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QCD Fokker-Planck equations with color diffusion

Alexei V. Selikhov* and Miklos Gyulassy
Department of Physics, Columbia University, New York, New York 10027
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Fokker-Planck equations are derived for QCD Wigner distributions taking into account quantum color dynamics. These equations show that the anomalously large color diffusion coefficient in a high T quark-gluon plasma leads to strong damping of collective color modes.

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The development of QCD transport theory [1–8] is required to link predicted properties of quark-gluon plasmas (QGP) [9] with experimental observables [10] from collisions of ultrarelativistic ($\sqrt{s} > 0.1$ A TeV) nuclei. Due to copious minijet production [11], the QGP produced in such collisions is initially out of local equilibrium, and the evolution must be calculated with an approach that takes into account the finite relaxation times. One approach, based on chromoviscous-hydrodynamics [2,12], describes the bulk dynamics in terms of the equation of state and transport coefficients such as the color and thermal conductivity and shear and bulk viscosity. However, a microscopic kinetic theory is required to estimate those coefficients. In this report, we derive a new set of Fokker-Planck type equations for QCD which describe the diffusion in both color and momentum space. Our approach is based on the kinetic theory formulation developed in [7] taking into account the full non-Abelian structure of the collision terms. We calculate not only the momentum relaxation time [13] that controls the friction and diffusion coefficients in momentum space but also the color relaxation time that controls the color diffusion coefficient introduced in [8]. We show that the same (divergent) color diffusion coefficient arises in both quantum and classical treatments of color dynamics. That divergence is caused by unscreened long range color magnetic fluctuations and is regulated by introducing a nonperturbative magnetic mass [14]. The kinetic equations derived here imply that long wavelength collective color modes in a QGP are more strongly damped by collisions (by a factor $1/\alpha_s$) than their Abelian counterparts. Non-Abelian plasmas are therefore also poor (color) conductors. The QCD Fokker-Planck equations derived here show that, unlike in electromagnetic plasmas, the divergent damping rates of hard partons computed diagrammatically (as in [14]) have considerable physical impact on the collective properties of QGP.

In kinetic theory, nonequilibrium physical systems are described by means of one-particle phase space distribution functions, which are statistical averages of appropriate Wigner operators. The gauge covariant quark, $\hat{Q}^+(p,x)$, and antiquark, $\bar{Q}(p,x) = \hat{Q}^-(-p,x)$, Wigner operators for SU(N) are $N \times N$ matrices in color spacc. They are related to the gauge covariant quark Wigner operator [1,3] in the Heisenberg representation via $\hat{Q}^{\pm}(p,x) = \theta(\pm p_0)\delta(p^2)\hat{W}(p,x)$, where

$$\hat{W}(p,x) = \int \frac{d^4y}{(2\pi)^4} e^{-\imath p\cdot y} \hat{\bar{\psi}}(x) e^{\frac{y}{2}\hat{D}^\dagger} \otimes e^{-\frac{y}{2}\hat{D}} \hat{\psi}(x) \ , \quad (1)$$

where $\hat{\psi}(x)$ is the N component quark field operator, and $\hat{D}_\mu = \partial_\mu + \imath g \hat{A}_\mu$, where $\hat{A}_\mu = \hat{A}^a_\mu t^a$ is the gluon field. Note that t^a are the $(N^2-1)\times(N^2-1)$ hermitian generators of SU(N) in fundamental representation, and the field tensor is $\hat{F}_{\mu\nu} = [\hat{D}_\mu, \hat{D}_\nu]/(ig)$. The covariant gluon Wigner operator $\hat{G}(p,x)$ is an $(N^2-1)\times(N^2-1)$ matrix in color space and is defined similarly [4,7].

Near equilibrium at high temperature $T \gg 200$ MeV, the typical momentum transfers, $k \sim gT$, in the plasma are perturbatively small compared to the average momenta, $p \sim 3T$. In that case, spin effects can be neglected in the first approximation, and the evolution can be treated in the eikonal approximation assuming approximate straight line trajectories. This physical picture forms the physical basis behind the hard thermal loop approximation [15] in high temperature pQCD. With spin effects neglected, the Wigner operator obeys the following dynamical equations in the semiclassical limit [3]:

$$p^\mu \hat{D}_\mu \hat{Q}^{\pm}(p,x) + g p^\mu \partial^\nu_p \frac{1}{2}\{\hat{F}_{\nu\mu}, \hat{Q}^{\pm}(p,x)\} = 0 \ , \quad (2)$$

where $\{,\}$ denotes the anticommutator. A similar equation holds for the gluon Wigner operator with generators t^a replaced by those in the adjoint representation [3,4,7]. The quark and gluon phase space densities are defined as quantum-statistical averages of the corresponding operators: $Q^{\pm}(p,x) = \langle\hat{Q}^{\pm}(p,x)\rangle$, $G(p,x) = \langle\hat{G}(p,x)\rangle$. In equilibrium,

$$Q^{\pm}_{\rm eq} = \frac{2N_f}{(2\pi)^3} 2\theta(\pm p_0)\delta(p^2)(\exp(\pm(p\cdot u)/T)+1)^{-1} \ ,$$

$$G_{\rm eq} = \frac{2}{(2\pi)^3} 2\theta(p_0)\delta(p^2)(\exp((p\cdot u)/T)-1)^{-1} \ , \quad (3)$$

where $u^\mu(x)$ is the local four velocity of the plasma at local temperature $T(x)$. For small deviations from these color neutral equilibrium distributions, we write

$$Q^{\pm}_{ij} = Q^{\pm}_{\rm eq}\delta_{ij} + \Delta Q^{\pm}_{ij} \ , \quad G_{ab} = G_{\rm eq}\delta_{ab} + \Delta G_{ab} \ . \quad (4)$$

*Permanent address: Kurchatov Institute, 123182 Moscow D-182, Russia.

The nonequilibrium deviations obey the linearized kinetic equations derived in Ref. [7]:

$$p^\mu \partial_\mu \Delta Q^\pm + g p^\mu \partial_p^\nu \Delta F_{\nu\mu} Q^\pm_{\rm eq} = \Delta C_1^\pm(p,x) + \Delta C_2^\pm(p,x)\,, \tag{5}$$

where the linearized collision term on the right-hand side has been decomposed into two parts. The first, which as we show below describes diffusion in momentum space, is given by

$$\Delta C_1^\pm(p,x) = \frac{\alpha_s^2}{2}\int dp' dk (k\partial_p)\Big[\mid p^\mu D_{\mu\nu} p'^\nu \mid^2 \delta(pk)\delta(p'k)(\varepsilon(p_0)k\partial_p - \varepsilon(p_0')k\partial_{p'})\Big(\mathcal{N}_{eq}(p')\delta^{ab}\{t^a,\{t^b,\Delta Q^\pm(p,x)\}\} + \frac{1}{2}\Delta\mathcal{N}^{\{ab\}}(p',x)\{t^a,\{t^b,Q^\pm_{\rm eq}(p)\}\}\Big)\Big]\ . \tag{6}$$

The second, which has no Abelian counterpart and, as shown below, describes diffusion in color space, is given by

$$\Delta C_2^\pm(p,x) = -2\alpha_s^2 \int dp' dk \mid p^\mu D_{\mu\nu} p'^\nu \mid^2 \delta(pk)\delta(p'k)\Big(\mathcal{N}_{eq}(p')\delta^{ab}\varepsilon(p_0)[t^a,[t^b,\Delta Q^\pm(p,x)]] + \frac{1}{2}\Delta\mathcal{N}^{[ab]}(p',x)\varepsilon(p_0')[t^a,\{t^b,Q^\pm_{\rm eq}(p)\}]\Big)\ . \tag{7}$$

In these linearized collision terms quantum statistics are neglected but dynamical polarization effects are included. In our notation $\varepsilon(p_0) = \theta(p_0) - \theta(-p_0)$, and the effective equilibrium density is $\mathcal{N}_{\rm eq}(p) = \frac{1}{2}(Q^+_{\rm eq} + Q^-_{\rm eq}) + NG_{\rm eq}$. This density controls the high temperature polarization tensor [2,5,6]

$$\Pi^{\mu\nu}(k) = -g^2 \int dp \frac{p^\mu k_\alpha (p^\alpha \partial_p^\nu - p^\nu \partial_p^\alpha)}{pk + \imath\epsilon p_0}\mathcal{N}_{\rm eq}(p) = \Pi_L Q_{\mu\nu} + \Pi_T P_{\mu\nu}\ , \tag{8}$$

where the longitudinal and transverse projectors are $Q_{\mu\nu} = \bar{u}_\mu \bar{u}_\nu / \bar{u}^2$ and $P_{\mu\nu} = \bar{g}_{\mu\nu} - Q_{\mu\nu}$, and $\bar{g}_{\mu\nu} = g_{\mu\nu} - k_\mu k_\nu / k^2$ with $\bar{u}_\mu = \bar{g}_{\mu\nu} u^\nu$. The longitudinal and transverse polarization functions are related to the gluon self-energy $\Pi^{\mu\nu}(k)$ through $\Pi_L = \Pi^{\mu\nu} Q_{\mu\nu}$ and $\Pi_T = \Pi^{\mu\nu} P_{\mu\nu}/2$. As noted in Ref. [2], the polarization tensor derived from kinetic theory coincides exactly with the gauge invariant high temperature one-loop result obtained diagrammatically in Ref. [15,16]. The polarization tensor in turn determines the medium modified (retarded) gluon propagator that appears in the kernel of the above collision integrals:

$$D_{\mu\nu}(k,u) = -\frac{Q_{\mu\nu}}{k^2 - \Pi_L} - \frac{P_{\mu\nu}}{k^2 - \Pi_T} + \zeta \frac{k_\mu k_\nu}{k^4}\quad , \tag{9}$$

where ζ is a gauge parameter. It is important to note that the collision terms are gauge independent because the eikonal mass shell conservation factors, $\delta(pk)$ and $\delta(p'k)$, insures that the convolution of eikonal vertex factors, p^μ and p'^ν, with the gauge fixing term vanishes.

The nonequilibrium color deviations in the collision term are defined as $\Delta\mathcal{N}^{\{ab\}} = \Delta\mathcal{N}^{ab} + \Delta\mathcal{N}^{ba}$ and $\Delta\mathcal{N}^{[ab]} = \Delta\mathcal{N}^{ab} - \Delta\mathcal{N}^{ba}$ with $\Delta\mathcal{N}^{ab} = {\rm Sp}[t^a t^b(\Delta Q^+ + \Delta Q^-)] + {\rm Tr}(T^a T^b \Delta G)$. The trace over color indices in the fundamental representation is denoted by Sp and the one in adjoint representation by Tr. $[{\rm Sp}(t^a t^b) = \delta^{ab}/2$, ${\rm Tr}(T^a T^b) = N\delta^{ab}$, $(T^a)^{bc} = -\imath f^{abc}]$.

We now rewrite the first collision term in Fokker-Planck form as

$$\Delta C_1^\pm(p,x) = \{t^a,\{t^a,(-\partial_p^\mu[a_\mu \Delta Q^\pm(p,x)] + \partial_p^\mu[b_{\mu\nu}\varepsilon(p_0)\partial_p^\nu \Delta Q^\pm(p,x)])\}\} + \delta c_1^\pm\ , \tag{10}$$

where $\{,\{,\}\}$ denotes a double anticommutator, and the momentum diffusion tensor and friction force vector are given by

$$b_{\mu\nu} = \int dp' B_{\mu\nu}(p,p')\mathcal{N}_{\rm eq}(p'),$$
$$a_\mu = \int dp' B_{\mu\nu}(p,p')\varepsilon(p_0')\partial^\nu_{p'}\mathcal{N}_{\rm eq}(p')\ . \tag{11}$$

The kernel is given by

$$B_{\mu\nu} = \frac{\alpha_s^2}{2}\int dk k_\mu k_\nu \mid p^\alpha D_{\alpha\beta}(k) p'^\beta \mid^2 \delta(pk)\delta(p'k)\ . \tag{12}$$

The tensorial structure of $b_{\mu\nu}$ is in general complicated, but the dominant term which leads to a (Rayleigh) friction coefficient proportional to velocity, has the structure

$$b_{\mu\nu} \approx b[p^\mu u^\nu + p^\nu u^\mu - g^{\mu\nu}(p\cdot u)]\ . \tag{13}$$

Neglecting quantum statistics, the friction force and diffusion tensor are related by the analog of the Einstein relation

$$a_\mu = -b_{\mu\nu}u^\nu/T = -bp_\mu/T\ . \tag{14}$$

We now relate $b = -T(au)/(pu)$ to the energy loss per unit length derived in [17]. Note first that in the high T limit, $\varepsilon(p_0)\partial_p^\mu \mathcal{N}_{\rm eq}(p) \approx -(u^\mu/T)\mathcal{N}_{\rm eq}(p)$, and thus from Eq. (8) it follows that

$${\rm Im}\,\Pi_{\mu\nu}(k) \approx \frac{\pi g^2 ku}{T}\int dp' p'_\mu p'_\nu \delta(p'k)\mathcal{N}_{\rm eq}(p')\ . \tag{15}$$

Noting next the identity

$$\int dk\ \delta(pk)\ {\rm Im}[pD(k)p] = -\int dk\ \delta(pk)\ pD(k)[{\rm Im}\,\Pi(k)]D^*(k)p\ , \tag{16}$$

we find that

$$\frac{4b}{T} = \frac{1}{C_2}\frac{dE}{dx} = -g^2 2\pi \int \frac{dk}{(2\pi)^4}\frac{ku}{pu}\mathrm{Im}\,[p^\mu D_{\mu\nu}(k)p^\nu]\,\delta(pk) \approx \frac{4\pi}{3}\alpha_s^2 T^2 \log(k^*/m_E) \ , \tag{17}$$

where the color electric screening mass is $m_E = gT\sqrt{(1+N_f/6)}$ and $k^* \ll 3T$ is a cutoff parameter separating the soft and hard momentum transfer scales. As discussed in [18], the above formula for energy loss is accurate only for low momentum transfers. Physically, this is also clear from our kinetic theory derivation which utilized the eikonal approximation. In practice, however, setting $k^* = 3T$ is adequate to logarithmic accuracy. The effects of hard collisions require of course an extension beyond the Fokker-Planck approximation. The consistency of the relation between b and dE/dx can be verified by multiplying Eqs. (5,10) in the plasma rest frame by p_0, taking the trace, and integrating over d^3x. We can also relate b to the gluon momentum relaxation time defined via

$$1/t_p = \langle 1/E\rangle dE/dx \approx (0.7/T)C_A(4b/T) \approx 4\alpha_s^2 T\log(1/\alpha_s) \ , \tag{18}$$

which is close to the numerical result obtained in Ref. [13].

The first term in Eq. (10) reduces in the Abelian case to the familiar Fokker-Planck term for QED plasmas [19,20]. The correction to the Fokker-Planck terms is given by

$$\delta c_1^{\pm} = \frac{1}{2}\partial_p^\mu \int dp' B_{\mu\nu}(p,p')[\varepsilon(p_0)\partial_p^\nu - \varepsilon(p_0')\partial_{p'}^\nu] \times \Delta\mathcal{N}^{\{ab\}}(p',x)\{t^a,\{t^b,Q_{\mathrm{eq}}^{\pm}(p)\}\} \ . \tag{19}$$

As in Abelian plasmas [20], the correction term can be generally neglected because it involves an integral over the small nonequilibrium deviations instead of the equilibrium density. The smallness of the correction term is caused [20] by constraints on the nonequilibrium deviations imposed by conservation laws of particle number, color current, and energy-momentum that make the first several moments of $\mathcal{N}^{\{ab\}}(p')$ vanish.

The new non-Abelian collision term, Eq. (7), can also be expressed in an analogous Fokker-Planck form

$$\Delta C_2^{\pm}(p,x) = -d^{ab}[t^a,[t^b,\Delta Q^{\pm}(p,x)]]\varepsilon(p_0) + \delta c_2^{\pm} \ , \tag{20}$$

The double commutator corresponds to the second-order term in the rotation of ΔQ in color space by random angles, θ_a, with $\langle\theta^a\rangle = 0$ but $\langle\theta^a\theta^b\rangle \propto \delta_{ab}$. Hence, the first term in (20) corresponds to diffusion in color space. The color diffusion tensor is a measure of the mean square fluctuations of the rotation angles in color space and is diagonal, $d^{ab} = d_c\delta_{ab}$, with the color diffusion coefficient, d_c, given by

$$d_c = 2\alpha_s^2 \int dp'dk \mid p^\mu D_{\mu\nu}p'^\nu \mid^2 \delta(pk)\delta(p'k)\mathcal{N}_{\mathrm{eq}}(p') = -2\pi g^2 T \int \frac{dk}{(2\pi)^4}\frac{\delta(pk)}{ku}\mathrm{Im}\,[p^\mu D_{\mu\nu}(k)p^\nu] \approx (pu)\alpha_s T\log(m_E/m_M) \ . \tag{21}$$

Note that we used Eqs. (15,16) again to express d_c in a form that is equivalent to the one derived in Ref. [8] starting from classical color dynamics [2,21]. As emphasized in [8] perturbative dynamic screening in a QGP is not enough to make the color diffusion coefficient converge. The momentum diffusion coefficient converges because of an extra two powers of $\omega = ku$ appearing in the integral. The divergence is due to long range unscreened color magnetic interactions.

In order to regulate that infrared divergence, we follow Ref. [14] and introduce formally a nonperturbative color magnetic screening mass, $m_M \sim (g^2T)$, via $\mathrm{Im}(k^2 - \Pi_T)^{-1} \approx -\mu m_E^2/[(\vec{k}^2 + m_M^2)^2 + (\mu m_E^2)^2]$ where $\mu = \omega/|\vec{k}|$. Note that d_c has dimensions of energy squared unlike that defined in Ref. [8] because we treat massless partons here. The color diffusion time, defined in this case by $t_c = (pu)/(Nd_c)$ is, however, identical! The coincidence of the quantum and classical color diffusion coefficient is one of the surprising results of the present derivation. We emphasize that the derivation here is also more general than in [8] because we treat both momentum space and quantum color dynamics simultaneously. The correction to the color diffusion term is given by

$$\delta c_2^{\pm} = -\alpha_s^2 \int dp'dk \mid p^\mu D_{\mu\nu}p'^\nu \mid^2 \delta(pk)\delta(p'k) \times\Delta\mathcal{N}^{[ab]}(p',x)\varepsilon(p_0')[t^a,\{t^b,Q_{\mathrm{eq}}^{\pm}(p)\}] \ . \tag{22}$$

This correction term can be neglected in the first approximation for reasons similar to the neglect of $\delta c_1^{\pm}$ in Eq. (19). The color trace of $\delta c_2^{\pm}$ vanishes, and the first color moment is small because of the constraint of color charge conservation.

Proceeding analogously with the kinetic equation for gluons [7], we obtain finally the QCD Fokker-Planck equations

$$\begin{aligned}
p^\mu\partial_\mu\Delta Q + gp^\mu\partial_p^\nu\Delta F_{\nu\mu}^a t^a Q_{\mathrm{eq}} &= -d_c[t^a,[t^a,\Delta Q]] + \partial_p^\mu((-a_\mu + b_{\mu\nu}\partial_p^\nu)\{t^a,\{t^a,\Delta Q\}\}) \\
p^\mu\partial_\mu\Delta\bar{Q} - gp^\mu\partial_p^\nu\Delta F_{\nu\mu}^a t^a \bar{Q}_{\mathrm{eq}} &= -d_c[t^a,[t^a,\Delta\bar{Q}]] + \partial_p^\mu((-a_\mu + b_{\mu\nu}\partial_p^\nu)\{t^a,\{t^a,\Delta\bar{Q}\}\}) \\
p^\mu\partial_\mu\Delta G + gp^\mu\partial_p^\nu\Delta F_{\nu\mu}^a T^a G_{\mathrm{eq}} &= -d_c[T^a,[T^a,\Delta G]] + \partial_p^\mu((-a_\mu + b_{\mu\nu}\partial_p^\nu)\{T^a,\{T^a,\Delta G\}\}) \\
\partial^\mu\Delta F_{\mu\nu}^a = \Delta j_\nu^a &= g\int dp p_\nu(\mathrm{Sp}(t^a(\Delta Q - \Delta\bar{Q})) + Tr(T^a\Delta G)) \ .
\end{aligned} \tag{23}$$

This system describes the transport of small deviations in a color neutral quark-gluon plasma in terms of the transport coefficients, d_c, a^μ, and $b^{\mu\nu}$, which are controlled by the two time scales, t_p and t_c computed above.

One of the interesting consequences of these equations is that they show that the damping of collective modes in a non-Abelian plasma is controlled by the (perturbatively divergent) parton damping rates. This can be seen by taking the color octet moments of the above equations. In that case the color diffusion terms transform into relaxation terms for the color octet deviations, e.g., $p\partial_x Q^a = -(pu)Q^a/t_c + \cdots$. Long wavelength collective color modes are therefore perturbatively over-damped. A finite damping rate requires going beyond the hard thermal loop approximation and introducing nonperturbative (quantal) effects such as the color magnetic mass or a self-consistent approach as discussed in [14]. The nonperturbative damping rate, $\gamma \sim O(g^2 \log(1/g)T)$, of color plasmons is only $O(g \log(1/g))$ times smaller than their natural frequencies, $\omega_{pl} \sim O(gT)$. This is a remarkable difference with respect to Abelian plasmas, where the divergent damping rates never enter at the level of semiclassical kinetic equations and collisional damping is controlled instead by the perturbatively smaller momentum relaxation rate, $1/t_p \sim O(g^4 \log(1/g))$. Finally we note that observable consequences of the non-Abelian Fokker-Planck transport in nuclear collisions are expected through the color conductive coupling between minijets and beam jets [12] and the spectrum of soft induced gluon radiation associated with jet quenching [22].

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[1] H.-Th. Elze and U. Heinz, Phys. Rep. **183** 81 (1989); St. Mrówczyński, *Quark-Gluon Plasma*, Advanced Series on Directions in High Energy Physics, edited by R. C. Hwa (World Scientific, Singapore 1990), Vol. 6.

[2] U. Heinz, Ann. Phys. (N.Y.) **168**, 148 (1986); **161**, 48 (1985).

[3] H.-Th. Elze, M. Gyulassy, and D. Vasak, Nucl. Phys. **B276**, 706 (1986); Phys. Lett. B **177**, 402 (1986).

[4] H.-Th. Elze, Z. Phys. **47**, 647 (1990).

[5] St. Mrówczyński, Phys. Rev. D **39**, 1940 (1989); Phys. Lett. B **188**, 129 (1987).

[6] Yu.E. Pokrovsky and A.V. Selikhov, Yad. Fiz. **52**, 229, 605 (1990) [Sov. J. Nucl. Phys. **52**, 146, 385 (1990); Nucl. Phys. **A525**, 669c (1991).

[7] A.V. Selikhov, Phys. Lett. B **268**, 263 (1991); B **285**, 398(E) (1992); Kurchatov Institute Report No. IAE-5526/1, 1992 (unpublished).

[8] A.V. Selikhov and M. Gyulassy, Phys. Lett. B **316**, 373 (1993).

[9] E.V. Shuryak, Phys. Rep. **61C**, 71 (1980); D.J. Gross, R.D. Pisarski, and L.G. Yaffe, Rev. Mod. Phys. **53**, 43 (1981); F.R. Brown *et al.*, Phys. Lett. B **251**, 181 (1990); Phys. Rev. Lett. **65**, 2491 (1990).

[10] See reviews and references in Proceedings of Quark Matter 91, edited by F. Plasil, Nucl. Phys. **A544**, 1c (1992).

[11] K. Kajantie, P. V. Landshoff, and J. Lindfors, Phys. Rev. Lett. **59**, 2527 (1987); J.P. Blaizot, A.H. Mueller, Nucl. Phys. **B289**, 847 (1987); X.-N. Wang and M. Gyulassy, Phys. Rev. Lett. **68**, 1480 (1992).

[12] K.J. Eskola and M. Gyulassy, Phys. Rev. C **47**, 2329 (1993).

[13] G. Baym *et al.*, Phys. Rev. Lett. **64**, 1867 (1990); H. Heiselberg and C.J. Pethick, Report No. NORDITA NBI-93-19 (unpublished).

[14] R.D. Pisarski, Phys. Rev. Lett. **63**, 1129 (1989). R.D. Pisarski, Report No. BNL-P-1/92 (unpublished).

[15] E. Braaten and R. Pisarski, Phys. Rev. Lett. **64**, 1338 (1990); Nucl. Phys. **B337**, 569 (1990).

[16] H.A. Weldon, Phys. Rev. D **26** 1394 (1982); V.V. Klimov, Yad. Fiz. **33**, 1734 (1981) [Sov. J. Nucl. Phys. **33**, 934 (1981)]; E.V. Shuryak, Zh. Eksp. Teor. Fiz. **74**, 408 (1978) [JETP **47** 212 (1978)].

[17] M. Thoma and M. Gyulassy, Nucl. Phys. **B351**, 491 (1991); M. Gyulassy, *et al.*, Nucl. Phys. **A538**, 37c (1992).

[18] St. Mrówczyński, Phys. Lett. B **269**, 383 (1991); E. Braaten and M.H. Thoma, Phys. Rev. D **44**, 1298 (1991); 2625.

[19] A. Lenard, Ann. Phys. (N.Y.) **10**, 390 (1960); R. Balescu, Phys. Fluids **3**, 52 (1960); W.B. Thompson and J. Hubbard, Rev. Mod. Phys. **32**, 714 (1960); J. Hubbard, Proc. Roy. Soc. **A260**, 114 (1961).

[20] R.L. Liboff, *Kinetic Theory* (Englewood Cliffs, Prentice Hall, 1990).

[21] S.K. Wong, Nuovo Cimento **65A**, 689 (1970).

[22] X.N. Wang and M. Gyulassy, Phys. Rev. Lett. **68**, 1480 (1992).