Fine structure of cluster decays

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Within the one level R-matrix approach, expressions are derived for the hindrance factors of cluster radioactive decays in which α particles or other light nuclei are emitted. The interior wave functions are supposed to be given by the shell model with effective residual interactions [e.g., the large scale shell model code OXBASH in the Michigan State University version for nearly spherical nuclei, or the enlarged superfluid model (ESM) recently proposed for deformed nuclei]. The exterior wave functions are calculated from a cluster-nucleus double-folding model potential obtained with the M3Y interaction. As examples of the cluster decay fine structure we analyzed the particular cases of α decay of ^{241}Am and ^{243}Cm, ^{14}C decay of ^{223}Ra, and ^{34}Si decay of ^{243}Cm. Good agreement with the experimental data is obtained.

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I. INTRODUCTION

Recently Hourani and co-workers [1] experimentally discovered the fine structure of the ^{14}C radioactivity [2–5]. The theoretical studies of alpha [9] (see also the review papers [6–8] and the references therein) and heavy cluster (e.g., ^{14}C) decay (see review papers [10–12, 5] and references therein) have very much in common. The theoretical models of heavy cluster decay are based, essentially, on Gamov's theory [13] which was the first success of quantum mechanics when applied to the α-decay phenomenon. The differences in approaches are related to the way of calculating the potential barrier defined by the (nuclear plus Coulomb) interaction potential acting between the emitted cluster and the residual nucleus. The decay energy always is taken to be equal to the experimental energy release of the decay [14]. All these theoretical treatments fit to a law for favored cluster transitions, analogous to the Geiger-Nuttal [15] law for favored α-decay, which emerges directly from the simplest JWKB expression of the penetrability determined by the square well plus Coulomb interaction potential.

The unfavored transitions do not follow the Geiger-Nuttal law, because of the large variations of the reduced widths [6–8, 20] which have a key role in the understanding of the decay process and require a precise knowledge of the structures of the initial and final quantum states. From such transitions we can learn much about the structure of atomic nuclei. In describing these transitions almost all the nowadays nuclear models fail, and it does not matter whether they are models for the structure of nuclear states or reaction mechanisms, or whether they are specific models, for the decay mechanism [6, 7, 23, 21, 22, 11, 5, 12].

The theoretical study of α decay has provided a basic test for our understanding of several fundamental quantum phenomena, such as tunneling through the potential barrier, the clusterization process [16, 17], and weak interaction models [18, 19]. However, in spite of the effort invested, a detailed description of the α particle emission is not yet available.

By contrast to the case of the γ or β decay, where the changes in the nuclear structure are small and may be treated within perturbation theory, α decay represents the simplest case of a series including phenomena like the heavy cluster decays [2, 4, 5] or fission, when the transition has dramatic effects, generating in fact two new nuclei. While the fine structure of α decay has been more or less understood [6, 7], few studies [24, 26, 25, 27, 28] of the fine structure of heavy cluster decay are available. In this case one should understand the mechanisms of heavy cluster decay, which in our opinion should be closer to α decay mechanism, then to the fission one.

It is the aim of this paper to calculate the hindrance factors for several α-, ^{14}C-, and ^{34}Si-cluster decays. The calculations will be performed within the one level R-matrix approximation analogously to the calculations done in Ref. [9] for the α decay of some ^{16}O excited states. The cluster residual nucleus scattering wave functions are generated by the Coulomb potential plus the realistic M3Y double folding potential [43, 34, 21, 22], in which one uses an effective interaction derived from the G-matrix elements based on the Reid soft-core NN potential [44] in the form assuming only the one pion exchange potential (OPEP) between the states with odd relative angular momentum [45]. The Pauli antisymmetrization kernel is used as proposed in Refs. [21, 22, 9].

Several favored and weak-hindered α transitions from the ground state of ^{241}Am to some states of ^{237}Np and from the ground state of ^{243}Cm to some states of ^{239}Pu are calculated by using for describing the initial and final nuclear states the enlarged superfluid model (ESM) [35, 36].

The situation is not as simple as previous one in the case of ^{14}C decay of Ra and Th isotopes.

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In the past few years, a wealth of new spectroscopic information has been obtained for the neutron-deficient Ra and Th isotopes. In this transitional region between spherical nuclei near the $N = 126$ shell closure and the well-deformed heavier isotopes unusual phenomena are observed, in particular the occurrence of a sequence of a very low-lying negative-parity states, strong $E1$ transitions and the existence of parity mixed doublets in several odd-A nuclei.

These phenomena have been described in the framework of two different models. The molecular cluster model [64, 65] takes the possibility of α clustering in the light actinides and treat the excitations in the framework of the interacting boson model as oscillations of the α-like bosons with respect to the residual core.

Alternatively, Leander, Sheline *et al.* [66] predicted reflection asymmetric ground-state deformations for a limited region of nuclei around ^{224}Th within a mean field including both the even and odd parity multipoles. In this model, the observed positive- and negative-parity states are regarded as a single rotational band of an octupole deformed core [67]. A doubling of all states with respect to their parities was predicted and the measured energies, electromagnetic transitions, spins, parities, and magnetic moments for ground state and some excited states of a series of Ra and Th isotopes were nicely reproduced [66].

Nuclei with strong octupole correlations, leading to reflection-asymmetric shapes, have particularly low-lying negative-parity states. From the systematics of the lowest negative parity excitations [66], one learns that the nuclei with strongest octupole correlations are the neutron-deficient nuclei around ^{224}Th (with negative parity excitations of the order of 100 keV [70]) and the neutron-rich nuclei around ^{146}Ba (with the energy of the lowest 1^- states of about 750 keV). The tendency toward maximal octupole coupling occurs just above the closed shells, i.e., for nuclei with Z or $N \approx 34, 56, 88, 134$, where (Nlj) intruder orbitals interact with $(N-1, l-3, j-3)$, natural parity states through the octupole component of the mean field.

The experimentally observed [1] transitions from the ground state of ^{223}Ra to some excited states of ^{209}Pb are estimated and discussed in comparison with previous works [24, 26, 25, 27, 28]. As a nuclear structure model for the ^{223}Ra we use a hybrid model discussed in detail latter.

The enlarged superfluid model of atomic nucleus [35] is used for predictions of several hindrance factors in the decay $^{243}\mathrm{Cm} \rightarrow {}^{34}\mathrm{Si} + {}^{209}\mathrm{Pb}$.

The main parameter of the R-matrix approach, the channel radius, will be chosen according to a new procedure developed in Ref. [9] eliminating in a large extent the ambiguities when using the matching of the internal and scattering radial wave functions for a definite decay channel.

II. HINDRANCE FACTORS

The experimental hindrance factor ($\mathrm{HF_{exp}}$) of any cluster decay is defined as a ratio between the Geiger-Nuttal [15] width divided by the width of the radioactive transition we are interested in [6]

$$\mathrm{HF_{exp}} = \frac{\Gamma_{GN}(Q)}{\Gamma(Q)}, \tag{1}$$

where Q stands for the energy release of the studied decay and [15]

$$\log_{10}\Gamma_{GN}(Q) = A + \frac{B}{\sqrt{Q}}. \tag{2}$$

The theoretical hindrance factor ($\mathrm{HF_{theo}}$) is defined by Eq. (1) in which the widths are replaced by their theoretical expressions. In the case of heavy deformed nuclei [7]:

$$\mathrm{HF_{theo}} = \frac{P_0(Q)\gamma_0^2}{\sum_l P_l(Q)\gamma_l^2} = \frac{\gamma_0^2}{\sum_l F_l\gamma_l^2}. \tag{3}$$

The factor γ_l^2 is the reduced width [20, 7, 6] and $P_l(Q)$ is the penetrability. Within the JWKB approximation P_l has the expression

$$P_l = 2R_c q_l(R_c) \exp\left(-\frac{2}{\hbar}\int_{R_c}^{r_0} q_l(r)dr\right) \tag{4}$$

in which "r_0" and "R_c" stand for the outer and inner turning points, respectively, and

$$q_l(r) = \sqrt{2m_0 A_{\mathrm{red}}(V_l^{\mathrm{Coul+nucl}} - Q)}, \tag{5}$$

where $V_l^{\mathrm{Coul+nucl}}$ is the sum of the Coulomb and nuclear one-body potential acting between the a cluster and the daughter nucleus when studying the radial part of the Schrödinger equation.

Thus

$$F_l = \exp\frac{2}{\hbar}\int_{R_c}^{r_0}[q_{l=0}(r) - q_l(r)]dr. \tag{6}$$

Usually [6, 11, 12] the Coulomb part of this potential is replaced by pointlike Coulomb potential while the nuclear part by a Saxon-Woods one. Within these simple prescriptions in the case of α decay the F_l function has the approximate expression [57]

$$F_l = \exp\left[-2.027l(l+1)Z^{-\frac{1}{2}}A^{-\frac{1}{6}}\right]. \tag{7}$$

III. REDUCED WIDTHS

In the case of axially deformed nuclei the reduced widths (see Ref. [29], Eq. (2)) can be written as

$$\gamma^2_{\lambda_c}(R_c; I_i K_i \pi_i \to I_f K_f \pi_f) = \frac{\hbar^2}{2\mu R_c^2} \frac{1}{(1+\delta_{K_i 0})(1+\delta_{K_f 0})} \times \binom{N}{N_a}\binom{Z}{Z_a} \Big| \sum_L [B^L_{l\, I_i I_f}(K_i - K_f)\, M_{L\, K_i - K_f}(K_i\pi_i \to K_f\pi_f) + (-1)^{I_i - L - K_f}\, \pi_f B^L_{l\, I_i I_f}(K_i + K_f) M_{L\, K_i + K_f}(K_i\pi_i \to K_f\pi_f)] \Big|^2, \tag{8}$$

where M_{LK} functions are the probability amplitudes to find the cluster a at the channel surface (r = R_c). The presence of two terms is due to the two terms in the nuclear wave function of the axially deformed nucleus

$$\Psi^{IMK\pi} = \frac{2I+1}{\sqrt{16\pi^2(1+\delta_{K0})}} [\chi_{K\pi} D^I_{MK}(\theta_i) + (-1)^{I-K} \pi \chi_{-K\pi} D^I_{M-K}(\theta_i)]. \tag{9}$$

The amplitude of the reduced width is related to the $f_{\rm int}$ function defined in Ref. [9] as

$$\gamma^{(I_i K_i \pi_i \to I_f K_f \pi_f)}_{L\ K}(R_c) = \sqrt{\frac{\hbar^2}{2\mu R_c}} f_{\rm int}^{I_i K_i \pi_i \to I_f K_f \pi_f; L\ K}(R_c), \tag{10}$$

$f_{\rm int}$ function is practically equal to the initial decaying state wave function projected onto the channel spin function [e.g., the function $R \cdot g_L^{I_i K_i \pi_i \to I_f K_f \pi_f}(R)$ introduced in Ref. [7]].

If the fragments would remain distinct in the internal region, then $\Phi^{I_i K_i \pi_i}_{A+a}$ could be represented as a cluster-like wave function [38], i.e., by an antisymmetrized product between a surface spin [20] function Φ_c and a radial function $f_{\rm int}$ depending on the relative distance. But as the fragments lose their identity, $f_{\rm int}$ should be extracted from the shell model state $\Phi^{I_i K_i \pi_i}_{A+a}$ by projection onto the channel c.

Until now, one can say there are known two different ways of extracting $f_{\rm int}$ from the initial state wave function. These ways assume two different decay mechanisms. The most known one simply assumes the cluster already formed with some probability in the structure of the model wave function describing the initial state [9]. The usual shell model with residual interactions in some specific conditions defines structures containing strong nucleon correlations, like superfluid Cooper pairs [35, 60, 36] or alpha condensates [62]. It may be useful also to construct models containing both fermions and α clusters [39, 40], however, such models could be fruitful in the case when the shell model with residual interactions fails in describing other processes or observables.

Another mechanism has been proposed recently [49]. This mechanism starts from an initial many-body Hartree-Fock configuration, i.e., a fermionic system without residual interactions between the fermions. Then by successive shape changes from this initial configuration (δ_0) the system may arrive to the final touching configuration (δ_n). One is considered a large amplitude collective motion along a path in the space of deformations, and describe it in terms of single coordinate, with the determinants ordered according to the value of that coordinate. This path is so chosen that at every step the system is deformed enough to put an integral number of particles above the spherical Fermi surface. The solution of this problem in the overlapping region is given by a linear superposition of Hartree-Fock determinants $\Phi_k(\delta_k)$ [$\Phi = \sum_k a_k \Phi_k(\delta_k)$], describing the minima between crossing of two single particle Fermi levels. The number of the deformation steps ($k = 0, 1, .., n$) is equal to the number of level crossings. The deformation is adiabatic, so that level crossings do not lead to excitations, but simply bring the system from one local minimum to the next one. The mixing is assumed to be due to a part of the residual interactions (mainly pairing interaction). The rest of the residual interactions may describe excitations at every deformation step. In such a way we may follow a particular contribution of a single particle orbital in the structure of the ground and low-lying excited states at every deformation step judging in this way about the fine structure of the cluster decay.

At this point one could calculate the $f_{\rm int}$, which should be proportional to the amplitude (a_{n-1}) of the next to the last configuration in the ground state. Here, of course, we assumed the high degree of orthogonality of the Φ_k wave functions and a large overlap of the channel state with the wave function of the last configuration (Φ_n), which is not completely true. In other words, this kind of $f_{\rm int}$ describes the amount of cluster configuration, in the initial nuclear system due to the large amplitude collective motion, which may be unimportant (and neglected) when describing other properties or processes involving that nuclear system.

In terms of a complete orthonormal set of radial oscillator wave functions $\mathcal{R}_{NL}$ the projected function $f_{\rm int}$ will be defined by the sum

$$f_{\rm int}(r) = \sum_N \theta^{I_i K_i \pi_i}_{Nc}\, r \mathcal{R}_{NL}(r, a_r), \tag{11}$$

where

$$\theta^{I_i K_i \pi_i}_{Nc} = \langle \mathcal{A}[\psi^a_{\rm int}(\psi^{I_f K_f \pi_f}_A)_{\rm int} Y_{LM} \mathcal{R}_{NL}] \mid (\Phi^{I_i K_i \pi_i}_{A+a})_{\rm int} \rangle \tag{12}$$

is the spectroscopic amplitude for the cluster a and $\mathcal{A}$ stands for the antisymmetrization operator. The basis functions R_{NL} are chosen as the radial eigenstates for

a particle having the reduced mass $A_{\rm red} = aA/(a+A)$ placed in an harmonic oscillator potential. The oscillator constant is $\alpha_r = \sqrt{A_{\rm red}}\alpha_0$, with $\alpha_0 = \sqrt{\frac{m_0\omega}{\hbar}}$, m_0 the nucleon mass, and ω the oscillator frequency for the shell model potential of the initial nucleus. The radial function $f_{\rm int}$ has a key role in the R-matrix calculation because its square at the channel radius gives the formation probability of the fragments

$$\gamma^2 = \frac{\hbar^2}{2m_0 A_{\rm red} R_c} \mid f_{\rm int}(R_c) \mid^2 \tag{13}$$

which are known as reduced widths [20].

Using this function, the boundary parameter B_c [20,9] may be simply estimated as

$$B_c = \rho \frac{(f_{\rm int})'}{f_{\rm int}}\Big|_{r_c} \tag{14}$$

with the prime denoting the derivative with respect to $\rho = kr$, $k = \sqrt{2m_0 A_{\rm red} E_{\rm c.m.}}/\hbar$, and $E_{\rm c.m.}$ the decay energy in the center-of-mass frame.

When the internal and the external wave functions for the resonance energy are joined at the channel surface, a functional relation appears between the phase shift, the boundary parameter, and the reduced widths defined by

$$\gamma = \sqrt{\frac{\hbar^2 r_c}{2m_0 A_{\rm red}}} \sum_N \theta_{Nc}^{\Phi_{A+a}^{I_i K_i \pi_i}} \mathcal{R}_{NL}. \tag{15}$$

Therefore the explicit formula for the calculus of the width using this approach is [50]

$$\Gamma = \frac{2}{\frac{P}{\gamma^2} + \frac{P\dot{S} - \dot{P}(S-B)}{P^2 + (S-B)^2} - \dot{\phi}}\Bigg|_{r_c}. \tag{16}$$

Here the overdot denotes the derivative with respect to the energy, $B = \rho G'/G$, $P = \rho/(F^2+G^2)$, $S = P\cdot(FF' + GG')$, $\rho = kr$, and $\tan\phi = F/G = (S-B)/P$. In particular, if at the channel radius G has a maximum ($B = 0$) and, as in our case, $\mid S \mid \ll P$, this formula takes the simple well-known form

$$\Gamma_0 = 2P\gamma^2. \tag{17}$$

The coefficients $\theta_{Nc}^{I_i K_i \pi_i}$ can be calculated by using one of the above procedures. In the following we shall use the procedure obtained by projecting the initial shell model wave function onto the cluster channel c as is done in the most of the R-matrix calculations [7]. Within this procedure the coefficients $\theta_{Nc}^{I_i K_i \pi_i}$ are complicated because it is difficult to perform separate integrals (even in the case of α decay), over the intrinsic coordinates of the fragments, but it was shown [51] that a simpler form may be obtained assuming that all centers of mass of the nuclei involved in the reaction have an harmonic oscillator motion with the same frequency. In this case, by introducing a complete set of a-body shell model wave functions $\mid \Psi_\beta\rangle$, the integral defining θ may be expressed in terms of the overlap integrals between states defining the mother ($\mid \Phi_{A+a}\rangle$), the daughter ($\mid \Psi_A\rangle$), and the cluster systems ($\mid \Psi_\beta\rangle$) represented in the same basis of oscillator states, the so-called cluster overlap [42], and overlap integrals between the states defining the cluster systems (Ψ_β) and the outgoing clusters ($\psi_{\rm int}^a$ which may have another oscillator frequency), the so-called intrinsic overlap integrals:

$$\theta_{Nc}^{I_i K_i \pi_i} = \left(\frac{A+a}{A}\right)^{N+L/2} \binom{A+a}{a}^{\frac{1}{2}} \sum_\beta \langle \Psi_A^{I_f K_f \pi_f} \Psi_\beta \mid\} \Phi_{A+a}^{I_i K_i \pi_i} \rangle \langle \psi_{\rm int}^a \mathcal{R}_{NL}(R_a, \sqrt{a}\alpha_0) Y_{LM} \mid \Psi_\beta \rangle. \tag{18}$$

The first factor $(\frac{A+a}{A})^{N+L/2}$ comes from the Moshinsky transformation for nonequal masses [53].

This approximation more or less good in the case of α decay [9] it is expected to be not very good in the case of heavy cluster decay, where the motion of the nucleons entering the mother, the daughter, and the cluster systems could be completely different at least concerning the oscillator frequency and the other global parameters that describe these systems.

The intrinsic functions of the nuclei a, A, and $A+a$ ($\psi_{\rm int}^a$, Ψ_A, and Φ_{A+a}) are expressed in terms of harmonic oscillator wave functions, with the oscillator constant ω_m chosen to fit the density radius [45,52]. The amplitude of the spectroscopic factor may be written as

$$\begin{aligned}\theta_{Nc}^{I_f K_f \pi_f} = \left(\frac{A+a}{A}\right)^{N+L/2} \binom{A+a}{a}^{\frac{1}{2}} \sum_{\beta_{\omega_a}^a} \sum_{\beta_{\omega_A}^A} \sum_{\beta_{\omega_{A+a}}^A} \sum_{\beta_{\omega_{A+a}}^a} \langle \psi_{\rm int}^a \mid \Psi_{\beta_{\omega_a}^a} \rangle \\ \times \langle \Psi_A^{I_f K_f \pi_f} \mid \Psi_{\beta_{\omega_A}^A} \rangle \langle \Psi_{\beta_{\omega_A}^A} \mid \Psi_{\beta_{\omega_{A+a}}^A} \rangle \langle \Psi_{\beta_{\omega_{A+a}}^A} \Psi_{\beta_{\omega_{A+a}}^a} \mid\} \Phi_{A+a}^{I_i K_i \pi_i} \rangle \langle \Psi_{\beta_{\omega_a}^a} \mathcal{R}_{NL}(R_a, \sqrt{a}\alpha_0) Y_{LM} \mid \Psi_{\beta_{\omega_{A+a}}^a} \rangle,\end{aligned} \tag{19}$$

i.e., the cluster overlap from Eq. (18) is

$$\begin{aligned}\langle \Psi_A^{I_f K_f \pi_f} \Psi_\beta \mid\} \Phi_{A+a} \rangle &= \langle \Psi_A^{I_f K_f \pi_f} \Psi_{\beta_{\omega_{A+a}}^a} \mid\} \Phi_{A+a}^{I_i K_i \pi_i} \rangle \\ &= \sum_{\beta_{\omega_A}^A} \sum_{\beta_{\omega_{A+a}}^A} \langle \Psi_A^{I_f K_f \pi_f} \mid \Psi_{\beta_{\omega_A}^A} \rangle \langle \Psi_{\beta_{\omega_A}^A} \mid \Psi_{\beta_{\omega_{A+a}}^A} \rangle \langle \Psi_{\beta_{\omega_{A+a}}^A} \Psi_{\beta_{\omega_{A+a}}^a} \mid\} \Phi_{A+a}^{I_i K_i \pi_i} \rangle\end{aligned} \tag{20}$$

while the intrinsic overlap from Eq. (18) is

$$\langle \psi^a_{\rm int}\mathcal{R}_{NL}(R_a,\sqrt{a}\alpha_0)Y_{LM} \mid \Psi_\beta\rangle = \langle \psi^a_{\rm int}\mathcal{R}_{NL}(R_a,\sqrt{a}\alpha_0)Y_{LM} \mid \Psi_{\beta^a_{\omega_{A+a}}}\rangle$$
$$= \sum_{\beta^a_{\omega_a}} \langle \psi^a_{\rm int} \mid \Psi_{\beta^a_{\omega_a}}\rangle \langle \Psi_{\beta^a_{\omega_a}}\mathcal{R}_{NL}(R_a,\sqrt{a}\alpha_0)Y_{LM} \mid \Psi_{\beta^a_{\omega_{A+a}}}\rangle \tag{21}$$

The two functions $\mid \Psi_{\beta^A_{\omega_A}}\rangle$; $\mid \Psi_{\beta^A_{\omega_{A+a}}}\rangle$ and $\mid \Psi_{\beta^a_{\omega_a}}\rangle$; $\mid \Psi_{\beta^a_{\omega_{A+a}}}\rangle$, respectively, differ in oscillator frequency only.

The first group of functions ($\mid \Psi_{\beta^A_{\omega_A}}\rangle$; $\mid \Psi_{\beta^A_{\omega_{A+a}}}\rangle$) are totally antisymmetrized A-nucleon single particle wave functions given by an average shell model (with no residual interaction) field.

The second group of functions ($\mid \Psi_{\beta^a_{\omega_a}}\rangle$ and $\mid \Psi_{\beta^a_{\omega_{A+a}}}\rangle$) are totally antisymmetrized a-nucleon (e.g., in the ^{14}C case, 6 protons and 8 neutrons) single particle wave functions given by an average shell model (with no residual interaction) field. They are given in terms of generalized coefficients of fractional parentage and, after a sequence of Moshinsky [78] transformations and orthogonal transformations for rearranging angular momenta, in terms of products of a $[\psi_{nlm}(\mathbf{r}\omega_m)]$ spatial harmonic oscillator wave functions and a spin function of a particles. $a-1$ of these harmonic oscillator wave functions are functions of Jacobi coordinates ($\boldsymbol{\rho}_k$). In the ^{14}C case the Jacobi coordinates are (see Fig. 1)

$$\rho_1 = \frac{\mathbf{r}_1-\mathbf{r}_2}{\sqrt{2}} \quad \rho_2 = \frac{\mathbf{r}_3-\mathbf{r}_4}{\sqrt{2}} \quad \rho_3 = \frac{\mathbf{r}_1+\mathbf{r}_2-\mathbf{r}_3-\mathbf{r}_4}{2},$$
$$\rho_4 = \frac{\mathbf{r}_5-\mathbf{r}_6}{\sqrt{2}} \quad \rho_5 = \frac{\mathbf{r}_7-\mathbf{r}_8}{\sqrt{2}} \quad \rho_6 = \frac{\mathbf{r}_5+\mathbf{r}_6-\mathbf{r}_7-\mathbf{r}_8}{2},$$
$$\rho_7 = \frac{\mathbf{r}_9-\mathbf{r}_{10}}{\sqrt{2}} \quad \rho_8 = \frac{\mathbf{r}_{11}-\mathbf{r}_{12}}{\sqrt{2}} \quad \rho_9 = \frac{\mathbf{r}_9+\mathbf{r}_{10}-\mathbf{r}_{11}-\mathbf{r}_8}{2}, \tag{22}$$
$$\rho_{10} = \frac{\mathbf{r}_{13}-\mathbf{r}_{14}}{\sqrt{2}} \quad \rho_{11} = \frac{-4\sum_{i=9}^{14}\mathbf{r}_i + 3\sum_{i=1}^{8}\mathbf{r}_i}{\sqrt{168}},$$
$$\rho_{12} = \frac{\sum_{i=1}^{4}\mathbf{r}_i - \sum_{i=5}^{8}\mathbf{r}_i}{2\sqrt{2}} \rho_{13} = \frac{\sum_{i=9}^{12}\mathbf{r}_i - 2\sum_{i=13}^{14}\mathbf{r}_i}{2\sqrt{3}},$$

and one of them is a function of $\mathbf{R} = \sqrt{14}\mathbf{R}_a$, where

$$\mathbf{R}_a = \frac{\sum_i \mathbf{r}_i}{14} \tag{23}$$

is the center-of-mass coordinate of the cluster (here ^{14}C).

Each relative oscillator wave function $[\psi_{n_{\rho_k},l_{\rho_k},m_{\rho_k}}(\boldsymbol{\rho}_k)]$ is obtained by a Moshinsky transformation in which a two particle ($\langle \mathbf{r}_1\mathbf{r}_2 \mid \Psi\rangle$) oscillator wave function is expressed in terms of a complete set of oscillator wave functions ($\langle \boldsymbol{\rho}_k\mathbf{R}_k \mid \Phi\rangle$), which are functions of the corresponding Jacobi and center-of-mass coordinates and in addition they have the same frequency as the original ($\langle \mathbf{r}_1\mathbf{r}_2 \mid \Psi\rangle$) wave function [54].

The indices ω_m from the functions $\mid \Psi_{\beta^N_{\omega_m}}\rangle$ are the shell model oscillator frequencies $\left(\hbar\omega = \frac{45}{A^{\frac{1}{3}}} - \frac{25}{A^{\frac{2}{3}}}\right)$ [42].

The overlap integral ($\langle \Psi_{\beta^N_{\omega_m}} \mid \Psi_{\beta^N_{\omega_{m'}}}\rangle$) is a function of $2(a-1)$ Moshinsky brackets, recoupling coefficients of angular momenta and products of single particle overlap integrals defined by

$$v_{nl} = \langle nlm(\mathbf{r},\omega_\beta) \mid n'lm(\mathbf{r},\omega_\alpha)\rangle = \frac{1}{2}\sqrt{\frac{4\Gamma(n+1)\Gamma(n'+1)}{\Gamma(n+l+\frac{3}{2})\Gamma(n'+l+\frac{3}{2})}}\left(\frac{1-\lambda}{1+\lambda}\right)^{n-n'}$$
$$\times F\left[-n-n';-n-n'-l-\frac{1}{2};\left(\frac{\lambda+1}{\lambda-1}\right)^2\right], \tag{24}$$

where

$$\lambda = \left(\frac{\beta}{\alpha}\right)^2 = \frac{\omega_\beta}{\omega_\alpha}. \tag{25}$$

Here

$$\beta(\alpha) = \sqrt{\frac{m_o\omega_{\beta(\alpha)}}{\hbar}}. \tag{26}$$

The intrinsic overlap integral for the ^{14}C decay differ from other cluster decays in using different models for the structure of the involved nuclei and in addition the overlap integrals (20) and (21) may contain more factors analogous to $v_{nl} = \langle \psi_{nlm}(\mathbf{r}\omega_m) \mid \psi_{n'lm}(\mathbf{r}\omega_{m'})\rangle$ given in Eq. (24).

In the present calculation the wave functions Φ_{A+a} and Ψ_A are given by the shell model OXBASH [42] or

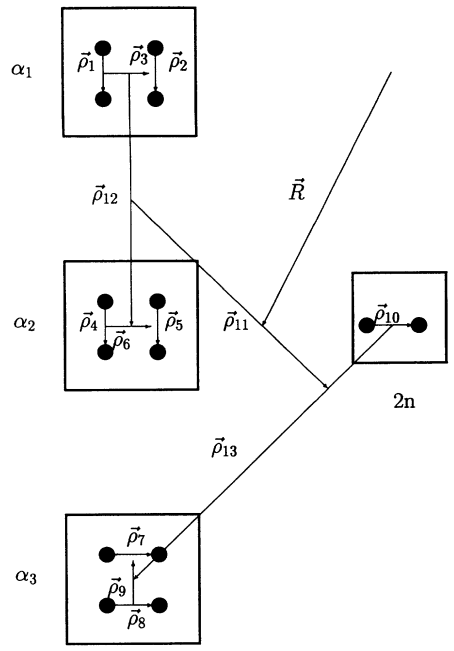

FIG. 1. The center of mass and intrinsic coordinates of the ^{14}C nucleus relative to the residual ^{209}Pb nucleus.

enlarged superfluid model (ESM) [35] and corresponds to the ground state of the initial nucleus and ground or excited states of the daughter nucleus, respectively.

The cluster overlap $\langle \Psi_A \Psi_\beta \mid \} \Phi_{A+a} \rangle$ is obtained by assuming a wave function Ψ_β constructed in the same model space as Φ_{A+a} and Ψ_A. The last factor $\langle \psi^a_{\rm int} \times \mathcal{R}_{NL}(\sqrt{a}R_a, \alpha_0)\ Y_{LM}(\Omega_a) \mid \Psi_\beta \rangle$ is calculated by expanding Ψ_β in terms of products of two proton and two neutron single particle wave functions. The expansion coefficients are products of two nucleon fractional parentage coefficients. This last factor is expressed in terms of overlap integrals, which for α decay are known as Mang's [7] overlap integrals. In the case of ^{14}C decay these overlap integrals are calculated by Florescu *et al.* [55, 56].

IV. NUMERICAL CALCULATIONS

As it is known, the major difficulty encountered in such calculation is a strong dependence on the channel radius, with large variations around the nodes of G, where the boundary parameter B [20, 9] becomes infinite and changes the sign. According to the previous suggestions [9, 49, 50] the channel radius should be chosen in the region of the last peak of the regular scattering wave function F_L inside the barrier. More precisely it considered the points where F_L decreases to one-half of the peak value [50], or accounting for the antisymmetrization effects, the point corresponding to the peak [49].

The antisymmetrization between the particle a and the residual nucleus affects the radial scattering wave functions, changing the function F_L in

$$\tilde{F}_L(r) = F_L(r) - \int_0^\infty dr' K_L(r,r') F_L(r'), \tag{27}$$

where the Pauli kernel [21, 22] $K_l(r,r')$ comes from the accurate antisymmetrization and normalization of the cluster-residual nucleus relative motion wave function, such that the $\tilde{F}_L(r)$ wave function does not contain spurious states. Its expression is

$$K_l(r,r') = \int_0^1 dx P_l(x) K(r,r',x). \tag{28}$$

Here $P_l(x)$ is the lth order Legendre polynomial, $x = \frac{(\mathbf{r}\cdot\mathbf{r}')}{r\cdot r'}$ and

$$\delta(\mathbf{r}-\mathbf{r}') - K(r,r',x) = \langle \mathbf{r} \mid 1 - K \mid \mathbf{r}' \rangle$$

$$= \langle \mathcal{A}(\delta(\mathbf{r}-\mathbf{R}_\alpha)\psi^a_{\rm int}\phi_A) \mid \mathcal{A}(\psi^a_{\rm int}\phi_A \delta(\mathbf{r}'-\mathbf{R}_\alpha)) \rangle. \tag{29}$$

If assuming for the A-nucleus ground-state wave function a Slater determinant we obtain

$$\delta(\mathbf{r}-\mathbf{r}') - K(r,r',x) = \langle \mathbf{r} \mid 1 - K \mid \mathbf{r}' \rangle$$

$$= \langle \delta(\mathbf{r}-\mathbf{R}_\alpha)\psi^a_{\rm int} \mid Q \mid \psi^a_{\rm int}\delta(\mathbf{r}'-\mathbf{R}_\alpha) \rangle$$

$$= \langle \delta(\mathbf{r}-\mathbf{R}_\alpha)\psi^a_{\rm int} \Big| \prod_{i=1}^{a}(1-P_i) \Big| \psi^a_{\rm int}\delta(\mathbf{r}'-\mathbf{R}_\alpha) \rangle \tag{30}$$

with $Q^2 = Q = Q^\dagger$ which projects out of the Fermi sea of the daughter nucleus. Using this approximation and expanding the ground state of the cluster nucleus in terms of Gaussian wave functions:

$$\psi^a_{\rm int} = \sum_k C_k \left(\frac{\beta_k}{\sqrt{\pi}}\right)^{3(a-1)} \exp\left(-\frac{\beta_k}{2}\sum_i \rho_i^2\right) \tag{31}$$

the kernel corresponds to a sum of terms corresponding to an exchange of s nucleons:

$$1 - K = \sum_s (-1)^s \binom{a}{s} K_s. \tag{32}$$

The expansion of $\psi^a_{\rm int}$ in terms of Gaussian wave functions is supported by the specific structure of $\psi^a_{\rm int}$. For α decay the ground state of the ^{4}He may be described by Eq. (31) with one term only. An expansion like that given in Eq. (31) may be still valid in the case of ^{14}C also, where 4 nucleons are in the $1s_{\frac{1}{2}}$ single particle state and the rest of 10 nucleons are, mainly, in the $1p_{\frac{3}{2}}$ and $1p_{\frac{1}{2}}$ states. Then, after the Moshinsky transformations, the oscillator relative wave functions $\psi_{n_{\rho_k} l_{\rho_k} m_{\rho_k}}(\rho_k)$ are mainly in the s state and we do not loose much neglecting the other contributions.

In the case of α particle:

$$K = 2K_p + 2K_n - K_{pp} - K_{nn} - 4K_{pn} + 2K_{pnn} + 2K_{npp} - K_{ppnn} \tag{33}$$

Moreover if the proton and neutron systems have the

same properties

$$K = 4K_1 - 6K_2 + 4K_3 - K_4 = \sum_{s=1}^{4} a_s K_s, \tag{34}$$

where s stands for the number of the exchanged nucleons. The general expression of the K_s kernel is

$$K_s(\mathbf{R}, \mathbf{R}')$$

$$= \langle \delta(\mathbf{R} - \mathbf{R}_\alpha)\phi_\alpha \mid \prod_{i=1}^{s} P_i \mid \phi_\alpha \delta(\mathbf{R}' - \mathbf{R}'_\alpha) \rangle, \tag{35}$$

where P_i projects onto the orbitals inside the Fermi sea of the daughter nucleus and may be taken as the non-local one fermion density operator for this nucleus. For medium and heavy nuclei this density operator can be written in the Slater approximation [63]

$$\langle x \mid P \mid x' \rangle = \rho(x, x') \approx \tfrac{1}{4}\rho(\mathbf{R})\rho_{\mathrm{Sl}}(\mathbf{s}), \tag{36}$$

where

$$\rho_{\mathrm{Sl}}(\mathbf{s}) = \hat{j}_1(k_F s) = \frac{3}{k_F s} j_1(k_F s). \tag{37}$$

Here

$$\mathbf{R} = \frac{\mathbf{r} + \mathbf{r}'}{2} \quad \mathbf{s} = \mathbf{r} - \mathbf{r}' \tag{38}$$

and the K_s operator can be written in the factorized form

$$K_s(\mathbf{R}, \mathbf{R}') = K_s^+(\mathbf{R}_+) K_s^-(\mathbf{R}_-), \tag{39}$$

where

$$K_s^+(\mathbf{R}_+) = \frac{1}{8}\left(\frac{\beta}{\sqrt{\pi}}\right)^9 \frac{1}{\rho_0^s} e^{4\beta^4 R_+^2} \int \prod_{i=1}^{4} d\mathbf{R}_i e^{-\beta^2 \sum_{i=1}^{4} R_i^2} \prod_{i=1}^{s} \rho(R_i) \delta\left(\mathbf{R}_+ - \frac{1}{4}\sum_{i=1}^{4} \mathbf{R}_i\right), \tag{40}$$

$$K_s^-(\mathbf{R}_-) = \left(\frac{\rho_0}{4}\right)^s e^{\beta^2 R_-^2} \int \prod_{i=1}^{4} d\mathbf{s}_i e^{-\frac{1}{4}\beta^2 \sum_{i=1}^{4} s_i^2} \prod_{i=1}^{s} \hat{j}_1(k_F s_i) \prod_{i=s+1}^{4} \delta(\mathbf{s}_i) \delta\left(\mathbf{R}_- - \frac{1}{4}\sum_{i=1}^{4} \mathbf{s}_i\right). \tag{41}$$

Here ρ_0 is the equilibrium density of the nuclear matter. For nuclear matter, for example, $K_s^+(\mathbf{R}_+) = 1$. After some calculations we have

$$K_s^+(\mathbf{R}_+) = \frac{1}{8}\left(\frac{\beta}{\sqrt{\pi}}\right)^9 e^{4\beta^4 R_+^2} \times \int \prod_{i=1}^{s} d\mathbf{R}_i \tilde{\rho}_1(R_i) f_s \left(\mathbf{R}_+ - \frac{1}{4}\sum_{i=1}^{s} \mathbf{R}_i\right), \tag{42}$$

$$K_s^-(\mathbf{R}_-) = e^{\beta^2 R_-^2} \times \int \prod_{i=1}^{s} d\mathbf{s}_i \tilde{\rho}_2(s_i) \delta\left(\mathbf{R}_- - \frac{1}{4}\sum_{i=1}^{4} \mathbf{s}_i\right) \tag{43}$$

with

$$\tilde{\rho}_1(R) = \frac{1}{\rho_0} e^{-\beta^2 R^2} \rho(R), \tag{44}$$

$$\tilde{\rho}_2(s) = \frac{\rho_0}{4} e^{-\frac{1}{4}\beta^2 s^2} \hat{j}_1(k_F s), \tag{45}$$

$$f_s(r) = \frac{1}{(2\pi)^3}\left(\frac{\sqrt{\pi}}{\beta}\right)^{3(4-s)} \left(\frac{64\pi\beta^2}{4-s}\right)^{\frac{3}{2}} e^{-\frac{16\beta^2 r^2}{4-s}}. \tag{46}$$

We recognize that the above integrals are typical s-folding integrals (s = 1,2,3,4) with a specific finite range interaction f_s for K^+ and with zero range interaction for K^-. The interaction f_s has the "norm" (volume integral) $(\frac{\sqrt{\pi}}{\beta})^{3(4-s)}$, and the mean square radius $\frac{3(4-s)}{32\beta^2}$, which for the particular case $s = 4$ (exchange of 4 nucleons) becomes a δ distribution.

The spherical s-folding integral is defined as

$$V_s(R) = \int \prod_{i=1}^{s} d\mathbf{R}_i \rho_i(R_i) v\left(\mathbf{R} - \sum_{i=1}^{s} \mathbf{R}_i\right). \tag{47}$$

The reduction of the numbers of integrals to one unit is equivalent to perform a double folding integral with a zero range interaction:

$$V_s(R) = \int \prod_{i=1}^{s-1} d\mathbf{R}_i \rho_i(R_i) U_{\rho_s v}\left(\mathbf{R} - \sum_{i=1}^{s-1} \mathbf{R}_i\right), \tag{48}$$

$$U_{\rho v}(\mathbf{x}) = \int d\mathbf{r}_1 d\mathbf{r}_2 \rho(\mathbf{r}_1) v(\mathbf{r}_2) \delta(\mathbf{x} - \mathbf{r}_1 - \mathbf{r}_2). \tag{49}$$

It is straightforward to generalize this procedure for heavy cluster kernels.

This solution is very close in the case of heavy nuclei to that given by the procedure developed by Kukulin, Neudatchin, and Smirnov (see Ref. [48] especially Eq. (35) of this reference). Numerical calculations shows that for all the states investigated the corrected function $\tilde{F}_L$ has only one dominant maximum in the internal region, at a radius r_{kernel} [9] very close to the previous proposed channel radii [49, 50]. For small radii $\tilde{F}_L$ almost van-

ishes, while near the maximum and at larger distances it becomes identical with F_L.

Clearly, the variations determined by different choices of the channel radius or of the interaction are large when calculating the absolute widths.

The kernel correction to the position of the last maximum is small, showing that the radius range considered is far enough to neglect the antisymmetrization effects on the scattering states, and is appropriate for the choice of the channel surface.

Further agreement with the data might be expected, if a more precise definition of the channel radius would be available. In fact, we know that in the internal region the relative wave function should be $f_{\rm int}$ rather than F_L, and only near the barrier this become inaccurate and must be replaced by the asymptotic solution G_L. Thus it appears natural to fix the channel radius near the last maximum of $f_{\rm int}$ instead of F_L, when both the internal and the scattering wave functions should be accurate, and such that $B_c = B$. When this condition is fulfilled, it becomes possible to extend continuously iG_L in the internal region by $f_{\rm int}^{\rm scaled} = \nu f_{\rm int}$, with $\nu = \mid iG_L(r_c)/f_{\rm int}(r_c) \mid$.

We solved this problem, analogously to the procedure proposed in Ref. [9], by fixing the channel radius r_c at the last maximum $r_f^{\max}$ of $f_{\rm int}$ and changing by a factor the nuclear folding potential $V_n(r)$ to $(1+\varepsilon)V_n(r)$ in order to have at $r_f^{\max}$ the furthest maximum of the irregular scattering solution G inside the Coulomb barrier. Such matching was preferred in order to have a simpler formula for the width (i.e., $B_c = B = 0$). The radius $r_f^{\max}$ is a little larger than $r_{\rm kernel}$ [9].

In this work, however, we are interested in calculating the hindrance factors (HF), i.e., quantities which have a weaker dependence on the channel radius than the widths themselves.

Before treating the neutron-deficient Ra and Th isotopes we wish to analyze some examples of well-treated cases of the fine structure of cluster decay. These examples could be the cases of the favored and weak unfavored alpha decays of ^{241}Am and ^{243}Cm ground states. There are three α transitions from ^{241}Am(g.s.) and two α transitions from ^{243}Cm(g.s.) with $F \leq 10$ only. Three are the α transitions from ^{241}Am(g.s.) leading to the first two states of the rotational band built on the $[523]\frac{5}{2}^-$(0.06 MeV) single quasiparticle state and the α transition leading to the $[523]\frac{5}{2}^- Q_{20}$ (0.7219 MeV) β vibrational state of ^{237}Np daughter nucleus. The other two transitions are the α transitions from ^{243}Cm(g.s.) leading to the first two states of the rotational band built on the $[622]\frac{5}{2}^+$(0.286 MeV) single quasiparticle state of ^{239}Pu daughter nucleus. All the other measured hindrance factors ($\rm HF_{exp}$) are much greater. From these data we may learn that, first, the structure of the ^{241}Am ground state is the same as the structure of the $[523]\frac{5}{2}^-$(0.06 MeV) single quasiparticle state of ^{237}Np daughter nucleus and, secondly, that the structure of the ^{243}Cm ground state is the same as the structure of the $[622]\frac{5}{2}^+$(0.286 MeV) single quasiparticle state of ^{239}Pu daughter nucleus. Such transitions between odd-A nuclei are called favored, analogously to the ground state to ground state α transitions between doubly even-A nuclei.

In these cases, within ESM [35], the pairing superfluidity dominates (see also Ref. [6]). In addition, we are dealing with a more or less clean structure of the mother and daughter states, which is described by a superposition of single quasiparticle states and a strong coupled single quasiparticle states with multipole (especially quadrupole) phonon states (see Tables I–III). These structures are in agreement with those predicted by the quasiparticle-phonon model [32, 33].

By using ESM [35] we calculated the hindrance factors ($\rm HF_{ESM}$) for the favored and some unfavored α decays from ^{241}Am to ground and some excited states ^{237}Np nucleus(see Tables IV–VII) and decays from ^{243}Cm to ground and some excited states ^{239}Pu nucleus(see Tables VIII and IX).

In these calculations the used ESM parameters are G_p = 0.143 MeV, G_n = 0.103 MeV, G_4 = 0.268 keV. The parameters of the average field are taken from Refs. [31, 60]. The used deformation parameters are β_{20} = 0.20 and β_{40} = 0.06 for ^{237}Np; β_{20} = 0.23 and β_{40} = 0.08 for ^{239}Pu; β_{20} = 0.24 and β_{40} = 0.06 for ^{241}Am and β_{20} = 0.22 and β_{40} = 0.08 for ^{243}Cm. The used particle-hole quadrupole and octupole parameters (see Ref. [35]) are $\kappa_{n\tau}^{\lambda\mu} = \kappa_{0\tau}^{2\mu}$ = 0.667 keV fm^{-4}, $\kappa_{n\tau}^{\lambda\mu} = \kappa_{1\tau}^{2\mu}$ = 0.062 keV fm^{-4}, $\kappa_{n\tau}^{\lambda\mu} = \kappa_{0\tau}^{3\mu}$ = 0.011 keV fm^{-6}, $\kappa_{n\tau}^{\lambda\mu} = \kappa_{1\tau}^{3\mu}$ = 0.001 keV fm^{-6}. The used particle-particle quadrupole parameter (see Ref. [35]) are $G_{n\tau}^{\lambda\mu} = G_\tau^{2\mu}$ = 15 eV fm^{-4}. The rest of the terms in Eq. (6) of Ref. [37] not mentioned

TABLE I. The calculated, within ESM [35], structure of some ground and excited states entering the α decay: ^{241}Am($\frac{5}{2}^-$ [523])(g.s.) $\rightarrow$ ^{237}Np.

Nucleus	I^π K	$E_{\rm exp}$ (MeV)	$E_{\rm theo}$ (MeV)	Structure
^{241}Am	$\frac{5}{2}^-$ $\frac{5}{2}$	0	0	**98.9**% [523] $\frac{5}{2}^-$ + 1.1% [523] $\frac{5}{2}^+ \otimes Q_{20}$
^{237}Np	$\frac{5}{2}^+$ $\frac{5}{2}$	0	0	**80.9**% [642] $\frac{5}{2}^+$ + 3.1% [642] $\frac{5}{2}^- \otimes Q_{20}$
^{237}Np	$\frac{5}{2}^-$ $\frac{5}{2}$	0.06	0.07	**90.9**% [523] $\frac{5}{2}^-$ + 0.89% [512] $\frac{5}{2}^-$ + 3.04% [523] $\frac{5}{2}^- \otimes Q_{20}$ + 1.5% [642] $\frac{5}{2}^+ \otimes Q_{30}$
^{237}Np	$\frac{5}{2}^-$ $\frac{5}{2}$	0.721	0.758	0.91% [523] $\frac{5}{2}^-$ + 0.09% [512] $\frac{5}{2}^-$ + **96.04**% [523] $\frac{5}{2}^- \otimes Q_{20}$ + 0.05% [642] $\frac{5}{2}^+ \otimes Q_{30}$
^{237}Np	$\frac{1}{2}^-$ $\frac{5}{2}$	0.281	0.358	**81**% [530] $\frac{1}{2}^-$ + 4.09% [530] $\frac{1}{2}^- \otimes Q_{20}$ + 6.04% [523] $\frac{5}{2}^- \otimes Q_{22}$ + 0.05% [642] $\frac{5}{2}^+ \otimes Q_{32}$

TABLE II. The calculated, within ESM [35], structure of some ground and excited states entering the α decay: ^{243}Cm(g.s.) $\rightarrow$ ^{239}Pu.

Nucleus	I^π K	$E_{\rm exp}$ (MeV)	$E_{\rm theo}$ (MeV)	Structure
^{243}Cm	$\frac{5}{2}^+$ $\frac{5}{2}$	0	0	**97**% [622] $\frac{5}{2}^+$ + 1.1% [633] + 0.1% [602] + 2.3% [752] $\frac{5}{2}^-$ $\otimes$ Q_{30} + 1.3% [734] $\frac{9}{2}^-$ $\otimes$ Q_{32}
^{239}Pu	$\frac{1}{2}^+$ $\frac{1}{2}$	0	0	**95.9**% [631] $\frac{1}{2}^+$ + 3.1 % [631] $\frac{1}{2}^+$ $\otimes$ Q_{20}
^{239}Pu	$\frac{5}{2}^+$ $\frac{5}{2}$	286.	310.	**95** % [622] $\frac{5}{2}^+$ + 2.1% [633] + 0.3% [752] $\frac{5}{2}^-$ $\otimes$ Q_{30} + 2.3% [734] $\frac{9}{2}^-$ $\otimes$ Q_{32}

TABLE III. The calculated, within ESM [35], structure of β-vibrational state of ^{238}Pu.

		Neutrons	
s	s'	$\psi_{ss'}$	$\phi_{ss'}$
[622]$\frac{3}{2}$	[622]$\frac{3}{2}$	-0.240	+0.113
[501]$\frac{1}{2}$	[501]$\frac{1}{2}$	-0.175	+0.138
[631]$\frac{1}{2}$	[631]$\frac{1}{2}$	+0.101	+0.798
[633]$\frac{5}{2}$	[633]$\frac{5}{2}$	-0.141	+0.460
[620]$\frac{1}{2}$	[620]$\frac{1}{2}$	-0.270	+0.129
[624]$\frac{7}{2}$	[624]$\frac{7}{2}$	-0.703	+0.209
[613]$\frac{7}{2}$	[613]$\frac{7}{2}$	-0.316	+0.101
[615]$\frac{9}{2}$	[615]$\frac{9}{2}$	-0.274	+0.102
[606]$\frac{13}{2}$	[606]$\frac{13}{2}$	-0.247	+0.135
[743]$\frac{7}{2}$	[743]$\frac{7}{2}$	-0.194	+0.490
[734]$\frac{9}{2}$	[734]$\frac{9}{2}$	-0.374	+0.162
[725]$\frac{11}{2}$	[725]$\frac{11}{2}$	-0.267	+0.104
[640]$\frac{1}{2}$	[620]$\frac{1}{2}$	-0.128	-0.111
[642]$\frac{3}{2}$	[622]$\frac{3}{2}$	-0.136	-0.101
[624]$\frac{7}{2}$	[613]$\frac{7}{2}$	+0.114	+0.109
		Protons	
s	s'	$\psi_{ss'}$	$\phi_{ss'}$
[615]$\frac{11}{2}$	[615]$\frac{11}{2}$	-0.205	+0.118
[624]$\frac{9}{2}$	[624]$\frac{9}{2}$	-0.313	+0.168
[633]$\frac{7}{2}$	[633]$\frac{7}{2}$	-0.537	+0.431
[642]$\frac{5}{2}$	[642]$\frac{5}{2}$	-0.239	+0.062
[651]$\frac{3}{2}$	[651]$\frac{3}{2}$	-0.205	+0.457
[505]$\frac{11}{2}$	[505]$\frac{11}{2}$	-0.415	+0.168
[514]$\frac{7}{2}$	[514]$\frac{7}{2}$	-0.528	+0.239
[512]$\frac{5}{2}$	[512]$\frac{5}{2}$	-0.289	+0.157
[523]$\frac{5}{2}$	[523]$\frac{5}{2}$	-0.978	+0.999
[521]$\frac{3}{2}$	[521]$\frac{3}{2}$	-0.484	+0.392
[530]$\frac{1}{2}$	[530]$\frac{1}{2}$	-0.255	+0. 744
[402]$\frac{3}{2}$	[402]$\frac{3}{2}$	-0.355	+0. 233

above have been neglected.

The expressions of the reduced widths within the superfluid model [60, 35, 30] are given in Ref. [29].

In these calculations we use the Coulomb potential plus the realistic M3Y double folding potential [43], in which one uses an effective interaction derived from the G-matrix elements based on the Reid soft-core NN potential [44] in the form assuming only OPEP force between the states with odd relative angular momentum [45]. This potential is obtained numerically, and then is interpolated by cubic spline functions to improve the accuracy of the numerical integration. The radial scattering wave functions are calculated at the experimental resonance energies using the Numerov algorithm. At a distance of 15 fm the nuclear folding potential V_n has practically no contribution, and the regular solution is normalized to have the asymptotic behavior of the Coulomb functions [46]. The value of the irregular solution at this distance is obtained from the Wronskian relation $F'_L G_L - F_L G'_L = 1$ and then the whole irregular solution is obtained integrating backward to the origin. However, at small distances the fragments interact strongly, and this asymptotic solution should be gradually replaced by the "internal" wave function supposed to describe the compound system before decay.

When the internal ($f_{\rm int}$) and the external (iG) wave functions for the resonance energy are joined at the channel surface, we have to fulfill the condition

$$\left.\frac{f'_{\rm int}}{f_{\rm int}}\right|_{r_c} = \left.\frac{G'}{G}\right|_{r_c} . \tag{50}$$

When this condition is fulfilled, it becomes possible to

TABLE IV. The calculated, within ESM [35], hindrance factors for favored α transitions from ^{241}Am(g.s.) to the members of the rotational band of ^{237}Np([523]$\frac{5}{2}^-$, E_x = 59.4 keV). These results are compared with the calculated hindrance factors (HF) by Mang, Poggenburg, and Rasmussen [8] and experimental data [57, 58].

E_f (keV)	$I_f^{\pi_f}$	HF$_{\rm exp}$	HF$_{\rm MPR}$	HF$_{\rm ESM}$
59.4	$\frac{5}{2}^-$	1.3	1.06	1.15
102.96	$\frac{7}{2}^-$	4.7	4.69	4.75
158.52	$\frac{9}{2}^-$	20	13	15
226.0	$\frac{11}{2}^-$	750	447	480
304.8	$\frac{13}{2}^-$	1560	775	825
395.2	$\frac{15}{2}^-$	1460	1739	1795

TABLE V. The calculated, within ESM [35], hindrance factors for weak unfavored α transitions from ^{241}Am(g.s.) to the members of the rotational band of ^{237}Np($[523]\frac{5}{2}^-$, E_x = 721.9 keV). These results are compared with the calculated hindrance factors (HF) by Mang, Poggenburg, and Rasmussen [8] and experimental data [57, 58].

E_f (keV)	$I_f^{\pi f}$	HF_{exp}	HF_{MPR}	HF_{ESM}
721.9	$\frac{5}{2}^-$	10	1	9
755.8	$\frac{7}{2}^-$	47	4	40
799.8	$\frac{9}{2}^-$		13	125
	$\frac{11}{2}^-$		447	4280
	$\frac{13}{2}^-$		775	8205
	$\frac{15}{2}^-$		1739	17095

TABLE VI. The calculated, within ESM [35], hindrance factors for unfavored α transitions from ^{241}Am(g.s.) to the members of the rotational band of ^{237}Np(g.s.). These results are compared with the calculated hindrance factors (HF) by Mang, Poggenburg, and Rasmussen [8] and experimental data [57, 58].

E_f (keV)	$I_f^{\pi f}$	HF_{exp}	HF_{MPR}	HF_{ESM}
0.0	$\frac{5}{2}^+$	690	879	975
33.2	$\frac{7}{2}^+$	760	1259	1405
75.9	$\frac{9}{2}^+$	2160	2521	2425
130.0	$\frac{11}{2}^+$	4300	5986	6428
	$\frac{13}{2}^+$		402392	452703
	$\frac{15}{2}^+$		21539	23095

TABLE VII. The calculated, within ESM [35], hindrance factors for unfavored α transitions from ^{241}Am(g.s.) to the members of the rotational band of ^{237}Np($[530]\frac{1}{2}^-$, E_x = 281 keV). These results are compared with the calculated hindrance factors (HF) by Mang, Poggenburg, and Rasmussen [8] and experimental data [57, 58].

E_f (keV)	$I_f^{\pi f}$	HF_{exp}	HF_{MPR}	HF_{ESM}
281.35	$\frac{1}{2}^-$		125216	158355
267.54	$\frac{3}{2}^-$	12700	3338	4405
357	$\frac{5}{2}^-$	2870	2915	3252
327	$\frac{7}{2}^-$	2190	1277	1828
485	$\frac{9}{2}^-$	1980	4175	4527
438	$\frac{11}{2}^-$	1450	1374	1522

TABLE VIII. The calculated, within ESM [35], hindrance factors for favored α transitions from ^{243}Cm(g.s.) to the members of the rotational band of ^{239}Pu ($[622]\frac{5}{2}^+$, E_x = 286 keV). These results are compared with the calculated hindrance factors (HF) by Mang, Poggenburg, and Rasmussen [8] and experimental data [57, 58].

E_f (keV)	$I_f^{\pi f}$	HF_{exp}	HF_{MPR}	HF_{ESM}
285.460	$\frac{5}{2}^+$	1.29	1.75	1.95
330.125	$\frac{7}{2}^+$	5.1	4.98	5.22
387.42	$\frac{9}{2}^+$	16	12	15
462	$\frac{11}{2}^+$	340	72	98
	$\frac{13}{2}^+$		63	102
	$\frac{15}{2}^+$		75	113

TABLE IX. The calculated, within ESM [35], hindrance factors for unfavored α transitions from ^{243}Cm(g.s.) to the members of the rotational band of ^{239}Pu($[631]\frac{1}{2}^+$ g.s.). These results are compared with the calculated hindrance factors (HF) by Mang, Poggenburg, and Rasmussen [8] and experimental data [57, 58].

E_f (keV)	$I_f^{\pi f}$	HF_{exp}	HF_{MPR}	HF_{ESM}
0	$\frac{1}{2}^+$	1840	6750	4393
7.861	$\frac{3}{2}^+$	540	422	822
57.276	$\frac{5}{2}^+$	1430	551	913
75.706	$\frac{7}{2}^+$	177	120	315
163.76	$\frac{9}{2}^+$	4090	131	451
194	$\frac{11}{2}^+$	475	799	1135

extend continuously G_L in the internal region by $f_{int}^{scaled} = \nu f_{int}$, with $\nu = G_L(r_c)/f_{int}(r_c)$.

To solve this problem we adopted the method of fixing the channel radius r_c at the last maximum r_f^{max} of f_{int} ($B_c = 0$), and changing the nuclear folding potential $V_n(r)$ to $(1+\varepsilon)V_n(r)$ in order to obtain the pole for a given α transition as proposed in Ref. [47]. No poles belonging to the new class mentioned in Ref. [47] were found in our cases.

The hindrance factors for the favored and some weak unfavored α decays of ^{241}Am to the members of two rotational bands built on the intrinsic excited states of ^{237}Np, namely, $\frac{5}{2}^-$; 0.06 MeV and $\frac{5}{2}^-$; 722 MeV states are with

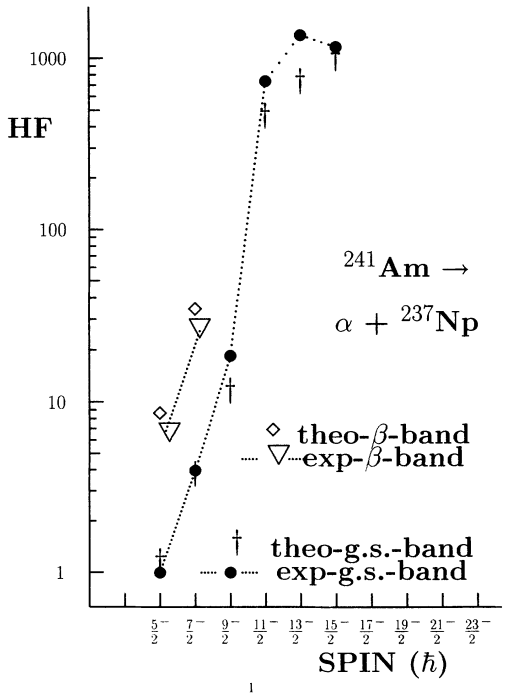

FIG. 2. The calculated within ESM hindrance factors for the favored α transition ^{241}Am ($\frac{5}{2}^-$ [523])(g.s.) → ^{237}Np($\frac{5}{2}^-$ [523]) (60 keV) and unfavored α transition from ^{241}Am($\frac{5}{2}^-$ [523])(g.s.) to the members of the rotational bands of ^{237}Np($\frac{5}{2}^-$ [523], 60 keV) single quasiparticle and ^{237}Np ($\frac{5}{2}^-$ [523] $\otimes$ Q_β 722 keV) β vibrational states.

more attention discussed and compared with the experimental data (see Fig. 2) [57, 58].

The explanation of small (close to unity) hindrance factors (HF), is based on the picture that the cluster (in this case an α particle) is built from the fermions just situated at the Fermi surface, where strong pairing correlations (or other collective, α and/or heavier cluster condensates [62, 35]) occur and, in addition, we may neglect the differences in structure of the parent and daughter states.

Let us analyze the ground state of the ^{241}Am nucleus. It is mainly a single quasiparticle $[523]\frac{5}{2}^-$ state. The corresponding Nilsson-like state does not participate in the α decay. Thus this state can be found in the structure of the daughter nucleus excited states. For instance, in the structure of the state lying at 60 keV excitation energy the contribution of the single quasiparticle $[523]\frac{5}{2}^-$ is almost 90% and in the structure of the state lying at 722 keV excitation energy the contribution of the $[523]\frac{5}{2}^- \otimes Q_{20}$ β vibrational state is almost 98%. Due to the fact that the α decay is a highly collective process we may neglect the differences in description of ^{241}Am($[523]\frac{5}{2}^-$) $\rightarrow \alpha +$ ^{237}Np($[523]\frac{5}{2}^-$ single quasiparticle state; $E_x = 60$ keV) and ^{241}Am($[523]\frac{5}{2}^-$) $\rightarrow \alpha +$ ^{237}Np($[523]\frac{5}{2}^- \otimes Q_{20}$ β vibrational state; $E_x = 722$ keV), because [60, 61] the β-vibrational state and the ground state are twin states. The situation is completely different when studying the transition [6] ^{241}Am ($[523]\frac{5}{2}^-$) $\rightarrow \alpha +$ ^{237}Np ($[642]\frac{5}{2}^+$) between the ground states. The initial and final single quasiparticle states are completely different in structure and in order to build an α cluster it is necessary to break up a Cooper pair. This process leads to the large (≥ 1000) hindrance factors (HF) (see Tables VI, VII, and IX).

The ^{223}Ra nucleus belongs [68] to the well-known region of nuclei with $Z \approx 88$ and $N \approx 134$, with strong octupole correlations in the ground and low-lying excited states, where the $1j_{\frac{15}{2}}$ intruder orbital interacts strongly with the $2g_{\frac{9}{2}}$ natural parity orbital. This situation is also determined by the fact that the energy distance between these orbitals coupled by the octupole field decreases with increasing the mass number. At the same time both the number and the magnitude of the matrix elements increase. This explains, for example, why the octupole coupling is stronger in the Ra-Th region than in other nuclear regions.

The hindrance factors for both the α and ^{14}C decays of the ground state of ^{223}Ra are very difficult to calculate at the moment, due to the not accurately known structure of the mother and daughter nuclei. Studying the experimental hindrance factor for α decays to ^{219}Rn ground and low-lying excited states [59] we learn that $\approx$10 transitions have small (≤ 100) hindrance factors (HF) and from these transitions five have $\mathrm{HF_{exp}} \leq 10$. The corresponding excited states have a very different structure and this tells us that the structure of the ground state of ^{223}Ra is not simple, as in the ^{241}Am case, and it may contain many more or less equal components of single quasiparticle or quasiparticle-phonon structure. An analogous situation one finds, when studying the structure of the ground and low-lying excited states of ^{219}Rn nucleus. This situation contradicts the somewhat hastened conclusions of the recent contributions [24, 26, 25, 27, 28], to the fine structure of ^{14}C decay. In the remarks of these authors one can understand that the hindrance factors are determined by the mother nucleus single-particle amplitudes and moreover, by the amplitude of the spherical orbital ($a^{\Omega}_{Nlj} = \langle Nlj \mid Nn_z\Lambda\Omega\rangle$) in the Nilsson-like orbital only. They do not analyze the effect of the residual interactions at least such like the pairing and/or multipole (especially octupole) correlations in the initial and final states. Of course, the hindrance factors increase when the products of the initial and final states amplitudes a^{Ω}_{Nlj} decrease [24], however, this may be a small component in a complex structure and in addition the rest of the factors occurring in every term of the spectroscopic factor may act constructively or destructively.

To understand this situation we construct a very simple model, which proves to deserve attention by itself and to suggest the highly nontrivial behavior of the realistic model.

Assume, for a moment, that the structure of the ground state of the ^{223}Ra nucleus consists of a spherical core and above the core there exists a deformed single particle orbital only. In this case the spectroscopic factor in the expression of the hindrance factor (HF)

$$\mathrm{HF}^{(I_iK_i\pi_i\rightarrow I_fK_f\pi_f)}_{l\ K_i-K_f} = \frac{\mid \sum_N \theta^{00^+(\mathrm{g.s.})\rightarrow 00^+(\mathrm{g.s.})}_{N0\ 0}\mathcal{R}_{N0}\mid^2}{\sum_l F_l \mid \sum_N \theta^{(I_iK_i\pi_i\rightarrow I_fK_f\pi_f)}_{Nl\ K_i-K_f}\mathcal{R}_{Nl}\mid^2} \quad (51)$$

may be factorized according to

$$\theta^{(K_iK_i\pi_i\rightarrow K_fK_f\pi_f)}_{Nl\ K_i-K_f} = C^{I_ilI_f}_{K_iKK_f} C_{K_i} C_{K_f} a^{\Omega_i=K_i}_{N_il_ij_i} a^{\Omega_f=K_f}_{N_fl_fj_f} \theta^{(j_i\pi_i\rightarrow j_f\pi_f)}_{\mathrm{spherical}}, \quad (52)$$

where C_{q_i}, C_{q_f} are the corresponding quasiparticle amplitude in the complex structures (see Eq. (5) from Ref. [29]) of the initial and final states, a^{Ω}_{Nlj} are the corresponding Nilsson-like amplitudes ($\chi_\Omega = \sum_{Nlj} a^{\Omega}_{Nlj}|Nlj\Omega\rangle$) and $\theta^{(j_i\pi_i\rightarrow j_f\pi_f)}_{\mathrm{spherical}}$ quantities, act as spectroscopic amplitudes between many-body spherical states. If considering a single $j^{(n)}$ shell, the main part of the $\theta^{(j_i\pi_i\rightarrow j_f\pi_f)}_{\mathrm{spherical}}$ quantities are the coefficients of fractional parentages $\langle j^{(n-a)}J^{\pi_i}_i; j^{(a)}0^+ \mid\} j^{(n)}, J^{\pi_i}_i\rangle$ entering the expansion of a totally antisymmetrized wave function and of given quantum numbers (spin, parity,...) describing the motion on n particles in terms of products of two antisymmetrized wave functions that describe $n-a$ and a, respectively, particles. These spectroscopic factors may be calculated within the restricted Kuo-Herling [42, 41] model space including four neutron orbitals ($N1i_{11/2}$, $2g_{9/2}$,$3d_{5/2}$,$1j_{15/2}$) and four proton orbitals ($P1h_{9/2}$, $2f_{7/2}$, $2f_{5/2}$, $3p_{3/2}$) above the shell closure at $Z = 82$ $N = 126$. The main quality of this calculation is to account for all possible configurations. Within this model space the structure of parent and daughter

nuclei consists in an inert ^{208}Pb core, and some active nucleons (15 for ^{223}Ra and one for ^{209}Pb). By using a diagonal interaction only we may produce the necessary wave functions of a given spin and parity ($\mid j_{i(f)}^{\pi_{i(f)}}\rangle$). The realistic interaction and a larger model space [41] necessary for realistic estimations of the spectroscopic factors are impossible to be used even when using the most modern computer due to the unrealistic necessary computer time and space. To calculate, for example, within this simple model the hindrance factor for the favored ^{14}C transition, i.e., ^{223}Ra(g.s.) $\rightarrow$ ^{14}C + ^{209}Pb ($\frac{11}{2}^+$, E_x = 779 keV) one needs to compute $\theta_{\rm spherical}^{[(\frac{11}{2}^+)^{223}{\rm Ra}\rightarrow(\frac{11}{2}^+)^{209}{\rm Pb}]}$.

This simple model determines essentially one dominant term in the spectroscopic amplitude. The real spectroscopic amplitude is a sum of many terms analogous to the above one and in addition the spherical spectroscopic amplitude $\theta_{\rm spherical}^{(j_i\pi_i\rightarrow j_f\pi_f)}$ should be replaced by a spectroscopic amplitude corresponding to a deformed core, which in reality is not as simple as for the spherical case. When having many terms in the sum cancellation effects may occur also.

Now assuming only one of these factors to be responsible for the entire hindrance factors, especially in the case of a large number of transferred nucleons, e.g., the case of ^{14}C decay, we think, it is a crude approximation.

For instance, when calculating the ratio (see Refs. [24, 26, 25, 27, 28]):

$$R\left(\frac{9}{11}\right)=\frac{{\rm HF}^{(\frac{3}{2}^+\rightarrow\frac{9}{2}^+,{\rm g.s.})}}{{\rm HF}^{(\frac{3}{2}^+\rightarrow\frac{11}{2}^+,779\ {\rm keV})}} \tag{53}$$

experimentally equal to 200 [1], one can say that, the spherical spectroscopic amplitudes $\theta_{Nl\,K}^{[(\frac{11}{2})_{^{223}{\rm Ra}}\rightarrow(\frac{11}{2})_{^{209}{\rm Pb}}]}$ and $\theta_{Nl\,K}^{[(\frac{9}{2})_{^{223}{\rm Ra}}\rightarrow(\frac{9}{2})_{^{209}{\rm Pb}}]}$ have been considered level independent quantities and moreover, they were considered equal constant quantities. In addition the initial and final levels have been considered single quasiparticle levels: $C_{1i_{\frac{11}{2}}\rightarrow\frac{3}{2}}^{^{223}{\rm Ra}}=1$, $C_{2g_{\frac{9}{2}}\rightarrow\frac{3}{2}}^{^{223}{\rm Ra}}=1$, $C_{1i_{\frac{11}{2}}\rightarrow\frac{3}{2}}^{^{209}{\rm Pb}}=1$, and $C_{2g_{\frac{9}{2}}\rightarrow\frac{3}{2}}^{^{209}{\rm Pb}}=1$. The final nuclear states have been assumed spherical single particle states $a_{1i_{\frac{11}{2}}}^{\frac{3}{2}}(^{209}{\rm Pb})=1$ and $a_{1g_{\frac{9}{2}}}^{\frac{3}{2}}(^{209}{\rm Pb})=1$. The only quantities used for estimating the above ratio [$R(\frac{9}{11})$] were the Nilsson-like coefficients $a_{1i_{\frac{11}{2}}}^{\frac{3}{2}}(^{223}{\rm Ra})$ and $a_{1g_{\frac{9}{2}}}^{\frac{3}{2}}(^{223}{\rm Ra})$. This prescriptions according to our formula for the hindrance factors (HF) may give in the calculations of Ref. [24] $R(\frac{9}{11})\leq 100$, while in the calculations of Ref. [28] $R(\frac{9}{11})\simeq 1$.

According to the predictions of Sheline and Ragnarsson [24] the ground state of the ^{223}Ra is mainly a single quasiparticle state determined by the Nilsson-like orbital $\Omega^\pi=\frac{3}{2}^+$ with a dominant contribution ($\simeq$ 52%) emerging from the positive parity single neutron orbital $1i_{11/2}$ and small admixtures from other positive parity neighboring $2g_{7/2}$ ($\simeq$ 4%), $2g_{9/2}$ ($\simeq$ 1.5%), orbitals and an intruder negative parity $1j_{15/2}$ ($\simeq$ 1.5%) orbital.

Another structure has been obtained in Refs. [69, 25], namely, the contributions of different spherical orbitals in the the ^{223}Ra ground-state Nilsson-like orbital are $1i_{11/2}$ ($\simeq$ 18%), $2g_{7/2}$ ($\simeq$ 4%), $3d_{5/2}$ ($\simeq$ 16%), $2g_{9/2}$ ($\simeq$ 42%), and $1j_{15/2}$ ($\simeq$ 1.5%).

It is not clear, however, in these calculations, what is the weight of a single quasiparticle and what are the weights of the more complex structure when the quasiparticle is coupled strongly with the octupole phonon, for instance. Moreover, it is not clear whether other phonon contributions may occur. In the next proposed for study case we shall see that these weights are essential in determining the hindrance factors.

We estimated the ratio $R(\frac{9}{11})$ by using a single quasiparticle dominance in the structure of the initial and final states and the Nilsson-like coefficients from Ref. [24]. We calculated, however, the spectroscopic amplitudes $\theta_{\rm spherical}^{(j_i\pi_i\rightarrow j_f\pi_f)}$ as mentioned in Sec. III, by using the shell model with no residual interactions for the initial and final states and considering the intrinsic overlap integrals level independent quantities. The only different quantities in the expression of $\theta_{\rm spherical}^{(j_i\pi_i\rightarrow j_f\pi_f)}$ were the cluster overlaps. Our estimation gives a large value for the above ratio [$R(\frac{9}{11})\simeq 1000$], showing how important are all quantities entering the hindrance factors for cluster decay. In these calculations we used the OXBASH code [42] and the REWIL [79] interaction within the ZBM [80] valence model space for the ground state of ^{14}C and then we eliminate the spurious states due to the center-of-mass motion. The $\psi_{\rm int}^{a}$ wave function has been expressed in terms of the phenomenological wave functions of three alpha clusters and two valence neutrons, which are functions of the relative Jacobi coordinates (see Fig. 1) [56].

The most discussed ratio [24, 26, 25, 27, 28],

$$R\left(\frac{15}{11}\right)=\frac{{\rm HF}^{(\frac{3}{2}^+\rightarrow\frac{15}{2}^+,1423\ {\rm keV})}}{{\rm HF}^{(\frac{3}{2}^+\rightarrow\frac{11}{2}^+,779\ {\rm keV})}} \tag{54}$$

within the prescriptions above mentioned is approximately 50, i.e., not very hindered.

A few more comments may be in order here. First of all our hybrid model with a spherical core and only one deformed orbital, when calculating the spectroscopic amplitudes is not to be taken too seriously for very complex structures. This should not be true even for structures close to single quasiparticle states, because the assumption of a spherical core is not realistic. On the other hand, when having realistic structures for both the initial and final states, calculations within shell models like OXBASH are practically impossible for nowadays computers. Therefore simple schemes as presented above would be useful.

There may be another explanation of the measured low HF [1] for the transition to the $\frac{15}{2}^-$, E_x = 1423 keV level in ^{209}Pb, namely, through a possible large parity mixing, let say some percentage, present in the ^{223}Ra parity mixed doublet due to the strong octupole correlations. The $\frac{3}{2}^+$ ground state of ^{223}Ra together with the first $\frac{3}{2}^-$ excited state determines a 50 KeV parity mixed doublet [71, 72, 19, 74]. The first $\frac{3}{2}^-$ excited state of ^{223}Ra is mainly a single quasiparticle state built on the Nilsson-like single particle deformed orbital $\frac{3}{2}^-$ emerging from the neutron $1j_{15/2}$ spherical orbital.

The necessary large parity admixture, in spite of predicted [66] strong octupole correlations, is not easy to be obtained. There are, however, evidences in this nuclear region [75–77] of large parity admixtures. Thus, the problem of parity mixing in ^{223}Ra could be an interesting problem not from the weak interaction point of view only. We may study the coupling between the positive and negative parity bands in order to find new insights concerning the structure of the nuclear states in the Ra-Th region.

Unfortunately it is not possible to evaluate the hindrance factors (HF) for the heavy cluster decays from the ground states of the nuclei in the Rn-Pa region before constructing an appropriate model for the states of these nuclei.

A better case for predictions could be the decay: ^{243}Cm (g.s.) $\rightarrow$ ^{34}Si + ^{209}Pb (ground and some excited states). From Table II we learn that within the ESM [35] the structure of the ^{243}Cm ground state contains contributions from three single quasiparticle states, namely, 97% [622], 1.1% [633], and 0.1% [602] emerging from $2g_{\frac{9}{2}}$, $1i_{\frac{11}{2}}$, and $3d_{\frac{5}{2}}$, respectively. According to Eq. (51) we can define the following ratios between the hindrance factors (HF):

$$R\left(\frac{11}{9}\right)=\frac{\mathrm{HF}^{(\frac{5}{2}^+\rightarrow\frac{11}{2}^+,779\ \mathrm{keV})}}{\mathrm{HF}^{(\frac{5}{2}^+\rightarrow\frac{9}{2}^+,\mathrm{g.s.})}},$$
$$R\left(\frac{5}{9}\right)=\frac{\mathrm{HF}^{(\frac{5}{2}^+\rightarrow\frac{5}{2}^+,1567\ \mathrm{keV})}}{\mathrm{HF}^{(\frac{5}{2}^+\rightarrow\frac{9}{2}^+,\mathrm{g.s.})}}, \tag{55}$$

which are equal to

$$R\left(\frac{11}{9}\right)\approx\frac{\sum_l F_l\mid C^{\frac{5}{2}l\frac{9}{2}}_{\frac{5}{2}2\frac{9}{2}}C_{[622]}a^{\frac{5}{2}[622]}_{2g_{\frac{9}{2}}}M(\frac{5}{2}[622]\rightarrow 2g_{\frac{9}{2}})\mid^2}{\sum_l F_l\mid C^{\frac{5}{2}l\frac{11}{2}}_{\frac{5}{2}3\frac{11}{2}}C_{[633]}a^{\frac{5}{2}[633]}_{1i_{\frac{11}{2}}}M(\frac{5}{2}[633]\rightarrow 1i_{\frac{11}{2}})\mid^2}\approx 114 \tag{56}$$

and

$$R\left(\frac{5}{9}\right)\approx\frac{\sum_l F_l\mid C^{\frac{5}{2}l\frac{9}{2}}_{\frac{5}{2}2\frac{9}{2}}C_{[622]}a^{\frac{5}{2}[622]}_{2g_{\frac{9}{2}}}M(\frac{5}{2}[622]\rightarrow 2g_{\frac{9}{2}})\mid^2}{\sum_l F_l\mid C^{\frac{5}{2}l\frac{5}{2}}_{\frac{5}{2}0\frac{5}{2}}C_{[602]}a^{\frac{5}{2}[602]}_{3d_{\frac{5}{2}}}M(\frac{5}{2}[602]\rightarrow 3d_{\frac{5}{2}})\mid^2}\approx 4150. \tag{57}$$

The quantities M replaces essentially the cluster overlap for the specific transition. For instance, the product $a^{\frac{5}{2}[602]}_{3d_{\frac{5}{2}}}M(\frac{5}{2}[602]\rightarrow 3d_{\frac{5}{2}})$ stands for the quantity

$$U_qV_q\sum_{\delta\delta'}\delta'\sum_{\nu_1\cdots\nu_{14}}\sum_{\omega_3\cdots\omega_{20}}A^{LM}_{\delta\delta'}(\nu_1\cdots\nu_{14}\mid qq\omega_3\cdots\omega_{20})\xi_{\delta\delta'}(\nu_1\cdots\nu_{14}\mid qq\omega_3\cdots\omega_{20}) \tag{58}$$

analogous to the quasiparticle contribution in the matrix element from Eq. (12) of Ref. [29] entering the α-decay rate of axially deformed odd-A nuclei.

The necessary used Nilsson-like coefficients are $a^{\frac{5}{2}[622]}_{2g_{\frac{9}{2}}}=0.62$, $a^{\frac{5}{2}[633]}_{1i_{\frac{11}{2}}}=0.67$, and $a^{\frac{5}{2}[602]}_{3d_{\frac{5}{2}}}=0.31$ very close to the values reported in Ref. [31]. By comparing the lifetimes for ^{34}Si decays from ^{243}Cm and ^{242}Cm: $\mathrm{T}_{\frac{1}{2}}$ [^{243}Cm (g.s) $\rightarrow$ ^{34}Si + ^{209}Pb (g.s.)] = $10^{27.0}$ sec [81] and $\mathrm{T}_{\frac{1}{2}}$ [^{242}Cm (g.s.) $\rightarrow$ ^{34}Si + ^{208}Pb (g.s.)] = $10^{23.24}$ sec [11] we can find the F for the first transition, namely, F[^{243}Cm (g.s.) $\rightarrow$ ^{34}Si + ^{209}Pb (g.s.)] is equal to the product of the ratio of lifetimes times the ratio of the penetrabilities corresponding to the ^{243}Cm and ^{242}Cm cluster decays, respectively. A direct calculation leads to HF $\approx$ 7. Thus the hindrance factors corresponding to the cluster transition to the 779 keV and 1567 keV levels in ^{209}Pb will be $\approx$ 800 and 30 000, respectively (see Table X). The transitions to the $\frac{15}{2}^-$, 1423 keV and $\frac{1}{2}^+$, 2032 keV levels in ^{209}Pb are estimated within the octupole or higher order contributions to the structure of ^{243}Cm ground state.

TABLE X. The calculated, within ESM [35], hindrance factors for favored and unfavored ^{34}Si transitions from ^{243}Cm(g.s.) to the excited states of ^{209}Pb.

E_f (keV)	I^{π}_f	$\mathrm{HF}_{\mathrm{theo}}$
0	$\frac{9}{2}^+$	≈ 7
779	$\frac{11}{2}^+$	≈ 800
1423	$\frac{15}{2}^-$	$\approx 10^7$
1567	$\frac{5}{2}^+$	$\approx 3\times 10^4$
2032	$\frac{1}{2}^+$	$\approx 10^5$

V. CONCLUSIONS

As examples of the cluster decay fine structure we analyzed the particular cases of α decays from ^{241}Am and ^{243}Cm, ^{14}C decay from ^{223}Ra, and ^{34}Si decay from ^{243}Cm. Good agreement with the experimental data is obtained in the case of α decay of ^{241}Am and ^{243}Cm.

Assuming for the structure of the ground state of the ^{223}Ra nucleus a hybrid model, with a spherical core and above the core only one deformed single particle orbital, we could factorize the spectroscopic amplitude for the ^{14}C decay into three factors; first one is the single quasiparticle weight into the structure of the ground state of the ^{223}Ra nucleus, the second one is the Nilsson-like amplitude of a spherical orbital into the deformed Nilsson-like orbital, and the last one is the spectroscopic am-

plitude of the ^{14}C decay from a *spherical configuration.* This last factor can be calculated by using an analogous recipe as given in Ref. [9] for the case of α decay. It may have large variations due to selection rules and internal structure of the core, when calculating its cluster overlap factor.

Our estimates of the hindrance factor differ in magnitude from previous estimations [24, 26, 25, 27, 28]. We overestimate the experimental hindrance factor corresponding to the ground state of ^{209}Pb, but we are closer to the experimental hindrance factor corresponding to the $\frac{15}{2}^-$ (1423 keV) state, when normalizing to the hindrance factor corresponding to the $\frac{11}{2}^+$ (779 keV) state.

If the hindrance factor corresponding to the $\frac{15}{2}^-$, 1423 keV state in ^{209}Pb could be explained by the large parity admixture of the $\frac{3}{2}^-$ first excited state in the ground state of ^{223}Ra then the problem of parity nonconservation in the ^{223}Ra parity mixed doublet becomes an interesting problem.

Additional experimental work on the ^{14}C fine structure decay of ^{223}Ra with higher resolution would be very valuable. This might allow the resolution of (1) the groups populating the $\frac{15}{2}^-$ and $\frac{5}{2}^+$ states in ^{209}Pb and (2) the groups leaving from ground $\frac{3}{2}^+$ and excited $\frac{3}{2}^-$ (50 keV) states of ^{223}Ra nucleus, in order to determine more conclusively the hindrance factor for populating the $\frac{15}{2}^-$ state in ^{209}Pb.

Predictions have been done for the hindrance factors corresponding to the ^{243}Cm (g.s.) $\rightarrow$ ^{34}Si + ^{209}Pb(g.s., 779 keV and 1567 keV excited states).

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