Cranking Bohr-Mottelson Hamiltonian applied to superdeformed bands in $A \sim 190$ region

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Level spins and transition energies of 26 superdeformed bands in the $A \sim 190$ region are studied with the rotational spectral formula derived from the cranking Bohr-Mottelson Hamiltonian proposed by us with satisfactory results.

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I. INTRODUCTION

Intensive investigations have been performed on the nuclear superdeformed (SD) states and several models have been proposed for their spin determinations [1–3]. All of them are empirical or semiempirical models adapted from the studies of the normal rotational bands. Their applications in the superdeformed states are not well justified and the studies throw little light on the structure of the SD states. Cheng-Li Wu *et al.* [4] have given a critical review of the problem. They have presented a convincing argument to show that any theoretical spin determination must be model dependent and that the present status of the nuclear model is not accurate enough to make a spin determination with an error less than $1\hbar$. Hence, it is worthwhile to develop more reliable models which may contribute towards the spin determination and the nuclear structure studies of the SD states.

Recently, we have developed a nuclear rotation-vibration model based on the cranking Bohr-Mottelson (BM) Hamiltonian [5]. The model has been successfully applied to the normal rotational bands of the well-deformed even-even nuclei. It is a well-founded model and can be applied to any quadrupole deformed nucleus with moderate rotation frequencies. The parameters appearing in the model have clear physical meaning and the nuclear structure information can be derived. The essence of the model will be presented in the second section, together with necessary modifications for the application of the model to the SD bands. Numerical results and discussions will be given in the third section.

II. SKETCH OF THE MODEL

In the cranked shell model [6], the Hamiltonian in the rotational frame of reference is given by

$$H' = H(a_0, a_2) - \boldsymbol{\omega} \cdot \mathbf{J}, \tag{1}$$

where $H(a_0, a_2)$ is the shell model Hamiltonian with the quadruple deformation parameters a_0,a_2. With suitable approximations, one obtains, for axisymmetrical quadrupole deformations, the cranking BM Hamiltonian

$$H' = -\frac{\hbar^2}{2B_0}\frac{\partial^2}{\partial a_0^2} - \frac{\hbar^2}{4B_2}\frac{\partial^2}{\partial a_2^2} - \frac{1}{2}B_1(3a_0^2 + 2a_2^2)\omega^2 + E(a_0, a_2)\,, \tag{2}$$

where $E(a_0, a_2)$ is the static energy surface, and B_0, B_1, and B_2 are the mass parameters [5].

In our former applications, a simple form for $E(a_0, a_2)$ was assumed and the eigenvalue equation easily solved. For the present case, we shall still assume mainly axisymmetrical deformation and $E(a_0, a_2)$ separately,

$$E(a_0, a_2) = U(a_0) + \frac{1}{2}C_2 a_2^2\,. \tag{3}$$

Since the SD states are well-known good rotors, it is reasonable to assume that the a_0-dependent part of the effective potential in Eq. (2), $V(a_0, \omega) = -\frac{3}{2}B_1\omega^2 a_0^2 + U(a_0)$, has a deep minimum at a certain value (depending on ω) of a_0, around some average value $\bar{a}_0$, hence it can be approximated by a Taylor's expansion around $\bar{a}_0$ up to the second-order terms

$$\begin{aligned} V(a_0, \omega) = {} & U(\bar{a}_0) - \tfrac{3}{2}B_1\bar{a}_0^2\omega^2 \\ & + [U'(\bar{a}_0) - 3B_1\bar{a}_0\omega^2](a_0 - \bar{a}_0) \\ & + \tfrac{1}{2}[U''(\bar{a}_0) - 3B_1\omega^2](a_0 - \bar{a}_0)^2\,, \end{aligned} \tag{4}$$

where $\bar{a}_0$ may be taken as the deformation at the minimum of $V(a_0, \omega_0)$ for certain value of ω_0 within the range of the rotational frequencies of the studied SD band, hence, $U'(\bar{a}_0) = 3B_1\bar{a}_0\omega_0^2$.

Inserting Eqs. (3) and (4) into Eq. (2), one obtains the eigenvalue of H',

$$\begin{aligned} E' = {} & (n_\beta + \tfrac{1}{2})\hbar\omega_\beta\left(1 - \frac{3B_1\omega^2}{B_0\omega_\beta^2}\right)^{1/2} \\ & + \left(n_\gamma + \frac{1}{2}\right)\hbar\omega_\gamma\left(1 - \frac{B_1\omega^2}{B_2\omega_\gamma^2}\right)^{1/2} \\ & - A\omega^2\left(1 - \frac{3B_1\omega^2}{B_0\omega_\beta^2}\right)^{-1}\,, \end{aligned} \tag{5}$$

where $\omega_\beta = \sqrt{C_0/B_0}$, $[C_0 = U''(\bar{a}_0)]$, $\omega_\gamma = \sqrt{C_2/2B_2}$ are the frequencies of the β and γ vibrations, n_β and n_γ are the corresponding quantum numbers, B_0, B_1, and B_2 are evaluated at the minimal deformations $\bar{a}_0$, $\bar{a}_2 = 0$, and

$$A = \frac{3}{2}B_1\bar{a}_0^2\left(1 - \frac{3B_1\omega_0^2}{B_0\omega_\beta^2}\right)^2 .$$

In the laboratory frame of reference, the energy of the collective motions is given by

$$\begin{aligned} E &= E' + B_1\omega^2\langle n_\beta, n_\gamma|3a_0^2 + 2a_2^2|n_\beta, n_\gamma\rangle \\ &= (n_\beta + \tfrac{1}{2})\hbar\omega_\beta\left(1 - \frac{3B_1\omega^2}{B_0\omega_\beta^2}\right)^{-1/2} + (n_\gamma + \tfrac{1}{2})\hbar\omega_\gamma\left(1 - \frac{B_1\omega^2}{B_2\omega_\gamma^2}\right)^{-1/2} + A\omega^2\left(1 + \frac{3B_1\omega^2}{B_0\omega_\beta^2}\right)\left(1 - \frac{3B_1\omega^2}{B_0\omega_\beta^2}\right)^{-2}, \end{aligned} \tag{6}$$

and the average projected angular momentum given by

$$\begin{aligned} \sqrt{I(I+1) - K^2} &= -\frac{1}{\hbar}\frac{\partial E'}{\partial\omega} \\ &= (n_\beta + \tfrac{1}{2})\frac{3B_1\omega}{B_0\omega_\beta}\left(1 - \frac{3B_1\omega^2}{B_0\omega_\beta^2}\right)^{-1/2} + (n_\gamma + \tfrac{1}{2})\frac{B_1\omega}{B_2\omega_\gamma}\left(1 - \frac{B_1\omega^2}{B_2\omega_\gamma^2}\right)^{-1/2} + \frac{2}{\hbar}A\omega\left(1 - \frac{3B_1\omega^2}{B_0\omega_\beta^2}\right)^{-2} . \end{aligned} \tag{7}$$

III. APPLICATION TO SD BANDS IN THE $A \sim 190$ REGION

For the SD states, we can, in general, assume $n_\beta = n_\gamma = 0$, otherwise they cannot remain as good rotors. A rough estimation of the various terms in right-hand side of Eqs. (6) and (7) shows that only the last terms give the most important contributions in both equations, and the contributions from other terms may be less than 1% or 2%. Hence the most important parameters are the effective moment of inertia $2A$ and the effective β vibration frequency $\omega' = \sqrt{B_0/B_1}\omega_\beta$. To reduce the number of parameters, we shall put $\omega_\beta = \omega_\gamma = \omega'$ and $B_0 = B_2$ in the first two terms of Eqs. (6) and (7). The simplified formulas are

$$\begin{aligned} E &= \tfrac{1}{2}\hbar\omega'\left[\left(1 - \frac{3\omega^2}{\omega'^2}\right)^{-1/2} + \left(1 - \frac{\omega^2}{\omega'^2}\right)^{-1/2}\right] \\ &\quad + A\omega^2\left(1 + \frac{3\omega^2}{\omega'^2}\right)\left(1 - \frac{3\omega^2}{\omega'^2}\right)^{-2} \end{aligned} \tag{6'}$$

and

$$\begin{aligned} &\sqrt{I(I+1) - K^2} \\ &= \tfrac{1}{2}\frac{\omega}{\omega'}\left[3\left(1 - \frac{3\omega^2}{\omega'^2}\right)^{-1/2} + \left(1 - \frac{\omega^2}{\omega'^2}\right)^{-1/2}\right] \\ &\quad + \frac{2}{\hbar}A\omega\left(1 - \frac{3\omega^2}{\omega'^2}\right)^{-2} . \end{aligned} \tag{7'}$$

From the numerical fit, it can be shown that ω' is large ($\hbar\omega' > 2$ MeV), hence any excited vibrational state will not remain a good rotor. On the left-hand side of Eq. (7') or (7), one may notice the presence of K^2 which may affect the value of I calculated from these equations. In the former papers [1–3], K is usually neglected. However, for the $A \sim 190$ region, where level spins are not very large, neglecting K is justified only for K value less than 3. For most of the cases, K values can be determined only through the detailed microscopic studies. With the exception of special cases, the effect of K in Eq. (7') is neglected. For a large K value, this simplification may lead to underestimation of the spin value by one or two units.

With the experimentally measured E_γ, the value of ω_I for a state with spin I can be calculated from

$$\omega_I = [E_\gamma(I+2) + E_\gamma(I)]/4. \tag{8}$$

For a given set of the ω_I value, parameters ω' and A may be determined by fitting Eq. (6') to the observed transition energies and the spin values calculated from Eq. (7'). The level spins are determined by taking the nearest integers (for even nuclei) or half integers (for odd nuclei) of the calculated values. With the determined spins, the energy spectrum can be obtained from Eqs. (6') and (7') by the least squares fit of the parameters ω' and A. The redetermined parameters differ only slightly from those used in the spin determination, but the fit of the spectrum is somewhat improved. Since Eqs. (6') and (7') are more accurate for small ω values, we shall use transitions 1–9 to determine the parameters, then all the transition energies are calculated with the parameter values thus determined.

We investigate 26 SD bands in the $A \sim 190$ region. With the exception of ^{189}Hg and ^{191}Hg(1) bands, the determined spins are the same as those given by Becker *et al.* [1,2] with the two-parameter Harris expansion and by Wu *et al.* [3], so the similar results are not repeated in this paper. In fact, it can be shown that Eqs. (6) and (7) reduce to the Harris formula for the range of the ω_I values in the $A \sim 190$ region.

For most of the investigated SD bands, the spin assignments are unambiguous and the spectra are well represented by Eqs. (6') and (7'). As is to be expected, the calculated spin values are insensitive to the number of

TABLE I. Calculated spin value I_1 of the second level in the observed SD band with various number (No.) of the experimental transitions included in the fit. The lowest level spin (exit spin) is given by $I_0 = I_1 - 2$.

	^{189}Hg[a]		^{191}Hg(1)[b]		^{192}Hg[c]	^{193}Tl(1)[d]
	I_1	I_1	I_1	I_1	I_1	I_1
No.	(K neglected)	($K = \frac{7}{2}$)	(K neglected)	($K = \frac{7}{2}$)	(K=0)	(K neglected)
5	16.87	17.23	17.19	17.54	10.12	11.55
6	16.94	17.30	17.05	17.40	10.15	11.49
7	17.10	17.45	16.95	17.30	10.15	11.50
8	17.10	17.45	16.93	17.28	10.13	11.46
9			16.95	17.30	10.15	11.49
10			16.99	17.34	10.19	11.49
11			17.08	17.43	10.22	11.52
12			17.10	17.45	10.24	11.54
13					10.27	11.56
14					10.30	
15					10.34	
16					10.38	
17					10.43	

[a] $E_\gamma(I_0 + 2 \to I_0) = 366.0$ keV, Ref. [10].
[b] $E_\gamma(I_0 + 2 \to I_0) = 350.6$ keV, Ref. [9].
[c] $E_\gamma(I_0 + 2 \to I_0) = 214.6$ keV, Ref. [11].
[d] $E_\gamma(I_0 + 2 \to I_0) = 228.1$ keV, Ref. [12].

TABLE II. Experimental and calculated transition energies, and assigned spins for ^{189}Hg, ^{192}Hg, and ^{191}Hg(1) bands. $E_\gamma(I)$ is the energy (in keV) of the transition from $I + 2$ to I.

^{189}Hg			^{192}Hg			^{191}Hg(1)			
$E_\gamma(I)$		Spin	$E_\gamma(I)$		Spin	$E_\gamma(I)$		Spin	$E_\gamma(I)$	Spin
exp[a]	cal[b]	I	exp[c]	cal[d]	I	exp[e]	cal[f]	I	cal[g]	I
	109.8	7/2		34.2	0		104.9	7/2	97.5	3
	153.4	11/2		79.8	2		146.6	11/2	140.6	5
	196.8	15/2		125.0	4		188.1	15/2	183.4	7
	239.9	19/2		169.8	6		229.4	19/2	225.9	9
	282.6	23/2	214.6	214.0	8		270.4	23/2	267.9	11
	324.8	27/2	257.7	257.3	10		311.0	27/2	309.4	13
366.0	366.6	31/2	299.9	299.9	12	350.6	351.2	31/2	350.4	15
407.7	407.7	35/2	341.1	341.5	14	390.5	390.9	35/2	390.8	17
448.7	448.3	39/2	381.6	382.1	16	430.3	430.2	39/2	430.5	19
488.9	488.3	43/2	420.8	421.5	18	469.6	469.0	43/2	469.5	21
527.8	527.6	47/2	459.1	459.9	20	508.1	507.2	47/2	507.8	23
566.4	566.2	51/2	496.3	497.0	22	545.3	544.8	51/2	545.3	25
604.2	604.1	55/2	532.4	533.0	24	582.1	581.8	55/2	582.0	27
640.6	641.3	59/2	567.9	567.8	26	617.8	618.2	59/2	618.0	29
			602.3	601.5	28	653.0	654.0	63/2	653.1	31
			635.8	634.0	30	687.4	689.1	67/2	687.4	33
			668.6	665.3	32	721.8	723.6	71/2	720.9	35
			700.6	695.6	34	754.3	757.4	75/2	753.7	37
			732.1	724.8	36					
			762.8	753.0	38					
			793.4	780.2	40					

[a] Reference [10].
[b] $K = 7/2$, $A = 0.04520\hbar^2$ keV^{-1}, $\hbar\omega' = 3190$ keV.
[c] Reference [11].
[d] $K = 0$, $A = 0.04334\hbar^2$ keV^{-1}, $\hbar\omega' = 2211$ keV.
[e] Reference [9].
[f] $K = 7/2$, $A = 0.04734\hbar^2$ keV^{-1}, $\hbar\omega' = 3340$ keV.
[g] K neglected, $I_0 = 15$, $A = 0.04572\hbar^2$ keV^{-1}, $\hbar\omega' = 2900$ keV.

transitions included in the fit. As a represenative example, the result of spin determination of ^{193}Tl(1) band is shown in Table I. For ^{189}Hg and ^{191}Hg(1) bands, the calculated spin values are very close to integers. Several alternatives may be taken for the spin assignments of these bands. One possible explanation is that the two bands are indeed the SD bands of some even nuclei with integer spins. Another possibility is that they might be the bands with large K values. An assignment of $K = 7/2$ which yields exit spin $I_0 = 31/2$ is also included in Table I. For comparison with a typical case, we have also listed the spin determination of ^{192}Hg band in the table. If we apply the process to less than nine transitions, the spins approach nicely to integer values. However, if all 17 transitions are included, the spin values become close to half integers. One of the possible explanations is that, in the derivation of Eqs. (6) and (7), we have tacitly introduced an average rotational frequency ω_0, hence the accuracy of the formulas may be affected when applied to a wide range of frequencies. Table II lists the calculated transition energies for the SD bands of ^{192}Hg, ^{189}Hg, and ^{191}Hg(1). For the last nucleus, different I and K assignments are compared. For the ^{192}Hg SD band, the agreement between theory and experiment is less satisfactory for the transitions with large spin values.

For the SD bands in this region, $\bar{a}_0 \sim 0.55$ [7–9]. From $A = \frac{3}{2}B_1\bar{a}_0^2\left(1 - 3\omega_0^2/\omega'^2\right)^2$, the mass parameter B_1 can be determined by taking ω_0 as the frequency of the medium level of the studied SD band (in fact, the effect of ω_0 is very slight because $\hbar\omega' > 2$ MeV for the SD bands in this region). The values of B_1 so obtained are shown in Fig. 1. From $\omega' = \sqrt{C_0/B_1}$, the rigidity parameter C_0 can be calculated and plotted in Fig. 2. For comparison, Figs. 1 and 2 also show the B_1 and C_0 values of the normally deformed (ND) states of the even-even nuclei in the rare-earth and actinide regions [5]. From Fig. 2, it can be seen that the C_0 values of the SD states are larger than those of the ND states. It implies that the

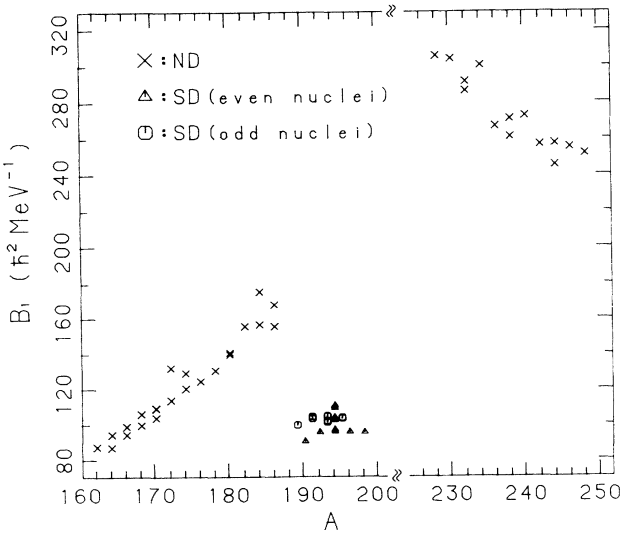

FIG. 1. The fitted B_1 values of the SD bands in the $A \sim 190$ region and the normally deformed (ND) bands of the even-even nuclei in the rare-earth and actinide regions.

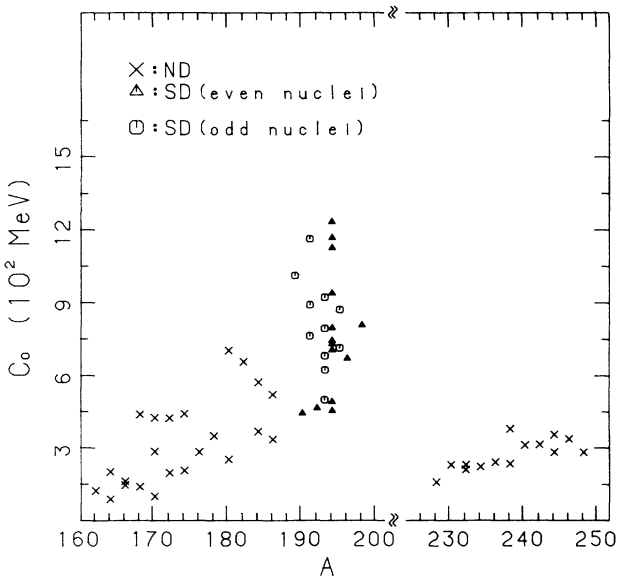

FIG. 2. The C_0 values of the same bands as in Fig. 1.

SD state has larger rigidity (designated as super-rigidity in Ref. [3]). No evident odd-even differences are found for the parameters B_1 and C_0 for the SD bands in the region. This is in accord with the decrease of the pairing effect for the large rotational frequency.

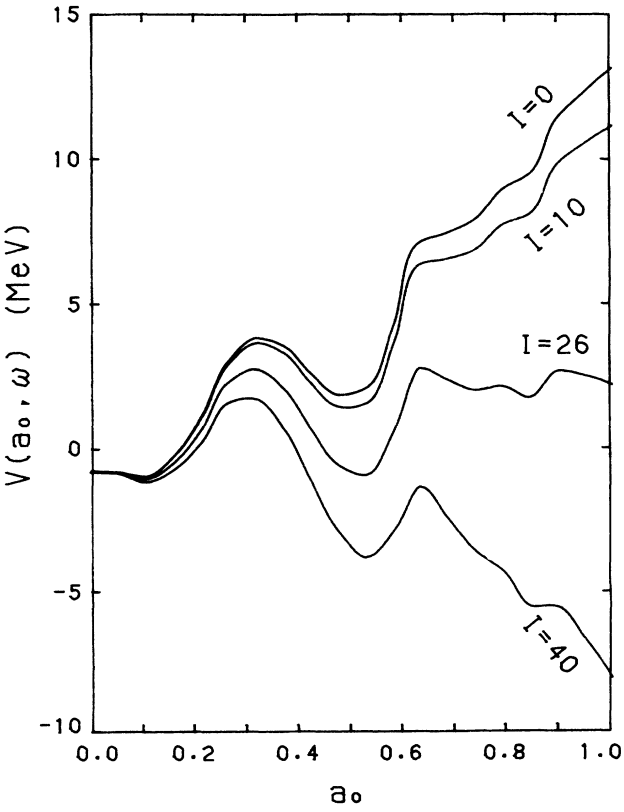

FIG. 3. The effective potential energy $V(a_0, \omega)$ when $I =$ 0, 10, 26, 40 for ^{192}Hg.

As a further demonstration of the validity of the present model, we have calculated the potential energy $U(a_0)$ for ^{192}Hg with the finite range liquid drop model [13] and the microscopic corrections derived from the standard Nilsson potential [14]. With the value of B_1 determined earlier, the effective potential $V(a_0,\omega)$ can be calculated. In Fig. 3, the $V(a_0,\omega)$ curves of ^{192}Hg with $I = 0$, 10, 26, 40 are plotted. It can be seen that there is a deep valley near $\bar{a}_0 \sim 0.52$ to produce SD state, which is close to the experimentally determined deformation $\bar{a}_0 \sim 0.55$ [9]. The spin value may extend to $I = 0$, as has been suggested by the authors of Refs. [3,15]. There is a shallow valley near $\bar{a}_0 \sim 0.85$ for the curve with $I = 26$. It may not be deep enough to form a stable hyperdeformed state. However, our calculation is less accurate for highly deformed states. From the potential energy curve, the value of C_0 can be obtained as 488 MeV which is in reasonable agreement with the value $C_0 = 467$ MeV obtained from fitting the SD band.

The SD bands in the $A \sim 150$ and 130 regions have higher rotational frequencies than those in the $A \sim 190$ region, hence, the higher perturbation terms in $V(a_0,\omega)$ may require consideration. This will be discussed in our future works.

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