Memory effects in relativistic heavy ion collisions

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We consider equilibration in relativistic nuclear dynamics starting from a nonequilibrium Green's-function approach. The widely used Boltzmann-Uehling-Uhlenbeck equation is obtained only as the Markovian limit (i.e., negligible memory time). The actual memory time in energetic nuclear collisions turns out to be $\sim$ 2–3 fm/c, which interferes substantially with the time scale of the relaxation process. The memory kernels of the collision process will be presented. Because of their more involved structure, depending sensitively on the kinematical regime, both less and more stopping power is observed in the reaction compared to the Markovian description.

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I. MOTIVATION

The present work is concerned with quantum transport equations for central energetic heavy ion collisions. Thereby we have in mind a regime of incident bombarding energies about 200 MeV/nucleon–2 GeV/nucleon where two-body collisions play an important role and where relativistic kinematics is needed, but which is (hopefully) below the regime where relativistic many-body effects (e.g. particle production) or QCD interfere. The treatment of the time-dependent relativistic mean fields is well understood, but the proper incorporation of the collision is still a much debated subject. Most present microscopic simulations adopt a Markovian (i.e., kinetic) approach in that they assume as the basic mechanism *instantanous* binary collisions of two quasifree, i.e., on-shell, nucleons. This is handled in practice by a Boltzmann-Uehling-Uhlenbeck collision term; see, e.g., [1]. Conclusions on extracting for example the equation of state or more direct observables like the collective flow [1] depend sensitively on a delicate interplay between the collision term and the mean field. Much has been done to understand medium effects on the cross section in the collision term, e.g., in a G-matrix approach [2] to equilibrated or momentum deformed, yet still *quasistatic* nuclear matter. There remains, however, the question of memory effects beyond the Markovian limit. If these turn out to be significant, the Boltzmann-Uehling-Uhlenbeck description may no longer be appropiate. This is the topic which we will discuss in this paper.

Memory effects, as were originally studied at rather low excitation energies [3–5], come into play if the collision time interferes with the typical time scale over which the mean field changes [3–5], either due to coherent motion or due to relaxation of occupation numbers. A finite collision or correlation time is generated by a destructive interference of the various scattering channels (including off shell) building up for times going more and more towards the past. Also the collision time is enhanced by Pauli blocking which reduces the available phase space (needed for the destructive interference). This goes as far that one sees large memory effects at low energies, e.g., for the damping of giant resonances, which oscillate at a rate of about 60 fm/c [4]. For heavy ion collisions at much higher energies, a rather rough estimate of the collision time in the "quasifree" regime is $\sim$ 2 fm/c, the time which a nucleon moves through the collision distance. This looks fairly short. But the mean field potentials may very well change at the same scale in energetic heavy ion collisions [6] as well as relaxation processes can be very fast there, as we will see.

II. SCHEMATIC VIEW OF MEMORY EFFECTS

Before proceeding to the full-scale calculation, we want to discuss the presently adressed memory effects schematically. To this end we consider a pure relaxation process which obeys the integral equation

$$\frac{\partial}{\partial T}\rho(T) = \int_0^\infty d\bar{t}\, M^{\rm in}(T,\bar{t})\,[1-\rho(T-\bar{t})] - \int_0^\infty d\bar{t} M^{\rm out}(T,\bar{t})\rho(T-\bar{t})\,, \qquad (1)$$

where $M^{\rm in/out}$ define two memory kernels which should vanish in the remote past $\bar{t} \to \infty$. The first term on the right-hand side specifies a backscattering ("in") rate, the other one a direct-scattering ("out") rate. The only process here is a relaxation and memory effects are caused by an interference of collision (i.e., memory) times with relaxation times. If one remembers that the Boltzmann-Uehling-Uhlenbeck collision term contains also three other distribution functions to be integrated over phase space, the memory kernels itselves have to be merely complex functionals of the whole distribution function in phase space at earlier times $t' < T$, i.e., $M^{\rm in/out}(\rho(t');T,\bar{t})$. The Markovian (or Boltzmann-Uehling-Uhlenbeck) limit of Eq. (1) is obtained by assuming that the distribution function ρ varies sufficiently slow compared to the range in time of the kernels:

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$$\left.\frac{\partial}{\partial T}\rho(T)\right|_B = \left[\Gamma^{\rm in}(T)[1-\rho(T)] - \Gamma^{\rm out}(T)\rho(T)\right] , \qquad (2)$$

where the rates are defined as the corresponding integrals

$$\Gamma^{\rm in/out}(T) := \int_0^\infty d\bar{t}\, M^{\rm in/out}(T,\bar{t}) \geq 0 \qquad (3)$$

and should be non-negative to be meaningful. To see the possible interference of the finite extension of the kernels we can perform a Taylor expansion of ρ in the past up to first order in Eq. (1). This yields

$$\left.\frac{\partial}{\partial T}\rho(T)\right|^{(1)} = \left(1+\frac{\tau_{\rm mem}(T)}{\tau_{\rm rel}(T)}\right)\left.\frac{\partial}{\partial T}\rho(T)\right|_B + \delta\Gamma^{\rm in}(T)[1-\rho(T)] - \delta\Gamma^{\rm out}(T)\rho(T) , \qquad (4)$$

where the upper index (1) stands for 1 order correction. The following abbrevations are introduced:

$$\begin{aligned} \tau_{\rm in/out}(T) &:= \frac{\int_0^\infty d\bar{t}\,\bar{t}\, M^{\rm in/out}(T,\bar{t})}{\Gamma^{\rm in}(T)+\Gamma^{\rm out}(T)} , \\ \tau_{\rm mem}(T) &:= \tau_{\rm in}(T)+\tau_{\rm out}(T) , \\ \tau_{\rm rel}(T) &:= \frac{1}{\Gamma_{\rm in}(T)+\Gamma^{\rm out}(T)} . \end{aligned} \qquad (5)$$

The first factor $\mathcal{Z}(T) := 1+\tau_{\rm mem}(T)/\tau_{\rm rel}(T)$ stems from the zeroth and first order expansions of the explicit ρ and $(1-\rho)$ dependence of Eq. (1). As just stated above, the memory kernels $M^{\rm in/out}$ are also functionals in $\rho(t')$, so that in addition they also should be subject of an Taylor expansion in the historical evoluion of ρ. The thus occurring corrections of first order we have indicated in (4) by $\delta\Gamma^{\rm in/out}(T)$ being proportional to the functional derivative $\delta M^{\rm in/out}(\rho(t'))/\delta\rho(t)$. At the moment, however, in this schematic view, we do not want to further stress the possible manifestation of these additional terms. The Markov assumption states that $\rho(T-\bar{t}) \approx \rho(T)$ over the range of the kernels, or in other words, the kernels should take a form $M^{\rm in/out} \sim \delta(\bar{t}^+)$ for the relevant time scale of relaxation, i.e., $|\tau_{\rm mem}| \ll \tau_{\rm rel}$. At this stage of approximation we obtain $\mathcal{Z}(T) \approx 1$ and regain the Boltzmann-Uehling-Uhlenbeck equation for $(\partial/\partial T)\rho(T)\,|_B$. The relaxation rates $(\partial/\partial)T\rho(T)$ are renormalized by the "memory factor" $\mathcal{Z}$ as soon as the here defined memory time $\tau_{\rm mem}$ starts to interfere with the relaxation time $\tau_{\rm rel}$. Note that strong oscillations in the kernels $M^{\rm in/out}$ may allow both signs for $\tau_{\rm mem}$. According to Eq. (4), one thus observes enhanced relaxation for $\tau_{\rm mem} > 0$, as well as reduced relaxation for $\tau_{\rm mem} < 0$ in comparison to the pure Markovian limit of the underlying collision process Eq. (1). The Taylor (or gradient) expansion breaks down altogether if $|\tau_{\rm mem}/\tau_{\rm rel}| \gtrsim 1$, and the full structure of $M^{\rm in/out}$ needs to be considered. Also, of course, the significance of the other correction terms in (4) have to be investigated. Nonetheless, the ratio $\tau_{\rm mem}/\tau_{\rm rel}$ remains a useful first indicator for memory effects.

III. TRANSPORT EQUATION

In this section we want to give a short account of the formalism restricted to the minimum required formulas. More detailed derivations are given in [12, 16]. Our aim here is mainly to obtain a transport equation of the type characterized by (1) and to provide the invoked memory kernels for describing the collision process, which both will be subject of numerical treatment and discussion in the next section. Thereby, being more or less exploratory in this work, we are trying to reveal the time structure of the quantum mechanical collision process in a highly excited relativistic Fermion system (e.g., in a heavy ion collision) and not to put too much effort to describe the in medium scattering amplitudes $\Gamma^{\rm in/out}$ as realistic as for example carried out in a G-matrix approach [2].

For our study we choose a simple, but fairly realistic model, the linear quantum hadrodynamical (QHD) Lagrangian [7] with σ and ω mesons. This model is usually treated in mean field approximation. We go beyond that by adding correlations in Born-collisional approximation [8] to implement the two-particle collision processes and to extract the memory kernels. We formulate the emerging equations of motion with the technique of real time Green's functions as introduced by Schwinger [9] and Keldysh [9]. Here time arguments are defined and ordered on a special time contour path C running from an initial time t_0 to $+\infty$ and back to t_0. This formalism has been sucessfully applied to derive quantum transport equations for nuclear dynamics with nonrelativistic interactions [10]. Relativistic physics requires changes [11, 12]: The instantanous interactions are replaced by free retarded meson propagators, now of course on the contour C, e.g.,

$$\begin{aligned} D^0(1-1') &:= i\langle\,|\mathcal{P}[\hat{\phi}_I(1)\hat{\phi}_I(1')]|\,\rangle \\ &:= \theta_C(t_1,t_{1'})\,D^{0>}(1-1') + \theta_C(t_{1'},t_1)\,D^{0<}(1-1') , \end{aligned} \qquad (6)$$

where $\mathcal{P}$ is the time ordering and θ_C is the step function on the path C. These propagators depend only on the difference of the coordinates. The retarded and advanced propagators along the physical time axis are obtained by the rearrangement

$$\begin{aligned} D^{0\,\rm ret}(1-1') &= \theta(t_1-t_{1'})[D^{0>}(1-1') - D^{0<}(1-1')] , \\ D^{0\,\rm av}(1-1') &= -\theta(t_{1'}-t_1)[D^{0>}(1-1') - D^{0<}(1-1')]; . \end{aligned} \qquad (7)$$

The equation of motion for the one-particle nucleonic Green's function $G(1,1')$ looks similar to the nonrelativistic case:

$$[i\gamma_\mu\partial_1^\mu - m_N - g_s\langle\hat{\phi}(1)\rangle - g_v(\gamma_\mu)\langle\hat{V}^\mu(1)\rangle]G(1,1') = \delta_C^4(1,1')\mathbf{1} + \int_{t_0}^{t_0} d2\,\Sigma_B(1,2)G(2,1')\,, \tag{8}$$

where

$$\Sigma_B(1,2) = \langle 1\bar{3}|V|\bar{1}3\rangle\langle\bar{2}\bar{4}|V|24\rangle\,[G(4,\bar{3})G(3,\bar{4})G(\bar{1},\bar{2}) - \text{exch}\,] \tag{9}$$

is the irreducible Born collision self-energy operator and the relativistic interaction is given by the meson propagators as

$$\langle 12|V|\bar{1}\bar{2}\rangle = \delta_C^4(1,\bar{1})\delta_C^4(2,\bar{2})\,\left[-g_s^2\mathbf{1}_{1\bar{1}}D_S^0(1-2)\mathbf{1}_{2\bar{2}} + g_v^2(\gamma^\mu)_{1\bar{1}}D^0_{\mu\nu;\,V}(1-2)\,(\gamma^\nu)_{2\bar{2}}\right]\,.$$

The integral in Eq. (8) runs over space and time along C and it is summed over spin and isospin indices related with point 2. This holds also for the repeated indices in the expression for the self-energy. Note that the collision term in Eq. (8) is causal in the sense that integral contributions for $t_2 \geq \max(t_1, t_{1'})$ always cancel. Explicit unfolding of the equation of motion in terms of the two portions $G^<$ and $G^>$ results in the relativistically generalized Kadanoff-Baym equations [8]. The retarded time structure of the interaction complicates a further explicit unfolding for the self-energy operator $\Sigma_B^{\gtrless}$ such that a rather messy and not very instructive expression emerges.

In the following we restrict our considerations only to *spatially* homogenous systems and we assume spin and isospin saturation. The one-particle distribution function $\rho(\mathbf{p},T)$ in momentum space is given by the trace over spin and isospin indices of the Green's function $G^<$ at equal times $t_1 = t_2 = T$, i.e.,

$$\rho(\mathbf{p},T) = -i\frac{1}{4}\mathrm{Tr}\{\gamma_0 G^<(\mathbf{p};T,T)\}\,.$$

The equation of motion for $\rho(\mathbf{p},T)$ is obtained by taking the difference of Eq. (8) and its adjoint. We employ $[G/\Sigma^{\gtrless}(1,1')]^\dagger = -\gamma_0 G/\Sigma^{\gtrless}(1,1')\gamma_0$ and obtain

$$\partial_T\,\rho(\mathbf{p},T) = \frac{1}{2}\,\mathrm{Re}\left(\int_{t_0}^{T} d\bar{t}\,\mathrm{Tr}\left\{\Sigma^<(\mathbf{p};T,\bar{t})G^>(\mathbf{p};\bar{t},T) - \Sigma^>(\mathbf{p};T,\bar{t})G^<(\mathbf{p};\bar{t},T)\right\}\right)\,. \tag{10}$$

We remark that the reduction to a density $\rho(\mathbf{p},T)$ destroys manifest covariance of the equations; it means to decide for one particular frame, usually the rest frame of the system. Also the assumption of spatial homogenity for such a nonequilibrated and thus time-dependent system can hold only in one particular Lorentz frame. Equation (10) is incomplete as the right-hand side requires the full knowledge of $G(\mathbf{p};t_1,t_2)$ whereas the left-hand side computes only the evolution of $\rho(\mathbf{p},T)$. We neglect the influence of damping on $G(\mathbf{p};t_1,t_2)$ and parametrize it purely in terms of a mean field propagation as

$$G^{\gtrless}(\mathbf{p};t_1,t_2) \approx \frac{\not{p}+m^*}{\mp 2iE_p^*}e^{-i(E_p^*+g_vV^0)(t_1-t_2)} \times \begin{cases}\rho(\mathbf{p},t_{\min}) \\ [1-\rho(\mathbf{p},t_{\min})]\end{cases}\,, \tag{11}$$

where $t_{\min} = \min(t_1,t_2)$ represents the lower time of the two arguments. This ansatz is consistent in the Born approximation in that it neglects only collisional effects of higher order in the collision term. It is basically identical to that introduced by Tohyama [5] for describing nonrelativistic heavy ion collisions at much lower incident bombarding energies ($E_{\mathrm{lab}}/A \sim 6$ MeV). It was emphasized by Lipavsky *et al.* [13] that such a quasiparticle parametrization (11) of the Green's functions bounded to $\rho(\mathbf{p},t_{\min})$ at its lower time argument is the only possible way to guarantee causality in the evolution. Information on $\rho(\mathbf{p},T)$ can only propagate from the past to the future, but not vice versa. We explicitly discarded the contributions of the antiparticles, which is in accordance with the σ-ω model describing valence-particle dynamics only. Antiparticle production might be less important in the intermediate energy regime. The mean field time evolution operator is slightly simplified by fixing the effective mass m^* and the single-particle energy $p_0 := E_p^*$ at the lower time $t_{\min}$. V_0 is the nonvanishing, but constant zeroth component of the mean vector field. By inserting (11) into (10) one ends up with

$$\partial_T\,\rho(\mathbf{p},T) = \int_{t_0}^{T} d\bar{t}\,M^{\mathrm{in}}(\mathbf{p},T;\bar{t})[1-\rho(\mathbf{p},\bar{t})] - \int_{t_0}^{T} d\bar{t}\,M^{\mathrm{out}}(\mathbf{p},T;\bar{t})\rho(\mathbf{p},\bar{t})\,, \tag{12}$$

where the memory kernels for in and out scattering are defined as

$$M^{\mathrm{in/out}}(\mathbf{p},T;\bar{t}) = \frac{1}{2}\mathrm{Re}\left(\mathrm{Tr}\left\{\frac{\not{p}+m^*}{\pm 2iE_p^*}\Sigma^{\lessgtr}(\mathbf{p};T,\bar{t})\right\} \times e^{-i(E_p^*+g_vV^0)(\bar{t}-T)}\right)\,. \tag{13}$$

In addition to a trivial substitution $\bar{t} \to T - \bar{t}$, Eq. (12) becomes identical to Eq. (1). Note that the occurrence of a finite memory is a natural outcome of the reduction of the many-body dynamics to the one-body level. The

exact evolution of the system was local in time, obeying a many-body dynamics $i\partial_T|\Phi_n(T)\rangle = \hat{H}|\Phi_n(T)\rangle$, where $\hat{H}$ represents the full Hamiltonian of the interacting particles.

For the further evaluation, we neglect the exchange part of $\Sigma_B^{\lessgtr}$ which is a fair approximation for a dense system. This allows one to apply the Langreth-Wilkins rules [14] for convolutions on the real-time path C connected in series, it reduces the number of terms (unfolded in $G^>$ and $G^<$) enormously, and it makes a numerical treatment feasible. Accordingly the direct amplitude of (9) can be rewritten into three parts with integrations solely on the physical time axis, where only one has the common Boltzmann-Uehling-Uhlenbeck structure of two scattering particles.

The remaining two terms contribute corrections to the production or absorption of real mesons on mass shell (containing terms $\sim D_0^{\gtrless}$ over the whole time axis). This kind of dissipation, interesting in itself, shall be omitted. These terms would vanish in the Markovian limit. Principally, they arise if the memory in these processes is taken into account. Because of the intrinsic time dependence of the one-particle distribtion function, such terms could manifest. A similar, but simpler reasoning would already appear in the treatment of the relativistic Fock diagram: Because of the assumed quasistationarity, the most elementary bremsstrahlung process will vanish in the Markovian treatment due to energy-momentum restriction of the on-shell particles (nucleons as well as mesons) [12]. However, the temporal change of the nucleon phase-space density may contain such high frequencies, at least principally, which would allow for such emissions.

Thus we are left with

$$
\begin{aligned}
M^{\rm in}(\mathbf{p},T;\bar{t}) = \mathrm{Re}\Bigg(& \sum_{\alpha,\beta=1}^{5} g_\alpha^2 g_\beta^2 \int_{t_0\to-\infty}^{\infty} dt_3 \int_{t_0\to-\infty}^{\infty} dt_4 \int \frac{d^3p_2\, d^3p_3\, d^3p_4}{(2\pi)^6}\, \delta^3(\mathbf{p}-\mathbf{p_2}-\mathbf{p_3}+\mathbf{p_4}) \\
& \times e^{-i(E^*_p - E^*_{p_2})(\bar{t}-T)}\, e^{-i\,(E^*_{p_4}-E^*_{p_3})\,(t_4-t_3)}\, D_\alpha^{0\,\rm ret}(\mathbf{p}-\mathbf{p_2};T,t_3)\, D_\beta^{0\,\rm adv}(\mathbf{p}-\mathbf{p_2};t_4,\bar{t}) \\
& \times \mathrm{Tr}\left\{ \Gamma_\beta \frac{\not{p}+m^*}{2E^*_p} \Gamma^\alpha \frac{\not{p}_2+m^*}{2E^*_{p_2}} \right\} \mathrm{Tr}\left\{ \Gamma_\beta \frac{\not{p}_4+m^*}{2E^*_{p_4}} \Gamma^\alpha \frac{\not{p}_3+m^*}{2E^*_{p_3}} \right\} \\
& \times \rho(\mathbf{p_2},\bar{t})\rho(\mathbf{p_3},\min(t_3,t_4))\,[1-\rho(\mathbf{p_4},\min(t_3,t_4))] \Bigg)\,,
\end{aligned}
\tag{14}
$$

where in compact notation $g_\mu = g_v$; $\Gamma_\mu = \gamma_\mu$ for $\mu = 1,\ldots,4$ and $g_5 = ig_s$; $\Gamma_5 = \mathbf{1}$. The kernel $M^{\rm out}$ is generated by the replacement $\rho \to (1-\rho)$. The meson propagators are the usual form of retarded and advanced propagators [11].

Considering (12) and (14), the Markovian approximation is performed by neglecting the historical evolution of the internal quantities, which means to shift all time arguments of the densities, the effective masses, and the single-particle energies to the actual time T, i.e., replacing $\rho(\mathbf{p},\bar{t}) \to \rho(\mathbf{p},T)$. From the explicit expression (14) it is then obvious that the semi-infinite time integration in (12) restricts the contributing processes to on-shell scattering, thus leading to the usual energy-conserving δ-function. One ends up with the Boltzmann-Uehling-Uhlenbeck (BUU) equation including the Pauli blocking and the spin- and isospin averaged Born cross sections of the QHD Lagrangian:

$$
\begin{aligned}
\partial_T\, \rho(\mathbf{p},T)\,|_B = & \int \frac{d^3p_2}{(2\pi)^3} \frac{d^3p_3}{(2\pi)^3} \int \frac{d^3p_4}{(2\pi)^3}\, \frac{1}{E^*_p(T)E^*_{p_2}(T)E^*_{p_3}(T)E^*_{p_4}(T)} \\
& \times (2\pi)^4\, \delta^3(\mathbf{p}-\mathbf{p_2}-\mathbf{p_3}+\mathbf{p_4})\, \delta(E^*_p(T) - E^*_{p_2}(T) - E^*_{p_3}(T) + E^*_{p_4}(T)) \\
& \times \Bigg\{ g_s^4\, \frac{1}{(E^s_{\vec{p}-\vec{p}_2})^2 - [E^*_p(T) - E^*_{p_2}(T)]^2}\, \frac{1}{(E^s_{\vec{p}-\vec{p}_2})^2 - [E^*_p(T) - E^*_{p_2}(T)]^2} \\
& \times [p_\mu p_2^\mu + m^{*2}(T)][p_{3,\,\mu} p_4^\mu + m^{*2}(T)] \\
& - 2g_s^2 g_v^2\, \frac{1}{(E^s_{\vec{p}-\vec{p}_2})^2 - [E^*_p(T) - E^*_{p_2}(T)]^2}\, \frac{1}{(E^v_{\vec{p}-\vec{p}_2})^2 - [E^*_p(T) - E^*_{p_2}(T)]^2} \\
& \times m^{*2}(T)\,(p_\mu + p_{2,\,\mu})(p_3^\mu + p_4^\mu) \\
& + 2g_v^4\, \frac{1}{(E^v_{\vec{p}-\vec{p}_2})^2 - [E^*_p(T) - E^*_{p_2}(T)]^2}\, \frac{1}{(E^v_{\vec{p}-\vec{p}_2})^2 - [E^*_p(T) - E^*_{p_2}(T)]^2} \\
& \times [2m^{*4}(T) - m^{*2}(T)(p_\mu p_2^\mu + p_{3,\,\mu} p_4^\mu) + p_\mu p_3^\mu p_{2,\,\nu} p_4^\nu + p_\mu p_4^\mu p_{2,\,\nu} p_3^\nu] \Bigg\} \\
& \times \{\, \rho(\mathbf{p_2},T)\, \rho(\mathbf{p_3},T)\, [1-\rho(\mathbf{p_4},T)]\, [(1-\rho(\mathbf{p},T)] \\
& \quad - [1-\rho(\mathbf{p_2},T)]\, [1-\rho(\mathbf{p_3},T)]\, \rho(\mathbf{p_4},T)\, \rho(\mathbf{p},T)\, \}\,.
\end{aligned}
\tag{15}
$$

The δ function in the four quasiparticle energies, together with the phase-space factor $\rho\rho(1-\rho)(1-\rho)-(1-\rho)(1-\rho)\rho\rho$, lead to the usual feature of the Boltzmann-Uehling-Uhlenbeck collision term, that it will vanish only for an equilibrated and stationary (hot) Fermi distribution.

It can be shown [12, 16] that the quasiparticle ansatz together with the Markov assumption immensely simplifies to disentangle even more involved Feynman diagrams. The basic reason is that assuming *quasistationarity* (and quasihomogenity), the diagrams can be worked out like in equilibrium with Fourier techniques. This is the general idea for all derivations of quantum kinetic equations [12, 15]. Yet quasistationarity and the implied neglection of the corrections due to the first order gradient expansion are not *a priori* justified for such a violent reaction expected in a relativistic heavy ion collision.

IV. MEMORY KERNELS — RESULTS AND DISCUSSION

To simulate a central heavy ion collision we model the equilibration by considering an infinitely extended, spatially homogenous system of two interpenetrating and counterstreaming Fermi fluids. We parametrize the initial anisotropy in momentum space by a two-center Fermi distribution

$$\rho(\mathbf{p},t_0) = \frac{1}{\exp\left\{\beta(E_p^* - v|p_z|) - \beta\mu\right\}+1} \leq 1 \qquad (16)$$

for the valence nucleons, where the effective inverse temperature $\beta = \gamma\beta_0$ and the effective chemical potential $\mu = \mu_0/\gamma$ have been introduced with $\gamma = 1/\sqrt{1-v^2}$. The velocity v provides the anisotropy in momentum space and represents the mean velocity of each streaming fluid in the center-of-mass frame. The energy is given by the expectation value of the mean field Hamiltonian within the counterstreaming Fermi fluid scenario. For low values of the velocity v, the c.m. energy rises like $E_{\mathrm{kin,cm}}/A = m^*(\gamma-1)$. For higher velocities the relativistic compression of the fluids gives an enhancement of the vector potential V^0, which further raises the energy. Then the incident bombarding energy in the laboratory frame is given by $2\left(\frac{(E_{\mathrm{tot,c.m.}}/A)^2}{m_N} - m_N\right)$. The chemical potential and the effective mass $m^* = m_N + g_s\langle\phi\rangle$ are determined in fixing the baryon density $\rho_B = 4\int d^3p\,\rho(\mathbf{p},t_0)$ to $2\gamma\rho_0$, where ρ_0 is the nuclear ground state density and γ accounts for the Lorentz contraction of the two oppositely boosted nuclei. The shape of the generalized Fermi surface reads, for zero temperature $\beta^{-1}=0$,

$$\frac{p_\perp^2}{\mu^2\gamma^2 - m^{*2}} + \frac{(|p_z| - \mu v\gamma^2)^2}{\gamma^2(\mu^2\gamma^2 - m^{*2})} = 1\ .$$

It represents two ellipsoids shifted in longitudinal momentum by $\mu v\gamma^2$, which may be overlapping or separated, depending on whether the shift is smaller or larger than the longitudinal axis $\gamma(\mu^2\gamma^2 - m^{*2})^{1/2}$. Here we employ the linear σ-ω model with the parameters $g_s = 9.57$, $m_s = 550\,\mathrm{MeV}$, $g_v = 11.67$, $m_v = 783\,\mathrm{MeV}$, which gives a small effective mass $m^*/m = 0.55$ for the nuclear ground state and $m^*/m \sim 0.3$ for the two-fluid model at $v \sim 0.7c$. With these parameters the two ellipsoids get detached for velocities larger than $v \sim 0.7c$ which roughly corresponds to a bombarding energy of 900 MeV/A. These parameters optimize the mean field within the σ-ω model and overestimate somewhat the scattering cross section compared to more detailed in-medium scattering amplitudes as computed within the G-matrix approach [2]. We nonetheless take it as a first reasonable input.

The numerical calculations exploit the axial symmetry of the simulation. The convolution integrations in Eq. (14) are performed with the convolution theorem and Fourier-Bessel transformations. Time integration is done with a predictor-corrector scheme and a time step of $0.15\,\mathrm{fm}/c$. The final accurancy of the results is on the 1 % level. The retardation of the meson fields is neglected in these first and exploratory calculations.

In this particular case, using the instantaneous approximation for the meson propagators in the memory kernels (14), the first order expansion as sketched in Sec. II takes the instructive form

$$\left.\frac{\partial}{\partial T}\rho(T)\right|^{(0)} = \mathcal{Z}^{\mathrm{in}}(T)\Gamma^{\mathrm{in}}(T)[1-\rho(T)] - \mathcal{Z}^{\mathrm{out}}(T)\Gamma^{\mathrm{out}}(T)\,\rho(T)\ , \qquad (17)$$

where the in and out renormalization factors are given by

$$\mathcal{Z}^{\mathrm{in/out}}(\mathbf{p},T) = 1 + \frac{\tau_{\mathrm{mem}}(\mathbf{p},T)}{\tau_{\mathrm{rel}}(\mathbf{p},T)} - \frac{1}{\Gamma_{\mathrm{in/out}}(\mathbf{p},T)}\frac{\partial}{\partial T}\left(\frac{\tau_{\mathrm{in/out}}(\mathbf{p},T)}{\tau_{\mathrm{rel}}(\mathbf{p},T)}\right) \qquad (18)$$

and where the first two terms, $1+\tau_{\mathrm{mem}}/\tau_{\mathrm{rel}}$, are the common renormalization of the scattering rates, as already outlined in Sec. II. The last term ($\equiv \delta\Gamma^{\mathrm{in/out}}$), which may differ for the in and out process, arises from the expansion of the kernels $M^{\mathrm{in/out}}(\rho)$

First, we want to investigate the time structure of the memory kernels $M^{\mathrm{in/out}}$ as such. To this end, we calculate them for a fixed mean field and distribution function, i.e., for fixed $\rho(\mathbf{p})$, m^*/m, and E_p^* accordingly. We show in Fig. 1 the results for a small center-of-mass velocity $v = 0.4c$ and zero temperature $\beta^{-1} = 0$. This corresponds to a bombarding energy of $E_{\mathrm{kin,Lab}} \simeq 200\,\mathrm{MeV}/A$. In the left upper part Fig. [1(a)] the density distribution $\rho(p_r, p_z = 0)$ and the Boltzmann-Uehling-Uhlenbeck rate $\dot{\rho}|_B$ along the radial axis p_r are depicted. As expected, the reaction is strongest at $p_r \sim 290$ MeV, slightly above the Fermi momentum where the on-shell inscattering is possible and unblocked. The right part of Fig. 1 shows the two memory kernel at that momentum. M^{out} exhibits a pronounced oscillation through zero, becoming negative after $\sim 1\,\mathrm{fm}/c$ in the past [compare Fig. 1(b)]. Visually the kernels stretch up to $\sim$ 2–4 fm/c. Because of the oscillation in M^{out}, the memory time defined in (5) turns out to be $\tau_{\mathrm{mem}} \sim -3\,\mathrm{fm}/c$ and is hence negative. The relaxation time is evaluated to be $2\,\mathrm{fm}/c$. In the left lower part Fig. [1(c)], both times are plotted along

the whole radial momentum axis p_r, showing that the Markovian condition $\left|\frac{\tau_{\rm mem}}{\tau_{\rm rel}}\right| \ll 1$ is clearly violated nearly everywhere, particularly at the most reactive Fermi surface. In general, the pattern of each kernel change with the position (p_r,p_z) in momentum space. For large momenta well above the Fermi surface, $M^{\rm in}$ displays several oscillations, whereas $M^{\rm out}$ turns out to become more smooth. This behavior is almost completely reverted around the Fermi level and even more in the interior of the density distribution. But the deep interior or the far outer space are physically not relevant, because the total scattering rates $\dot{\rho}|_B$ are vanishingly small [see Fig. 1(a)]. In addition, to demonstrate also the breakdown of the gradient expansion more explicitly, we have depicted in Fig. 1(d) the two renormalization factors $\mathcal{Z}^{\rm in/out}$ and also their common contribution $\mathcal{Z} = 1 + \tau_{\rm mem}/\tau_{\rm rel}$ along the radial axis. This were taken from a dynamical integration of the Boltzmann-Uehling-Uhlenbeck equation after some short evolving time $T = 2$ fm/c, so that the time derivatives in (18) could be extracted (the pure BUU dynamics compared to the full, non-Markovian evolution is shown in Fig. 4). One recognizes that the negative renormalization physically does not make sense, and so the process clearly *must* be non-Markovian. Nonetheless trying to give arguments along the first order expansion one is tempted to reason that the actual relaxation in momentum space should be considerably slowed down compared to the dynamics treated in the Markovian limit.

As a further and extreme example we consider in Fig. 2 a very violent reaction of $v = 0.9c$, where the two ellipsoides are separated. This would correspond to a bombarding energy of $\simeq 4$ GeV/nucleon which goes beyond the limits of our approach. The reaction rate $\dot{\rho}|_B$ has a maximum of about $0.45c$/fm at $p_r \sim 410$ MeV. The characteristic oscillations of $M^{\rm out}$ almost disappear [Fig. 2(a)] and the memory time $\tau_{\rm mem} \sim 0.4\,{\rm fm}/c$ turns out to be small, but positive. Now, although $\dot{\rho}|_B$ is moderate, the corresponding relaxation rate Γ_B reaches $5\,c$/fm, and thus the critical ratio becomes $\frac{\tau_{\rm mem}}{\tau_{\rm rel}} = 1.7$ [Fig. 2(b)]. The Boltzmann-Uehling-Uhlenbeck equation is invalidated again.

The relaxation in the process will reduce the velocity $\pm v$ and thermalize the distribution. In order to investigate the typical memory kernels at later stages, we also consider cases with temperatures $\beta^{-1} = 50$–150 MeV in the parametrization (16). The results are summarized in Fig. 3 by plotting the critical ratio $\frac{\tau_{\rm mem}}{\tau_{\rm rel}}$ for several velocities v and different temperatures at the gridpoint p_r $(p_z = 0)$ with largest scattering rate $\dot{\rho}|_B$ of each case. A ratio $\frac{\tau_{\rm mem}}{\tau_{\rm rel}} \geq 0.5$ is encountered for high temperatures and/or velocities, both related to high incident energies; here the relaxation should be enhanced. The Markov assumption is not valid and even the gradient expansion may fail in some cases. An interesting aspect is the possibility to obtain more stopping power in heavy collisions arising from the occurrence of constructive dynamical interference during the collision process. For intermediate energies ≤ 1 GeV/nucleon one may observe a reduction of the relaxation in the first stages of the evolution because $\frac{\tau_{\rm mem}}{\tau_{\rm rel}} \leq -0.5$ at small incident temperatures $\beta^{-1} \approx 0$. But soon larger temperatures are reached and the process should behave Markovian, because the oscillations in the still extended kernels cancel to a small memory time $\tau_{\rm mem}$. In a nonrelativistic energy domain of $\lesssim 400$ MeV/nucleon $(v \lesssim 0.5\,c)$ we observe $\frac{\tau_{\rm mem}}{\tau_{\rm rel}} \leq -1.0$ and thus the reaction as described in our model will be

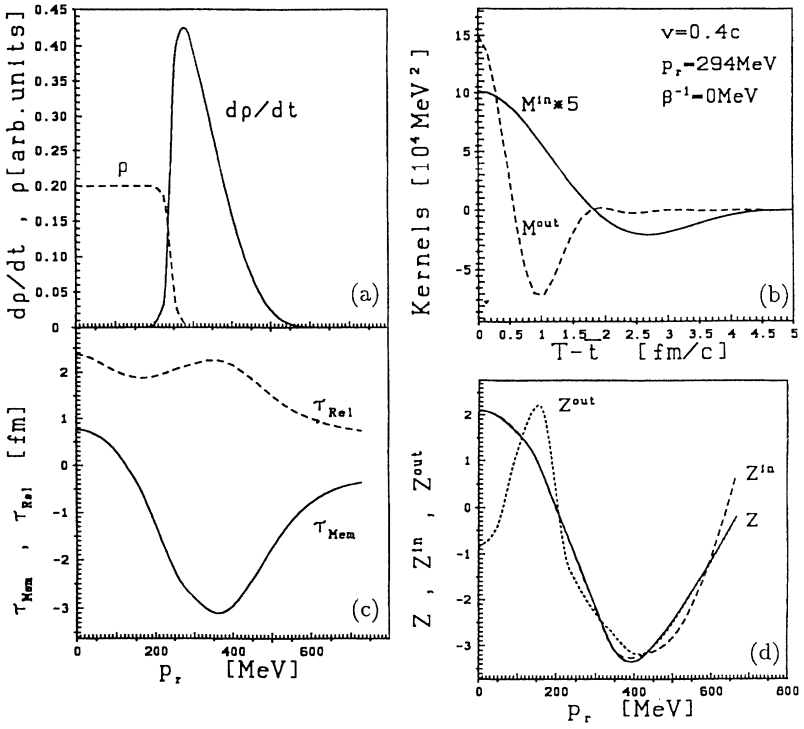

FIG. 1. $v/c = 0.4$, $m^* = 271.7$ MeV, and $\mu_{\rm eff} = 365.1$ MeV. (a) The distribution function $\rho(p_r,p_z{=}0)$ (in arbitrary units, going from 1 to 0) and the Boltzmann-Uehling-Uhlenbeck scattering rate $\partial_T \rho(p_r,p_z{=}0)|_B$ are shown along the radial axis of momentum space. (b) The memory kernels $M^{\rm in/out}$ as function of time in the past. (c) The memory time $\tau_{\rm mem}(p_r,p_z{=}0)$ (solid line) and the relaxation time $\tau_{\rm rel}(p_r,p_z{=}0)$ (dashed line) as function of radial momentum. (d) The renormalization factors $\mathcal{Z}^{\rm in/out}$ and $1 + \tau_{\rm mem}/\tau_{\rm rel}$ (compare text) are extracted from a dynamical simulation of the Boltzmann-Uehling-Uhlenbeck equation as presented in Fig. 4.

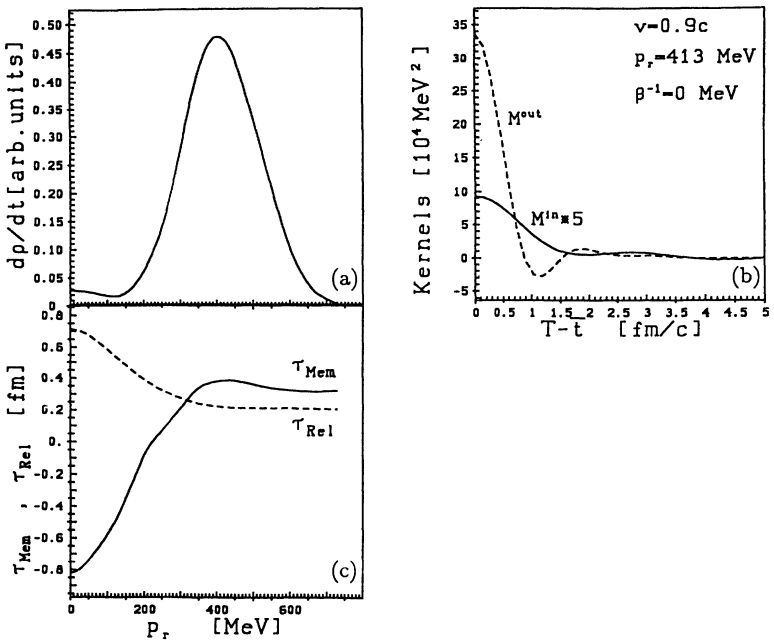

FIG. 2. $v/c = 0.9$, $m^* = 262.9$ MeV, und $\mu_{\text{eff}} = 164$ MeV. (a) The scattering rate $\partial_T \rho(p_r, p_z{=}0)|_B$ is shown along the radial axis of momentum space. (b) The memory kernels $M^{\text{in/out}}$ as function of time in the past, the memory time $\tau_{\text{mem}}(p_r, p_z{=}0)$ (solid line). (c) The relaxation time $\tau_{\text{rel}}(p_r, p_z{=}0)$ (dashed line) as function of radial momentum is depicted.

significantly slowed down due to the memory effects and does not behave Markovian.

Finally, we investigate a *complete* time evolution according to Eqs. (12,14), neglecting again the retardation of the meson propagators. This has been done for the initial configuration with $v = 0.4c$ and temperature $\beta^{-1} \approx 0$. The proper initialisation requires that one runs the dynamics with fixed mean field and distribution some "preparation time" from $\bar{t} = t_0 = -10\,\text{fm}/c$ to $\bar{t} = T_0 = 0$ in order to build up properly the memory kernels and to describe complete collisions as in a Boltzmann-Uehling-Uhlenbeck equation (this is connected to the question of energy conservation which will be discussed in a separate publication [16]). At $T = T_0 = 0$, the full dynamics is released and relaxation can go its way. A comparison of the full evolution with the Boltzmann-Uehling-Uhlenbeck case is shown in form of density contour plots in Fig. 4(a). Because of the axial symmetry of the system only the upper half of the (p_r, p_z) plane is depicted. One clearly sees a much delayed relaxation in the full treatment; the reaction is slowed down significantly. This is also reflected in the evolution of the quadrupole moment of the distribution function. Calculated from the density by $Q = \frac{1}{2}\left(3\langle p_z^2\rangle/\langle p^2\rangle - 1\right)$, it is a measure of the deviation of the system from equlibrium. Figure 4(b) shows $Q(T)$ according to the pure BUU- and the full evolution. An enhancement of the relaxation time by a factor of 2–3 is clearly visible.

This result agrees with the study of Danielewicz [17], who has solved numerically the complete nonrelativistic Kadanoff-Baym equations in a similar homogenous system for a bombarding energy $\sim$ 400 MeV/nucleon. It has been argued there that the "slowing down" is due to the Heisenberg uncertainty in energy between proceeding, independent collisions. From our point of view, these results may be reinterpreted by means of the non-Markovian charater of the transport process and the corresponding memory kernels having here $\frac{\tau_{\text{mem}}}{\tau_{\text{rel}}} < 0$.

It is to be remarked that we have used coupling parameters g_s and g_v which are adjusted to describe the mean field. But the absolute strength of the collision term depends also very sensitively on these couplings, $\sim g^4$. To account for a more realistic description of heavy ion collisions, one should employ for the collision term such coupling constants which (more directly) reproduce the (in-medium) cross section of nucleons. This will be pursued in a furthcoming publication [16]. It suffices for the present purposes to estimate how the results scale with g.

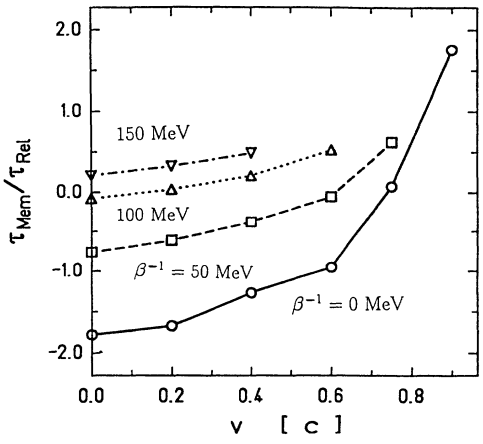

FIG. 3. The ratio of memory time versus relaxation time $\tau_{\text{mem}}/\tau_{\text{rel}}$ is summarized for different velocities and temperatures at the momentum grid point p_r with the highest Boltzmann-Uehling-Uhlenbeck scattering rate. Exclusively, the couplings of the linear QHD model were employed. Note that the Markov assumption is only guaranteed for some special configurations.

The memory time $\tau_{\rm mem}$ will change little if we take care to leave the mean field unchanged, because the shape of the kernels is not sensitively affected by any rescaling. But the relaxation time scales as the inverse cross section, $\tau_{\rm rel} \sim \sigma^{-1} \sim g^{-4}$. Hence reducing the effective cross section will, of course, enhance the relaxation time $\tau_{\rm rel}$ while the memory time $\tau_{\rm mem}$ remains nearly unaffected. Thus the whole dynamics may come bit closer to the Markovian limit.

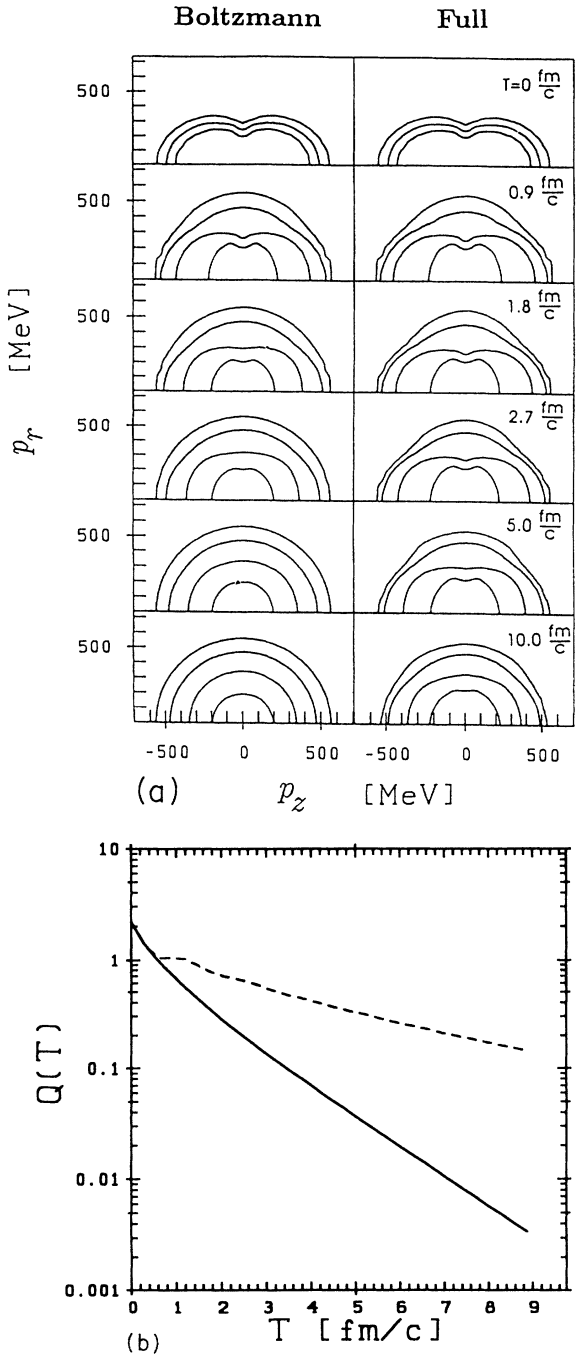

FIG. 4. The time evolution of a momentum anisotropical system with $v = 0.4c$ for the Boltzmann-Uehling-Uhlenbeck equation and the full one including memory are compared. (a) The lines from the inner to the outer refer to a constant density distribution in momentum space of 0.9, 0.5, 0.05, and 0.005, respectively. (b) Additionally we show the relaxation of the quadrupole moment of the momentum distribution function for the pure BUU (solid line) and the full dynamics (dashed line). A strong deacceleration in the transport process containing memory is found.

V. SUMMARY AND OUTLOOK

The intention of our work was to investigate the significance of memory effects in transport theories for energetic heavy ion collisions and its consequences for the equilibration times.

Memory effects are the inevitable result of a reduction of the multiparticle dynamics on, e.g., the one-body level, which induces phase correlations into the history of the system [18]. Their interference back in the past time gives raise to a characteristic correlation or collision time. Large phase spaces for the intermediate correlations give rise to large destructive interference and thus to very short memory times. This is the hope beyond the Boltzmann-Uehling-Uhlenbeck-type descriptions where the intermediate collisions are treated as being instantanous.

For the theoretical description we employed the techniques of real-time Green's functions for nonequilibrium dynamics leading to a transport equation posessing an in- and out-scattering contribution. The Boltzmann-Uehling-Uhlenbeck description is obtained as the Markovian limit of this equation. However, the Markov assumption is only valid if the characteristic time scale of the relaxation in phase space is large compared to the correlation time. As a quick and easy measure for the memory effects we introduced the memory time as first integral moment of the memory kernel in the collision term. The ratio of memory time to relaxation time serves as a criterion for the validity of the Markovian approximation.

We have calculated the memory kernel for a relativistic σ-ω model where the heavy ion collision is simulated in a model of two counterstreaming homogenous nuclear fluids. We find that the memory times depend sensitively on the energy in the system. It is very large for low energies and ranges around a few fm/c for the typical relativistic collisions. Thus they still interfere with the relaxation times which are of the same order. Moreover, it is an interesting feature that the memory times can have both signs, positive or negative, due to the oscillatory pattern of the memory kernels. The the correction from memory effects can act in both directions, reduced dissipation for negative memory times and enhanced dissipation for positive memory times. Both cases are observed in practice depending on the kinematical situation.

In our exploratory study the (in-medium) cross sections are probably overestimated and should be refined to make more quantitative predictions [16]. The overall cross section will influence relaxation time $\tau_{\rm rel}$. The shape of the memory kernels and its spreadings are not much affected by a tuning of the scattering rates which means that the memory times $\tau_{\rm mem}$ are robust against a rescaling of the cross section. But it is to be noted that a more realistic cross section needs to fitted up the

details of the angular distribution and changing the angular distribution of the cross section has effects which cannot be easily deduced from scaling arguments. Altogether we expect that the observed interference of the two time scales will still persist and lead to subsequent modifications compared to the Boltzmann-Uehling-Uhlenbeck limit. Thus one should reanalyze the memory effects with carefully fitted cross sections. A good first indicator is the simple ratio $\tau_{\rm mem}/\tau_{\rm rel}$ which then asks for more detailed analysis of all contributions if it turns out not to be very small.

It was found that the shapes of the memory kernels follow very general pattern in spite of the complicated microscopic expressions which have been used to generate them. It is another important issue for future investigations to related these pattern to simple phase-space arguments. This may allow eventually much simpler and nonetheless microsopic estimates of the memory effects which could turn out to be useful also in other areas where cascadelike descriptions are used (parton cascades, quark-gluon dynamics, hadron dynamics).

To conclude, we find that a full treatment of a relativistic quantum collision term in the interior of a typical central heavy ion collision can produce relaxation rates which differ significantly from the standard Boltzmann-Uehling-Uhlenbeck treatment. The changes go in different directions depending on the kinematical regime: An enhanced collision rate can be expected at very high bombarding energies whereas a reduced rate is found at lower energies. The Markovian limit is recovered for thermalized systems only in the final stages of the collision.

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[1] G. Peilert, A. Rosenhauer, J. Aichelin, H. Stöcker, and W. Greiner, Phys. Rev. C **39**, 1402 (1989); J. Jaenicke, J. Aichelin, N. Ohtsuka, R. Linden, and A. Faessler, Nucl. Phys. **A536**, 201 (1989).

[2] A. Bohnet, N. Ohtsuka, J. Aichelin, R. Linden, and A. Faessler, Nucl. Phys. **A494**, 349 (1989); B. ter Haar and R. Malfliet, Phys. Lett. B **172**, 10 (1986); Phys. Rev. C **36**, 1611 (1987).

[3] P. Grangé, H.A. Weidenmüller, and G. Wolschin, Ann. Phys. (N.Y.) **136**, 190 (1981).

[4] H.L. Yadav, P.G. Reinhard, and C. Toepffer, Nucl. Phys. **A458**, 301 (1986).

[5] M. Tohyama, Phys. Lett. **163B**, 14 (1985); Phys. Rev. C **36**, 187 (1987).

[6] R.Y. Cusson, P.-G. Reinhard, H. Stöcker, J.J. Molitoris, M.R. Strayer, and W. Greiner, Phys. Rev. Lett. **55**, 2786 (1985); J.J. Bai, R.Y. Cusson, J. Wu, P.-G. Reinhard, H. Stöcker, and M.R. Strayer, Z. Phys. A **326**, 269 (1987).

[7] J.D. Walecka, Ann. Phys. (N.Y.) **83**, 491 (1974).

[8] L.P. Kadanoff and G. Baym, *Quantum Statistical Mechanics* (Benjamin, New York, 1962).

[9] J. Schwinger, J. Math. Phys. **2**, 407 (1961); L.V. Keldysh, Zh. Eksp. Teor. Fiz. **47**, 1483 (1965) [Sov. Phys. JETP **20**, 1018 (1965)].

[10] C.Y. Wong and H.H.K. Tang, Phys. Rev. C **20**, 1419 (1979); P. Danielewicz, Ann. Phys. (N.Y.) **152**, 239 (1984); W. Botermans and R. Malfliet, Phys. Rep. **198**, 115 (1990).

[11] B. Bezzerides and D.F. DuBois, Ann. Phys. (N.Y.) **70**, 10 (1972).

[12] S. Mrowczynski and U. Heinz, Ann. Phys. (N.Y.) **229** (1994).

[13] P. Lipavsky, V. Spicka, and B. Velicky, Phys. Rev. B **34**, 6933 (1986).

[14] D. Langreth and J.W. Wilkins, Phys. Rev. B **6**, 3189 (1972).

[15] S.R. DeGroot, W.A. Van Leeuwen, and Ch.G. Van Weert, *Relativistic Kinetic Theory* (North-Holland, Amsterdam, 1980).

[16] C. Greiner, K. Wagner, and P.-G. Reinhard (unpublished); C. Greiner, Ph.D. thesis, University Erlangen, 1992; K. Wagner, Ph.D. thesis, University Erlangen, 1993.

[17] P. Danielewicz, Ann. Phys. (N.Y.) **152**, 305 (1984).

[18] E. Fick and G. Sauermann, *The Quantum Statistics of Dynamic Processes*, Springer Series in Solid-State Sciences Vol. 86 (Springer, New York, 1990).