PHYSICAL REVIEW C VOLUME 49, NUMBER 4 APRIL 1994

New effective internucleon forces in microscopic α-cluster model

Akihiro Tohsaki
Department of Fine Materials Engineering, Shinshu University, Ueda, 386, Japan
(Received 4 October 1993)

We present new effective internucleon forces responsible for the microscopic α-cluster model. We introduce the two- and three-body operators with finite ranges, which play an essential role in accounting for the bulk properties of nuclei in a broad range of masses including nuclear matter. The proposed forces are suitable for analyzing the elastic α-α scattering.

PACS number(s): 21.60.Gx, 21.30.+y, 21.65.+f, 25.55.Ci

The microscopic cluster model has played an important role in understanding the property of light nuclei from both bound and scattering states [1,2]. Its framework is based simultaneously on the full account of the Pauli principle among all of the nucleons and the effective internucleon force (EINF). It is well known that the interaction between clusters obtained from a microscopic point of view strongly depends upon what kind of EINF is used. However, until now there has been no appropriate EINF which can reproduce nuclear saturation properties over the wide mass number region. On the other hand, the realistic internucleon forces based on the meson theory have successfully explained the properties of $(0s)$-shell nuclei. We can enumerate the Reid potential [3], the Paris potential [4], and the Bonn potential [5]. Nevertheless, we need still more steps to apply them to multicluster structures or scattering phenomena between nuclei, since these problems are closely concerned with the model wave functions considered.

We have known only two kinds of EINF's which can guarantee the saturation properties for nuclear matter, namely the Brink-Boeker force [6] and the Skyrme-type force [7]. The other EINF's such as the Volkov force [8], the Hasegawa-Nagata force [9], and the Minesota force [10] usually adopted by cluster-model study for light nuclei could not prevent nuclear matter from collapsing. However, the Brink-Boeker force and the Skyrme-type force behave totally different from each other in satisfying the saturation property for nuclear matter. The former has the strong Majorana mixture in the short-range force, which does not act on the binding energy of α particle but leads to normal density of nucleons in nuclear matter. Therefore, it can reproduce the bulk properties of α particle and nuclear matter at the same time. Nevertheless, since this force behaves very badly for heavy nuclei beyond the $(0s)$-shell region, most nuclear physicists have avoided using it in studying the cluster model. On the other hand, the latter normally includes the zero-range three-body operator to conserve the density of nucleons. Unfortunately, this point makes it impossible to guarantee an appropriate size of α particle given by electron scattering. Besides, we have hesitated to use this force for scattering phenomena between light nuclei because it also includes the zero-range two-body operator, which leads to too many hard peripheral effects.

The EINF with two-body operators alone, which can fulfill the saturation properties of α particles and ^{16}O nuclei simultaneously, generally gives an excess of repulsion between two α particles [11]. In other words, no usual EINF allows us to explain the bulk properties for both two separated nuclei and two fused nuclei in the microscopic cluster model. Therefore, we have no reasonable EINF even for light two-nucleus system. We are afraid that the cluster model approaches for light nuclei up to now have included some incorrect conclusions by using the traditional EINF's such as Volkov force and so on.

The main aim of this paper is to propose new kinds of EINF's responsible for the microscopic α-cluster model by satisfying overall saturation properties for nuclei in a broad range of masses from α particle to nuclear matter. In order to do this, we examine new kinds of EINF covering double closed shell nuclei up to ^{40}Ca and nuclear matter. Next we shed light on elastic α-α scattering from a microscopic point of view using new EINF's to be proposed. In our previous paper [12], we investigated the properties of the EINF by focusing on the ^{16}O-^{16}O interaction. In this case, we introduced not only two-body operators with finite ranges which can give a softness to nucleus-nucleus interaction but also zero-range three-body operator like the Skyrme force in order to guarantee the saturation property for nuclear matter. However, such kinds of EINF shows serious defects, namely, lack of accordance with both the experimental size of α particle and the empirical incompressibility of nuclear matter. In short, using it, we cannot remove such weak points as the Skyrme-type force has. Therefore, it may be very important to investigate how well the EINF including three-body operators with finite ranges would behave in nucleus-nucleus interaction.

We begin with the microscopic Hamiltonian as follows:

$$\mathcal{H} = -\frac{\hbar^2}{2M}\sum_i \nabla_i^2 - T_G + \sum_{i<j} V_{ij}^C + \sum_{i<j} V_{ij}^{(2)} + \sum_{i<j<k} V_{ijk}^{(3)}, \tag{1}$$

where the first term stands for the kinetic energy operator, the second is c.m. energy one to be removed, and the third means the Coulomb energy one. The last two terms just corresopnd to the EINF which is divided into the two-body and three-body operators. Here, we assume the superposition of the Gaussian functions with the EINF, whose operators have finite ranges such as

$$V_{ij}^{(2)} = \sum_n v_n^{(2)} \exp\left\{-\left(\frac{r_{ij}}{r_n^{(2)}}\right)^2\right\}(W_n^{(2)} + M_n^{(2)}P_{ij}) \quad (2)$$

and

$$V_{ijk}^{(3)} = \sum_n v_n^{(3)} \exp\left\{-\left(\frac{r_{ij}}{r_n^{(3)}}\right)^2 - \left(\frac{r_{jk}}{r_n^{(3)}}\right)^2\right\} \times (W_n^{(3)} + M_n^{(3)}P_{ij})(W_n^{(3)} + M_n^{(3)}P_{jk}), \quad (3)$$

where $v_n^{(k)}$ and $r_n^{(k)}$ are the strength and the range parameters in the k-body operators, respectively. The P_{ij} means the Majorana exchange operator defined by $-P_{ij}^{\sigma}P_{ij}^{\tau}$. The parameters $W_n^{(k)}$ and $M_n^{(k)}$ are also the strengths for the Wigner and Majorana forces. Here, since we concentrate our interest on the $4N$ nuclei, the Bartrett and Heisenberg parts are omitted, having no contribution. It is well known that the relation of $W_n^{(k)} + M_n^{(k)}$ should satisfy the unity. These formulas can be easily understood as a natural extension from zero-range two- and three-body operators.

As a preliminary attempt, we present two kinds of parameter sets arranged in Tables I and II. For simplicity, the range parameters are assumed to be common for two- and three-body operators. The longest range is set to adjust the one-pion exchange effects between two nucleons. For the effective two-body operator, the short-range part acts as the repulsion in the innermost region and the middle range part bears the attractive effect in nucleon-nucleon force. At present, we have no criterion for the character of the three-body operator except that it cannot show attractive effects, in order to guarantee the saturation property for nuclear matter. Following the condition mentioned above, the other parameters concerned with the strengths and the exchange characters are chosen so as to reproduce the bulk properties for double closed shell nuclei up to ^{40}Ca and nuclear matter simultaneously. The wave functions for double closed shell nuclei and nuclear matter are represented by the harmonic oscillator and the plane wave, respectively. The total binding energies for the double closed shell nuclei are calculated as the function with the size parameter a written by $\phi_l(r) = NL_l(\sqrt{a/2r})\exp(-ar^2)$, where L_l is the Laguerre polynomial and N the normalization factor. The binding energy per nucleon in the nuclear matter is estimated in terms of the Fermi momentum as well. The calculated results are listed in Table III, where the giant monopole excitations for the double closed shell nuclei and the incompressibility for the nuclear matter are also estimated. In this table, for comparison we add the properties obtained from the Brink-Boeker (BB) and the Skyrme 2 forces modified by Vautherin and Brink (SII) [13]. Here, the giant monopole state energy is defined by

$$\hbar\Omega = \sqrt{\frac{\hbar^2}{M}\frac{K}{\langle r^2\rangle}} \quad (4)$$

with

$$K = \frac{4a^2}{A}\frac{d^2E(a)}{da^2}\bigg|_{a=\in\min\{E(a)\}}, \quad (5)$$

where $\langle r^2\rangle$ means the rms radius of nucleus and A is the mass number of nucleus. We can see from this table that the proposed EINF's nicely overcome the disadvantages in the traditional EINF's. For instance, the size of α particle is fitted to the empirical value which has never been achieved by the SII force. The bulk properties for ^{16}O and ^{40}Ca also behave well under the proposed EINF's, but are hardly fitted under the BB force. The empirical incompressibility [14] of nuclear matter is wonderfully reproduced by the proposed EINF's as well; these are the first appropriate EINF's which can explain the empirical incompressibility of nuclear matter. The finite-range three-body operator not only plays an important role in moderating the hardness of the nuclear matter, but also gives α particle of the correct size in contrast with those for the zero-range force.

Next, it is very important to apply the proposed EINF's to the elastic α-α scattering. This is because the EINF responsible for the microscopic α-cluster model should inevitably reproduce the experimental phase shifts for α-α scattering [15]. However, when we use the EINF's with two-body operators alone which can satisfy the binding energies for $(0p)$-shell nuclei, the α-α interaction will be absolutely repulsive in contrast with the experimental data. In Fig. 1, we show the calculated phase shifts for the elastic α-α scattering together with those for the BB and SII forces. Here, we use the resonating group method when obtaining the phase shifts. The results are compared with the experimental data up to $l = 6$ for the proposed EINF's and up to $l = 4$ for the BB

TABLE I. Parameter set for the EINF 1 proposed (F1 force).

n	$r_n^{(k)}$ (fm)	$v_n^{(2)}$ (MeV)	$M_n^{(2)}$	$v_n^{(3)}$ (MeV)	$M_n^{(3)}$
1	2.5	−5.00	0.750	−0.31	0.0
2	1.8	−43.51	0.462	7.73	0.0
3	0.7	60.38	0.522	219.00	1.909

TABLE II. Parameter set for the EINF 2 proposed (F2 force).

n	$r_n^{(k)}$ (fm)	$v_n^{(2)}$ (MeV)	$M_n^{(2)}$	$v_n^{(3)}$ (MeV)	$M_n^{(3)}$
1	2.5	−5.00	0.750	0.05	0.0
2	1.4	−102.49	0.420	16.10	0.0
3	0.7	237.66	0.508	219.00	−1.167

TABLE III. Physical quantities for double closed shell nuclei together with nuclear matter.

		F1	F2	BB	SII	Expt.
α	$E_{\rm bin}$ (MeV)	27.5	27.0	27.4	26.0	28.3
	a (fm^{-2})	0.50	0.50	0.50	0.41	0.51
^{16}O	$E_{\rm bin}$ (MeV)	123.0	124.0	93.0	121.4	127.6
	a (fm^{-2})	0.35	0.35	0.31	0.32	0.34
	$E_{\rm mp}$ (MeV)	23.3	27.7	23.2	32.7	26.4
^{40}Ca	$E_{\rm bin}$ (MeV)	334.0	340.2	250.8	325.5	342.1
	a (fm^{-2})	0.26	0.27	0.25	0.26	0.27
	$E_{\rm mp}$ (MeV)	20.8	24.1	19.0	27.0	23.5
NM	E/A (MeV)	17.0	16.7	15.7	16.0	16.0
	k_f (fm^{-1})	1.27	1.29	1.45	1.30	1.30
	K (MeV)	309	332	193	342	300

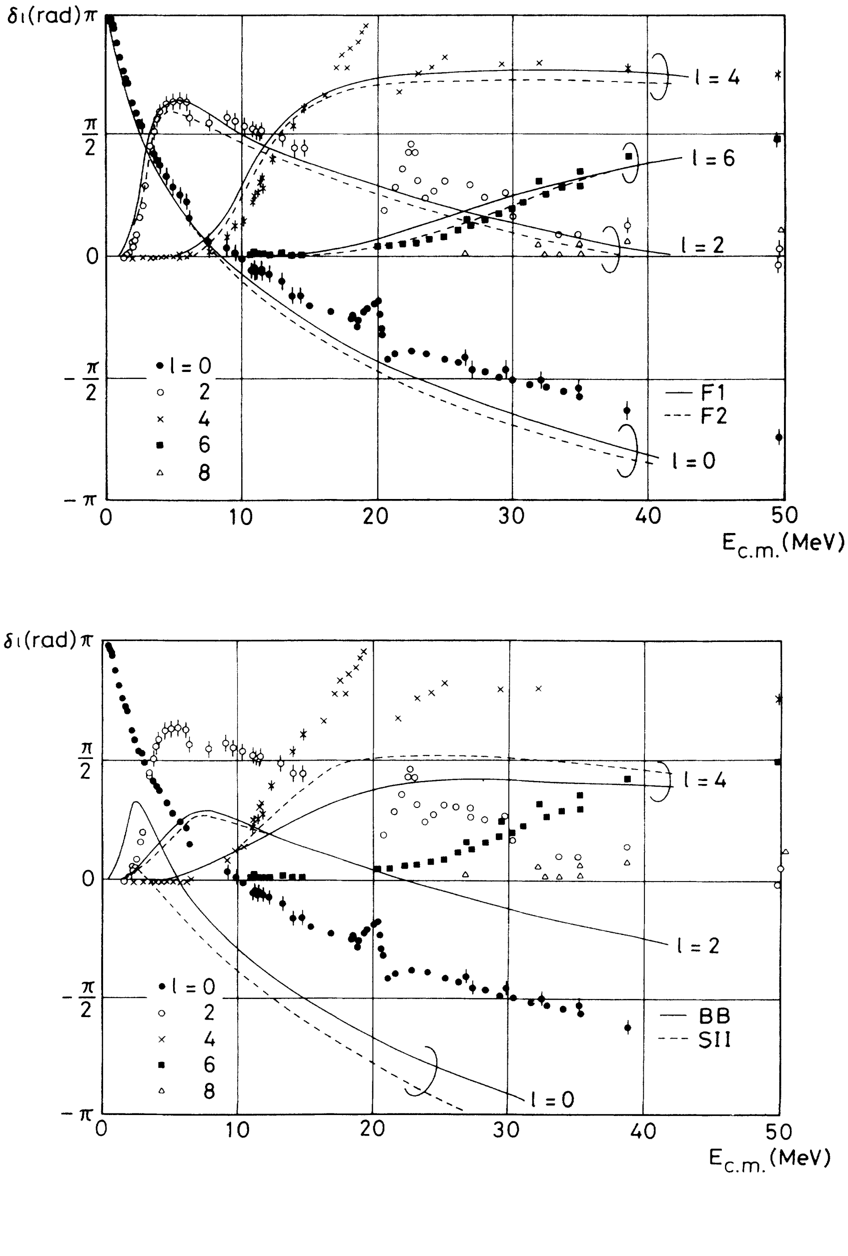

FIG. 1. The calculated phase shift of the elastic α-α scattering. The upper part corresponds to the results for the proposed forces, and the lower part is those for the BB and SII forces.

TABLE IV. Total binding energies for ^{12}C and ^{20}Ne.

		F1	F2	BB	SII	Expt.
^{12}C	E_{bin} (MeV)	75.2	74.6	55.5	87.5	92.6
	a (fm^{-2})	0.36	0.36	0.31	0.33	0.38
^{20}Ne	E_{bin} (MeV)	153.0	153.8	119.5	143.8	158.7

and SII forces. The proposed EINF's tend to be a little repulsive in accordance with the experimental data, and greatly improve the description of the α-α interaction by removing the disadvantages in the BB and SII forces. The behavior of the relative motion strongly depends on the property of two-body interaction. The defects in the BB force mainly come from the poor reproduction of the binding energy of $(0p)$-shell nuclei. Although the binding energy for α particle and ^{16}O are fairly well reproduced by the SII force, the extremely strong repulsion originates in the zero-range two-body operator. Therefore, the finite-range two-body operator plays an essential role in explaining the behavior of the phase shifts for α-α scattering.

We also show whether the proposed EINF's are applicable to ^{12}C and ^{20}Ne nuclei or not. The wave function for ^{12}C is assumed to be the SU3-(04) state corresponding to the zero-distance limitation from the 3α-cluster configuration. In fact, we should treat dynamically the 3α-cluster state, but this subject is too heavy to be carried out in this preliminary work. On the other hand, the ground state of ^{20}Ne is described by α+^{16}O states with different size parameters which can agree with the experimental data. We solve this problem by the resonating group method using the same computational program as we do in analyzing the elastic α-α scattering. In Table IV, we list the total binding energies as a preliminary result. We can see the following features: (i) The total binding energy for ^{20}Ne can be very well reproduced by the proposed EINF's. (ii) In contrast, the bulk property for ^{12}C can never be explained from the SU3-(04) state. The 3α configuration may not be expected to overcome this defect as well, because it does not make the SU3-(04) configuration change drastically. This problem remains unsolved for the present.

We conclude that the proposed EINF's definitely remove the disadvantages of the BB and SII force. In particular, the three-body operator with finite ranges plays an essential role in accounting for the bulk properties for nuclei in a broad range of masses as well as for nuclear matter. If we introduce the Bartrett and the Heisenberg terms in the proposed EINF's, the experimental data for the deuteron, ^{3}H, and ^{3}He will be fitted as well.

The author thanks not only T. Matsuse and A. Arima for useful discussions on the early stage of this work but also D. Baye, M. Kruglanski, P. Hodgson, and R. Tamagaki for their fruitful comments on both light nuclei and nuclear matter.

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