Influence of nuclear structure on the characteristics of light pionic atoms

A. Cieplý and R. Mach
Nuclear Physics Institute, 250 68 Řež, Czech Republic
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An extended version of the standard pion-nucleus optical potential is presented using the momentum-space formulation of the multiple scattering theory. This potential is used for studying the influence of nonlocal effects and nuclear structure on the characteristics of light pionic atoms. A method is suggested to treat the long-range two-nucleon correlations involved in the second-order optical potential. Particularly, the spin-orbital Fermi motion correction to the first-order optical potential, the center-of-mass correlations, and the spin-isospin corrections to the second-order optical potential are discussed in some detail. The results are presented in comparison with the experimental data for the $1s$ and $2p$ levels of light pionic atoms up to ^{44}Ca.

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I. INTRODUCTION

During the last decade special attention has been paid to solve a bulk of problems that arose in the description of π-atomic characteristics. The theoretical calculations based on the standard pion-nucleus optical potential developed by Ericson and Ericson [1] were found not to reproduce the newly measured data obtained by using very precise experimental techniques [2,3]. A lot of work has been done to refine the Ericson-Ericson potential and its parameter set without significant improvement of the situation [4–8]. The principal importance of finite-range effects associated with pion nonlocal interaction in nuclear interior was shown recently [9], but the authors have not attempted to reproduce the experimental data fully. There was also an interesting phenomenological extension of the optical potential [10]; however, the physical content of the model appears to be dubious [11]. Thus the problems persist and many "anomalous phenomena" have not been solved up to now.

At the same time the alternative theoretical approaches were presented trying to bring the calculated results into better agreement with experiment. They are based mostly on the fundamental ideas taken from physical models which were successfully tested in related fields and deploy the relativistic description of both the nuclear structure and pion-nucleus dynamics in a more realistic fashion. Here we mention only two of them, the mean-field approach developed by Birbrair *et al.* [12] and the extended version of the Δ-hole model adopted for pionic atoms by Oset and co-workers [13,14]. The last work [14] goes even beyond the standard Δ-hole model and treats the Δ and nucleon pole on the same footing.

Unfortunately, all the aforementioned models presume a many-body character of the nucleus and their use is to some extent doubtful in the case of very light nuclei. Further, the models do not involve more subtle effects caused by nuclear structure of individual nuclei and the dependence on the nuclear mass number A enters the optical potential in a very simple way. To be more specific, we note that the pion-nucleus optical potential has been expressed only as a function of the nuclear density and its powers. On the other hand, the new very precise experimental data for pionic atoms [15–17] and also for π scattering on polarized nuclei [18] can hardly be reproduced by means of such a simplified theoretical input. In our opinion, other nuclear ingredients are needed to enter the potential as well.

In this paper we present a detailed analysis of the characteristics of light pionic atoms ($A \leq 44$). Our aim is just to emphasize the significance of nuclear structure effects in the pion-nucleus interaction. We show that a more careful construction of the optical potential results in a better description of the experimental data. Of course, we do not intend to solve all the problems accummulated so far and especially the well known anomaly associated with the small absorption widths of the $3d$ levels [19] is not touched in the present paper.

In the following section we start with the optical potential in momentum space formulated in the framework of Galileo-invariant multiple scattering theory. Performing the angle transformation from πN to π-nucleus center-of-mass system (CMS) frame the first-order optical potential receives also a spin-dependent contribution due to nucleonic Fermi motion. The corresponding term is omitted in the majority of existing calculations. It is repulsive, reflects the nuclear spin-orbital interaction, and can play an important role particularly for nuclei with half-filled valence shells.

Further, the first-order optical potential is extended by a phenomenological term standing for pion annihilation and reemission. We assume that the pion absorption takes place on noninteracting nucleon pairs immersed in a nuclear environment. This is an analog of the impulse approximation, which is usually used in constructing the first-order optical potential. Kinematical and dynamical consequences of such an assumption are carefully studied. We end up with an expression which receives contribution from two-nucleon long-range correlation function and reduces to the conventional ρ^2 term only in the limiting case.

Having constructed the optical potential we study the

role of the long-range recoil correlations and of the spin-orbital ($\boldsymbol{\sigma}\cdot\boldsymbol{l}$) term in Sec. III. The results are presented for the $1s$ and $2p$ levels of light pionic atoms in comparison with the available experimental data. We also discuss in detail the role of the spin-isospin corrections to the second-order optical potential and their influence on the characteristics of light pionic atoms. Some conclusions and outlooks are summarized in the last section.

II. THE OPTICAL MODEL

A. First-order optical potential

To construct the pion-nucleus optical potential we use the standard multiple scattering procedure formulated by Watson many years ago [20]. Here we follow Ref. [21] and analyze in detail the first-order optical potential describing the pion-nucleus strong interaction. Presuming the validity of closure and impulse approximations it can be obtained in the form

$$\langle \mathbf{Q}' \,|\, U^{(I)}(E) \,|\, \mathbf{Q}\rangle = \frac{A}{(2\pi)^3}\int d^3v\, d^3\xi'\, d^3\xi\; e^{i\frac{A-1}{2A}\mathbf{q}\cdot(\boldsymbol{\xi}'+\boldsymbol{\xi})}\; e^{i\mathbf{v}\cdot(\boldsymbol{\xi}'-\boldsymbol{\xi})} \times Sp[\rho(\boldsymbol{\xi}';\boldsymbol{\xi})\langle \mathbf{q}_f \,|\, t(e) \,|\, \mathbf{q}_i\rangle]\,, \tag{1}$$

where the transferred momentum

$$\mathbf{q} = \mathbf{Q}' - \mathbf{Q} = \mathbf{q}_f - \mathbf{q}_i\,,$$

$\rho(\boldsymbol{\xi}';\boldsymbol{\xi})$ is the nuclear density matrix normalized to 1, and the symbolic notation

$$Sp\,[\rho(\boldsymbol{\xi}';\boldsymbol{\xi})\,\langle \mathbf{q}_f \,|\, t(e) \,|\, \mathbf{q}_i\rangle] = \sum_{\sigma_z',\tau_z',\sigma_z,\tau_z} \psi^*_{\sigma_z',\tau_z'}(\boldsymbol{\xi}')\langle \mathbf{q}_f\,\sigma_z'\,\tau_z' \,|\, t(e) \,|\, \mathbf{q}_i\,\sigma_z\,\tau_z\rangle \times \psi_{\sigma_z,\tau_z}(\boldsymbol{\xi}) \tag{2}$$

is used for the summation over the target nucleon spin and isospin projections. In the pion-nucleus CMS frame (A-c.m.) the final (initial) pion momentum and reaction energy are denoted by $\mathbf{Q}'$ ($\mathbf{Q}$) and E, respectively. The transformation of kinematic variables from A-c.m. to pion-nucleon CMS frame (2-c.m.) is performed by means of the following formulas [22]:

$$\mathbf{q}_f = \mathbf{Q}' - \frac{A-1}{2A}\frac{\mu}{M}(\mathbf{Q}'+\mathbf{Q}) + \frac{\mu}{M}\mathbf{v} \equiv \mathbf{q}_0' + \frac{\mu}{M}\mathbf{v}\,, \tag{3a}$$

$$\mathbf{q}_i = \mathbf{Q} - \frac{A-1}{2A}\frac{\mu}{M}(\mathbf{Q}'+\mathbf{Q}) + \frac{\mu}{M}\mathbf{v} \equiv \mathbf{q}_0 + \frac{\mu}{M}\mathbf{v}\,, \tag{3b}$$

$$e = E - \frac{1}{2M}\frac{\mu}{\mathcal{M}}\frac{A}{A-1}\left[\mathbf{v} - \frac{A-1}{2A}(\mathbf{Q}'+\mathbf{Q})\right]^2 - \epsilon_B\,. \tag{3c}$$

Here the reduced masses are defined as

$$\mu = \frac{m\,M}{m+M}\quad,\quad \mathcal{M} = \frac{m\,M_A}{m+M_A}$$

with μ, M, and M_A standing for the mass of pion, nucleon, and nucleus, respectively. The correction ϵ_B represents the nucleon-core binding energy and momentum $\mathbf{v}$ is the average of relative nucleon-core momentum in the initial and final states. The integration over d^3v in Eq. (1) reflects the Fermi motion of nucleons within the nucleus. In practical calculations we use approximations that lead to a more static picture of the π-N subsystem propagation. First, we set

$$-\frac{1}{2M}\frac{\mu}{\mathcal{M}}\frac{A}{A-1}v^2 - \epsilon_B = 0 \tag{4}$$

in Eq. (3c). Then the potential matrix $U^{(I)}$ is evaluated for the effective nucleon momenta [23]

$$\mathbf{p} = -\frac{\mathbf{Q}}{A} + \frac{A-1}{2A}(\mathbf{Q}'-\mathbf{Q})\,,$$
$$\mathbf{p}' = -\frac{\mathbf{Q}'}{A} - \frac{A-1}{2A}(\mathbf{Q}'-\mathbf{Q}) \tag{5}$$

in the initial and final states, respectively. Thus only the terms up to order of $\frac{m}{M} \sim \frac{1}{7}$ retain in our calculations. Using this approximation one avoids, without any serious loss of accuracy, the numerical averaging over the nucleonic Fermi motion normally needed in Eq. (1).

For the pion-nucleon scattering matrix we take the parametrization

$$\langle \mathbf{q}_f \,|\, t(e) \,|\, \mathbf{q}_i\rangle = -\frac{2\pi}{\mu}\left\{f_0(\theta) + i\,\boldsymbol{\sigma}\cdot[\mathbf{q}_i\times\mathbf{q}_f]\,f_1(\theta)\right\}\,,$$
$$f_S(\theta) = a_{S0} + a_{S1}\mathbf{t}\cdot\boldsymbol{\tau}\,, \tag{6}$$

where $\mathbf{t}$ is the pion isospin operator, $\boldsymbol{\sigma}$ and $\boldsymbol{\tau}$ stand for spin and isospin Pauli matrices of target nucleon, respectively, and $a_{ST} \equiv a_{ST}(\cos\theta)$ ($S = 0,1$; $T = 0,1$) are usual combinations of the pion-nucleon partial amplitudes $f^j_{LI}(q_f, q_i; e)$ taken from Ref. [24]. Here $\cos\theta = \hat{\mathbf{q}}_f\cdot\hat{\mathbf{q}}_i$, L is the πN orbital momentum, $I = \frac{1}{2}$ or $\frac{3}{2}$ is the isospin of the πN system, and the total angular momentum $j = L \pm \frac{1}{2}$. The separable potential model is used to define the off-shell extrapolation of the amplitudes $f^j_{LI}(q_f, q_i; e)$. It has already been mentioned [25] that our off-shell prescription of the πN amplitudes f^j_{LI} can be put into close correspondence with the finite-range model by Kalbermann *et al.* [9].

Splitting the πN t matrix into three parts

$$\langle \mathbf{q}_f \,|\, t(e) \,|\, \mathbf{q}_i\rangle = -\frac{2\pi}{\mu}\left\{f_0(\theta) + i\,\boldsymbol{\sigma}\cdot[\mathbf{q}_0\times\mathbf{q}_0']\,f_1(\theta) + \frac{\mu}{M}i\,\boldsymbol{\sigma}\cdot[\mathbf{v}\times\mathbf{q}]\,f_1(\theta)\right\}\,, \tag{7}$$

the first-order optical potential can be decomposed into corresponding terms

$$U^{(I)}(E) = -\frac{2\pi}{\mathcal{M}}\left[V_N^{(1)}(E) + V_N^{(J)}(E) + V_N^{(\sigma l)}(E)\right] . \quad (8)$$

Using the formula

$$i\,\mathbf{v}\,e^{i\mathbf{v}\cdot(\boldsymbol{\xi}'-\boldsymbol{\xi})} = \frac{1}{2}(\nabla_{\xi'} - \nabla_{\xi})\,e^{i\mathbf{v}\cdot(\boldsymbol{\xi}'-\boldsymbol{\xi})}$$

in the expression for the last component, the Wigner-Eckart theorem for $V_N^{(J)}$, we obtain in the limit $|\mathbf{v}| \longrightarrow 0$

$$\langle \mathbf{Q}' | V_N^{(1)}(E) | \mathbf{Q} \rangle = \frac{\mathcal{M}}{\mu} A \left[a_{00} + \frac{N-Z}{A} a_{01}\right] F(\mathbf{q}) , \quad (9a)$$

$$\langle \mathbf{Q}' | V_N^{(J)}(E) | \mathbf{Q} \rangle = \sum_T 2i\,\mathbf{J}\cdot[\mathbf{Q}\times\mathbf{Q}']\,\tilde{a}_{1T}\,(2\mathbf{t}\cdot\mathcal{T})^T \frac{(0\|\{j_0(q'\xi)\,\mathbf{s} - \sqrt{2\pi}\,j_2(q'\xi)\left[Y_2(\hat{\boldsymbol{\xi}})\otimes\mathbf{s}\right]_1\}\,[\tfrac{1}{2}\boldsymbol{\tau}]^T\|0)}{(0\|\,\mathbf{J}\,[\mathcal{T}]^T\,\|0)} , \quad (9b)$$

$$\langle \mathbf{Q}' | V_N^{(\sigma l)}(E) | \mathbf{Q} \rangle = \frac{A-1}{A}\frac{\mathcal{M}}{M} q^2 A \langle \Psi_0 | \frac{j_1(q'\xi)}{q'\xi} f_1(\theta)\,(\boldsymbol{\sigma}\cdot\boldsymbol{l}) \,|\, \Psi_0 \rangle \quad (9c)$$

after some simple manipulations (see [21] as well). Here we denoted

$$\tilde{a}_{1T} = \frac{\mathcal{M}}{\mu}\frac{|\mathbf{q}_0\times\mathbf{q}_0'|}{|\mathbf{Q}\times\mathbf{Q}'|}\,a_{1T}$$

and the nuclear form factor

$$F(\mathbf{q}) = \int d^3\xi\, e^{i\frac{A-1}{A}\mathbf{q}\cdot\boldsymbol{\xi}}\,\rho(\boldsymbol{\xi};\boldsymbol{\xi}) = \int d^3r\, e^{i\mathbf{q}\cdot\mathbf{r}}\rho(\mathbf{r}) . \quad (10)$$

We hope there is no confusion due to the same symbols used here for the nuclear density matrix as a function of nucleon-core relative distances $\boldsymbol{\xi}', \boldsymbol{\xi}$ and its diagonal matrix elements $\rho(\mathbf{r})$, in which $\mathbf{r}$ corresponds to the position of nucleon with respect to the nuclear center of mass. In Eqs. (9) $\mathbf{J}$ ($\mathbf{s}$) and $\mathcal{T}$ ($\frac{1}{2}\boldsymbol{\tau}$) are the nuclear (nucleon) spin and isospin operators, respectively. Further, $j_l(q'\xi)$ are the spherical Bessel functions with $\mathbf{q}' = \frac{A-1}{A}\mathbf{q}$ and

$$\boldsymbol{l} \equiv \boldsymbol{l}_\xi = -i\,[\boldsymbol{\xi}\times\nabla_\xi] ,$$

$$\left[Y_2(\hat{\boldsymbol{\xi}})\otimes\mathbf{s}\right]_{1\gamma} = \sum_{\alpha\beta}\begin{bmatrix} 2 & 1 & 1 \\ \alpha & \beta & \gamma \end{bmatrix} Y_{2\alpha}(\hat{\boldsymbol{\xi}})\,\mathbf{s}_\beta .$$

Before concluding this section we briefly summarize the main features of the optical potential $U^{(I)}(E)$. It is constructed fully microscopically and no free parameter enters Eqs. (9). The term $V_N^{(1)}(E)$ is expressed in the factorized form as a coherent sum of free πN amplitudes and is proportional to the Fourier transform of nuclear density, which we assume to have the same form for both protons and neutrons. At variance with the classical first-order optical potential introduced in the coordinate space by Ericson and Ericson [1] the potential $V_N^{(1)}(E)$ contains the finite-range effects of the πN interaction and the kinematics of the transformation from 2-c.m. to A-c.m. is treated in a natural, very precise way. On the other hand, note that the Lorentz-Lorenz renormalization is not considered in our approach. Nevertheless, the pionic atom data are not very sensitive to the nonlinearity of the optical potential and one can always absorb the Lorentz-Lorenz effect into the phenomenological second-order term discussed below. Another point worth mentioning is the mixing of amplitudes with different L in our potential when they are transformed from 2-c.m. to A-c.m. and decomposed into π-nucleus partial waves (see Ref. [26] for more details). However, the real effect due to the nonequivalence $\hat{\mathbf{Q}}'\cdot\hat{\mathbf{Q}} \neq \hat{\mathbf{q}}_f\cdot\hat{\mathbf{q}}_i$ is not too significant in the calculations, as well.

The $V_N^{(J)}(E)$ and $V_N^{(\sigma l)}(E)$ terms depend strongly on the shell structure of the nucleus exhibiting the characteristic features of individual nuclei in this way. The second term of our potential (8) is proportional to the nuclear spin and represents a negligible correction of the order of $1/A$ to the previous one. It also does not contribute to the s-wave characteristics of pionic atoms with $J = 0, \frac{1}{2}$. For these reasons we have not included the term $V_N^{(J)}(E)$ in the present calculations keeping only $V_N^{(1)}(E)$ and $V_N^{(\sigma l)}(E)$ in our first-order optical potential.

The last contribution to the potential $U^{(I)}$ is the Fermi motion spin-orbital term $V_N^{(\sigma l)}(E)$. It has its origin in the spin-flip part of the elementary πN amplitude and arises due to its transformation from 2-c.m. to A-c.m. also reflecting the nucleonic Fermi motion. One can easily see that the $(\boldsymbol{\sigma}\cdot\boldsymbol{l})$ term contributes to the potential $U^{(I)}$ even in the case of spin zero nuclei. Provided that admixtures of higher configurations are neglected, the matrix elements of $V_N^{(\sigma l)}(E)$ are evaluated in the nuclear ground state and are equal to zero for magic nuclei (^{4}He, ^{16}O, ^{40}Ca, ...) reaching its maximal magnitude for nuclei with half-filled valence shells. Of course, $V_N^{(\sigma l)}(E)$ is by a factor of $\frac{\mu}{M}\frac{\nu_v l_v}{A}$ smaller than the leading term in Eq. (8), where ν_v is the number of valence nucleons (holes) and l_v stands for their orbital momentum. Nevertheless, the

contributions from a given nuclear subshell add constructively to the small isoscalar part of $V_N^{(1)}(E)$, so the $(\boldsymbol{\sigma}\cdot\boldsymbol{l})$ correction can grow to be rather important for subshells with higher l_v and deformed nuclei in the middle of a valence shell. Unfortunately, the actual contribution of the $V_N^{(\sigma l)}(E)$ term to the π-atomic characteristics is difficult to estimate as their A dependence is not simple.

For a purpose of comparison with the common coordinate space formulation we also present the Fourier transform of Eq. (9c). Omitting the correction factor $\frac{A-1}{A}$ connected with the CMS motion it can be rewritten in the form

$$V_N^{(\sigma l)}(r) = \frac{\mathcal{M}}{M}\frac{1}{r^2}\frac{d}{dr}\left[r\sum_v \rho_v(r)[(n_v+p_v)a_{10} + (n_v-p_v)a_{11}]\langle\boldsymbol{\sigma}\cdot\boldsymbol{l}\rangle_v\right], \qquad (11)$$

where the sum extends over all valence subshells with radial density distributions $\rho_v(r)$ normalized to 1. The occupation numbers of protons (neutrons) are denoted by p_v (n_v) and $\langle\boldsymbol{\sigma}\cdot\boldsymbol{l}\rangle_v = l_v$ or $-(l_v+1)$ for angular momentum $j_v = l_v \pm \frac{1}{2}$. Almost the same expression for the term in question was obtained recently by Birbrair and Gridnev [27] in the framework of relativistic mean-field approach but it differs from that published earlier by Friedman and Gal [4]. The last one is probably in error.

In order to avoid a possible confusion, we recall the reader that Birbrair and Gridnev obtained another spin term [$\rho_k(r)$ in their notation], which is of "relativistic" origin [27]. The term $\rho_k(r)$ does not vanish for spin saturated nuclei and depends critically on the off-mass-shell extrapolation of the πN amplitude [28]. Further, the term $\rho_k(r)$ was claimed to be responsible for the anomalous pion-nucleus repulsion in the s wave. However, the formalism used in Ref. [28] for obtaining $\rho_k(r)$ was seriously questioned by Koch and Oset recently [29]. Particularly, the authors of Ref. [29] have proven that within the relativistic impulse approximation and for spin saturated systems there is no contribution from the spin-flip part of the elementary πN amplitude. In our opinion, this result seems to be irrefutable. Of course, $\rho_k(r)$ does not have any counterpart in our optical potential.

It is readily seen from Eq. (11) that the potential $V_N^{(\sigma l)}(r)$ is an oscillating function of the relative pion-nucleus distance and its volume integral vanishes. Since the pion-nucleus wave function in the relative s state is almost constant inside the nucleus, the contribution of the Fermi motion term to the $1s$-level energy shifts is expected to be smaller than that to the $2p$-level shifts. Further, the coefficients a_{ST} are real, thus our correction $V_N^{(\sigma l)}(E)$ will affect mainly energy shifts leaving the level widths almost intact. Finally, the angular behavior of the $(\boldsymbol{\sigma}\cdot\boldsymbol{l})$ term is controlled by $q^2 = Q'^2 + Q^2 - 2QQ'P_1(\cos\theta)$. If the positive sign of the matrix element of the spin-orbital operator (see Sec. II C below), $a_{10} > 0$, and the coefficient at $P_1(\cos\theta)$ are combined, we can conclude that $V_N^{(\sigma l)}(E)$ yields a repulsive correction to the energy of the π-atomic $2p$ level. The same holds also for the $1s$ level, because the momenta squared are negative in the case of pion-nucleus bound state.

Unfortunately, the proper investigation of effects caused by the $(\boldsymbol{\sigma}\cdot\boldsymbol{l})$ term was missed in the literature for a long time supposing that its contribution to the leading part $V_N^{(1)}(E)$ of the pion-nucleus optical potential is negligible. Including the $V_N^{(\sigma l)}(E)$ term in our recent calculations [30] we have demonstrated the vital role of this correction in accounting for the very precise data measured for the $2p$ levels of pionic atoms in the region of "deformed" s-d shell nuclei. Here we would like to show an improved evaluation of the $(\boldsymbol{\sigma}\cdot\boldsymbol{l})$ term making use of the realistic shell-model wave functions and put our results into the context of a more complete analysis.

B. Second-order optical potential and nuclear correlations

Processes associated with pion annihilation are beyond the scope of the nonrelativistic multiple scattering theories and are usually taken into account on a phenomenological level. The leading mechanism involves at least two nucleons ($\pi NN \longrightarrow NN \longrightarrow \pi NN$) and its contribution to the pion-nucleus optical potential is usually postulated as the famous ρ^2 term [31]

$$\langle\mathbf{Q}'|V_N^{(2)}(E)|\mathbf{Q}\rangle = A(A-1)\left[dB_0 + \frac{C_0}{d}\mathbf{Q}'\cdot\mathbf{Q}\right] \times \frac{v_0(Q')\,v_0(Q)}{[v_0(Q_0)]^2}\,G(\mathbf{q}), \qquad (12)$$

where B_0 and C_0 are phenomenological parameters, which characterize pion interaction with nucleon pair in s and p wave, respectively. Further, $G(\mathbf{q})$ is the Fourier transform of the nuclear density squared,

$$G(\mathbf{q}) = \int d^3r\, e^{i\mathbf{q}\cdot\mathbf{r}}\,\rho^2(\mathbf{r}) \qquad (13)$$

and $d = 1 + m/2M$ is a kinematical transformation factor commonly used in the coordinate-space calculations. In this section we show that the ρ^2 term (12) can easily be generalized to include the long-range nuclear correlations. Such a modification turns out to be essential in the case of the lightest nuclei.

Let us start with the second-order optical potential assumed in the form

$$\langle\mathbf{Q}'|U^{(II)}(E)|\mathbf{Q}\rangle = A(A-1)\,\langle 0|\,t^s_{\pi NN}(E)\,|0\rangle, \qquad (14)$$

where

$$t^s_{\pi NN}(E) = t_{\pi NN}(E) - t_{\pi N}(E)\,|0\rangle\langle 0|\,G_A(E)\,|0\rangle\langle 0|\,t_{\pi N}(E) \qquad (15)$$

is the short-range part of the full πNN scattering matrix $t_{\pi NN}(E)$ and $G_A(E)$ stands for the Green's function of the π-nucleus system. The second-order contribution of the first-order optical potential was subtracted in (15) to avoid the double counting. Due to the momentum

mismatch $\Delta \approx 360$ MeV/c in the pion annihilation $\pi NN \longrightarrow NN$, the scattering matrix $t^s_{\pi NN}(E)$ is expected to vanish for internucleon separations larger than $r \approx 0.3$ fm.

Introducing further the Jacobi coordinates $\mathbf{r}$, $\mathbf{R}$, $\boldsymbol{\xi}_3, \ldots, \boldsymbol{\xi}_{A-1}$, where $\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$ and

$$\mathbf{R} = (\mathbf{r}_3 + \mathbf{r}_4 + \cdots + \mathbf{r}_A)/(A-2) - (\mathbf{r}_1 + \mathbf{r}_2)/2 ,$$

we have

$$\langle \mathbf{Q}' \,|\, U^{(II)}(E) \,|\, \mathbf{Q} \rangle = \frac{A(A-1)}{(2\pi)^9} \int d^3v\, d^3r'\, d^3r\, d^3R'\, d^3R\, d^3\eta'\, d^3\eta\, e^{i\frac{A-2}{2A}\mathbf{q}\cdot(\mathbf{R}'+\mathbf{R})} \times e^{i\mathbf{v}\cdot(\mathbf{R}'-\mathbf{R})}\, e^{-i\boldsymbol{\eta}'\cdot\mathbf{r}'} \rho_2(\mathbf{R}', \mathbf{r}'; \mathbf{r}, \mathbf{R}) \langle \mathbf{q}_2' \boldsymbol{\eta}' \,|\, t^s_{\pi NN}(z) \,|\, \mathbf{q}_2 \boldsymbol{\eta} \rangle\, e^{i\boldsymbol{\eta}\cdot\mathbf{r}} , \tag{16}$$

where

$$\rho_2(\mathbf{R}', \mathbf{r}'; \mathbf{r}, \mathbf{R}) = \int \prod_{j=3}^{A-1} d^3\xi_j \, \langle 0 \,|\, \boldsymbol{\xi}_{A-1}, \ldots, \boldsymbol{\xi}_3, \mathbf{R}', \mathbf{r}' \rangle \times \langle \mathbf{r}, \mathbf{R}, \boldsymbol{\xi}_3, \ldots, \boldsymbol{\xi}_{A-1} \,|\, 0 \rangle$$

and

$$\mathbf{q}_2' = \mathbf{Q}' - \frac{A-2}{2A}\frac{m}{2dM}(\mathbf{Q}' + \mathbf{Q}) + \frac{m}{2dM}\mathbf{v} \equiv \mathbf{q}_{2,0}' + \frac{m}{2dM}\mathbf{v} , \tag{17a}$$

$$\mathbf{q}_2 = \mathbf{Q} - \frac{A-2}{2A}\frac{m}{2dM}(\mathbf{Q}' + \mathbf{Q}) + \frac{m}{2dM}\mathbf{v} \equiv \mathbf{q}_{2,0} + \frac{m}{2dM}\mathbf{v} , \tag{17b}$$

$$z = E - \frac{1}{4M}\frac{m}{d\mathcal{M}}\frac{A}{A-2}\left[\mathbf{v} - \frac{A-2}{2A}(\mathbf{Q}' + \mathbf{Q})\right]^2 . \tag{17c}$$

In what follows, the motion of the two-nucleon subsystem is treated in the effective momentum approximation, i.e., we set $\mathbf{v}=0$ in Eqs. (17) in analogy with the derivation of the first-order optical potential. Now the second-order optical potential reads as

$$\langle \mathbf{Q}' \,|\, U^{(II)}(E) \,|\, \mathbf{Q} \rangle = \frac{A(A-1)}{(2\pi)^6} \int d^3r'\, d^3r\, d^3R\, d^3\eta'\, d^3\eta\, e^{i\frac{A-2}{A}\mathbf{q}\cdot\mathbf{R}} \times e^{-i\boldsymbol{\eta}'\cdot\mathbf{r}'} \rho_2(\mathbf{R}, \mathbf{r}'; \mathbf{r}, \mathbf{R}) \langle \mathbf{q}_{2,0}' \boldsymbol{\eta}' \,|\, t^s_{\pi NN}(z_0) \,|\, \mathbf{q}_{2,0} \boldsymbol{\eta} \rangle\, e^{i\boldsymbol{\eta}\cdot\mathbf{r}} . \tag{18}$$

Further, let us write the two-nucleon density in the form

$$\rho_2(\mathbf{R}, \mathbf{r}'; \mathbf{r}, \mathbf{R}) = \rho_2^s(\mathbf{r}', \mathbf{r})\, \rho_2^l(\mathbf{R}, \mathbf{r}'; \mathbf{r}, \mathbf{R}) , \tag{19}$$

where $\rho_2^s(\mathbf{r}',\mathbf{r})$ reflects the NN correlations associated with the short-range NN repulsion and $\rho_2^l(\mathbf{R},\mathbf{r}';\mathbf{r},\mathbf{R})$ stands for the long-range part of the two-nucleon density as provided, e.g., by the shell model. In designing the ansatz of Eq. (19) we were inspired, of course, by the Jastrow model. We also assume that within the region r $(r') \leq 0.3$ fm the shell-model density is a slowly warying function of $\mathbf{r}$ and $\mathbf{r}'$. Therefore, we take $\rho_2^l(\mathbf{R}, \mathbf{r}'; \mathbf{r}, \mathbf{R}) \simeq \rho_2^l(\mathbf{R}, 0; 0, \mathbf{R})$ in evaluating Eq. (18). Then the resulting expression is

$$\langle \mathbf{Q}' \,|\, V_N^{(2)}(E) \,|\, \mathbf{Q} \rangle = A(A-1)\left[\overline{B}_0 + \overline{C}_0\, \mathbf{q}_{2,0}' \cdot \mathbf{q}_{2,0}\right] \frac{v_0(q_{2,0}')\, v_0(\mathbf{q}_{2,0})}{[v_0(Q_0)]^2}\, G_0(\mathbf{q}) , \tag{20}$$

where

$$G_0(\mathbf{q}) = \int d^3R\, e^{i\frac{A-2}{A}\mathbf{q}\cdot\mathbf{R}}\, \rho_2^l(\mathbf{R}, 0; 0, \mathbf{R}) \tag{21}$$

and we have used the parametrization

$$-\frac{2\pi}{\mathcal{M}}\left(\overline{B}_0 + \overline{C}_0\, \mathbf{q}_{2,0}' \cdot \mathbf{q}_{2,0}\right) \frac{v_0(q_{2,0}')\, v_0(q_{2,0})}{[v_0(Q_0)]^2} = \frac{1}{(2\pi)^6} \int d^3r'\, d^3r\, d^3\eta'\, d^3\eta\, e^{-i\boldsymbol{\eta}'\cdot\mathbf{r}'} \rho_2^s(\mathbf{r}', \mathbf{r}) \langle \mathbf{q}_{2,0}' \boldsymbol{\eta}' \,|\, t^s_{\pi NN}(z_0) \,|\, \mathbf{q}_{2,0} \boldsymbol{\eta} \rangle\, e^{i\boldsymbol{\eta}\cdot\mathbf{r}} \tag{22}$$

for a phenomenological construct, which combines the short-range part of the πNN amplitude and the short-range part of the two-nucleon density. The angular transformation was taken into account using the appropriate formulas (17) in the scalar product $\mathbf{q}'_{2,0}\cdot\mathbf{q}_{2,0}$. The parameters $\overline{B}_0$, $\overline{C}_0$ are related to those, introduced in Eq. (12) by the relations $\overline{B}_0 \approx dB_0$, $\overline{C}_0 \approx dC_0$. [Here the sign $\approx$ means equality for $G_0(\mathbf{q}) \longrightarrow G(\mathbf{q})$ and $A \longrightarrow \infty$.] In comparison with the ρ^2 term in Eq. (12), our model also receives a contribution from the nuclear recoil and Pauli principle correlations via the function $G_0(\mathbf{q})$.

1. CMS correlations

Let us further investigate the role of nuclear recoil correlations in more detail. At first, it is instructive to rewrite Eq. (21) as

$$G_0(\mathbf{q}) = \frac{1}{(2\pi)^3}\int d^3p_1\, d^3p_2\, \delta(\mathbf{p}_1+\mathbf{p}_2-\mathbf{q})\, D(\mathbf{p}_1,\mathbf{p}_2)\,, \tag{23}$$

where $D(\mathbf{p}_1,\mathbf{p}_2)$ is the diagonal in nucleon momenta $\mathbf{p}_1$ and $\mathbf{p}_2$ two-nucleon density

$$\begin{aligned} D(\mathbf{p}_1,\mathbf{p}_2) = \int \prod_{j=3}^{A} d^3p_j\, \delta\Big(\sum_{j=1}^{A}\mathbf{p}_j\Big) \\ \times\langle 0_{\text{sh.m.}} \mid \mathbf{p}_1,\mathbf{p}_2,\ldots,\mathbf{p}_A\rangle \\ \times\langle \mathbf{p}_A,\ldots,\mathbf{p}_2,\mathbf{p}_1 \mid 0_{\text{sh.m.}}\rangle \end{aligned} \tag{24}$$

and $\mid 0_{\text{sh.m.}}\rangle$ stands for the nuclear ground-state wave function in the shell model. Following the prescription of Feshbach *et al.* [32], the two-nucleon density matrix can be rewritten as

$$\begin{aligned} D(\mathbf{p}_1,\mathbf{p}_2) = \frac{1}{F_{\text{c.m.}}(\mathbf{p}_1+\mathbf{p}_2)} \\ \times\Big[C^{(M)}(\mathbf{p}_1,\mathbf{p}_2) + F^{(M)}(\mathbf{p}_1)F^{(M)}(\mathbf{p}_2)\Big]\,, \end{aligned} \tag{25}$$

where the correlation function $C^{(M)}(\mathbf{p}_1,\mathbf{p}_2)$ and form factor $F^{(M)}(\mathbf{p}) = F(\mathbf{p})F_{\text{c.m.}}(\mathbf{p})$ are model quantities calculated using an independent particle model and $F(\mathbf{p})$ is the nuclear form factor of Eq. (10) as extracted from the electron scattering measurements. The center-of-mass motion is assumed to be in the lowest harmonic oscillator state, so we will use the form

$$F_{\text{c.m.}}(\mathbf{q}) = e^{-\frac{a^2q^2}{4A}}$$

to evaluate the CMS corrections present in Eq.(25). Here a denotes the harmonic oscillator constant.

The long-range correlations involve the center-of-mass correlations and those due to the Pauli exclusion principle. The range of these correlations is of the order of nuclear radius ($\approx 1.2A^{1/3}$ fm) and the reverse Fermi momenta (≈ 0.7 fm), respectively. In the region of pion-nucleus interaction at very low energies the CMS correlations prevail the other ones and we expect the role of Pauli principle correlations to be suppressed in the case of pionic atoms. If one retains only the CMS correlations, for which the function $C^{(M)}(\mathbf{p}_1,\mathbf{p}_2)$ vanishes, the two-nucleon density matrix of Eq. (25) is made up merely of the nuclear form factors. Then one can obtain a simple formula

$$G_0(\mathbf{q}) = \frac{1}{(2\pi)^3}\int d^3p\, F(\mathbf{p})\, F(\mathbf{q}-\mathbf{p})\, e^{\frac{a^2}{2A}(\mathbf{p}\cdot\mathbf{q}-p^2)}\,. \tag{26}$$

Constraining the center-of-mass motion to one point in space, $F_{\text{c.m.}}(\mathbf{q}) = 1$, our model converges to the common ρ^2 term

$$G_0(\mathbf{q}) \longrightarrow \frac{1}{(2\pi)^3}\int d^3p\, F(\mathbf{p})\, F(\mathbf{q}-\mathbf{p}) = G(\mathbf{q})\,.$$

This is just what one expects in the limit $A \longrightarrow \infty$. It is also instructive to express Eq. (26) in terms of nuclear densities as

$$\begin{aligned} G_0(\mathbf{q}) = \left(\frac{A}{2\pi a^2}\right)^{\frac{3}{2}} e^{\frac{a^2q^2}{8A}} \int d^3r_1\, d^3r_2\, e^{i\mathbf{q}\cdot\frac{\mathbf{r}_1+\mathbf{r}_2}{2}} \\ \times e^{-\frac{A}{2a^2}(\mathbf{r}_1-\mathbf{r}_2)^2}\rho(\mathbf{r}_1)\,\rho(\mathbf{r}_2) \end{aligned} \tag{27}$$

from which the nonlocality due to the nuclear recoil effect is easily seen.

2. Spin-isospin corrections

The standard second-order optical potential (12) is constructed omitting the spin and isospin structure of the elementary pion-two nucleon amplitude. Correspondingly, the parametrization adopted in Eq. (22) reflects only the spin-isospin independent part of the full πNN amplitude. In fact, this can be written in the more general form [50]

$$\begin{aligned} f_{\pi NN} = \; & \{B_0 + B_1(\boldsymbol{\sigma}^i\cdot\boldsymbol{\sigma}^j) + B_2(\boldsymbol{\tau}^i\cdot\boldsymbol{\tau}^j) + \left[B_{30} + B_{31}(\boldsymbol{\sigma}^i\cdot\boldsymbol{\sigma}^j)\right]\left[(\mathbf{t}\cdot\boldsymbol{\tau}^i) + (\mathbf{t}\cdot\boldsymbol{\tau}^j)\right] \\ & + B_4(\boldsymbol{\sigma}^i\cdot\boldsymbol{\sigma}^j)(\boldsymbol{\tau}^i\cdot\boldsymbol{\tau}^j) + \left[B_{50} + B_{51}(\boldsymbol{\sigma}^i\cdot\boldsymbol{\sigma}^j)\right]\left[(\mathbf{t}\cdot\boldsymbol{\tau}^i)(\mathbf{t}\cdot\boldsymbol{\tau}^j) + (\mathbf{t}\cdot\boldsymbol{\tau}^j)(\mathbf{t}\cdot\boldsymbol{\tau}^i)\right] \\ & + [p\text{-wave part}]\}\delta(\mathbf{r}^i-\mathbf{r}^j)\,\delta\left(\mathbf{r}-\tfrac{\mathbf{r}^i+\mathbf{r}^j}{2}\right), \end{aligned} \tag{28}$$

where the superscripts i and j distinguish the two interacting nucleons. Imaginary part of the parameters B_n can either be extracted from the available πNN data [1] or calculated making use of some simple model predictions [50]. Of course, the parametrization of the πNN amplitude used here is not quite general, because spin-vector and spin-tensor terms cannot be completely ruled out. The analysis of such terms could be a subject of further work.

To get the second-order optical potential one has to average the πNN amplitude over the nuclear ground-state wave functions, e.g., via Eq. (22). In the early work of Ericson and Ericson [1] the authors replaced the delta function $\delta(\mathbf{r}^i - \mathbf{r}^j)$ in Eq. (28) by the unity operator and averaged the remaining spin-isospin part over the nuclear ground state. Within their formulation the s-wave parameter B_0 is to be replaced by an effective equivalent

$$B_0^{\text{eff}} = B_0 + \Delta B_0(A) \tag{29}$$

in the second-order optical potential. The lengthy expression for the A-dependent correction can be found in Appendix B of Ref. [1]. It vanishes for heavier nuclei but cannot be neglected in the region of the lightest nuclei with $A \leq 10$. The resulting formulas were further exploited by Tauscher and Schneider [42], who studied the influence of the corresponding $1/A$ corrections to the pion-nucleus optical potential. Another drawback of the formalism used in Refs. [1,42] is the independent treatment of spin and isospin variables in evaluating the nuclear matrix elements of $f_{\pi NN}$. Therefore, the antisymmetry of the nuclear wave function is not properly taken into account in their formalism.

The recent approach by Germond and Lombard is much more convincing [50]. They studied the imaginary part of the s-wave pion-nucleus optical potential and elaborated the two-particle spin-isospin operators preserving the delta function in Eq. (28). Nevertheless, the analysis was restricted on situations, where the nucleus ground-state wave function is time-reversal invariant. In their approach the "corrected" parameter B_0^{eff} turns out to be A independent.

Following the line drawn by Germond and Lombard we evaluated the matrix elements $\mathcal{G}_n$ of the spin-isospin operators, which are multiplied by corresponding B_n's in Eq. (28), e.g.,

$$\mathcal{G}_0 = \sum_{i\neq j} \langle \Psi_0 | \delta(\mathbf{r}^i - \mathbf{r}^j) \delta\left(\mathbf{r} - \frac{\mathbf{r}^i + \mathbf{r}^j}{2}\right) | \Psi_0 \rangle = A(A-1) G_0(\mathbf{q}),$$

$$\mathcal{G}_1 = \sum_{i\neq j} \langle \Psi_0 | (\boldsymbol{\sigma}^i \cdot \boldsymbol{\sigma}^j)\, \delta(\mathbf{r}^i - \mathbf{r}^j)\, \delta\left(\mathbf{r} - \frac{\mathbf{r}^i + \mathbf{r}^j}{2}\right) | \Psi_0 \rangle \text{ , etc.}$$

In our approach, the antisymmetry of the nuclear wave function was fully taken into account, $\mathcal{A} \mid \Psi_0\rangle = - \mid \Psi_0\rangle$, as well as the fact that $\delta(\mathbf{r}^i - \mathbf{r}^j)$ projects out of the nuclear wave function its part symmetric with respect to the transposition of the coordinates $\mathbf{r}^i$ and $\mathbf{r}^j$. Using the identities

$$\begin{aligned} \sum_{i\neq j} \langle \Psi_0 |\, f_{\pi NN}\, | \Psi_0 \rangle &= \frac{1}{2} \sum_{i\neq j} \langle \Psi_0 |\, f_{\pi NN}(1 - \mathcal{A})\, | \Psi_0 \rangle \\ &= \frac{1}{2} \sum_{i\neq j} \langle \Psi_0 |\, f_{\pi NN} \left(1 - \frac{1}{4}(1 + \boldsymbol{\sigma}^i \cdot \boldsymbol{\sigma}^j)(1 + \boldsymbol{\tau}^i \cdot \boldsymbol{\tau}^j)\right) | \Psi_0 \rangle \end{aligned} \tag{30}$$

we arrive at

$$\mathcal{G}_1 + \mathcal{G}_2 = -2\mathcal{G}_0 \,, \tag{31a}$$

$$\mathcal{G}_{31} = -3\mathcal{G}_{30} \,, \tag{31b}$$

$$\mathcal{G}_4 = -3\mathcal{G}_0 \,. \tag{31c}$$

In addition, neglecting the isotensor term inherent in the combination $(\mathbf{t} \cdot \boldsymbol{\tau}^i)(\mathbf{t} \cdot \boldsymbol{\tau}^j) + (\mathbf{t} \cdot \boldsymbol{\tau}^j)(\mathbf{t} \cdot \boldsymbol{\tau}^i)$, we have

$$\mathcal{G}_{50} = \frac{4}{3}\mathcal{G}_2 \,, \tag{31d}$$

$$\mathcal{G}_{51} = \frac{4}{3}\mathcal{G}_4 = -4\mathcal{G}_0 \,. \tag{31e}$$

Assuming further the same symmetry structure of the nuclear wave function in both the spin and isospin spaces (i.e., $\mathcal{G}_2 = \mathcal{G}_1$) one can combine the expressions (31) to get

$$\sum_{i\neq j} \langle \Psi_0 |\, f_{\pi NN}\, | \Psi_0 \rangle = B_0^{\text{eff}}\, \mathcal{G}_0 + (B_{30} - 3B_{31})\, \mathcal{G}_{30} \,, \tag{32}$$

where only the s-wave part has been kept in consideration. The resulting formula for the effective isoscalar parameter B_0^{eff},

$$B_0^{\text{eff}} = B_0 - B_1 - B_2 - 3B_4 - \frac{4}{3}B_{50} - 4B_{51} \tag{33}$$

is the same as that obtained recently by Germond and Lombard. However, the present derivation is not restricted only to the 0^+ nuclei and its validity appears to be much more general. The ^{3}He and ^{3}H nuclei represent the important examples to that our formalism can be fully applied. In fact, even if the spin and isospin structure of a nucleus were different, one could still make some conclusions from the relations (31), the validity of which is quite general.

C. Nuclear structure input

Here we give some details of our calculation procedure. At first, in constructing the optical potential $V_N(E)$ we

TABLE I. The nuclear charge radii adopted to fix the parameters b of harmonic oscillator density distributions.

Nucleus	^{3}He	^{4}He	^{6}Li	^{7}Li	^{9}Be	^{10}B	^{11}B	^{12}C	^{13}C	^{14}N	^{16}O	^{18}O
R_{ch} [fm]	1.88	1.673	2.49	2.39	2.519	2.45	2.42	2.45	2.44	2.54	2.71	2.784

introduced the relativistic kinematics in the standard way [25]. Further, the strong-interaction optical potential $\langle \mathbf{Q}' \mid V_N(E) \mid \mathbf{Q} \rangle$ was supplemented with the electromagnetic terms (Coulomb potential, finite size, and vacuum polarization corrections) and incorporated into the Lippmann-Schwinger equation to be solved relativistically in the momentum space. Using the matching procedure of Vincent and Phatak [33] we calculate the complex energies of the bound pion-nucleus system looking for zeros of the corresponding Jost function. The method was described in detail in our previous paper [25].

In the present calculations we use the harmonic oscillator density for all nuclei up to ^{18}O. Taking into account the center-of-mass motion and the finite size of proton, we parametrize the nuclear density as

$$\rho(r) = \rho_0 \left(1 + \alpha \frac{r^2}{b^2}\right) e^{-\frac{r^2}{b^2}} . \tag{34a}$$

Here, ρ_0 is the normalization factor and the parameters are

$$b^2 = \frac{A-1}{A} a^2 = \frac{2(A-1)}{5A-11}(R_{ch}^2 - \langle r_p^2 \rangle) ,$$
$$\alpha = \begin{cases} \frac{2}{9}(A-4) & \text{for } A > 4, \\ 0 & \text{for } A \le 4 , \end{cases} \tag{34b}$$

where R_{ch} (r_p) denotes the nuclear (proton) charge radius and $r_p = 0.81$ fm. The values of R_{ch} were taken from Ref. [34] and are presented in Table I. Finally, using the corresponding formula for the nuclear form factor and Eqs. (13) and (26), the functions $G(\mathbf{q})$ and $G_0(\mathbf{q})$ can readily be expressed in an analytical form.

For heavier nuclei ($A > 18$) we use the symmetrized Fermi density [35]

$$\rho(r) = \rho_0 \frac{\sinh\left(\frac{c}{b}\right)}{\cosh\left(\frac{c}{b}\right) + \cosh\left(\frac{r}{b}\right)} \tag{35a}$$

which is known to describe well the electron scattering data in a broad interval of transferred momenta. The parameters b and c were fitted to reproduce the nuclear charge form factors

$$F_{ch}(\mathbf{q}) = F(\mathbf{q})\, F_N(\mathbf{q}) \quad , \quad F_N(\mathbf{q}) = \left[1 + \frac{q^2}{0.71\,\mathrm{GeV}^2}\right]^{-2}$$

making use of the constraint

$$R_{ch}^2 - \langle r_p^2 \rangle = \frac{1}{5}\left[7(\pi b)^2 + 3c^2\right] . \tag{35b}$$

The values we use in our analysis are given in Table II. The nuclear densities (35a) lead to analytical expressions for $F(\mathbf{q})$ and $G(\mathbf{q})$. This speeds up the numerical calculations to some extent.

To evaluate the matrix elements needed in Eq. (9c) we use the harmonic oscillator model to construct the nuclear distributions $\rho_v(r)$. Since only the valence subshells with $l_v > 0$ contribute to the sum in Eq. (11) and $\rho_v(r)$ has the same form for both the $j_v = l_v + \frac{1}{2}$ and $j_v = l_v - \frac{1}{2}$ subshells we can separate the radial integration in all cases we are interested in and write the matrix elements in the form

$$A \langle \Psi_0 \mid \frac{j_1(q'r)}{q'r} (\boldsymbol{\sigma} \cdot \boldsymbol{l})(\mathbf{t} \cdot \boldsymbol{\tau})^T \mid \Psi_0 \rangle = I(q') \langle (\boldsymbol{\sigma} \cdot \boldsymbol{l})(\mathbf{t} \cdot \boldsymbol{\tau})^T \rangle , \tag{36}$$

where $I(q')$ stands for radial integral and the angular part is defined by

$$\langle (\boldsymbol{\sigma} \cdot \boldsymbol{l})(\boldsymbol{t} \cdot \boldsymbol{\tau})^T \rangle = \sum_v \left[n_v + (-1)^T p_v\right] \langle \boldsymbol{\sigma} \cdot \boldsymbol{l} \rangle_v . \tag{37}$$

Because the nuclear subshells with $j_v = l_v + \frac{1}{2}$ tend to be occupied first, the term $\langle (\boldsymbol{\sigma} \cdot \boldsymbol{l})(\boldsymbol{t} \cdot \boldsymbol{\tau})^0 \rangle$ is always positive. The isovector term may be negative but not the complete matrix element of Eq. (9c) in which the isoscalar part prevails. The corresponding values we have used in our analysis are given in Table III. They were ob-

TABLE II. The nuclear charge radii and parameters of the symmetrized Fermi density.

Nucleus	c [fm]	b [fm]	R_{ch} [fm]
^{19}F	2.629	0.507	2.891
^{20}Ne	2.773	0.521	3.004
^{22}Ne	2.882	0.471	2.951
^{23}Na	2.875	0.498	3.007
^{24}Mg	2.984	0.484	3.039
^{27}Al	3.260	0.397	3.035
^{28}Mg	3.134	0.477	3.114
^{32}S	3.291	0.520	3.300
^{35}Cl	3.630	0.457	3.384
^{40}Ca	3.690	0.487	3.479
^{44}Ca	3.757	0.481	3.510

TABLE III. The angular part of the isoscalar and isovector matrix elements of the spin-orbital operator.

Nucleus	$\langle (\boldsymbol{\sigma} \cdot \boldsymbol{l})(\mathbf{t} \cdot \tau)^0 \rangle$	$\langle (\boldsymbol{\sigma} \cdot \boldsymbol{l})(\mathbf{t} \cdot \tau)^1 \rangle$
^{10}B	3.87 (6.00)	0.00
^{12}C	3.59 (8.00)	0.00
^{14}N	3.70 (4.00)	0.00
^{18}O	2.44 (4.00)	2.44
^{24}Mg	8.60 (16.0)	0.00
^{27}Al	13.20 (22.0)	1.20
^{28}Si	14.40 (24.0)	0.00
^{32}S	14.72 (24.0)	0.00
^{35}Cl	9.00 (15.0)	−1.80
^{44}Ca	12.00 (12.0)	12.00

tained using Cohen-Kurath wave functions [36] for p-shell nuclei and the wave function from Ref. [37] in the case of ^{18}O. The occupation numbers of protons and neutrons beyond the ^{16}O core were calculated by Brown *et al.* [38] for even-even nuclei. We use the numbers given in Table 3 and extrapolate the values of our elements (37) for the odd nuclei ^{27}Al and ^{35}Cl, as well. In the brackets we present the values that one obtains if the simple model of closed nuclear subshells is adopted [30], in which the $j_v = l_v + \frac{1}{2}$ states are occupied first.

III. RESULTS AND DISCUSSION

Using our momentum-space code PIATOM [25], we have calculated the characteristics (strong-interaction energy shifts ΔE_N and absorption widths $\Gamma_{\rm ABS}$) of light pionic atoms. Our parameters B_0 ($\overline{B}_0$) and C_0 ($\overline{C}_0$) were fixed to reproduce the experimental data of π-atoms ^{16}O [39,42] ($1s$ and $2p$ levels) and ^{40}Ca [40,41] ($2p$ level). We have found

$$B_0 = (-0.096 + i\,0.048)\,m^{-4}\,, \qquad C_0 = (-0.152 + i\,0.076)\,m^{-6} \tag{38a}$$

and

$$\overline{B}_0 = (-0.110 + i\,0.055)\,m^{-4}\,, \qquad \overline{C}_0 = (-0.151 + i\,0.077)\,m^{-6} \tag{38b}$$

when the calculations were performed using the second-order potential of Eqs. (12) and (20), respectively. These values have been used in all the calculations to be discussed below.

In the present analysis we have included all the available data up to ^{24}Mg where the pionic $1s$ level had been measured and most of those measured for the $2p$ level in the region we are interested in ($A \leq 44$). The selection of experimental data was performed involving those collected in the article of Tauscher and Schneider [42] and those published later [15–17,39–41,43–47]. When more data were available for one π-atomic level, we always took the weigh average of them. For the sake of completeness it should be mentioned that we have slightly increased (to be 1–2 % of the corresponding magnitude) the error bars of some shifts ΔE_N with respect to those published in original papers. We proceed so to take into account the uncertainties in extracting the electromagnetic corrections from measured transition energies.

In Figs. 1 and 2, the results of our calculations are presented in comparison with the experimental data. Figure 1 visualizes the influence of the recoil correlations on the characteristics of the π-atomic $1s$ levels. The presented results were obtained exploiting the standard optical potential, the phenomenological part of which is defined by Eqs. (12) and (38a) and using the potential with the improved second-order part parametrized by means of Eqs. (20) and (38b). The correction $V_N^{(\sigma l)}(E)$ was omitted on this stage of our calculations. As we expected, the long-range center-of-mass correlations influence significantly only the characteristics of very light ($A \leq 10$) pionic atoms yielding a negligible effect for heavier pionic atoms. The most important point is that the characteristics of the pionic atom ^{4}He are quite well reproduced in our present calculations, the fact which cannot be achieved by applying any version of the standard optical potential. On the other hand, the worse description of the ^{3}He data does not represent a crucial problem in the analysis. In fact, the open channel of charge exchange reaction in the virtual state may give an additional contribution to the calculated shift and width in this case [48]. Inclusion of such processes stands beyond the framework of the formalism presented here and can be a subject of further investigations.

The description of the $1s$-level characteristics is generally rather good and similar to the results of other analyses. Particularly, the isospin dependence of the strong-interaction shifts $\Delta E_N(1s)$ is nicely reproduced in our model. One should notice that this achievement is not a result of the fitting procedure, because the isospin structure of our optical potential fully reflects the isospin structure of the elementary πN amplitude. The complex analyses of light pionic atoms are not numerous in the

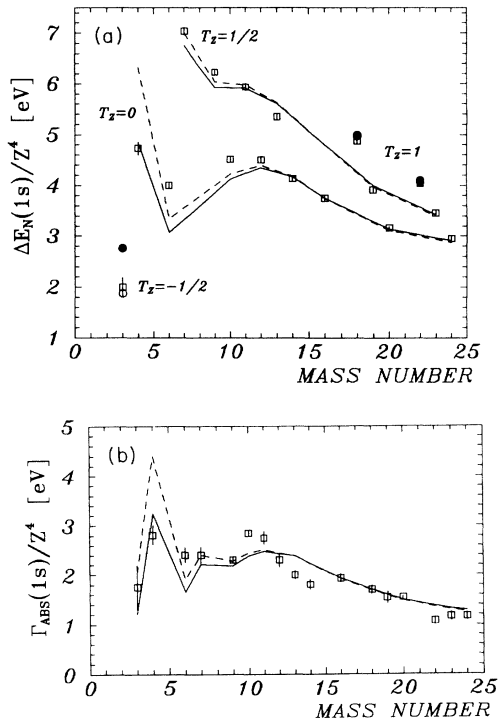

FIG. 1. The calculated characteristics of the pionic $1s$ levels are connected by the full and dashed lines (or denoted by full and empty circles) that correspond to the use of the optical potential with the second-order term proportional to $G(\mathbf{q})$ and $G_0(\mathbf{q})$, respectively. Our results are presented in comparison with the available experimental data. (a) The strong-interaction shifts. The data for nuclei with different isospin are displayed separately. In the case of pionic ^{3}He, the shift is presented with the opposite sign. (b) The absorption widths.

literature, thus the systematic comparison with them is difficult. Many years ago Tauscher and Schneider studied the $1s$-level characteristics and incorporated the spin-isospin $1/A$ corrections in an attempt to explain the discrepancy that one observes between the calculated and measured data of very light pionic atoms. Unfortunately, their approach was based on the formula given by Ericson and Ericson [1] which has been found to be inaccurate. We will come back to that point at the end of this section. Here we mention that a slightly better description of the shifts $\Delta E_N(1s)$ can be achieved if the A^2 dependence [instead of the $A(A-1)$ factor] is assumed in the second-order optical potential [49]. It would push the theoretical results up in the region $A \leq 10$ and bring the calculated characteristics of the ^{6}Li π atom into close agreement with the experimental data. On the other hand, the results obtained for ^{4}He would be quite off then. Moreover, we do not see any microscopical justification of the A^2 factor in question.

Besides the discrepancies observed for $6 \leq A \leq 10$ in Fig. 1(a), other problems seem to occur in the description of the absorption widths in the neighborhood of the ^{12}C π-atom and another one is represented by the "anomalously" small experimental width of the $1s$ level in pionic ^{22}Ne. We have no real explanation for these deviations at present.

The strong-interaction energy shifts and absorption widths of the pionic $2p$ levels are shown in Fig. 2. Because

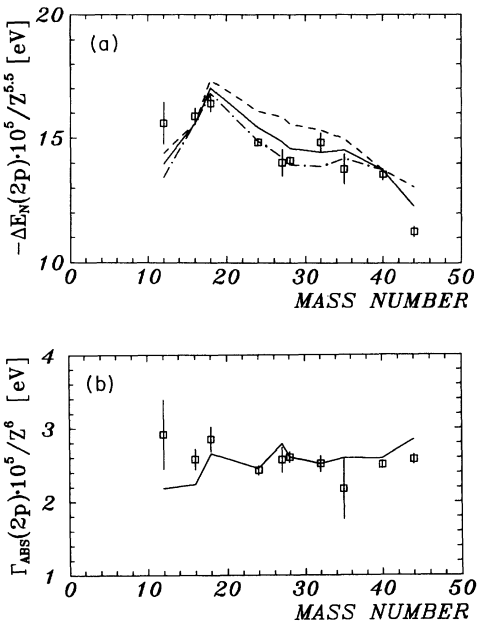

FIG. 2. Comparison of the calculated and measured characteristics of the pionic $2p$ levels. The full (dashed) line connects the results obtained when the $(\boldsymbol{\sigma}\cdot\boldsymbol{l})$ term was included (omitted) in our calculations. The dot-dashed line corresponds to the calculations with the $(\boldsymbol{\sigma}\cdot\boldsymbol{l})$ correction evaluated using the simple model of closed nuclear subshells. (a) The strong-interaction shifts. (b) The absorption widths. Only the results including the $(\boldsymbol{\sigma}\cdot\boldsymbol{l})$ term are shown. The dashed and dot-dashed lines would be quite close to the full one in this figure.

the effects due to the recoil correlations turn out to be negligible in the region of heavier nuclei, only the results obtained with the standard pion-nucleus optical potential are presented and we concentrate our attention to the role of the $(\boldsymbol{\sigma}\cdot\boldsymbol{l})$ correction. The latter one influences the absorption widths $\Gamma_{\rm ABS}(2p)$ given in Fig. 2(b) to a small extent (about 1% of their magnitudes). This is the reason why only the results obtained including the $(\boldsymbol{\sigma}\cdot\boldsymbol{l})$ term in the optical potential are shown in Fig. 2(b).

As one can see, the $2p$-level shifts are affected by the Fermi motion correction to relatively large extent. In the region of deformed s-d shell nuclei ($24 \leq A \leq 32$) the effect represents some $5-7\%$ of the total strong-interaction level shift and brings the theoretical calculations to much better agreement with experimental data. It is encouraging to see that the theoretical line follows the trend observed particularly for the most precise data taken from the PSI measurement [16]. One gets an even better description of the $2p$-level shifts if the approximate model of closed nuclear subshells is used to calculate the matrix elements of Eq. (36) [30]. It is visualized by the dot-dashed line in Fig. 2(b). However, the succes is partly fortuitous, since the quantities $\langle(\boldsymbol{\sigma}\cdot\boldsymbol{l})(\mathbf{t}\cdot\boldsymbol{\tau})^T\rangle$ represent upper bounds rather than realistic values in this case. The present model (full line in Fig. 2) does not fit the experimental data fully; nevertheless, it gives quite a reasonable description of the observed π-atomic characteristics for the $2p$ levels. The only real deviation occurs in the case of ^{44}Ca; however, it can easily be removed by making the nuclear distributions of protons and neutrons different [41]. Generally, the neutron excess and the difference between the distributions of protons and neutrons start to play a non-negligible role for heavier nuclei.

For the sake of completeness, we have also studied the effects caused by the $(\boldsymbol{\sigma}\cdot\boldsymbol{l})$ term to the characteristics of the pionic $1s$ level. Incorporating the spin-orbital correction into the optical potential we have found that the results obtained for the absorption widths $\Gamma_{\rm ABS}(1s)$ and strong-interaction shifts $\Delta E_N(1s)$ are affected only on the level of 1% or 2% of their magnitude, respectively [30]. Such an effect is mostly under the experimental precision and can be neglected. The shifts $\Delta E_N(2p)$ exhibit a much larger effect, which is in agreement with the remarks made in Sec. II A. We expect the effects due to the $(\boldsymbol{\sigma}\cdot\boldsymbol{l})$ term to be even more significant for the heavier deformed nuclei having the valence nucleons in the subshells with higher angular momenta.

In concluding this section we would like to make some comments on the spin-isospin corrections to the second-order optical potential, which were often expected to improve the description of light pionic atoms. As we have already shown, the spin-isospin operators contribute effectively to the isoscalar part of the πNN amplitude and the parametr B_0 should be replaced by $B_0^{\rm eff}$ defined in Eq. (33). The imaginary part of the parameters B_n can be calculated within the framework of a simple rescattering model developed by Bertsch and Riska [51]. The analytical expressions relating $\mathrm{Im}\,B_n$ to the s-wave πN scattering lengths were given in Ref. [50]. We have adopted $b_0 = \frac{1}{3}(a_1 + 2a_3) = -0.0093m^{-1}$ and

TABLE IV. The imaginary part of the parameters B_n.

Parameter	$\mathrm{Im}\,B_0$	$\mathrm{Im}\,B_1$	$\mathrm{Im}\,B_2$	$\mathrm{Im}\,B_{30}$	$\mathrm{Im}\,B_{31}$	$\mathrm{Im}\,B_4$	$\mathrm{Im}\,B_{50}$	$\mathrm{Im}\,B_{51}$
Value in m^{-4}	0.0323	0.0107	−0.0320	−0.0183	−0.0060	−0.0108	0.0139	0.0047

$b_1 = \frac{1}{3}(a_3 - a_1) = -0.091m^{-1}$ [24] to get the results shown in Table IV.

The effective isoscalar parameter $\mathrm{Im}\,B_0^{\mathrm{eff}} = 0.049m^{-4}$ appears to be in surprisingly good agreement with our phenomenological value $\mathrm{Im}\,\overline{B}_0/d = 0.051m^{-4}$ given in Eq. (38b). On the other hand, Germond and Lombard obtained a value of $\mathrm{Im}\,B_0^{\mathrm{eff}} = 0.071m^{-4}$. However, their analysis was based on the $\vec{d}p \longrightarrow {}^3\mathrm{He}\,\pi^0$ reaction, data of which are less precise than those on the low-energy πN interaction. Furthermore, the value $\mathrm{Im}\,B_0$ obtained by fitting the π-atomic data varies within some 20% depending on the concrete parametrization of the total optical potential and on the fitting procedure used by different authors. Thus, the agreement of our fitted $\mathrm{Im}\,B_0$ and calculated $\mathrm{Im}\,B_0^{\mathrm{eff}}$ can be rather fortuitous.

Another interesting point worth mentioning is the strong suppression of the isospin dependence in the $\pi NN \longrightarrow NN \longrightarrow \pi NN$ process. The combination $\mathrm{Im}\,(B_{30} - 3B_{31})$ gives a value proportional to the isoscalar scattering length b_0 squared. Since its value almost vanishes, the only possible isospin dependence of the absorption part of the optical potential comes from the tensor part of the matrix elements $\mathcal{G}_{50}$ and $\mathcal{G}_{51}$. Observing the apparent isospin independence of the measured absorption widths in our Figs. 1(b) and 2(b), one can safely assume this correction to be practically negligible in the region of pionic atoms we are restricted to.

At variance with the observation of Tauscher and Schneider [42] the correct evaluation of the spin-isospin corrections leads to the effective value of B_0^{eff} that is practically A independent. The slight variations of B_0^{eff} can occur due to possible nonequivalence $\mathcal{G}_1 \neq \mathcal{G}_2$ and due to a weak isospin dependence. Unfortunately, these corrections are not large enough to remove the discrepancy observed between the calculated and measured strong-interaction shifts in the region of very light pionic atoms. The origin of it remains obscure.

IV. SUMMARY

We have presented a momentum-space formulation of pion-nucleus optical potential constructed fully microscopically in the first order. It receives a contribution due to the nucleonic Fermi motion when the spin-dependent part of the πN amplitude is transformed from the pion-nucleon to the pion-nucleus center-of-mass frame. It gives a contribution to the small isoscalar part of the potential (usually denoted by b_0) causing an additional repulsion. We have shown that the discrepancy between the experimental data and theoretical results for pionic $2p$ levels in the region of deformed s-d shell nuclei is mostly accounted for by this correction reflecting the shell structure of the nuclear wave function. We expect even more pronounced consequences for heavier deformed nuclei and levels with higher orbital momenta.

Another manifestation of nuclear structure effects is associated with the nuclear correlations. We have presented a suitable method to incorporate the long-range two-particle correlations into the second-order optical potential. It leads to an additional nontrivial A dependence in the optical potential in comparison with the common ρ^2 term. We have studied the influence of the CMS correlations on the characteristics of very light pionic atoms. Our new results for the ^{4}He pionic atom turn out to be in nice agreement with the experimental data. On the other hand, the shifts $\Delta E_N(1s)$ and absorption widths $\Gamma_{\mathrm{ABS}}(1s)$ of the surrounding pionic atoms are not reproduced quite well. The situation cannot be improved by means of the spin-isospin corrections to the second-order optical potential as some earlier analyses seemed to indicate. We do not see any obvious mechanism, which could account for the other deviations in the region of very light pionic atoms.

The present formulation of the optical potential is very suitable for studying the nonlocal features of the pion-nucleus interaction. The Fermi motion $(\boldsymbol{\sigma} \cdot \boldsymbol{l})$ correction and the CMS correlations discussed in this paper represent only two examples of such effects. Taking them into account, we were able to reproduce the characteristics of pionic atoms much better, especially for very light nuclei and nuclei with nonclosed nuclear shells.

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