Coulomb and nuclear excitation in intermediate-energy heavy-ion collisions

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Following previous works where the field of application of Glauber methods were extended to heavy-ion scattering in the energy range of 30−400 MeV/nucleon, we develop here a formalism which includes both Coulomb and nuclear excitation processes and explicitly accounts for the effects arising from the energy difference in different channels. First order inelastic excitations are described in the framework of a distorted wave eikonal approximation; in this connection nuclear phase shifts and form factors are described in terms of microscopic nucleon-nucleon interactions while the Coulomb excitation is described through a proper manipulation of phenomenological form factors. Several specific examples are discussed. The interest is then focused on second-order processes, more specifically on the effects introduced in the elastic channel by the coupling to the inelastic channels, and the corresponding Coulomb and nuclear polarization potentials are derived and discussed. On the ground of these achievements a general formalism is built up, capable of describing coupled-channel problems at any order of scattering. Due to the eikonal propagation, the multichannel multistep series leads to simple algebraic forms for each partial-wave scattering matrix, which involve powers of a channel matrix whose elements only depend on the impact parameter.

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I. INTRODUCTION

In the last years a large amount of experimental data has become available in the field of heavy-ion grazing collisions at intermediate energies. These include elastic and inelastic scattering, stripping and pickup reactions, spin and isospin exchange. These grazing collisions often involve strong coupling to collective states and are usually described in the framework of the optical model and coupled-channels formalism [1]. In these approaches it is not always easy to separate the structure aspects from those pertaining to the scattering mechanisms. This is, for example, reflected in the strong dependence of the optical and coupling potentials on the bombarding energy. In addition, the geometrical features arising from the coupling of the large number of relative motion partial waves with the channel angular momenta generate rather involved computational problems.

While resorting to these approaches is somehow inescapable at low bombarding energies due to the dominance of mean-field features of the problem, it is a long standing achievement that at higher energies it can be assumed that the dynamics of the scattering process is dominated by nucleon-nucleon collisions. In this connection, the Glauber model [2] has been widely used to describe reactions induced by light ions at high energy. The only ingredients for these descriptions are the densities and transition densities of the colliding nuclei and the elementary N-N scattering amplitudes. Owing to its appealing simplicity, it is clearly worthwhile to assess the applicability of this method to intermediate energy heavy-ion scattering processes. Such a project was carried out by DeVries *et al.* [3] for the analysis of total reaction cross sections down to bombarding energies of a few MeV/nucleon. Though this is a rather simple quantity, these achievements can be viewed as a suggestion to exploit the possibilities of Glauber methods in more complicated heavy-ion scattering processes.

In a series of papers [4–8] the authors have developed a method for the description of heavy-ion grazing collisions at intermediate energies ($E/A \simeq 30-400$ MeV) in the presence of strong channel coupling. In these developments the starting point has been the extended version of the Glauber theory set up by Czyż and Maximon [9] and by Formánek [10] which includes the description of nucleus-nucleus scattering. The basic ingredients that this method requires to lead to actual calculations is the knowledge of nuclear correlation functions of any order, and the evaluation of the related multidimensional integrals. In this rigid formulation this is not a practicable approach. Also in the reductive hypothesis of totally uncorrelated wave functions the combinatorial aspects of the classification of the sequences of the elementary collisions make the method unpractical already for relatively light colliding system, as for example the case of α-^{12}C [7]. As an alternative method the authors have suggested introducing a truncated basis of nuclear states between two successive microscopic collisions [7]. As in the standard coupled-channel approaches the basis is restricted to those states which reliably are expected to play a relevant role in the considered reaction. In the framework of the Glauber theory, this coupled-channel description leads to an algebrization of the scattering problem: the contribution of each impact parameter to the scattering amplitude is in fact expressed in terms of suitable products of elements of a channel matrix which depend on the impact parameter only.

Although a variety of processes have been satisfactorily described by the above formalism, such as elastic and inelastic scattering [4–7], charge exchange [11] and transfer reactions [12], and including features as the optical and polarization potentials [8], it is not possible to introduce in the formalism the Coulomb excitation and the effects

related to the excitation energy of the inelastic channels. In an alternative approach Feshbach and Hüfner [13] start from the eikonalization of a scattering problem directly formulated within a coupled-channel scheme. While finite Q-value effects have been explicitly taken into account, the Coulomb excitation processes have not been considered. On the other hand, Coulomb excitation theories developed for the description of high-energy scattering problems take into account the nuclear interaction only through a strong absorption radius [14, 15]. In the intermediate energy range we are interested in, the Coulomb and nuclear excitation are often of comparable importance, so that an unified perspective is necessary. This perspective has been pursued in some recent papers (see, e.g., Ref. [16]), which have, however, concentrated their interest only on first-order inelastic scattering processes.

The aim of our present work is to give a systematic description of elastic and inelastic heavy-ion intermediate energy processes at any order of scattering, in the framework of a nucleus-nucleus multiple-scattering theory. An explicit coupled-channel scheme is introduced and eikonal propagation is assumed for the nucleus-nucleus relative motion. Such an approach allows one to introduce in a natural way both Coulomb excitation and Q-value effects. In the limiting case in which these features are disregarded one obtains the formalism previously developed in Ref. [7].

We start by considering in Sec. II an eikonalized distorted-wave Born approximation in which both the Coulomb and the nuclear forces are taken into account to describe first-order inelastic excitation. Several examples of analysis of experimental data are displayed, which correspond to different patterns of Coulomb-nuclear interference. We also introduce a method for describing the Q-value effects for the nuclear excitation without giving up the possibility of exploiting some phenomenological features of the Glauber theory. In Sec. III the interest is focused on second-order processes, more specifically on the effects introduced in the elastic channel by the coupling to the inelastic channels. In this connection we derive and discuss the corresponding Coulomb and nuclear polarization potentials. The general formalism is developed in Sec. IV. The main points of the method can be summarized as follows. In a first stage a nucleus-nucleus Hamiltonian is explicitly introduced in which the interaction between the two nuclei is described through effective N-N interactions [13, 8], and then the formalism is extended to include the Coulomb interaction. The Lippmann-Schwinger equation describing this problem is expanded in a Born series. The Green's function describing the propagation of the two nuclei between two successive collisions is described in the eikonal approximation. Using as a guideline some basic features obtained in the description of the first- and second-order processes, we proceed to describe the main features of the dynamics of the multistep scattering process through a form-factor matrix in the channel space, depending only on the impact parameter. As a final result the scattering amplitude of any process is expressed by simple algebraic manipulations of products of this matrix.

II. FIRST-ORDER TRANSITIONS IN THE DISTORTED WAVE EIKONAL APPROXIMATION

A. The Coulomb excitation

In this section we are dealing with the first-order Coulomb excitation in heavy-ion collisions at intermediate energies. For simplicity we consider the case of a structureless projectile and discuss the transition between the ground state and a state LM of the target. By separating outside the charge radius R_C the Coulomb interaction V_C in the monopole-monopole term $V_C^0 = Z_P Z_T e^2/r$ and the monopole-multipole interaction $\mathcal{V}_C$,

$$\mathcal{V}_C = \sum_{\lambda=1,\mu}^{\infty} \frac{4\pi Z_P e}{(2\lambda+1)} \frac{1}{r^{L+1}} Y^*_{\lambda,\mu}(\hat{r}) \mathcal{M}(\lambda, -\mu) \ , \tag{2.1}$$

the first-order scattering amplitude in the distorted wave approximation can be written in the form

$$f^C_{0\to LM}(\theta,\phi) = -\frac{m}{2\pi\hbar^2} \int d\mathbf{r} \ \Psi^{(-)*}(\mathbf{k}_f, \mathbf{r}) F^C_{LM}(\mathbf{r}) \Psi^{(+)}(\mathbf{k}_i, \mathbf{r}) \ , \tag{2.2}$$

where

$$F^C_{LM}(\mathbf{r}) = \langle LM|\mathcal{V}_C|00\rangle = Q_L \frac{1}{r^{L+1}} Y^*_{LM}(\hat{r}) \tag{2.3}$$

is the corresponding electric 2^L-pole form factor. The coupling constant is

$$Q_L = \frac{4\pi Z_P e}{(2L+1)^{3/2}} [B_{0\to L}(EL)]^{1/2} \ , \tag{2.4}$$

where the reduced transition probability is commonly expressed through the phenomenological 2^L-pole deformation parameter β^C_L according to

$$B_{0\to L}(EL) = |\langle L||\mathcal{M}(EL)||0\rangle|^2 = \left[\frac{3Z_T e}{4\pi}(R_C)^L \beta^C_L\right]^2 . \tag{2.5}$$

Since we are considering the excitation process at relatively high bombarding energy, an appropriate description of the relative motion is obtained within the eikonal approximation. In this approximation the distorted waves have the expressions

$$\Psi^{(-)*}(\mathbf{k}_f, \mathbf{r}) \simeq \Psi^{(-)*}_{\text{eik}}(\mathbf{k}_f, \mathbf{r}) = e^{-\mathbf{k}_f\cdot\mathbf{r}} \exp\left(-\frac{i}{\hbar v} \int_z^{\infty} dz' V^0_C(\mathbf{b}, z') \right) , \tag{2.6}$$

$$\Psi^{(+)}(\mathbf{k}_i, \mathbf{r}\ ) \simeq \Psi^{(+)}_{\text{eik}}(\mathbf{k}_i, \mathbf{r}\ ) = e^{\mathbf{k}_i\cdot\mathbf{r}} \exp\left(-\frac{i}{\hbar v} \int_{-\infty}^{z} dz' V^0_C(\mathbf{b}, z') \right) . \tag{2.7}$$

Here and in the following the quantities depending on the relative motion coordinate $\mathbf{r}$ will be evaluated according to the prescription described in Ref. [4]. This amounts to evaluating the integrals along a straight line trajectory with impact parameter $b' = r_0(b)$ where $r_0(b)$ is the distance of closest approach for a Coulomb trajectory. This prescription has been successfully applied in a series of papers (see, e.g., Refs. [6, 17]); an improved version, which also includes the effect of the nuclear interaction on the point of closest approach, was given in Ref. [18]. On choosing the z axis perpendicular to the transferred momentum $\mathbf{q}$ and separating $\mathbf{r}$ into its components $(\mathbf{b}, z)$, the eikonal Coulomb phase shift is given [19] by

$$\chi_C(b) = -\frac{i}{\hbar v}\int_{-\infty}^{\infty} dz\; V_C^0(\mathbf{r}) = 2i\eta \ln(kb) \tag{2.8}$$

and one gets

$$\Psi_{\text{eik}}^{(-)*}(\mathbf{k}_f, \mathbf{r})\Psi_{\text{eik}}^{(+)}(\mathbf{k}_i, \mathbf{r}\,) = e^{i\mathbf{q}\cdot\mathbf{b}} e^{i(k_{i_z} - k_{f_z})z} e^{\chi_C(b)} \tag{2.9}$$

(note that when adding the phase shifts we disregarded the difference between the velocities v and v_L in the entrance and final channels). The scattering amplitude in the eikonal distorted wave approximation (DWEA) assumes then the form

$$f^C_{0\to LM}(\theta,\phi) = -\frac{m}{2\pi\hbar^2}\, Q_L \int d^2b\; e^{i\mathbf{q}\cdot\mathbf{b}} e^{\chi_C(b)} \int dz\; e^{i\Delta k_z z} \frac{1}{r^{L+1}} Y^*_{LM}(\hat{r})\,, \tag{2.10}$$

with $\Delta k_z = k_{i_z} - k_{f_z}$. By introducing now the adimensional z-integrated form factor through the definition

$$e^{-iM\phi_b}\mu^C_{LM}(b) = -\frac{i}{\hbar v}\int dz F^C_{LM}(\mathbf{r}) e^{i\Delta k_z z}\,, \tag{2.11}$$

the expression for the inelastic scattering amplitude becomes

$$f^C_{0\to LM}(\theta,\phi) = -ike^{-iM\phi}\int db\; b\; J_M(qb) e^{\chi_C(b)} \mu^C_{LM}(b)\,. \tag{2.12}$$

The integrated form factor $\mu^C_{LM}(b)$ can be obtained in the closed form [14]

$$\mu^C_{LM}(b) = -\frac{i}{\hbar v}\,(\Delta k_z)^L Q_L C_{LM} K_M(\Delta k_z b)\,, \tag{2.13}$$

with

$$C_{LM} = \left(\frac{2L+1}{\pi}\right)^{1/2} \frac{(-1)^{(L-M)/2}}{[(L+M)!(L-M)!]^{1/2}}\,. \tag{2.14}$$

The function K_M is the modified Bessel function of order M. This expression can be further simplified in the limit of low excitation energy and/or high collision energy. Note that since one can approximate $\Delta k_z = k_{i_z} - k_{f_z} \simeq \omega/v$, $\hbar\omega$ being the excitation energy $(E_f - E_i)$, one obtains $\Delta k_z \to 0$ in both of these limits. The functions $K_M(\Delta k_z b)$ can therefore be approximated by their lowest-order expression

$$K_M(\Delta k_z b) \simeq \frac{1}{2}(|M|-1)!\left(\frac{2}{\Delta k_z b}\right)^{|M|} \tag{2.15}$$

in both cases. As a consequence, according to Eq. (2.13), in this limit only the terms with $M = \pm L$ survive and one gets

$$\mu^C_{LM}(b) = -\frac{i}{\hbar v}\,\delta_{L|M|} Q_L C_{LM} \frac{(L-1)!}{2}\left(\frac{2}{b}\right)^L\,. \tag{2.16}$$

This implies that in these cases the emerging excited target is completely polarized along the z axis and the scattering amplitude is simply given by

$$f^C_{0\to L,M=\pm L}(\theta,\phi) = 2^{L-1}(L-1)! Q_L C_{LM} e^{-iM\phi}\int db\; b\; J_M(qb) e^{\chi_C(b)} \frac{1}{b^L}\,. \tag{2.17}$$

This complete polarization of the excited target along the incident momentum axis is easily interpreted in terms of classical dynamics. In the sudden limit $(\Delta k_z \to 0)$ the collision time is short compared to the characteristic time of the intrinsic motion. The internal degrees of freedom of the target are therefore frozen during the excitation process, and due to the symmetry of the problem the total impulse has no component along the incident momentum axis. The resulting angular momentum is consequently aligned with this axis.

Low impact parameters, where the multipole expansion (2.1) is no longer valid, are naturally suppressed by the cutoff introduced by the nuclear absorption. Due to the sensitivity of the results to the choice of the cutoff we prefer to avoid the use of an *ad hoc* radius R_{abs} and to postpone the actual evaluation of the Coulomb excitation cross sections to the next section where the effects of the nuclear forces are properly accounted for. Note also that our description is intended to cover energy ranges where retardation effects and relativistic kinematics can be disregarded. These issues are discussed for example in Ref. [20].

B. Nuclear interactions

1. *Phenomenological description*

Let us introduce now, in addition to the Coulomb field, the nuclear potential V_N, again separated into the mean optical potential V_N^0 and the coupling term $\mathcal{V}_N$. In the DWEA considered above, the expression for the inelastic scattering amplitude assumes the form

$$f_{0\to LM}(\theta,\phi) = -ike^{-iM\phi}\int db\; b\; J_M(qb)e^{\chi_N(b)+\chi_C(b)}(\mu^C_{LM}(b)+\mu^N_{LM}(b))\;, \tag{2.18}$$

where the adimensional z-integrated form factor $\mu^N_{LM}(b)$ is connected to the nuclear form factor through the equation

$$e^{-iM\phi_b}\mu^N_{LM}(b) = -\frac{i}{\hbar v}\int dz\; F^N_{LM}(\mathbf{r})e^{i\Delta k_z z}\;, \tag{2.19}$$

which is the nuclear counterpart of Eq. (2.11). More precisely, the quantity $\chi_N(b)$ in Eq. (2.18) is the elastic nuclear phase shift, which is obtained in the eikonal approximation integrating the phenomenological nuclear potential V^0_N in the form

$$\chi_N(b) = -\frac{i}{\hbar v}\int_{-\infty}^{\infty} dz\; V^0_N(\mathbf{r})\;. \tag{2.20}$$

The inelastic nuclear form factor $F^N_{LM}(\mathbf{r})$ is a spherical tensor of rank L of the form

$$F^N_{LM}(\mathbf{r}) = \langle\Phi^T_{LM}\Phi_{00}|\mathcal{V}_N|\Phi^T_{00}\Phi_{00}\rangle = F^N_L(r)Y^*_{LM}(\hat{r})\;, \tag{2.21}$$

and in the standard collective model associated with the excitation of vibrational states its radial part is directly related to the nuclear potential by the expression [1]

$$F^N_L(r) = \beta^N_L R_T\frac{dV^0_N(r)}{dr} \tag{2.22}$$

in terms of the nuclear dynamic deformation parameter β^N_L and the target nuclear radius R_T.

2. *Microscopic description*

In a microscopic description based on a modified Glauber approach [7,8] the phase shift can be calculated in terms of densities of the two colliding nuclei and the nucleon-nucleon profile function in the form

$$\chi_N(b) = \int d\mathbf{r}_P d\mathbf{r}_T\;\rho^P_0(\mathbf{r}_P)\;\gamma_{NN}(|\mathbf{b}-\mathbf{s}_P+\mathbf{s}_T|)\;\rho^T_0(\mathbf{r}_T); \tag{2.23}$$

the profile function is defined as the two-dimensional Fourier transform of the elementary scattering amplitude

$$\gamma(b) = \frac{1}{2\pi ik_{NN}}\int e^{-i\mathbf{b}\cdot\mathbf{q}}f_{NN}(\mathbf{q})d\mathbf{q}\;, \tag{2.24}$$

and $\mathbf{s}_P, \mathbf{s}_T$ are the projections of the coordinate vectors of the nucleons on the plane perpendicular to the z axis. An expression of the phase shift directly related to the phenomenological nucleon-nucleon scattering amplitude is obtained through the Fourier transform

$$\chi_N(b) = \int_0^\infty q\;dq\;\hat{\rho}^P_0(q)\;f_{NN}(q)\;\hat{\rho}^T_0(q)\;J_0(qb), \tag{2.25}$$

where the caret denotes Fourier transformation.

Accordingly, a microscopic representation of $\mu^N_{LM}(b)$ involving the transition density $\delta\hat{\rho}^{LM}_T$ can be obtained in the form

$$\begin{aligned}\mu^N_{LM}(b) &= \int d\mathbf{r}_P d\mathbf{r}_T\;\rho^P_0(\mathbf{r}_P)\;\gamma_{NN}(|\mathbf{b}-\mathbf{s}_P+\mathbf{s}_T|)\;\delta\rho^T_{LM}(\mathbf{r}_T)\\ &= \frac{B_{LM}}{ik_{NN}}\int_0^\infty dq\;\hat{\rho}^P_0(q)\;f_{NN}(q)\;\delta\hat{\rho}^T_{LM}(q)\;J_M(qb)\;,\end{aligned} \tag{2.26}$$

where B_{LM} is a geometrical factor given by

$$B_{LM} = (-1)^M\;[(2L+1)/4\pi]^{1/2}\;[(1+(-1)^{L-M})/2]\;\frac{[(L-M)!(L+M)!]^{1/2}}{(L-M)!!(L+M)!!}. \tag{2.27}$$

This is originated by the assumed form

$$\delta\rho^T_{LM}(\mathbf{r}) = \delta\rho^T_L(r)\;Y^*_{LM}(\hat{r}) \tag{2.28}$$

for the transition density of the target.

The direct derivation of the μ^N factor in the present DWEA formalism goes as follows [8]. One introduces as a first step the coordinate representation of the nucleon-nucleon scattering operator, obtained through the Abel transform of the profile:

$$t_{NN}(r) = \frac{2\hbar v_{NN}}{\pi r}\frac{d}{dr}\int_r^\infty\frac{\gamma_{NN}(b)}{(b^2-r^2)^{1/2}}\;b\;db\;. \tag{2.29}$$

The microscopic form factor is then defined in terms of matrix elements of this interaction between antisymmetric nuclear states normalized to unity:

$$\begin{aligned}F^N_{LM}(\mathbf{r}) &= \langle\Phi^T_{LM}\Phi_{00}|\mathcal{V}_N|\Phi^T_{00}\Phi_{00}\rangle\\ &= \int d\mathbf{r}_P d\mathbf{r}_T\;\rho^P_0(\mathbf{r}_P)\;t_{NN}(|\mathbf{r}-\mathbf{r}_P+\mathbf{r}_T|)\\ &\quad\times\delta\rho^T_{LM}(\mathbf{r}_T).\end{aligned} \tag{2.30}$$

In the framework of the DWEA we use again the definition (2.19) to obtain the related z-integrated form factor $\mu^N(b)$. The z-variable integral introduced by the relative motion wave functions implies performing the integration

$$\int dz\, e^{i\Delta k_z\, z} t_{NN}(|\mathbf{r}-\mathbf{r}_P+\mathbf{r}_T|)\ . \tag{2.31}$$

If we are in the limit $\Delta k_z \simeq 0$ the above relation reduces to the inverse of the Abel transform of Eq. (2.29) and gives directly $\gamma(b)$, so that the z integral of the form factor gives Eq. (2.26).

A similar procedure in the eikonal approximation leads from the folding potential [8]

$$V(r) = \int \rho_0^P(\mathbf{r}_P)\rho_0^T(\mathbf{r}_T) t_{NN}(|\mathbf{r}-\mathbf{r}_P+\mathbf{r}_T|) d\mathbf{r}_P d\mathbf{r}_T \tag{2.32}$$

to the phase shift (2.23).

Note that a peculiar character of the nuclear form factor so defined, originating from the high-energy structure of t_{NN}, is its locality in $\mathbf{r}$. We furthermore remark that we have displayed the above formalism in the case of central interaction. The treatment of more general interactions involving spin, isospin, and spin-orbit dependence is straightforward for processes treated in first order (cf. Ref. [2]). More elaborated developments are needed for higher-order descriptions [21].

3. *Q-value effects*

The appealing form (2.26) for the nuclear form factor in the impact parameter representation, which involves only immediately significative quantities, has been obtained at the cost of disregarding the effects of the difference between the momentum in the initial and final channel. While this approximation cannot be accepted at all in Coulomb excitation, it may be less dangerous in dealing with short-range nuclear forces: this approximation, in fact, is equivalent to assume that $e^{i\Delta k_z z} \simeq e^{i(\omega/v)z} \simeq 1$ in this short interaction range, an approximation expected to be valid at intermediate energies. One can however account in an approximate way for this Q-value dependence in the case of relatively lower bombarding energies.

Let us reconsider the microscopic form factor (2.30). The complex nature and the energy dependence of the interaction makes this microscopic form factor richer than the corresponding phenomenological one. However, it can be reasonably assumed to have the structure

$$F^N_{LM}(\mathbf{r}) = F^N_L(r) e^{-iM\phi} Y^*_{LM}(\theta,0)\ , \tag{2.33}$$

as in the phenomenological case. One can accept, furthermore, that the two form factors will have a similar radial behavior in the decaying tail, i.e., in the region where the inelastic process occurs without being absorbed. So, for suitably large values of the impact parameter, the radial form factor can be attributed the explicit exponential form

$$F^N_L(r) = F^N_{LM} e^{-r/a}\ . \tag{2.34}$$

It follows, in this scheme, that the dependence on the channel excitation energy of the function $\mu^N_{LM}(b)$, induced by the z integration involving $e^{i\Delta k_z z}$, can be described in the form

$$\mu^N_{LM}(b,\Delta k) \simeq A_{(LM)}(b,\Delta k)\, \mu^N_{LM}(b,\Delta k=0)\ , \tag{2.35}$$

where $\mu^N_{LM}(b,\Delta k = 0)$ is the quantity defined in Eq. (2.26) and $A_{(LM)}$ is the adiabaticity correction factor defined by the relation

$$A_{(LM)}(b,\Delta k) = \frac{I_{LM}(b,\Delta k)}{I_{LM}(b,\Delta k=0)}\ , \tag{2.36}$$

with

$$I_{LM}(b,\Delta k) = \int_{-\infty}^{\infty} e^{-r/a} Y^*_{LM}(\hat{r}) e^{i\Delta k_z z} dz. \tag{2.37}$$

The parentheses in the indices LM in (2.35) is related to the fact that in different approximations the dependence on L and/or M could be absent. By expressing the spherical harmonics in terms of $\sin\theta = b/\sqrt{b^2+z^2}$ and $\cos\theta = z/\sqrt{b^2+z^2}$ the resulting one-dimension integral is easily performed numerically. According to the above discussion, the adiabaticity correction factor can be assumed to be the same both in the microscopic and in the phenomenological descriptions. Note that in any case this correction factor is valid only for $b \geq R_{\rm abs}$; for lower values, on the other hand, the form factor gives no contribution at all because of the strong absorbtion.

A simple analytical form is obtained by introducing a further approximation to the form factor. This amounts to approximating the angular part of the nuclear interaction by the value it has at the point of maximum approach, i.e., $Y_{LM}(\theta,0) \simeq Y_{LM}(\pi/2,0)$. Under this assumption one gets

$$I_{LM}(b,\Delta k) = H_{LM} I(b,\Delta k)\ , \tag{2.38}$$

where

$$H_{LM} = 2e^{-iM\phi} Y_{LM}(\pi/2,0) \tag{2.39}$$

and

$$I(b,\Delta k) = \int_0^\infty e^{-r/a}\cos(\Delta K_z z) dz = H_{LM} \frac{b}{\sqrt{1+(a\Delta k)^2}} K_1\left(\frac{b}{a}\sqrt{1+(a\Delta k)^2}\right)\ , \tag{2.40}$$

consequently,

$$A(b,\Delta k) = \frac{I(b,\Delta k)}{I(b,\Delta k=0)} = \frac{1}{\sqrt{1+(a\Delta k)^2}} K_1\left(\frac{b}{a}\sqrt{1+(a\Delta k)^2}\right) \Big/ K_1\left(\frac{b}{a}\right)\ . \tag{2.41}$$

Note that the dependence of the adiabaticity factor on LM disappears in this approximation. Examples of adiabaticity factors are given in Fig. 1, where the corresponding factors for Coulomb excitation are also shown. The figure confirms the expected feature that the damping effect due to the finite Q value is drastically reduced in the nuclear case in comparison with the Coulomb case.

By taking into account that we are interested in regions in which $b \gg a$, so that the argument of the Bessel function K_1 is much larger than 1, the dependence of the adiabaticity factor on Δk can be further simplified by expanding this function in its asymptotic series and retaining the first term, i.e., $K_1(z) \simeq \sqrt{\pi/2z}e^{-z}$. This leads to the simple final result

$$A(b,\Delta k) \simeq \left(1 - \frac{3}{4}(a\Delta k)^2\right) e^{\frac{-b}{2a}(a\Delta k)^2}. \qquad (2.42)$$

It is worth noticing that the dependence on Q value, ruled by the Gaussian function, is the same as the one derived in the framework of the semiclassical time-dependent description of the inelastic scattering (cf. Ref. [22]). We underline, however, that in our reference frame, with the beam axis as quantization axis, there is no mingling of Q-value effects with those originating from the value of the magnetic quantum number. In our case the perturbative expansion of $Y_{LM}(\theta,0)$ around $\theta = \pi/2$ gives no correction in first order, while the second-order term must be evaluated numerically. In spite of these apparent hindrances, the approximation is rather accurate, as apparent from the examples given in Fig. 2, where these expressions are compared with the "exact" expressions (2.36). In Fig. 3 the dependence of the adiabatic factor on the impact parameter b is shown. Its rather smooth behavior and the limited window in b contributing to the nuclear excitation suggest to use, in the expression for the scattering amplitude, a unique factor $A(b,\Delta k) \equiv A(b_g,\Delta k)$ corresponding to the grazing value b_g. This can be also seen from Fig. 4(a) where angular distributions of nuclear excitation of different 2^+ levels in lead have been evaluated. One can note that an overall scaling factor is sufficient to relate the different cross sections. On the converse [see Fig. 4(b)], the Q-value effects are dramatically manifested in the case of Coulomb excitation of the same levels.

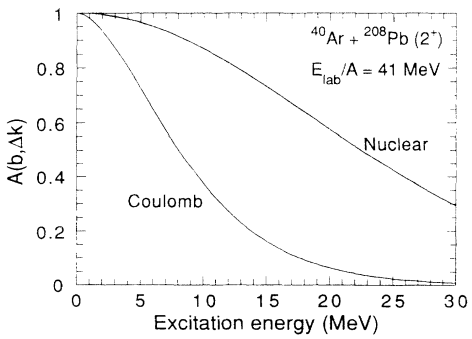

FIG. 1. Adiabaticity factors for the Coulomb and nuclear excitation of a quadrupole mode in lead as a function of the excitation energy. The bombarding energy per nucleon is 41 MeV.

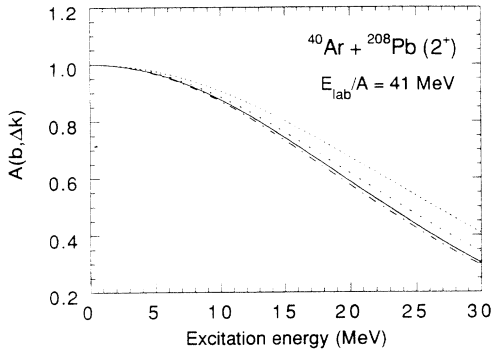

FIG. 2. Nuclear adiabaticity factor as a function of the excitation energy of a quadrupole state. The slope parameter a has been taken equal to 0.6 fm, the impact parameter b = 14 fm, and the bombarding energy is 41 MeV/nucleon. The dotted and dashed curves, corresponding to M = 0, 2, respectively, have been calculated using expression (2.36). The solid and the dot-dashed lines were calculated using the approximated formulas (2.42) and (2.41), respectively.

C. Applications

Examples of Coulomb excitation cross sections at different bombarding energies are given in Figs. 5 and 6. The nuclear interaction is considered only for the distortion and absorption effects through the nuclear phase shift χ_N, disregarding for the moment the contribution of the nuclear excitation. For example, this prescription in the case of the angular integrated cross section for Coulomb excitation amounts to using the expression

$$\sigma_{0\to L} = k^2 \int db\, b\, e^{2\mathrm{Re}\chi_N(b)} |\mu^C_{LM}(b)|^2 . \qquad (2.43)$$

We note that the use of the realistic smooth transmission coefficient through the nuclear phase shift inhibits the possibility of obtaining closed analytical expressions, at variance with the results derived in the sharp-cutoff assumption [14, 15].

In Fig. 5 the reaction ^{40}Ar+ ^{208}Pb is considered. We display the Coulomb excitation (CE) angular distribution at different bombarding energies for the giant quadrupole resonance (GQR), in order to evidence the forward focusing effect with increasing energy. The angular range $\Delta\theta$ in which the cross section is appreciably concentrated is in good agreement with the predictions based on semiclassical considerations [14, 15], i.e., $\Delta\theta = 2Z_P Z_T e^2/RE$. For the same reaction, in Fig. 6 we display, as a function of the bombarding energy, the total integrated CE cross sections for different modes. More precisely, for each multipolarity $L \leq 4$ a vibrational low-lying mode and a giant mode are considered with coupling strengths resulting from the random phase approximation (RPA) calculation [23]. For the dipole mode only, the giant isovector resonance has been considered. For each mode the cross section reaches a maximum and then decreases, with the value of the maximum depending on the excitation energy as a consequence of the adiabaticity condition.

To evidence the importance of the Q value at low bombarding energies we display in Fig. 7 both the cross sections obtained with the exact expression (2.13) for the

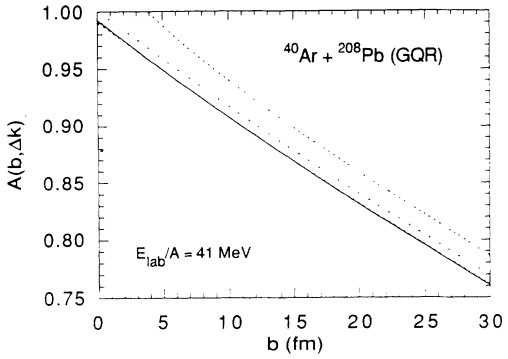

FIG. 3. Nuclear adiabaticity factor for the excitation of a quadrupole state as a function of the impact parameter. The slope parameter a has been taken equal to 0.6 fm, and the bombarding energy per nucleon equal to 41 MeV. The excitation energy is fixed equal to 10.8 MeV. As in Fig. 2, the dotted and dashed curves, corresponding to $M = 0$, 2, calculated using expression (2.36). In this case the solid curve stands for both expressions (2.41) and (2.42) that coincide.

form factor (solid lines) and those obtained with the approximated expression (2.16) (dashed lines) for the case of the two quadrupole modes. As foreseen, the results obtained with the approximate and the exact expressions start to agree at higher energies for the high-lying GQR.

In order to compare the predictions of our method with

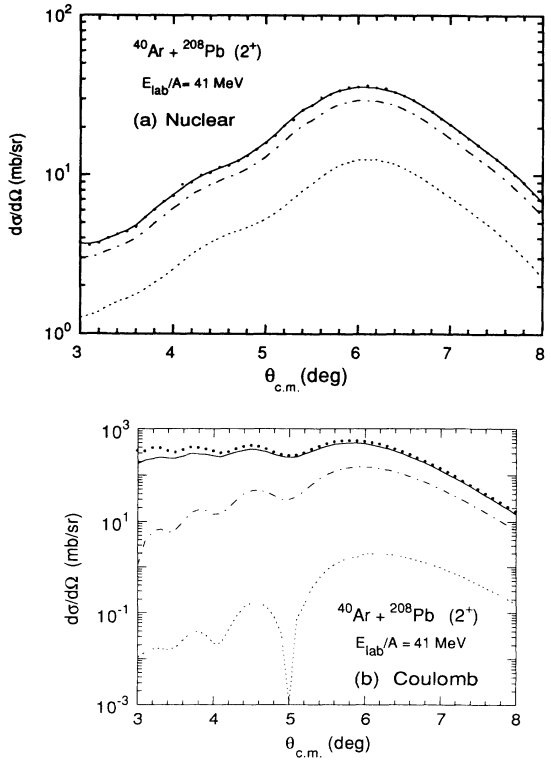

FIG. 4. Adiabaticity effect on the pure nuclear (a) and the pure Coulomb (b) excitation cross sections. The solid lines correspond to $E^* = 4.1$ MeV, the dot-dashed lines to $E^* = 11$ MeV, and the dotted lines to $E^* = 24$ MeV. The dots over the solid curve correspond to sudden limit, i.e., $E^* = 0$ MeV.

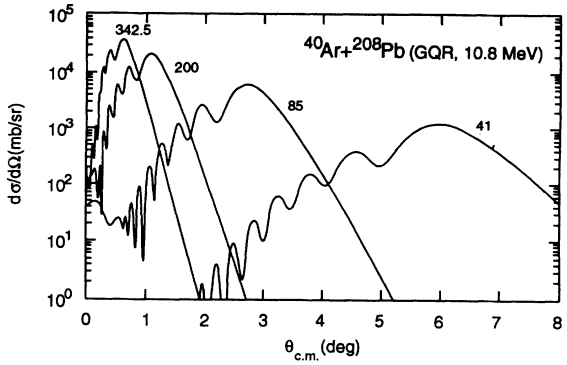

FIG. 5. Angular distributions for the Coulomb excitation of the GQR in the reaction ^{40}Ar + ^{208}Pb at different bombarding energies. The value of E_{lab}/A is used to label the different curves.

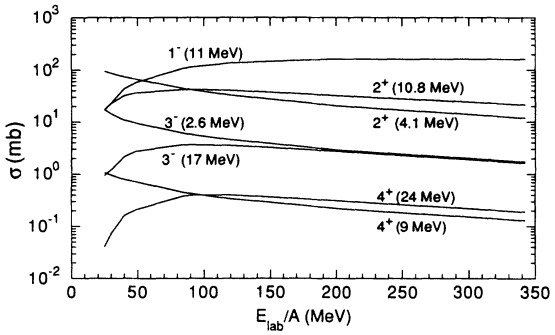

FIG. 6. Coulomb excitation integrated cross sections for the reaction ^{40}Ar + ^{208}Pb leading to different modes as a function of the bombarding energy per nucleon.

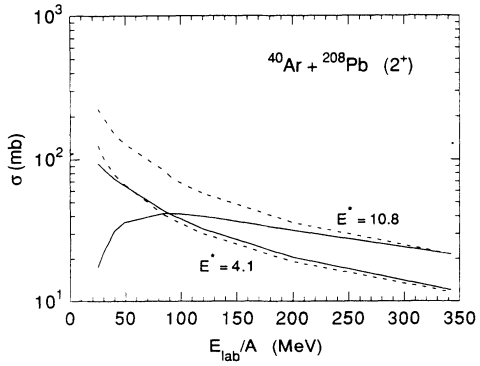

FIG. 7. Coulomb excitation integrated cross sections for the low-lying quadrupole vibration and GQR in the reaction ^{40}Ar + ^{208}Pb. Solid lines have been obtained using the exact expression (2.13) for the inelastic form factor. Dotted lines correspond to those obtained using the approximate expression (2.16).

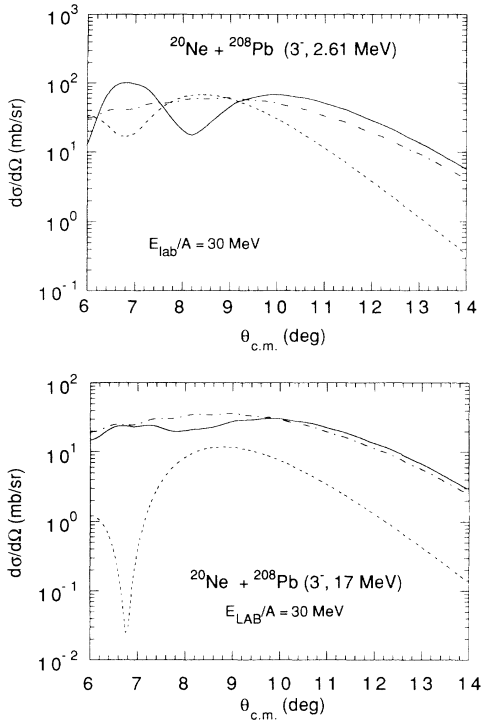

FIG. 8. Inelastic angular distribution for the excitation of two octupole states in lead bombarded with ^{20}Ne at 30 MeV/nucleon. Dashed and dot-dashed curves correspond to the use of only Coulomb or only nuclear excitation, respectively. The solid line displays the interference pattern when both contributions are included.

the experimental data, we include now the nuclear excitation. As an example we consider the case of the excitation of the octupole modes in lead in the reaction ^{20}Ne + ^{208}Pb. As a preliminary we display in Fig. 8 the result of a purely theoretical calculation; the separated contributions to the cross section obtained with only nuclear (dot-dashed) or Coulomb excitation (dashed) are also evidenced. The different adiabaticity factors for the Coulomb and the nuclear contributions makes the

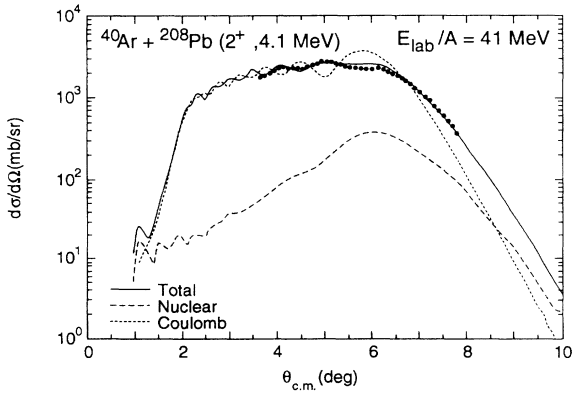

FIG. 9. Angular distribution for the ^{40}Ar + ^{208}Pb inelastic scattering exciting the quadrupole vibration. The bombarding energy is 41 MeV/nucleon. Experimental data are taken from Ref. [31]. For the inelastic form factor we used β_2^C = 0.07, β_2^N = 0.08.

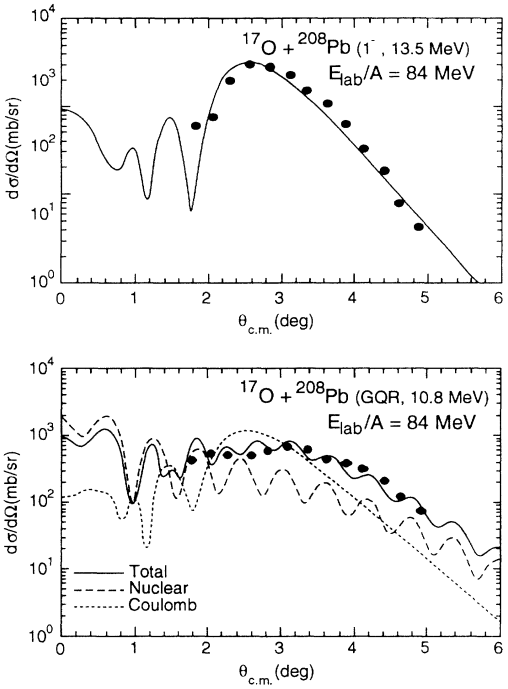

FIG. 10. Angular distribution for the ^{17}O + ^{208}Pb inelastic scattering exciting the giant dipole resonance (a) and GQR (b). The bombarding energy is 84 MeV/nucleon. Experimental data are taken from Ref. [32]. For the inelastic form factors we used β_1^C = 0.059 (a) and β_2^C = 0.060, β_3^N = 0.065 (b).

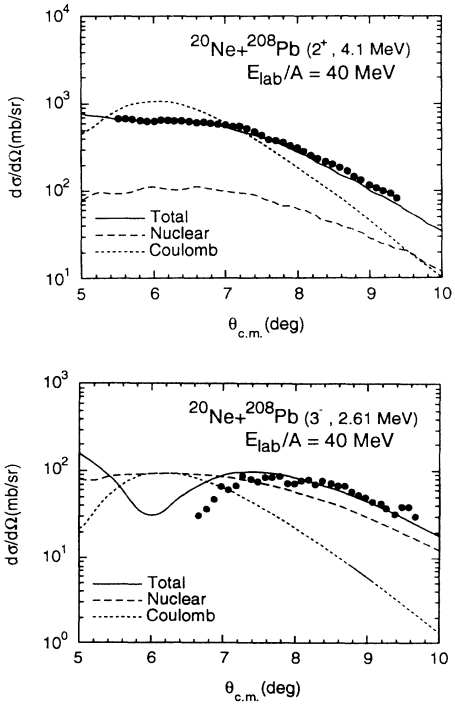

FIG. 11. Angular distribution for the ^{20}Ne + ^{208}Pb inelastic scattering exciting the low vibrational quadrupole (a) and octupole (b) modes. The bombarding energy is 40 MeV/nucleon. Experimental data are taken from Ref. [24]. For the inelastic form factors we used β_2^C = 0.069, β_2^N = 0.073 (a) and β_3^C = 0.085, β_3^N = 0.087 (b).

interference pattern strongly dependent on the excitation energy. The two contributions are comparable (and therefore lead to strong interference) in the case of low excitation energy [Fig. 8(a)], while the nuclear contribution becomes dominant in the latter case (and hence weak interference) [Fig. 8(b)]. We should also keep in mind, however, that the balance between the Coulomb and the nuclear contributions depends also on the multipolarity of the excited state, being the Coulomb form factor strongly L dependent in front of a nuclear coupling practically insensitive to the multipolarity.

In Fig. 9 an actual comparison with the experimental data is exhibited for the excitation of the 2^+ state in lead at 4.1 MeV in the reaction ^{40}Ar + ^{208}Pb. As in the previous figure, the expression (2.18) has been used, with Coulomb and nuclear phase shifts given by Eq. (2.8) and (2.20) and with z-integrated form factors given by Eq. (2.13) and (2.35). We note the fact that the only necessary inputs of the calculation are the densities and transition densities of the colliding systems and the microscopic N-N scattering amplitude at the corresponding energy. Densities and N-N scattering amplitudes can be taken from the phenomenological data, while for the transition density we have assumed the standard collective description with a normalization adjusted to the experimental $B(EL)$. The calculation therefore avoids all uncertainties connected with the choice of the ion-ion optical potential and form factor.

Other examples of analysis of experimental data, performed by applying the same procedure, are given in Figs. 10 and 11. Note that in the case of Fig. 10(a), because of the isovector nature of the excited state, the excitation process is mainly due to the Coulomb field. In the octupole case illustrated in Fig. 11(b), where the agreement between theory and experiment is of lower quality, results of similar nature were obtained in the analog distorted wave Born approximation (DWBA) analysis given in Ref. [24].

III. SECOND-ORDER ELASTIC SCATTERING AND POLARIZATION POTENTIALS

A. The Coulomb excitation

In this section we consider the modifications on the elastic scattering originated from the coupling to inelastic channels. We shall first consider the Coulomb excitation. The second-order elastic scattering amplitude for the sequential excitation and deexcitation of an intermediate state LM can be written as

$$f^{II}_{0\rightleftharpoons LM}(\theta) = -\frac{m}{2\pi\hbar^2}\int d\mathbf{r}\, d\mathbf{r}'\, \Psi^{(-)*}_{\rm eik}(\mathbf{k}_f,\mathbf{r})F^*_{LM}(\mathbf{r})G_{\rm eik}(\mathbf{k}_L;\mathbf{r},\mathbf{r}')F_{LM}(\mathbf{r}')\Psi^{(+)}_{\rm eik}(\mathbf{k}_i,\mathbf{r}')\,, \tag{3.1}$$

where $G_{\rm eik}(\mathbf{k}_L;\mathbf{r},\mathbf{r}')$ is the eikonalized propagator of the colliding nuclei in channel L:

$$G_{\rm eik}(\mathbf{k}_L;\mathbf{r},\mathbf{r}') = -\frac{i}{\hbar v}\delta(\mathbf{b}-\mathbf{b}')\theta(z-z')e^{ik_{L,z}(z-z')}\exp\left(-\frac{i}{\hbar v}\int_{z'}^{z}V^0_C(\mathbf{r}'')dz''\right)\,, \tag{3.2}$$

k_L is the relative linear momentum in the excited channel. As in the previous section, in the phase shift integrals we disregard velocity variations; this allows us to add the partial contributions to the phase shifts arising from the distorted waves and from the Green's function. Furthermore, it is useful to extract from the form factors the ϕ dependence by writing

$$F_{LM}(r,\theta,\phi) = e^{-iM\phi}\tilde{F}_{LM}(r,\theta) = e^{-iM\phi}\tilde{F}_{LM}(b,z)\,, \tag{3.3}$$

where $r=\sqrt{b^2+z^2}$ and $\theta=\arctan(b/z)$. Note that the function $\tilde{F}_{LM}$ so defined is real for real couplings, such as the ones arising from Coulomb interactions. The final expression for scattering amplitude is therefore

$$f^{II}_{0\rightleftharpoons LM}(\theta) = \frac{ik}{2\pi\hbar^2 v^2}\int d\mathbf{b}\, e^{i\mathbf{q}\cdot\mathbf{b}}e^{\chi_C(b)}\int_{-\infty}^{\infty}dz\int_{-\infty}^{z}dz'\,\theta(z-z')e^{i\Delta k_z z}\tilde{F}_{LM}(b,z)e^{-i\Delta k_z z'}\tilde{F}_{LM}(b,z'). \tag{3.4}$$

The integrals of the type entering in the above equation can be usefully put into the form

$$\int_{-\infty}^{\infty}dz f(b,z)\int_{-\infty}^{z}dz' g(b,z') = \frac{1}{2}\left[\int_{-\infty}^{\infty}dz f(b,z)\right]\left[\int_{-\infty}^{\infty}dz' g(b,z')\right] + \int_{-\infty}^{\infty}dz\int_{-\infty}^{z}dz'\,[g(b,z)f(b,z')-g(b,z')f(b,z)]\,. \tag{3.5}$$

According to Eqs. (3.4), (3.3), and (2.19) one obtains

$$\int_{-\infty}^{\infty}dz\, e^{i\Delta k_z z}\tilde{F}_{LM}(b,z)\int_{-\infty}^{z}dz'\, e^{-i\Delta k_z z'}\tilde{F}_{LM}(b,z') = -\frac{\hbar^2 v^2}{2}\left[\mu^C_{LM}(b)\right]^2 + \int_{-\infty}^{\infty}dz\int_{-\infty}^{z}dz'\tilde{F}_{LM}(b,z)\tilde{F}_{LM}(b,z')[e^{i\Delta k_z(z-z')}-e^{-i\Delta k_z(z-z')}]. \tag{3.6}$$

Since

$$e^{i\Delta k_z(z-z')} - e^{-i\Delta k_z(z-z')} = 2i\sin[\Delta k_z(z-z')] \simeq 2i\sin[\omega(z-z')/v] \tag{3.7}$$

and the functions $\tilde{F}_{LM}$ are real, one immediately recognizes that the $(\mu^C_{LM})^2$ term is real and negative while the integral in the right hand side of the above equation is purely imaginary. In the energy regime at which we are working, where $\omega/v \simeq 0$, this term is vanishing. By summing the direct elastic and the $(\mu^C_{LM})^2$ term of the inelastic transition, one gets therefore for the elastic scattering amplitude the final expression

$$\begin{aligned} f_{\rm el}(\theta) &= f^I_{\rm el}(\theta) + \sum_M f^{II}_{0\rightleftarrows LM}(\theta) \\ &= \frac{ik}{2\pi}\int d\mathbf{b}\; e^{i\mathbf{q}\cdot\mathbf{b}}\left[1 - e^{\chi_C(b)}\left\{1 + \frac{1}{2}\sum_M \left(\mu^C_{LM}(b)\right)^2\right\}\right] . \end{aligned} \tag{3.8}$$

In order to introduce the polarization potential the following procedure can be used. We first put Eq. (3.8) in the form

$$f_{\rm el}(\theta) = \frac{ik}{2\pi}\int d\mathbf{b}\; e^{i\mathbf{q}\cdot\mathbf{b}}\left[1 - e^{\chi_C(b)+\chi'_C(b)}\right] , \tag{3.9}$$

where the additional phase shift

$$\chi'_C(b) = \ln\left[1 + \frac{1}{2}\sum_M \left(\mu^C_{LM}(b)\right)^2\right] \tag{3.10}$$

has been defined. In the eikonal approximation, potentials and corresponding phase shifts are connected through the Abel transform [2]. Due to the additivity properties of this transform, the polarization potential (i.e., the correction to the bare potential V^0_C) is directly obtained [8] by inverting $\chi'_C(b)$ and one has

$$\Delta V_C(r) = -i\frac{\hbar v}{\pi r}\frac{d}{dr}\int_r^\infty \frac{\chi'_C(b)}{(b^2-r^2)^{1/2}}\, b\, db . \tag{3.11}$$

Note that this polarization potential is purely imaginary and negative for pure Coulomb interaction as a consequence of the fact that $\chi'_C(b)$ is a real, negative and monotonically increasing function of b.

In the weak coupling limit the additional phase shift can be approximated according to

$$\chi'_C(b) \simeq \frac{1}{2}\sum_M \left(\mu^C_{LM}(b)\right)^2 . \tag{3.12}$$

If we further disregard excitation energy effects, from Eq. (2.16) one gets

$$\frac{1}{2}\sum_M \left[\mu^C_{LM}\right]^2 = \left[\mu^C_{L,M=L}\right]^2 = -\frac{1}{(\hbar v)^2}\frac{4\pi Z_T^2 e^2}{(2L+1)^2}[B_{0\to L}(EL)]\frac{[(L-1)!]^2}{(2L)!}\frac{2^{2L}}{b^{2L}} . \tag{3.13}$$

The Abel transform of the above function is analytically known, and in this approximation the polarization potential assumes the simple dependence

$$\Delta V_C(r) = -i\frac{3\pi Z_T^2 e^2[B_{0\to L}(EL)][(L-1)!]^2 2^{2L}}{\hbar v(2L+1)^2(2L)!}\frac{1}{r^{2L+1}} . \tag{3.14}$$

In the particular case of quadrupole states, the expression reads

$$\Delta V_C(r) = -i\frac{2\pi Z_T^2 e^2 B_{0\to L}(E2)}{25\hbar v}\frac{1}{r^5} . \tag{3.15}$$

Note that our procedure, which gives directly a local potential via the Abel transform of the phase shift, differs from other approaches used in similar contexts. As a result our final expression, although similar, does not exactly coincide with the high-energy limit of the polarization potential obtained in Ref. [1], whose starting point is the second-order Feshbach potential and resorts then to localization procedures. Alternative forms of the polarization potential have been derived within semiclassical descriptions of the scattering process [22, 25]. Also in this connection one observes that the results obtained by different approaches are very similar, but not coincident.

B. The nuclear interaction and Coulomb-nuclear interference

Due to the linearity embodied in the eikonal propagation, the addition of a nuclear mean potential amounts to adding a nuclear phase shift in the Green's function. The z-integrated form factor is now given by the sum of the form factors of Eqs. (2.13) and (2.35) related to $\mathcal{V}_C$ and $\mathcal{V}_N$ respectively, as discussed in Sec. II B. Note however that, although the structure of the second order term for the nuclear case has again the form displayed by Eq. (3.6), some simplifications may not occur. In this case, in fact, the commutator-like expression in the last member of that equation may contribute also in the case in which $\Delta k_z = 0$, when not commuting operators, such as spin-orbit interactions or others, are active. Problems arising in this connection have been discussed, e.g., by Glauber [2] and by Mandelzweig and Wallace [21]. We will only deal in the following with states whose nuclear excitation is only due to the central part of the interaction.

With this proviso the extension of Eq. (3.8) in order to take into account both Coulomb and nuclear interactions is straightforward and gives the scattering amplitude

$$f_{\rm el}(\theta) = \frac{ik}{2\pi}\int d\mathbf{b}\; e^{i\mathbf{q}\cdot\mathbf{b}}\left[1 - e^{\chi_C(b)+\chi_N(b)}\left\{1+\frac{1}{2}\sum_M\left(\mu^C_{LM}(b)+\mu^N_{LM}(b)\right)^2\right\}\right] . \tag{3.16}$$

Elastic angular distributions for the reaction ^{40}Ar+^{238}U are displayed in Fig. 12. The first-order cross section is plotted in full line. The dotted and dashed lines, accounting for the coupling with the Coulomb and nuclear+Coulomb excitation and deexcitation of octupole states, respectively, exhibit important reductions with respect to the first cross section.

By introducing in analogy to Eq. (3.10) the phase shift

$$\chi'(b) = \ln\left[1+\frac{1}{2}\sum_M\left(\mu^C_{LM}(b)+\mu^N_{LM}(b)\right)^2\right] , \tag{3.17}$$

Eq. (3.16) can be put into the form

$$f_{\rm el}(\theta) = \frac{ik}{2\pi}\int d\mathbf{b}\; e^{i\mathbf{q}\cdot\mathbf{b}}\left[1 - e^{\chi_C(b)+\chi_N(b)+\chi'(b)}\right] . \tag{3.18}$$

This phase shift can now be inverted in the same way as in Eq. (3.11) to give the total polarization potential ΔV which is in general complex. We recall the fact that in the pure Coulomb case the polarization potential in our eikonal limit is purely imaginary, due to the vanishing of the real part arising from the commutator term in Eq. (3.5). The nuclear μ^N, on the other hand, has in general a complex character arising from the "dressed" interaction described by t_{NN}, or γ, or by the microscopic scattering amplitudes f_{NN}. Therefore, even neglecting the contribution from the commutator part, the related polarization potential includes both refraction and absorption components. As an example, in addition to the two separate nuclear and Coulomb polarization potentials we have the interference term, which due to the opposite character of nuclear and Coulomb forces tend to cancel the previous term. This is illustrated in Fig. 13 for the reaction ^{40}Ar+^{238}U at 41 MeV/nucleon. In Fig. 13(a) the real part of the total polarization potential (full line) is compared with the contribution arising from the nuclear interaction. In Fig. 13(b) the total imaginary polarization potential is displayed together with the pure nuclear (dashed line) and pure Coulomb (dotted line) components. A further comparison with the immaginary part of the nuclear optical potential shows the long range absorptive effects produced by the couplings.

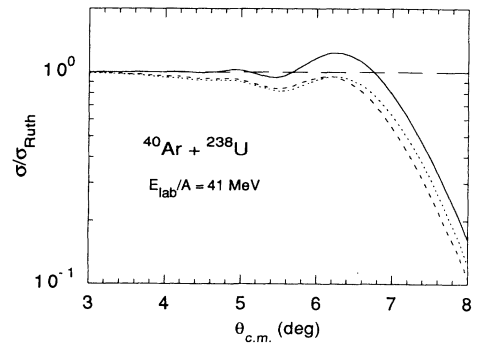

FIG. 12. Effect of the coupling on the elastic scattering cross section. The solid line corresponds to the "pure" elastic channel. The dashed line includes the coupling to the Coulomb excitation channel and the dotted line includes both Coulomb and nuclear excitations. The long dashed line guides the eye to the pure Rutherford case.

IV. THE MULTISTEP-MULTICHANNEL PROBLEM

In the present section we extend to any order of scattering the methods displayed in Secs. II and III for the description of first- and second-order scattering, respectively. Our aim is to describe multistep scattering processes in which the nuclear and the Coulomb forces play a comparable role and the bombarding energy is sufficiently high to justify an eikonal description of the relative motion of the two colliding nuclei. The formalism is

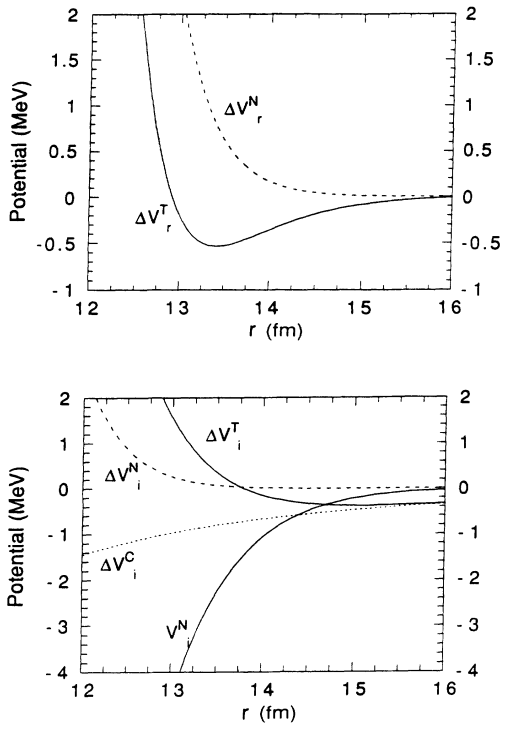

FIG. 13. Polarization potentials for the reaction ^{40}Ar +^{238}U at 41 MeV/nucleon. In Fig. 13(a) the real part of the total polarization potential (full line) is compared with the contribution arising from the nuclear interaction. In Fig. 13(b) the total imaginary polarization potential is displayed together with the pure nuclear (dashed line) and pure Coulomb (dotted line) components. A further comparison with the immaginary part of the nuclear optical potential shows the long range absorptive effects produced by the couplings.

developed in the framework of standard scattering theory, but we use effective microscopic interactions to describe the nucleus-nucleus interaction. Disregarding the Coulomb excitation processes, our final result is formally equivalent to the equations obtained by Feshbach and Hüfner [13]. In a nucleon–nucleus Hamiltonian they replace the two-body interactions by the nucleon–nucleon t_{NN} scattering operators; the scattering problem is then solved by the eikonalization of the coupled–channel system of differential equations derived from the above introduced Hamiltonian. We prefer to attack the problem in the framework of the Lippmann-Schwinger equation and then eikonalize the Green's function describing the nucleus-nucleus multiple scattering mechanism. The results so obtained can be also compared with those previously obtained by the authors in Ref. [7], provided the Q-value effects and Coulomb interactions are disregarded.

To make the exposition of our formalism clearer we keep separated the developments which can be given in the framework of standard scattering theory (Sec. IV A) from the ones in which eikonal methods are massively used (Sec. IV B). Simple formal applications of the method are given in Sec. IV C.

A. Preliminary developments in standard scattering theory

1. The Hamiltonian and the Lippmann-Schwinger equation

We assume that the complete projectile-target Hamiltonian can be written in the form

$$H = H_P + H_T + \mathcal{T} , \tag{4.1}$$

where

$$\mathcal{T} = \sum_{A=1}^{A_P} \sum_{\alpha=1}^{A_T} t_{A\alpha} \tag{4.2}$$

and $t_{A\alpha}$ is the effective t_{NN} operator describing the interaction between the αth nucleon of the target with the Ath nucleon of the projectile. In agreement with Ref. [26], this is a generalization of the nucleon-nucleus Hamiltonian used in Ref. [13] to the description of the scattering of composite objects. As discussed in Sec. II B 2, with the definition of t_{NN} through Eqs. (2.24) and (2.29), the optical potential and the inelastic form factors can be obtained by folding this interaction and correspond to those obtained from our Glauber-like coupled-channel approach [8].

The starting point for our developments is the Lippmann-Schwinger equation

$$T = \mathcal{T} + \mathcal{T}\mathcal{G}_0 T , \tag{4.3}$$

where

$$\mathcal{G}_0^{(+)} = \frac{1}{E^+ - K - H_P - H_T} \tag{4.4}$$

is the unperturbed propagator. The symbol (+) has the standard meaning of designating outgoing boundary conditions [27]; in the following it will be omitted, except when it appears as a label of the energy. The quantity K is the kinetic energy operator of the relative motion.

The uncoupled state of the target and projectile is defined by the relation

$$\Phi_n = \Phi_i{}^T \Phi_j{}^P . \tag{4.5}$$

The states Φ^T and Φ^P are supposed to be separately antisymmetric and are normalized so that

$$\langle \Phi_p | \Phi_q \rangle = \delta_{pq} . \tag{4.6}$$

To avoid formal complications arising from the angular momentum algebra we shall assume that the projectile is always in its ground state with zero angular momentum and that the target states are identified by a quantum number $n = L_n, M_n$, so that in the following we shall write

$$\Phi_n = \Phi_{L_n, M_n}{}^T \Phi_0{}^P . \tag{4.7}$$

The energies are defined so that for the ground state Φ_0 one has $\epsilon_0 = 0$.

2. Introducing the Coulomb interaction and optical potential

In Refs. [28, 29] a formalism has been built up to describe the nucleon-nucleon effective interaction in terms of a t_{NN} operator which embodies both the nuclear and the electromagnetic forces. In heavy-ion scattering processes Coulomb excitation is conveniently accounted for in the framework of the phenomenological description outlined in the previous sections, so we prefer to assume a more empirical philosophy.

Our basic assumption is that the complete transition operator can be defined by the equation

$$T = \mathcal{V} + \mathcal{V}\mathcal{G}_0 T , \tag{4.8}$$

where the full interaction is

$$\mathcal{V} = \mathcal{T} + V_c \tag{4.9}$$

[this approximation can be viewed as a further simplification of Eq. (2.49) of Ref. [29]]. With reference to the Born expansion

$$T = \mathcal{V} + \mathcal{V}\mathcal{G}_0\mathcal{V} + \mathcal{V}\mathcal{G}_0 V \mathcal{G}_0 \mathcal{V} + \cdots \tag{4.10}$$

of the above equation, our assumption can by rephrased by saying that each step of the nucleus-nucleus collision is ruled by the effective nuclear operator $\mathcal{T}$ plus the Coulomb potential V_c. The ultimate justification of this asymmetry relies on the fact that, owing to the weaker character of the Coulomb forces, we can treat them at the first order while the nuclear forces must be summed to any order of nucleon-nucleon collision.

In the Glauber theory and in our previous works [7, 8] the phase shift related to the nuclear direct elastic scattering emerges quite naturally from the nucleus-nucleus multiple scattering process. It has been furthermore shown that this phase shift is associated with the folding optical potential

$$\mathcal{V}_N^{\rm opt} = \langle\Phi_0|\mathcal{T}|\Phi_0\rangle \ . \tag{4.11}$$

Since in the present approach the Coulomb phase shift cannot be associated to a multiple N-N scattering process, it is useful to introduce directly into the formalism the complete optical potential

$$\mathcal{V}^{\rm opt} = \langle\Phi_0|\mathcal{T} + V_c|\Phi_0\rangle = \mathcal{V}_0. \tag{4.12}$$

The formal solution of Eq. (4.8) can be written

$$T = \mathcal{V} + \mathcal{V}\frac{1}{E^+ - K - H_{TP} - \mathcal{V}}\mathcal{V}; \tag{4.13}$$

by introducing the optical potential one obtains

$$T = \mathcal{V} + \mathcal{V}\frac{1}{E^+ - K - H_{TP} - \mathcal{V}_0 - \mathcal{V}_R}\,\mathcal{V}\ , \tag{4.14}$$

where

$$\mathcal{V}_R = \mathcal{V} - \mathcal{V}_0 \tag{4.15}$$

is the residual interaction. In the following we shall consider the Born expansion of this operator in terms of the optical Green's function

$$\mathcal{G}_{\rm opt} = \hat{\mathcal{G}}_0 = \frac{1}{E^+ - K - H_P - H_T - \mathcal{V}_0}\ . \tag{4.16}$$

In this connection it is useful to introduce the spectral representation

$$\langle\mathbf{r}\hat{\mathcal{G}}_0\mathbf{r}'\rangle = \sum_n \Phi_n\rangle\langle\Phi_n\hat{G}_n(\mathbf{r},\mathbf{r}'), \tag{4.17}$$

expressed in terms of the states of the nuclei [see Eq. (4.7)] and of the optical Green's function describing their relative motion.

3. *The multistep expansion*

The above formalism can be further developed by the introduction of the two-potential method. Accordingly the scattering amplitude for the transition from the initial channel i to the final channel f can be written

$$f_{if} = \delta_{if}f_{\rm el} + \hat{f}_{if}\ , \tag{4.18}$$

where f_{el} describes the direct elastic scattering produced by the optical potential $\mathcal{V}_0$, and $\hat{f}_{if}$ in our problem is the multiple scattering term

$$\hat{f}_{if} = -\frac{m}{2\pi\hbar^2}\langle\Psi^{(-)}(\mathbf{k}_f)\Phi_f|\hat{T}|\Phi_i\Psi^{(+)}(\mathbf{k}_i)\rangle; \tag{4.19}$$

the functions $\Psi^{\pm}$ are scattering states of the optical potential. The multiple scattering operator $\hat{T}$, defined by the integral equation

$$\hat{T} = \mathcal{V}_R + \mathcal{V}_R\hat{\mathcal{G}}_0\hat{T}, \tag{4.20}$$

can be given the expansion

$$\hat{T} = \mathcal{V}_R + \mathcal{V}_R\hat{\mathcal{G}}_0\mathcal{V}_R + \mathcal{V}_R\hat{\mathcal{G}}_0\mathcal{V}_R\hat{\mathcal{G}}_0\mathcal{V} + \cdots\ . \tag{4.21}$$

In terms of this perturbative series one can write

$$\hat{f}_{if} = \sum_{n=1}\hat{f}_{if}^{(n)}\ , \tag{4.22}$$

where

$$\hat{f}_{if}^{(n)} = -\frac{m}{2\pi\hbar^2}\langle\Psi^{(-)}(\mathbf{k}_f)\Phi_f|\underbrace{\mathcal{V}_R\hat{\mathcal{G}}_0\mathcal{V}_R\hat{\mathcal{G}}_0\mathcal{V}_R\hat{\mathcal{G}}_0\cdots\hat{\mathcal{G}}_0\mathcal{V}_R}_{n\ \text{factors}\ \mathcal{V}_R}|\Phi_i\Psi^{(+)}(\mathbf{k}_i)\rangle. \tag{4.23}$$

On representing the optical Green's function according to its spectral expansion [see Eq. (4.17)] one gets

$$\hat{f}_{if}^{(n)} = -\frac{m}{2\pi\hbar^2}\sum_{q_1,q_2,\cdots,q_{n-1}}\langle\Psi^{(-)}(\mathbf{k}_f)\Phi_f\mathcal{V}_R\Phi_{q_1}\rangle\hat{G}_{q_1}\langle\Phi_{q_1}\mathcal{V}_R\Phi_{q_2}\rangle\cdots\langle\Phi_{q_{n-2}}\mathcal{V}_R\Phi_{q_{n-1}}\rangle\hat{G}_{q_{n-1}}\langle\Phi_{q_{n-1}}\mathcal{V}_R\Phi_i\Psi^{(+)}(\mathbf{k}_i)\rangle. \tag{4.24}$$

It is clear from the structure of the residual interaction that in the present formalism the diagonal matrix elements involving the ground state are zero, i.e.,

$$\langle\Phi_0|\,(\mathcal{V} - \mathcal{V}_0)\,|\Phi_0\rangle \equiv 0. \tag{4.25}$$

With the further standard assumption about the diagonal matrix elements

$$\langle\Phi_m|\mathcal{V}|\Phi_m\rangle = \langle\Phi_0|\mathcal{V}|\Phi_0\rangle\ , \tag{4.26}$$

one gets

$$\langle\Phi_m|\,(\mathcal{V} - \mathcal{V}_0)\,|\Phi_m\rangle \equiv 0. \tag{4.27}$$

In the following we shall assume that Eq. (4.27) is always satisfied. As a consequence, the optical potential is the same in all the channels.

B. The eikonal multistep multichannel approximation (EMMA)

The relations from Eq. (4.18) to Eq. (4.24) are direct consequences of Eq. (4.8) in standard scattering theory and take into account features such as multiple scattering processes, strong and Coulomb coupling, and excitation energy effects. On the grounds of these results we are going now to develop a general method suitable to describe problems in which, beside the above features, the eikonal propagation is assumed to play an important role. More precisely, we assume that the distorted waves and the Green's function are correctly described in the eikonal approximation while the elementary interactions are assumed to satisfy relations (2.24) and (2.29). The next paragraph is devoted to reobtain the results of Secs. II

and III starting from Eq. (4.24) plus the eikonal assumptions, and to introduce a more general notation for the z-integrated form factors to be used in the setting up of the general formalism.

1. The first- and second-order scattering terms

The first-order scattering term in Eq. (4.24) can be written in the form

$$\hat{f}_{if}^{(1)}(\theta,\phi) = -\frac{m}{2\pi\hbar^2}\int d\mathbf{r}\ \Psi_{\rm eik}^{(-)*}(\mathbf{k}_f,\mathbf{r})\langle\Phi_{LM}^T\Phi_{00}^P|\mathcal{V}_R|\Phi_{00}^T\Phi_{00}^P\rangle\Psi_{\rm eik}^{(+)}(\mathbf{k}_i,\mathbf{r}), \tag{4.28}$$

coincident with the sum of the Coulomb and nuclear DWEA expressions discussed in Sec. II. By defining the optical potential $\mathcal{V}_0(r)$ appearing in the exponential of the eikonal distorted waves as the folding of the interaction according to Eq. (4.12) one obtains the phase shift

$$\chi(b) = \chi_C(b) + \chi_N(b) \tag{4.29}$$

introduced in Eqs. (2.8) and (2.23). The explicit integration on the z variable in Eq. (4.28) gives the final form for the first-order scattering term

$$\hat{f}_{if}^{(1)}(\theta,\phi) = -ik\int db\ b\ e^{i\mathbf{q}\cdot\mathbf{b}}e^{\chi_C(b)+\chi_N(b)}\mathbf{D}_{if}(\mathbf{b})\ , \tag{4.30}$$

where

$$\begin{aligned}\mathbf{D}_{nn'}(\mathbf{b}) &= -\frac{i}{\hbar v}\int dz\ F_{nn'}^C(\mathbf{r})e^{i\Delta k_z z} + \int\rho_{00}^P(\mathbf{r}_A^P)\ \rho_{nn'}^T(\mathbf{r}_\alpha^T)\ \gamma(\mathbf{b}-\mathbf{s}_\alpha^T+\mathbf{s}_A^P)\ d\mathbf{r}_A^P d\mathbf{r}_\alpha^T \\ &= -\frac{i}{\hbar v}\int dz\ F_{nn'}^C(\mathbf{r})e^{i\Delta k_z z} + \frac{1}{2\pi i k_{NN}}\int\hat{\rho}_{00}^P(\mathbf{q})\ \hat{\rho}_{nn'}^T(\mathbf{q})\ f_{NN}(q)\ e^{i\mathbf{q}\cdot\mathbf{b}}d\mathbf{q}\ ;\end{aligned} \tag{4.31}$$

the factors μ previously introduced to couple the ground to excited states have been now replaced by the general matrix $\mathbf{D}(\mathbf{b})$. Explicit expressions for the transition densities $\rho_{nn'}$ can be obtained in the framework of specific nuclear models, as, for example, the collective Bohr-Mottelson or the Tassie models. The quantity pertaining to the first integral is the generalization of Eq. (2.13) to the case in which both the initial and the final states of the target carry angular momentum, while the second integral is the partner of Eq. (2.26). Note that the integral involving nuclear interactions can be multiplied by an adiabaticity factor $A_{nn'}(b,\Delta k)$ as discussed in Sec. II B to account for finite Q-value effects.

As a second step, we can consider the explicit form assumed by the second-order term of the scattering amplitude

$$\hat{f}_{if}^{(2)} = -\frac{m}{2\pi\hbar^2}\langle\Psi^{(-)}(\mathbf{k}_f)\Phi_f|\mathcal{V}_R\hat{\mathcal{G}}_0\mathcal{V}_R|\Phi_i\Psi^{(+)}(\mathbf{k}_i)\rangle\ , \tag{4.32}$$

when the eikonal assumption is introduced. The optical Green function $\hat{G}_{q_i}$ is approximated in the form

$$\hat{G}_{q_i}(\mathbf{r},\mathbf{r}') \simeq G_{\rm eik}(\mathbf{k}_{q_i};\mathbf{r},\mathbf{r}\,') = -\frac{i}{\hbar v}\delta(\mathbf{b}-\mathbf{b}')\theta(z-z')\exp[ik_{q_i,z}(z-z')]\exp\left(-\frac{i}{\hbar v}\int_{z'}^{z}\mathcal{V}_0(\mathbf{r}'')dz''\right)\ . \tag{4.33}$$

If the form factors are evaluated according to the prescriptions discussed above, the procedure displayed in Sec. III allows one to put the second-order scattering amplitude in the general form

$$\hat{f}_{if}^{(2)} = -\frac{ik}{2\pi}\int d^2b\ e^{i\mathbf{q}\cdot\mathbf{b}}\ e^{\chi_C(b)+\chi_N(b)}\sum_q\mathbf{D}_{fq}(\mathbf{b})\mathbf{D}_{qi}(\mathbf{b}), \tag{4.34}$$

where the sum is extended to all the intermediate states assumed to play a relevant role in the process examined.

2. The general equations

The general result arising from the above procedure is the algebrization of the scattering problem for each impact parameter: Eq. (4.24) has been in fact transformed in a simple integral of products of elements of the matrix $\mathbf{D}(\mathbf{b})$. More precisely, the general term of the nth order of $\hat{f}_{if}$ can be written

$$\hat{f}_{if}^{(n)} = -\frac{m}{2\pi\hbar^2}\int d^2b e^{i\mathbf{q}\cdot\mathbf{b}}e^{\chi_C(b)+\chi_N(b)}\,(T_2)_{if}^{(n)}\,(\mathbf{b}) \tag{4.35}$$

where

$$\left(\hat{T}\right)_{if}^{(n)} = \frac{iv}{n!}\sum_{q_1,q_2,\cdots q_{n-1}}\mathbf{D}_{fq_1}(\mathbf{b})\mathbf{D}_{q_1q_2}(\mathbf{b})\cdots\mathbf{D}_{q_{n-1}i}(\mathbf{b}) = \frac{iv}{n!}\{[\mathbf{D}(\mathbf{b})]^n\}_{if} \tag{4.36}$$

and therefore, on summing to all the orders, one gets for $\hat{f}_{if}$

$$\hat{f}_{if} = -\frac{ik}{2\pi}\int d^2b\; e^{i\mathbf{q}\cdot\mathbf{b}} e^{\chi_C(b)+\chi_N(b)} \sum_{n=1}\frac{1}{n!}\{[\mathbf{D}(\mathbf{b})]^n\}_{if}$$
$$= -\frac{ik}{2\pi}\int d^2b\; e^{i\mathbf{q}\cdot\mathbf{b}} e^{\chi_C(b)+\chi_N(b)} \left\{e^{\mathbf{D}(\mathbf{b})} - \mathbf{I}\right\}_{if} . \tag{4.37}$$

The factorials take origin from an ordered product when the operators are assumed to commute or when commutators are disregarded (see, e.g., Ref. [30]). Since in the eikonal approximation the elastic potential–scattering amplitude has the form

$$f_{el} = \frac{ik}{2\pi}\int d\mathbf{b}\; e^{i\mathbf{q}\cdot\mathbf{b}}\left[1 - e^{\chi_C(b)+\chi_N(b)}\right], \tag{4.38}$$

the scattering amplitude can be written in the final form it assumes in the EMMA

$$f_{if} = \delta_{if} f_{\mathrm{el}} + \hat{f}_{if} = \frac{ik}{2\pi}\int d\mathbf{b}\; e^{i\mathbf{q}\cdot\mathbf{b}}\left[\delta_{if} - e^{\chi_C(b)+\chi_N(b)}\left\{e^{\mathbf{D}(\mathbf{b})}\right\}_{if}\right]. \tag{4.39}$$

This expression is the generalization to any order of scattering of the equations set up in Secs. II and III to describe first- and second-order processes, respectively. Our model is essentially the association of this formula with the representation of the **D** matrix given by Eq. (4.31), and the adiabaticity factor given by Eq. (2.36).

With reference to the formalism developed in Ref. [7], we observe that in the present approach the diagonal elements in the matrix **D** are identically zero since the direct elastic scattering has been extracted from the beginning to generate the optical phase shift. With this proviso, the present formalism becomes identical to the previous one when Coulomb interaction and finite Q value are disregarded.

More general equations can be written by introducing z-ordering operators (see Ref. [13]) which parallel the time-ordering operators in semiclassical time-dependent approaches [30]. A thorough discussion on these problems in connection with the Glauber theory can be found in Ref. [21].

3. *The polarization potential*

The polarization potential for any problem capable of being framed in the EMMA formalism can be obtained, in principle, according to the procedure displayed in Sec. III. As a first step, one writes the elastic-scattering amplitude appearing in Eq. (4.39) in the form

$$f_{00} = \frac{ik}{2\pi}\int d\mathbf{b}\; e^{i\mathbf{q}\cdot\mathbf{b}}\left[1 - e^{\chi_C(b)+\chi_N(b)+\chi'(b)}\right] \tag{4.40}$$

or, in other terms, one should provide an explicit representation of the type

$$\left\{e^{\mathbf{D}(\mathbf{b})}\right\}_{00} = e^{\chi'(b)} , \tag{4.41}$$

for the "elastic" element of the exponential matrix. The second step amounts to evaluating the Abel transform of the additional phase shift $\chi'(b)$. This gives the polarization potential according to the relation

$$\Delta\mathcal{V}(r) = -i\frac{\hbar v}{\pi r}\frac{d}{dr}\int_r^\infty \frac{\chi'(b)}{(b^2-r^2)^{1/2}}\, b\, db . \tag{4.42}$$

In general to find an explicit form for $\chi'(b)$ is not an easy task. This quantity can be obtained in a perturbative context, as discussed in Sec. III, or in particular coupling schemes, as described in the next subsection.

C. Schematic examples

Let us consider as an example the bordered interaction matrix [13, 7] originating from the case in which the ground state is directly coupled with all the excited states and no other coupling is active. Recalling that the matrix **D** is symmetric, it is easy to verify that in this case one gets

$$\{[\mathbf{D}(\mathbf{b})]^n\}_{00} = [C(b)]^n \qquad \text{even } n \tag{4.43}$$

for the "elastic matrix element" and

$$\{[\mathbf{D}(\mathbf{b})]^n\}_{0i} = \mathbf{D}_{0i}[C(b)]^{n-1} \qquad \text{odd } n \tag{4.44}$$

for the inelastic ones, where

$$C(b) = \left[\sum_i (\mathbf{D}_{0i})^2\right]^{1/2} . \tag{4.45}$$

The scattering amplitudes, according to Eq. (4.39), will take on the form

$$f_{00} = \frac{ik}{2\pi}\int d\mathbf{b}\; e^{i\mathbf{q}\cdot\mathbf{b}}\left[1 - e^{\chi_C(b)+\chi_N(b)}\cosh C(b)\right] , \tag{4.46}$$

$$f_{0i} = -\frac{ik}{2\pi}\int d\mathbf{b}\; e^{i\mathbf{q}\cdot\mathbf{b}}\left[e^{\chi_C(b)+\chi_N(b)}\,[\mathbf{D}_{0i}(b)/C(b)]\sinh C(b)\right] , \tag{4.47}$$

for the elastic and the inelastic scattering, respectively. These results coincide in their structure with the ones obtained in Ref. [7] for the same coupling scheme, in the limit $A_P\ A_T \gg 1$.

According to the discussion in the previous subsection, in the problem we are considering the polarization potential can be obtained by inverting the phase shift

$$\chi'(b) = \ln[\cosh C(b)] \tag{4.48}$$

through Eq. (4.42).

Another soluble problem is obtained when the coupling matrix elements are assumed to have the form

$$\mathbf{D}_{i,i\pm1}(b) = \nu(b)\ , \tag{4.49}$$

i.e., only the two diagonals adjacent the principal diagonal have nonzero elements. For the sufficiently interesting problem in which the ground state is coupled to two excited states according to the scheme (4.49), one gets a closed form for the scattering amplitudes in terms of hyperbolic functions. For the elastic and the two inelastic channels one obtains, respectively,

$$f_{00} = \frac{ik}{2\pi}\int d\mathbf{b}\ e^{i\mathbf{q}\cdot\mathbf{b}}\left[1 - \frac{1}{2}e^{\chi_C(b)+\chi_N(b)}\ [1+\cosh(\sqrt{2}\nu(b))]\right], \tag{4.50}$$

$$f_{01} = \frac{ik}{2\pi}\int d\mathbf{b}\ e^{i\mathbf{q}\cdot\mathbf{b}}\left[e^{\chi_C(b)+\chi_N(b)}\ \left(\frac{1}{\sqrt{2}}\sinh[\sqrt{2}\nu(b)]\right)\right], \tag{4.51}$$

$$f_{02} = \frac{ik}{2\pi}\int d\mathbf{b}\ e^{i\mathbf{q}\cdot\mathbf{b}}\left[e^{\chi_C(b)+\chi_N(b)}\frac{1}{2}(\cosh[\sqrt{2}\nu(b)]-1)\right]. \tag{4.52}$$

V. CONCLUDING REMARKS

The interplay of nuclear and Coulomb interactions has always been one of the most interesting features in inelastic excitations induced by heavy ions. The different dependence of the two contributions on the bombarding energy, on the mass and charge of the colliding systems, on the excitation energy, and on the multipolarity of the populated state does in fact offer the possibility of a wide spectrum of different situations. While at low bombarding energies around the Coulomb barrier the two processes have been usually described on equal footing, at high energy one has often concentrated on either effect. For example in the case of collisions induced by very heavy systems, where the Coulomb excitation of the dipole modes is dominant, the effect of the nuclear interactions has been limited to the introduction of an absorption radius. On the other hand, in the case of relatively light projectiles, all the dynamics of the process at high energy have been described in terms of the individual nucleon-nucleon collisions, along approaches which are essentially derived from the Glauber model and traditionally neglect, for example, the effect of finite Q values. In this paper we have displayed a general framework, intended to cover the intermediate energy regime, where the contributions of the two processes are mingled together. Along the eikonal approximation we combine to all orders the underlying microscopic nuclear multiple scattering with the macroscopic Coulomb interactions, profitting from the results already existing in both fields.

As a direct consequence of the eikonal propagation the traditional system of differential coupled equations reduces, for each impact parameter, to a simple algebraic expression involving a matrix whose elements are obtained by integrating along straight lines densities, transition densities, and microscopic effective interaction. It should be further possible, in analogy to previous developments, to combine this algebrization of the relative motion with the specific intrinsic algebra describing the dynamical symmetry associated with the nuclear states. Analytic expressions should in that case be derived for the scattering amplitude if the coupling matrix has a suitable geometrical structure, as in the examples displayed in Sec. IV C.

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