [26]. Shown are the published TRIUMF and not the unrenormalized data as used by Herrmann and Nakayama in [27]. These TRIUMF data agree well with the complete potential model calculation where the t matrices are exactly treated with their on- and half off-shell values as they are implied by theory. An approximation, where all t matrices are set to their on-shell limit, i.e., $t(q,k;E(k)) \to t(k,k;E(k))$ in the single-scattering

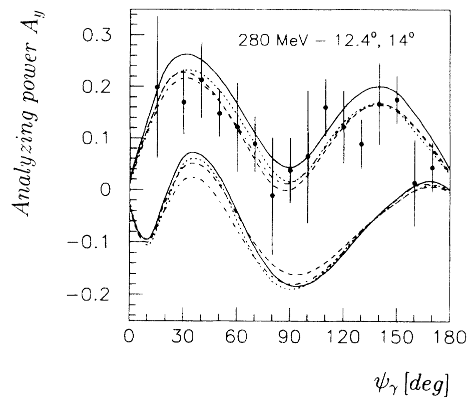

FIG. 2. Theoretical $pp\gamma$ analyzing powers based on Nijmegen-II inversion (solid), Paris (dash-dotted), Bonn-B (dashed), and Bonn-B phase shifts inversion (dotted) potentials. The upper curves with data [26] are exact and the lower curves are on-shell approximated results.

terms, forms the second set of this test. For several potentials, which are not comparably successful in their on-shell predictions, this has been used in the past to demonstrate the off-shell sensitivity of bremsstrahlung and their examples are convincing [19,28]. We add, to this generally accepted feature with our calculation, that all comparable successful on-shell potentials give closely the same results in the approximation set as well as in the exactly treated set. Particular appreciation requires therefore the closeness of the nonlocal Bonn-B with the local inversion potential partner. Any other splitting within potentials is due to slight differences between potentials and their on-shell predictions. It is most unfortunate for the whole bremsstrahlungs program that any sizeable splitting between nonlocal, local-equivalent, and within potentials cannot be predicted. If we permit a few percent uncertainty for the bremsstrahlung potential model and two nucleon data, there seems little left to be learned from these experiments, since all data are well described from any good potential. In the following subsections are shown results of an analysis of classical and most recent data with the purpose to enter a caveat for future experiments.

A. Results for *pp* bremsstrahlung

The features of a mostly on- and off-shell $pp\gamma$ measurement can be seen in Harvard's [29] (Fig. 3) and Indiana University Cyclotron Facility's (IUCF's) [30] (Fig. 4) experiments. Both cover the whole allowed noncoplanar kinematic region and give triple differential cross sections for fixed angles of all three emerging particles. The Harvard experiment was done at medium energy (157 MeV) and we choose here the largest measured symmetric proton angle pair at 35°. IUCF used a medium projectile energy of 294 MeV and their proton angles vary from 4.8° to 12.0°. In Harvard's experiment is the medium photon energy, in the initial cms, 35, 36, and 37 MeV for any low, medium, and high noncoplanarity angle Φ. Values for the three angle bins of the IUCF experiment are 124, 126, and 128 MeV.

All Harvard data are analyzed in their own geometry [29] and are averaged over noncoplanarity bins $\Phi \in \{[0\text{–}1],[1\text{–}2],[2\text{–}3]\}$. With a possible exception at the highest noncoplanarity angle in forward scattering, all data are very well reproduced by any of the used potentials. The Harvard cross sections are dominated by P and coupling PF-matrix elements. Practically, Paris, Nijmegen-II, and Arndt inversion potentials are equivalent and only Bonn-B yields a 3–4 % larger cross section. We note, however, that all the potential differences are much smaller than experimental errors and noncoplanarity does not affect the conclusions we draw from these results.

IUCF's data have been obtained as a by-product of a pion production measurement near threshold and the nominal incident proton energy of 294 MeV is a luminosity weighted average of 31 different bombarding energies in the interval between 278 and 325 MeV. Only events with photon energies between 20 and 120 MeV were detected. As a consequence, the cross section vanishes for small proton and medium photon angles θ_γ. Furthermore, azimuthal angles of emerging particles were not constrained and thus observables are averaged within kinematically permitted noncoplanarities. These data require a careful averaging of the theory since we find

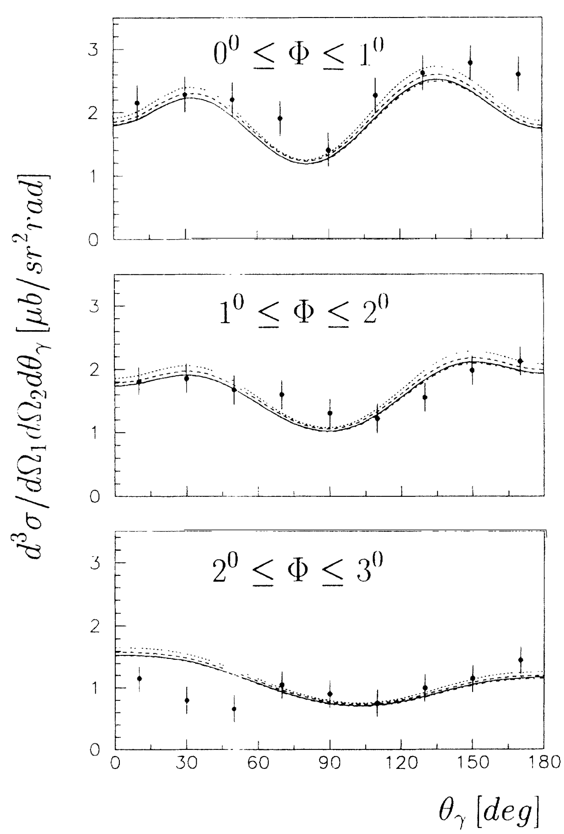

FIG. 3. Noncoplanar $pp\gamma$ cross sections at 157 MeV and proton angles $\theta_1 = \theta_2 = 35°$. Calculations are shown for inversion potentials from Nijmegen-II (solid), Arndt-FA91 (dotted), Bonn-B (dashed), and boson exchange potentials Paris (dash-dotted).

cross sections are highly nonlinear in their dependence on proton and photon polar angles. Finally, we simulate here an angular resolution of 10° in the photon angle θ_γ estimated in [30]. This is approximately done in three steps. First the calculated cross section averaged over a bin ($\theta_{\rm low} < \theta_\gamma < \theta_{\rm high}$) is converted to a relative number of events multiplying it by the solid angle factor $2\pi(\cos\theta_{\rm high} - \cos\theta_{\rm low})$. These events are then smoothed assuming a Gaussian distribution for the actual position of each. The result is finally reconverted to the corrected cross section. The correction effect is obviously largest at forward and backward photon angles where the solid angle factor is minimal. This aspect was neglected in a preliminary analysis [21] and leads to considerable improvements in the fit of the data.

In general, IUCF's data are well reproduced by the potential model, and only a discrepancy for large proton and medium photon angles where we underestimate the data remains unexplained. With all efforts and many trials of incident energy weighting we could not improve the fit. Our conclusion to this misfit is suggesting a repetition of the experiment with an improved detection of all experimental boundaries. In our defense of theory against experiment we bear in mind that these data imply predominantly $^1S_0$ t matrices. This channel, however, has the least uncertainty in the phase shifts and correspondingly all potentials and t matrices are in very close agreement. These results repeat the conclusions we have drawn to results shown above in Figs. 1 and 2 about

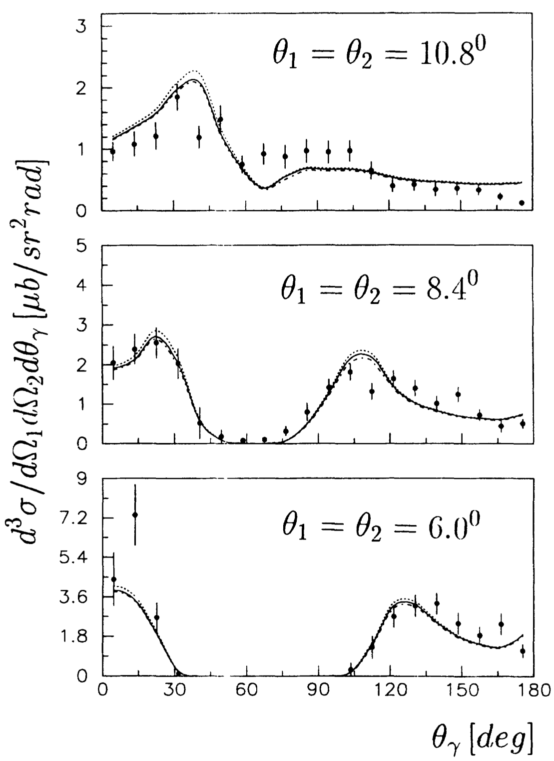

FIG. 4. IUCF measured cross sections at 294 MeV with averaging and bin overlaps as described in [30]. Calculations are shown for inversion potentials from Nijmegen-II (solid), Arndt-FA91 (dotted), Bonn-B (dashed), and boson exchange potentials Paris (dash-dotted).

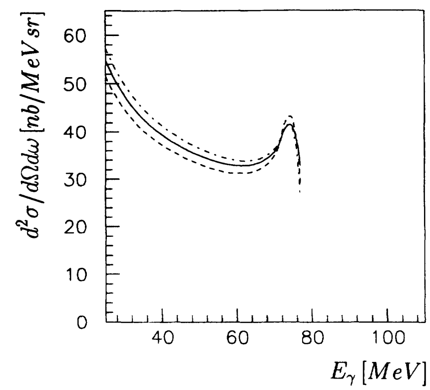

FIG. 5. Result assumes a fixed incident neutron energy of 170 MeV and uses the Nijmegen-II inversion (solid), Paris (dash-dotted), and Bonn-B potentials.

small potential differences and the general insensitivity of bremsstrahlung to half off-shell t matrices.

B. Results for *np* bremsstrahlung

Neutron-proton bremsstrahlung measurements are difficult to perform, so only recently have relatively accurate experimental data become available. We choose here the Saclay experiment [31] where inclusive cross sections $d^2\sigma/d\Omega d\omega$ were measured for an incident neutron energy and estimated width of 170±35 MeV, at 90° fixed photon angle and emerging proton energies ranging between 25 and 110 MeV.

As the photon energy varies in the inclusive experiment and the off-shell dominance enters by increasing E_γ we reach a maximum when $p' = 0$ in Eq. (21). A comparison of single energy curves in Fig. 5 shows indeed that the splitting between Paris and Nijmegen inversion potential results is larger for $np\gamma$ than for $pp\gamma$. However, the results get closer in the extreme off-shell domain where the SD channels dominate the $np\gamma$ cross section [17]. The mentioned difference in Fig. 5 is thus more likely a consequence of improvements of newest potentials since in the last decade np data were improved considerably and

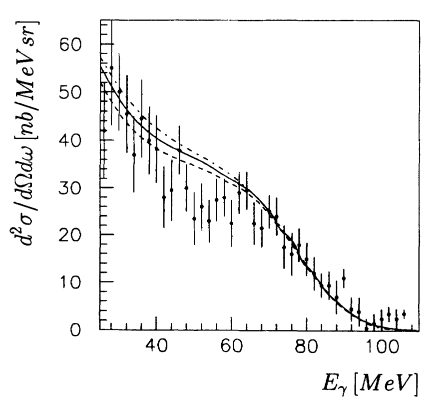

FIG. 6. Complete model results for inclusive $np\gamma$ cross sections at 170 ± 35 MeV based on Nijmegen-II inversion (solid), Bonn-B (dashed), and Paris (dash-dotted) genuine potentials. All theoretical result are averaged with a Gaussian energy distribution of 35 MeV (full width at half maximum) around 170 MeV with data from [31].

Nijmegen-II phase shifts should be better than older ones used to fit the Paris potential.

All features of single energy calculations remain unchanged when a bombarding energy distribution is included whose results are shown in Fig. 6. For high photon energies is any potential prediction practically the same, and our calculations are fully consistent with results of Herrmann and Nakayama [20]. It is in good agreement with data except for photon energies between 50 and 70 MeV where the experiment suggests a local minimum in the cross section. Of this latter point we could recently show [22] that a minimum arises simply by assuming, instead of a symmetric Gaussian, an asymmetric energy distribution for the incident neutron, e.g., a low energy cutoff in the Gaussian distribution at $\sim$140 MeV.

III. CONCLUSIONS

The bremsstrahlung potential model in complete form gives good predictions for $pp\gamma$ and $np\gamma$ data below 300 MeV. From a theoretical point of view, there are no principal objections against inclusive measurements as long as any experimental conditions, such as energy distributions, angular resolutions, etc. are well defined and specified. All theoretical results of modern potentials are largely equivalent. Model dependent off-shell effects of NN forces yield no distinctive features in bremsstrahlung and with local inversion potentials it is possible to incorporate any aspect of two nucleon data.

ACKNOWLEDGMENTS

We would like to thank Dr. Holger Kohlhoff for his help and for letting us use the inversion potentials contained in his thesis. This work was supported in part by COSY, KFA–Jülich, Grant No. 41126865.

[1] M.I. Sobel and A.H. Cromer, Phys. Rev. **132**, 2698 (1963).

[2] M.L. Halbert, in *Proceedings of the Gull Lake Symposium on the Two-Body Force in Nuclei*, edited by S.M. Austin and G.M. Crawley (Plenum, New York, 1972).

[3] M. Lacombe *et al.*, Phys. Rev. C **21**, 861 (1980).

[4] R. Machleidt, in *Advances in Nuclear Physics*, edited by J.W. Negele and E. Vogt (Plenum, New York, 1989), Vol. 19; J. Haidenbauer, K. Holinde, and M.B. Johnson, Phys. Rev. C **45**, 2055 (1992).

[5] I.E. Lagaris and V.R. Pandaripande, Nucl. Phys. **A359**, 331 (1981); R.B. Wiringa, R.A. Smith, and T.L. Ainsworth, Phys. Rev. C **29**, 1207 (1984).

[6] V.G.J. Stoks and J.J. de Swart, Phys. Rev. C **47**, 761 (1993); private communication.

[7] M.K. Srivastava and D.W.L. Sprung, in *Advances in Nuclear Physics*, edited by M. Baranger and E. Vogt (Plenum, New York, 1975), Vol. 8.

[8] K. Chadan and P.C. Sabatier, *Inverse Problems in Quantum Scattering Theory* (Springer, New York, 1989).

[9] Th. Kirst *et al.*, Phys. Rev. C **40**, 912 (1989).

[10] H. Kohlhoff, M. Küker, H. Freitag, and H.V. von Geramb, Phys. Scr. **48**, 239 (1993); H. Kohlhoff, *Nukleon-Nukleon Potentiale und Quanteninversion der S-Matrix*, thesis, Hamburg, 1993; inversion potentials are retrievable by anonymous FTP from I04KTHA.DESY.DE.

[11] H.V. von Geramb, in *Quantum Inversion Theory and Applications*, edited by H.V. von Geramb, Lecture Notes in Physics Vol. 427 (Springer, Berlin, 1993).

[12] J.L. Friar, B.F. Gibson, and G.L. Payne, Phys. Rev. C **37**, 2869 (1988); J.L. Friar, G.L. Payne, V.G.J. Stoks, and J.J. deSwart, Los Alamos National Laboratory report, 1993.

[13] V.R. Pandaripande, Nucl. Phys. **A553**, 191c (1993).

[14] T.-Y. Saito and I.R. Afnan, Flinders University report, 1993.

[15] V.R. Brown and J. Franklin, Phys. Rev. C **8**, 1706 (1973).

[16] R.L. Workman and H.W. Fearing, Phys. Rev. C **34**, 780 (1986).

[17] K. Nakayama, Phys. Rev. C **39**, 1475 (1989).

[18] V. Herrmann, J. Speth, and K. Nakayama, Phys. Rev. C **43**, 394 (1991).

[19] V. Herrmann and K. Nakayama, Phys. Rev. C **45**, 1450 (1992).

[20] V. Herrmann and K. Nakayama, IKP-KFA Jülich, Annual Report, 1992.

[21] M. Jetter, H. Freitag, and H. V. von Geramb, Nucl. Phys. **A553**, 655c (1993); Phys. Scr. **48**, 228 (1993).

[22] M. Jetter, *Nukleon-Nukleon und Nukleon-Antinukleon Bremsstrahlung im Potentialmodell*, thesis, Hamburg, 1993.

[23] R.A. Arndt, Phys. Rev. D **45**, 3995 (1992); SAID interactive dial in system.

[24] H.W. Fearing, Phys. Rev. Lett. **42**, 1394 (1979).

[25] V. Herrmann and K. Nakayama, Phys. Lett. B **251**, 6 (1990).

[26] K. Michaelian *et al.*, Phys. Rev. D **41**, 2689 (1990).

[27] V. Herrmann and K. Nakayama, Phys. Rev. C **46**, 2199 (1993).

[28] A. Katsogiannis and K. Amos, Phys. Rev. C **47**, 1376 (1993).

[29] B. Gottschalk, W.J. Shlaer, and K.H. Wang, Nucl. Phys. **A94**, 491 (1967).

[30] B. v. Przewoski *et al.*, Phys. Rev. C **45**, 2001 (1992).

[31] F. Malek *et al.*, Phys. Lett. B **266**, 255 (1991).