Nucleon solution of the Faddeev equation in the Nambu–Jona-Lasinio model

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Given the phenomenological success of the Nambu–Jona-Lasinio model in describing the meson physics in the low-energy limit, it is tempting to find the fully relativistically structured nucleon solution in the same model under the similar approximation employed in the mesonic sector. To achieve this goal we need to solve a relativistic Faddeev equation. The factorizability of the two-body T matrix reduces the three-body Faddeev equation to a tractable two-body Bethe-Salpeter equation. The reduced equation is then solved numerically. Our result indicates that the nucleon consists of three loosely bound constituent quarks.

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I. INTRODUCTION

One of the most important feature of QCD is the chiral symmetry and its dynamical breaking, which is expected to dictate the low-energy hadronic physics. There exists work, such as the QCD sum rule [1], the instanton liquid model [2], and an explicit lattice QCD simulation via cooling technique [3], directly or indirectly confirming this expectation. The Lagrangian introduced by Nambu and Jona-Lasinio (NJL) [4] a long time ago conveniently mimics such an essential aspect of QCD in the low-energy limit. Models based on the NJL type of Lagrangians have been demonstrated to be very successful in describing the low-energy mesonic physics [5]. On the other hand, due to technical reasons, these models are much less effective in describing low-energy physics involving baryons. It is very often that extra assumptions beyond these models have to be used in order to make concrete predictions in the baryonic sector.

While there is very little doubt that the NJL type of models could support bound baryonic states, the direct approach in solving a three-body problem has only been attempted recently [6] with approximations apparently quite different from that of employed in the mesonic sector. An important point is that approximations in the baryonic sector have to be consistent with chiral symmetry, for example, the nucleon solution should approximately satisfy the Goldberger-Treiman relation [7]. Otherwise the very essence of the NJL model, the chiral symmetry, is ruined by the *ad hoc* approximations. Other indirect attempts in finding the nucleon solution in the NJL-like models, such as the nontopological soliton approach [8], the bosonization approach [9], and undoubtedly others can be found in the literature.

In this paper we undertake the task of finding a nucleonlike solution in the NJL type of models. First we derive the three-body Faddeev equation in the valence constituent quark approximation by ignoring the three-body irreducible graphs. Due to the heaviness of the constituent quark, this approximation is expected to be good at low energies, as shown in the mesonic sector. By observing that the two-body diquark T matrix has a separable form, the Faddeev equation can be reduced to an effective two-body Bethe-Salpeter equation with an energy dependent interaction. Then the reduced problem is solved numerically, without any further approximations. Although we cannot explicitly show that our solution respects the exact chiral symmetry, in contrast with the meson solutions in the Hartree-Fock approximation, we believe that our work is a step forward in the right direction. As long as we can find a weakly bound nucleonlike state of three constituent quarks, the chiral symmetry should be well protected, since the chiral symmetry is exact at the constituent quark level [5].

Due to the lack of confinement in the NJL type of models certain caution has to be observed when one tries to use these models to approximate hadronic states, especially those that are relatively close to the constituent quark threshold. On the other hand, it is also important to realize that the constituent quark singularities (in the sense of dispersion relations) in color singlet channels in the NJL type of models should not always be dismissed lightly. When one only wants to address the static properties of hadrons such as masses and charge radii and so on, not those related to decays, the relevance of the constituent quark singularity should be judged by dynamical calculations. A good example is the heavy quarkonium structure such as the $b\bar{b}$ system. As shown explicitly by Jaffe [10], although the existence of confinement indeed turns all quark singularities into hadron singularities, the structure of heavy quarkonium systems is essentially determined by the nonconfining Coulomb potential, as one would intuitively expect. It is in this spirit that we try to approximate the nucleon state in terms of nonconfining quarks under the assumption that the chiral symmetry breaking is the dominating force.

This paper is organized as follows. In Sec. II we first introduce the model we explicitly consider and then

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briefly review the two-body sector to fix parameters in the model. In Sec. III the derivation of the three-body Faddeev equation and its reduction to the effective two-body equation are presented. The numerical technique involved in solving the reduced fully relativistic Bethe-Salpeter equation, based on the work of Rupp and Tjon [11], is recapitulated and then applied to our case in Sec. IV. A summary and some outlook follows in Sec. V.

II. TWO-BODY SECTOR

The Lagrangian we consider is the two flavored Nambu–Jona-Lasinio model given by

$$\mathcal{L} = \bar{\psi} i\gamma_\mu \partial^\mu \psi + G_1[(\bar{\psi}\psi)^2 + (\bar{\psi} i\gamma_5 \tau_a \psi)^2] - G_2[\bar{\psi}\gamma_\mu(\lambda_A/2)\psi]^2, \tag{2.1}$$

where ψ is the quark field, τ_a $(a = 1,2,3)$ and λ_A $(A = 1,2,\ldots,8)$ are the generators of the flavor $SU_f(2)$ and color $SU_c(3)$ groups, respectively. Small current quark masses are ignored for simplicity. Since the coupling constants G_1 and G_2 have negative mass dimension, this model is not renormalizable. An appropriate ultraviolet cutoff procedure has to be specified in order to make the model well defined. In this work we insert a form factor $g(k) = g(-k)$, whose functional form will be eventually taken to be a four-momentum cutoff Λ in Euclidean space for convenience, at every fermion vertex in the loop integrals.

The justification of using Eq. (2.1) to model the low-energy physics of the strong interaction and its phenomenological success in mesonic channels were well studied in the literature. A recent review can be found in Ref. [5]. Although our primary goal is to find three-body baryonic solutions in this model, it is adequate to recapitulate the essential features of this model in the meson sector, which is used to fix all the parameters but the one in the model. Then the two-body T matrix in the scalar-isoscalar diquark channel, which consists of an essential component of the three-body Faddeev equation, will be derived.

A. Meson channel

The most important feature that makes the model resemble QCD at low-energy domain is that the NJL model and QCD share the same chiral symmetry and its dynamical breaking. The manifestation of this phenomenon in the NJL model is that the massless quarks acquire dynamical masses through the following self-consistent gap equation, when only the fermion bubble chain graphs are included, or in the Hartree-Fock approximation,

$$1 = i(a_1 G_1 + a_2 G_2) \int \frac{d^4k}{(2\pi)^4} \frac{4g(k)}{k^2 - m^2}, \tag{2.2}$$

where m is the constituent quark mass, which is related to the fermion condensate $\langle\bar{\psi}\psi\rangle$ by $m = -(a_1 G_1 + a_2 G_2)\langle\bar{\psi}\psi\rangle/(N_c N_f)$. For $N_c = 3$ and $N_f = 2$, $a_1 = 13$, and $a_2 = 8/3$.

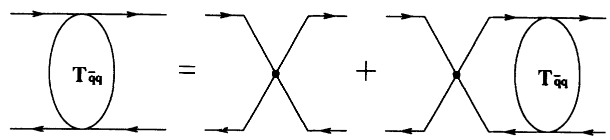

FIG. 1. Feynman graphs for $T_{\bar{q}q}$ in pseudoscalar channel.

As a consequence of the chiral symmetry breaking the pion emerges as the massless Goldstone boson, which manifests itself explicitly as the massless pole in the quark-antiquark two-body $T_{\bar{q}q}$ matrix in pseudoscalar channel. If again only the fermion bubble chain graphs are retained or in the RPA approximation, $T_{\bar{q}q}$ given by Fig. 1 can be readily calculated. The residue of $T_{\bar{q}q}$ matrix at this pole, Γ_π^a, has the form

$$\Gamma_\pi^a = g_{\pi\bar{q}q}[\mathbb{1}_C \otimes \tau^a \otimes i\gamma_5], \tag{2.3}$$

where $g_{\pi\bar{q}q}$ is the pion-quark-antiquark coupling constant. The pion decay constant, f_π, is defined through the axial-vector current matrix element,

$$i f_\pi p_\mu \delta_{ab} \equiv \left\langle 0 \left| \bar{\psi}\gamma_\mu\gamma_5 \frac{\tau_a}{2}\psi \right| \pi_b(p) \right\rangle. \tag{2.4}$$

Using the chiral Ward identity, or the Goldberger-Treiman relation at the quark level, $f_\pi g_{\pi\bar{q}q} = m$, one can easily find

$$f_\pi^2 = 4N_c m^2 \int \frac{d^4k}{(2\pi)^4} \frac{i g(k)}{[k^2 - m^2]^2}. \tag{2.5}$$

In arriving at the above result the on-shell condition $p^2 = m_\pi^2 = 0$ has been used.

There are three parameters in the model, two couplings G_1, and G_2 and the cutoff Λ. By equating f_π and m or $\langle\bar{\psi}\psi\rangle$ to the phenomenological values through Eqs. (2.2) and (2.5) we can fix two of them, which we pick to be Λ and $G \equiv a_1 G_1 + a_2 G_2$. This more or less fixes the theory in the mesonic sector. The last parameter $\eta \equiv G_1/G_2$ is left free to vary.

B. Diquark channel

If we use the same fermion bubble chain approximation in the quark-quark sector, we can easily calculate the corresponding T matrix. In the color $\bar{3}$ scalar-isoscalar channel the T matrix has the structure, when ignoring the mixing with other channels (for example, the color $\bar{3}$ vector-isoscalar channel)

$$T_{qq}^{ab,dc}(p) = iR(p)[\lambda_A^{[\,]} \otimes \tau_2 \otimes C\gamma_5]^{ab}[\lambda_A^{[\,]} \otimes \tau_2 \otimes C\gamma_5]^{dc}, \tag{2.6}$$

where $\lambda_A^{[ij]} \equiv (\lambda_A^{ij} - \lambda_A^{ji})/2$, $C \equiv i\gamma_0\gamma_2$ is the charge conjugation matrix, and a and b label all the color, fla-

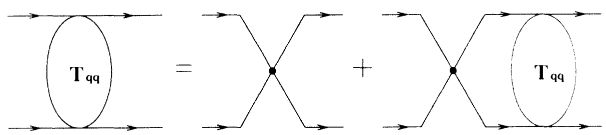

FIG. 2. Feynman graphs for T_{qq} in scalar-isoscalar diquark channel.

vor, and Dirac indices (see Fig. 2). The scalar function $R(p)$ can be obtained straightforwardly by summing the fermion bubble chain, yielding $R(p) = G'/[1 - G'J(p)]$ with $G' = (b_1G_1 + b_2G_2)/4$ and

$$J(p) = 4i \int \frac{d^4k}{(2\pi)^4} g^2(k)\, \mathrm{Tr}[C\gamma_5 S_F(k+p/2) C\gamma_5 \times S_F^T(-k+p/2)],$$

where Tr denotes the trace in Dirac space and $S_F(k)$ and $S_F^T(k)$ are the constituent quark propagator and its transpose (in Dirac space), respectively. Furthermore we have $b_1 = 4$ and $b_2 = 8/3$.

Whether there exists diquark bound states in this model depends on whether $R(p)$ develops poles in the timelike region. As shown in [12] it is possible by varying η to find a bound diquark state in this channel. It should be emphasized that the existence of such a diquark bound state is not a necessary condition for the existence of a three-quark bound state, though it might be useful to utilize the diquark concept phenomenologically to explain certain scaling violations in lepton-nucleon experiments. In this paper the diquark state is merely an intermediate device in setting up the three-body Faddeev equation. The phenomenological relevance of the diquark will not be pursued here.

III. THREE-BODY SECTOR

Given the fundamental four-fermion vertex by the Lagrangian and the quark-quark two-body T_{qq} matrix, and ignoring the three-body irreducible graphs, the three-body T matrix can be solved from the Faddeev equation by iterating the fundamental vertex and the two-body T_{qq} matrix. Throwing away the three-body irreducible graphs is in some sense equivalent to ignoring the nonvalence constituent quark loops in the iteration process. Due to the heavyness of the constituent quark mass (300–400 MeV) this approximation is justified in the low-energy region. Of course, one should realize that we do not invoke more approximations here. Essentially the same kind of approximation was used in the mesonic and diquark cases.

A. Faddeev equation

Since we are only interested at the moment in the three-body bound state, we only need to consider the homogeneous Faddeev equation. If the full three-body amplitude $\Gamma^{f,d}$ (with f and d being the external flavor and Dirac indices) is decomposed as a sum of three partial amplitudes Γ_i $(i = 1,2,3)$, with

$$\Gamma_1^{f,d} \equiv \epsilon_{c_1c_2c_3} \tau_2^{f_2f_3} (C\gamma_5)^{d_2d_3} \delta^{f_1f} \Gamma^{(1)}_{d_1d}(p_1,p_2,p_3), \quad (3.1)$$

and similarly for Γ_2 and Γ_3 by cyclically permuting $(1,2,3)$, then these partial amplitudes satisfy the following integral equation,

$$\Gamma^{(3)}(p_1,p_2,p_3) = 2g(p_1-p_2)iR(p_1+p_2)\Big\{ \int \frac{d^4p_1'}{(2\pi)^4} g(p_1'-p_2')[C\gamma_5 S_F^T(p_2')C\gamma_5 S_F(p_1')]\Gamma^{(1)}(p_1',p_2',p_3) + \int \frac{d^4p_2'}{(2\pi)^4} g(p_1'-p_2')[C\gamma_5 S_F^T(p_1')C\gamma_5 S_F(p_2')]\Gamma^{(2)}(p_1',p_2',p_3)\Big\}. \quad (3.2)$$

The notation of the above equation is depicted in Fig. 3. The factor of 2 in Eq. (3.2) arises from the color sum $\epsilon_{ac_1c_2}\epsilon_{bc_1c_2} = 2\delta_{ab}$. Although formally similar to the nonrelativistic Faddeev equation, Eq. (3.2) is exact within the approximation mentioned above. An analogous equation with scalar particles was considered by Rupp and Tjon in a different context [11]. Since we explicitly included the color, flavor, and Dirac structures in the definition of the two-body T_{qq} matrix and three-body amplitudes $\Gamma^{f,d}_{1,2,3}$, the recoupling-coefficient matrix has already been automatically taken into account in Eq. (3.2).

B. Reduction to an effective Bethe-Salpeter equation

If the two-body T_{qq} matrix involved has a general form, it would be a formidable task to find the solution for

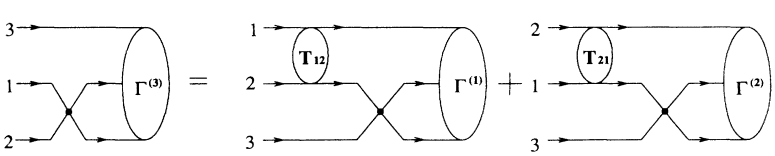

FIG. 3. Faddeev equation for the nucleon.

Eq. (3.2). The crucial observation is that the T_{qq} matrix has a factorized form and hence we are only dealing with the so-called separable situation. The separability of the two-body interaction leads to a reduction of the three-body problem to an effective Bethe-Salpeter equation. As a matter of fact, this reduction has already been hinted by the explicit form of Eq. (3.2). More concretely, the three-body amplitudes can be written as

$$\Gamma^{(1)}_{dd'}(p_1,p_2,p_3) = \Psi_{dd'}(p_1)g(p_2-p_3)R(p_2+p_3), \quad (3.3)$$

and similarly for $\Gamma^{(2,3)}$, with Ψ satisfying

$$\Psi(p_3) = 4i\int \frac{d^4p_1'}{(2\pi)^4} g(p_1'-p_2')R(p_2'+p_3)g(p_2'-p_3) \times[C\gamma_5 S_F^T(p_2')C\gamma_5 S_F(p_1')]\Psi(p_1'), \quad (3.4)$$

as a matrix equation in Dirac space. When deriving the above equation the quarks are treated as identical particles.

Diagrammatically, Eq. (3.4) can be represented by Fig. 4, which looks like a boson (with propagator R) coupling to a third quark to form a three-body bound state. However, this ought to be distinguished from identifying the "boson" as the diquark bound state. The reduction of the three-body Eq. (3.2) to the effective two-body Eq. (3.4) does not depend on whether the diquark channel has a pole, but rather on the separatability of T_{qq} matrix.

Introducing the equal-mass Jacobi momentum variables q and q',

$$p_3 \equiv \frac{P}{3} - q, \quad p_1' \equiv \frac{P}{3} - q', \quad (3.5)$$

where the total momentum P is given by

$$P \equiv p_1 + p_2 + p_3 = p_1' + p_2' + p_3, \quad (3.6)$$

we find that the reduced Bethe-Salpeter equation can be written as

$$\Psi(P,q) = \frac{i}{4\pi^4}\int d^4q' V(q,q';P)R(\tfrac{2}{3}P+q')K\Psi(P,q'), \quad (3.7)$$

where we have defined an energy dependent interaction

$$V(q,q';P) = \frac{g(p_1'-p_2')g(p_2'-p_3)}{(p_1'^2-m^2)(p_2'^2-m^2)}. \quad (3.8)$$

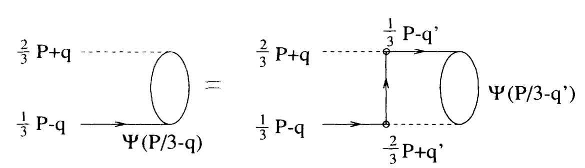

FIG. 4. Reduced effective Bethe-Salpeter equation for the nucleon.

The Dirac structure of the kernel is contained in the operator

$$K = [C\gamma_5(\gamma^T p_2' + m)C\gamma_5(\gamma p_1' + m)]. \quad (3.9)$$

Using well-known properties of the charge conjugation operator C, this simplifies to

$$K = (\gamma p_2' + m)(\gamma p_1' + m). \quad (3.10)$$

In view of Eqs. (3.5) and (3.6) the various momenta present in Eq. (3.7) can be expressed in terms of the Jacobi variables q, q', and total momentum P.

C. Decomposition of the reduced amplitudes

To see the Dirac structure more clearly let us reduce the operator K into the Pauli form. Using the ρ-spin notation of Ref. [13] for the upper and lower components of four-spinors, we get for the matrix elements $K(\rho,\rho')$ (with $\rho,\rho'=\pm$),

$$\begin{aligned} K(+,+) &= (p_{20}'+m)(p_{10}'+m) - \boldsymbol{\sigma}\cdot\mathbf{p}_2'\boldsymbol{\sigma}\cdot\mathbf{p}_1', \\ K(+,-) &= -(p_{20}'+m)\boldsymbol{\sigma}\cdot\mathbf{p}_1' - (m-p_{10}')\boldsymbol{\sigma}\cdot\mathbf{p}\,{}_2', \\ K(-,+) &= (p_{10}'+m)\boldsymbol{\sigma}\cdot\mathbf{p}_2' - (m-p_{20}')\boldsymbol{\sigma}\cdot\mathbf{p}_1', \\ K(-,-) &= (-p_{20}'+m)(-p_{10}'+m) - \boldsymbol{\sigma}\cdot\mathbf{p}_2'\boldsymbol{\sigma}\cdot\mathbf{p}_1'\cdot \end{aligned} \quad (3.11)$$

The simplest approximation which can be made is to neglect the lower components, i.e., the kernel is replaced by $K(+,+)$. The resulting eigenvalue equation becomes in this case

$$\frac{i}{4\pi^4}\int d^4q' V(q,q';P)R(\tfrac{2}{3}P+q')[(p_{20}'+m)(p_{10}'+m) - \boldsymbol{\sigma}\cdot\mathbf{p}\,{}_2'\boldsymbol{\sigma}\cdot\mathbf{p}\,{}_1']\chi(q') = \lambda\chi(q), \quad (3.12)$$

where a physical bound-state solution corresponds to the eigenvalue $\lambda = 1$. Assuming we are in the overall three-quark c.m. system $P = (\sqrt{s},\mathbf{0})$, we see that there are two classes of solutions to Eq. (3.12):

$$\begin{aligned} \chi_1 &= \Phi_1(q_0,|\mathbf{q}|), \\ \chi_2 &= \boldsymbol{\sigma}\cdot\mathbf{q}\;\;\Phi_2(q_0,|\mathbf{q}|). \end{aligned} \quad (3.13)$$

These classes are not coupled to each other in the integral equations. This is due to parity and angular momentum conservation. χ_1 is an s-wave solution with ($l=0, s=1/2, j=1/2$) and χ_2 is a p-wave ($l=1, s=1/2, j=1/2$). In view of the simple form of the χ's the angular integration in Eq. (3.12) can be explicitly carried out. As a result, we obtain a two-dimensional integral equation of the form

$$\chi_n(q) = \frac{i}{2\pi^3}\int_{-\infty}^{\infty} dq_0' \int_0^{\infty} q'^2 dq' V_n(q,q';P) \times R(\tfrac{2}{3}P+q')\chi_n(q'), \quad (3.14)$$

where $q \equiv |\mathbf{q}|$ and

$$V_n(q,q';P) = \int_{-1}^{1} dx \frac{g(p_1'-p_2')g(p_2'-p_3)}{(p_2'^2-m^2)(p_1'^2-m^2)} \times \mathrm{Tr}_2[O_n K(+,+)] \qquad (3.15)$$

with $x = \cos(\theta_{\mathbf{qq'}})$ and Tr_2 is the trace to be taken in Pauli space. Furthermore, the operator O_n is given by $1/2$ and $(\boldsymbol{\sigma}\cdot\mathbf{q}')/(2\mathbf{q}^2)$ for $n = 1$ and 2, respectively.

This analysis can be extended to the full equation. From Eq. (3.12) we see that the Pauli spin dependence in Ψ can be either the unit operator or $\boldsymbol{\sigma}\cdot\mathbf{q}$. In view of parity conservation there are also two classes of solutions, which are given by four spinors of the form

$$\Psi_1 = \begin{pmatrix} \phi_1(q_0,q) \\ \boldsymbol{\sigma}\cdot\mathbf{q}\ \ \phi_2(q_0,q) \end{pmatrix} \qquad (3.16)$$

and

$$\Psi_2 = \begin{pmatrix} \boldsymbol{\sigma}\cdot\mathbf{q}\ \ \phi_3(q_0,q) \\ \phi_4(q_0,q) \end{pmatrix}. \qquad (3.17)$$

The vertex functions Ψ_1 and Ψ_2 are again not coupled to each other. With this form for Ψ a partial wave decomposed set of coupled integral equations can be derived. Inserting Eq. (3.16) in Eq. (3.7) we get

$$\phi_n(q) = \frac{i}{2\pi^3}\sum_{m=1}^{2}\int_{-\infty}^{\infty} dq_0' \int_0^{\infty} q'^2 dq' V_{nm}(q,q';P) \times R(\tfrac{2}{3}P+q')\phi_m(q') \qquad (3.18)$$

with $n = 1, 2$ and

$$V_{nm}(q,q';P) = \int_{-1}^{1} dx \frac{g(p_1'-p_2')g(p_2'-p_3)}{(p_2'^2-m^2)(p_1'^2-m^2)} K_{nm}. \qquad (3.19)$$

The explicit expression for the matrix K_{nm} can be determined by noting that

$$\Psi_1 = \left(\frac{1+\gamma_0}{2}\phi_1 - \boldsymbol{\gamma}\cdot\mathbf{q}\frac{1+\gamma_0}{2}\phi_2\right)w, \qquad (3.20)$$

where w is a four-spinor with every component equal to 1. The operator $(1+\gamma_0)/2$ is in the three-quark c.m. system nothing else but the projection operator $\Omega = (M_N + \gamma P)/(2M_N)$. With this we can now calculate K_{nm} by projecting out the Dirac form on Ω and $\boldsymbol{\gamma}\cdot\mathbf{q}$. In so doing we get for the Dirac part of the kernel

$$K_{1m} = \tfrac{1}{2}\mathrm{Tr}[K\kappa_m(\mathbf{q}')],$$
$$K_{2m} = \frac{\mathrm{Tr}[\boldsymbol{\gamma}\cdot\mathbf{q}K\kappa_m(\mathbf{q}')]}{\mathrm{Tr}[\boldsymbol{\gamma}\cdot\mathbf{q}\kappa_2(\mathbf{q})]}, \qquad (3.21)$$

where $\kappa_1 = \Omega, \kappa_2 = \boldsymbol{\gamma}\cdot\mathbf{q}\,\Omega$. Equation (3.21) can be evaluated in a straightforward way. We find

$$K_{11} = -q_0q_0' + q_0\left(m + \frac{M_N}{3}\right) - \mathbf{q}'^2 - \mathbf{q}\cdot\mathbf{q}' + \left(m + \frac{M_N}{3}\right)^2,$$
$$K_{12} = -\left(q_0 - 2\frac{M_N}{3}\right)\mathbf{q}'^2 + \left(q_0' + m - \frac{M_N}{3}\right)(\mathbf{q}\cdot\mathbf{q}'), \qquad (3.22)$$
$$K_{21} = q_0' - m - \frac{M_N}{3} - \left(q_0 + 2\frac{M_N}{3}\right)\frac{\mathbf{q}\cdot\mathbf{q}'}{\mathbf{q}^2},$$
$$K_{22} = \mathbf{q}'^2 + \left[-q_0\left(q_0' - m + \frac{M_N}{3}\right) - \mathbf{q}'^2 + \left(m - \frac{M_N}{3}\right)^2\right]\frac{\mathbf{q}\cdot\mathbf{q}'}{\mathbf{q}^2}.$$

In a similar way the coupled set of equations can be derived for Ψ_2. It should be noted that possible solutions of this type correspond to p-wave-like states and as a result are expected not to be the ground state of the three-quark system due to the centrifugal term. Since we are interested in this paper in the nucleon, it is natural to confine ourselves to the solutions of the s-wave type, given by Ψ_1.

IV. CALCULATIONS

Following Ref. [11] the resulting integral equations can be studied by performing a Wick rotation of the q_0 and q_0' variables to the complex plane. Assuming that the diquark system supports a bound state at M_{qq}, we find that at the threshold point of quark-diquark scattering a pinching singularity can occur in the kernel of Eq. (3.18) at $q_0 = \hat{q}_0 = \frac{1}{3}(2m - M_{qq})$. It can readily be verified that in the triquark bound-state region, corresponding to $\sqrt{s} < m_q + M_{qq}$, the q_0 and q_0' variables can be rotated to a path going through the point $\hat{q}_0$ and parallel to the imaginary axis without encountering any singularities in the kernel. In so doing we implicitly assume that eventual singularities in the form factors $g(q)$ do not cross the imaginary q_0 axis. Furthermore the arguments of the form factors are approximated by neglecting the $\hat{q}_0$ dependence. The resulting Euclidean form of the integral equation is regular in the bound-state region and as a result it can in principle be solved by standard discretization procedures. Because of the actual size of the resulting matrix equations we have adopted the method described in Ref. [11]. The perturbation series is determined by iterating the equations, while the occurring two-dimensional integrals are evaluated using standard Gaussian quadratures. From this series the energy of the bound state is determined using the ratio method of Malfliet and Tjon [14]. It should noted that, as a by-product, the corresponding wave function can also be found in this way.

There are several parameters in the model: the cutoff Λ and the coupling constants G_1 and G_2. The overall mass scale can be set by the choice of the the cutoff mass Λ. There are two constraints we would like to satisfy.

TABLE I. Diquark masses needed to get a nucleon solution at $M_N = 939$ MeV in various approximations.

m (MeV)	Λ (MeV)	$M_{qq}{}^{a}$ (MeV)	$M_{qq}{}^{b}$ (MeV)	$M_{qq}{}^{c}$ (MeV)
375	750	579.0	570.9	576.8
400	739	572.8	554.7	564.7
450	728	577.0	531.5	547.8

[a]Static limit.
[b]One channel defined by Eq. (3.12).
[c]Two channel defined by Eq. (3.18).

TABLE II. Comparison of predictions of the nucleon mass in various approximations. The value of M_{qq} is fixed so that $M_N = 939$ MeV in the full two-channel calculation.

m (MeV)	Λ (MeV)	M_{qq} (MeV)	$M_N{}^{a}$ (MeV)	$M_N{}^{b}$ (MeV)
375	750	576.8	936.4	945.3
400	739	564.7	928.2	950.0
450	728	547.3	892.7	957.9

[a]Static limit.
[b]One channel defined by Eq. (3.12).

From the pion decay constant $f_\pi = 93$ MeV, we can determine according to Eq. (2.5) the constituent quark mass m. Taking a value of $\Lambda = 750$ MeV we find $m = 375$ MeV. Decreasing Λ, for instance, to 739 MeV, the quark mass increases to $m = 400$ MeV. Secondly, from the self-consistent mass gap equation, the value of $G \equiv 13G_1 + 8/3G_2$ is fixed. As a consequence the only free parameter is the ratio G_1/G_2, which can be used as the parameter to determine the diquark mass. In Fig. 5 the diquark mass dependence on this ratio is shown.

Once the parameters of the model have been fixed we may study the three-quark bound state. In Table I we list the diquark masses needed to get a nucleon solution at $M_N = 939$ MeV in three approximations, nonrelativistic (or static limit) $K_{11} \to 4m^2$; one channel defined by Eq. (3.12) and two-channel defined by Eq. (3.18). As one can see, a stable nucleon solution always requires the scalar-isoscalar diquark state to lie below two-quark threshold. The binding of the three-quark system clearly depends on the choice of the diquark energy. In Fig. 6 are shown for two cases of Λ the results of the calculated mass of the three-quark ground state as a function of the diquark mass (solid line). Also plotted are the results when we only keep the s-wave components of the three-quark wave function (dot-dash line) and the static limit of $K_{11} \to 4m^2$ (dash line). From this we see that at lower diquark masses the static limit predicts a substantially deeper binding than the full two-channel result and hence it can be an unreliable approximation.

To have a feeling on the quality of the static and one channel approximations we list in Table II the nucleon masses with diquark mass fixed at the value where the two-channel calculation would yield $M_N = 939$ MeV. It is clear from Table II that the relativistic forms give rise to less attraction, leading to a slightly higher-lying ground state, though there is no qualitative difference from the static approximation [6]. From these results we may conclude that over a wide range of M_{qq} a stable nucleon solution indeed exists in the considered NJL model. Increasing the diquark mass leads to a weakening of the quark-quark interaction and as result the nucleon mass increases.

In the range of diquark mass we considered, the existence of the nucleon solution near its experimental value is required to be about $150 - 300$ MeV binding in the scalar-isoscalar diquark. This kind of diquark clustering is also observed in a recent instanton model calculation by Schäfer *et al.* [15] in the nucleon channel and qualitatively confirmed by the lattice simulation through cooling [16]. Since the NJL type of models are practically

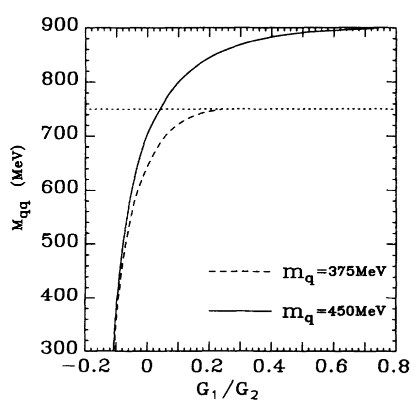

FIG. 5. Scalar-isoscalar diquark mass as a function of G_1/G_2 for two values of constituent quark masses. The horizontal lines indicate the quark-quark thresholds.

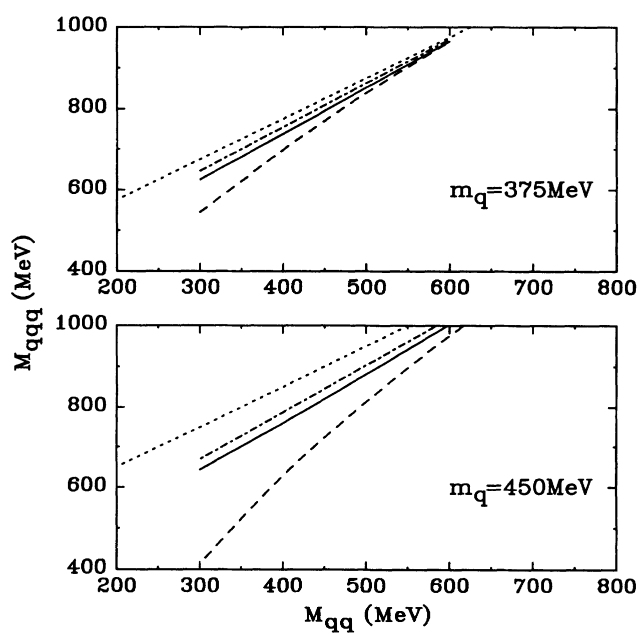

FIG. 6. Nucleon mass as a function of the diquark mass, with the dash line corresponding to static limit $K_{11} \to 4m^2$, dot-dash line corresponding to one channel defined by Eq. (3.12), solid line corresponding to full two-channel defined by Eq. (3.18) solutions, respectively. The dotted line indicates the quark-diquark scattering threshold.

effective theories for these instanton models, the similar diquark clustering in our case may not be a mere coincidence.

V. SUMMARY AND OUTLOOK

We have been able to demonstrate that the NJL type of models can easily accommodate the nucleonlike state under similar types of approximations employed in the mesonic sector. Although we were not able to explicitly show, from the derived Faddeev equation, that the solution of the nucleon state satisfies the Goldberger-Treiman relation, we indeed found that the nucleon be a loosely bound state of the constituent quarks. In order to have the nucleon solution as a true bound state a bound diquark in the scalar-isoscalar channel is necessary in our model.

There are clearly some interesting questions which can be addressed in such a model. Using the wave function corresponding to the three-quark bound state, the properties of the various form factors for the nucleon can in principle be studied. It is also of interest to examine possible Δ isobar states in the same model we have considered. The mass splitting between the baryon decuplet and octet constitutes a nontrivial test of the NJL type of models, while the mass splittings within the same baryon multiplets are less stringent due to the fact that the latter splittings mainly come from quark masses. Since the dominant diquark configuration in the Δ should be vector isovector, the resulting three-body bound state could be a resonance rather than a bound state. In principle, the states lying beyond the constituent quark threshold in the NJL model should have no connection with hadrons due to the explicit confining nature of QCD. However, when the resonance becomes strong enough it may again indicate that the chiral symmetry breaking is still playing the dominating role. The recent instanton model study [15] and lattice simulation with cooling [16], where the effect of confinement is explicitly or implicitly absent, strongly suggest that the ρ meson and Δ baryon are likely to be such resonating constituent quark states, as long as we are only concerned with their masses and certain matrix elements, but not their widths. The numerical method we used in the nucleon case needs to be modified if the Δ lies in the continuum. A more careful examination of the compatibility of the Faddeev equation and the chiral symmetry could provide useful insight on how the Goldberger-Treiman relation at the nucleon level is realized. Finally, also pion-nucleon and nucleon-nucleon low-energy scattering processes can in principle be studied. It is easy to anticipate that the meson-exchange potential could merge between nucleons, if only the valence quark lines are included at each instance in the Feynman graphs. Furthermore, since the nucleon in this model is a loosely bound state of three constituent quarks, it is very likely that there are anomalous singularities in the scattering processes, which could potentially modify the one-meson exchange nuclear force picture in the low-energy limit. The role of the anomalous singularity in the form factor for loosely bound states in similar models was studied recently [17].

After submitting this paper for publication, a related work has appeared [18] very recently.

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