Exact solutions and constrained Hartree-Fock spectra in a soluble triaxial quasispin model

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It is shown that in a soluble triaxial quasispin model the good quasispin states $|JK\rangle$ and $|J\rangle$ can be established on basis SU(2)×SU(2) by means of the diagonalization method. The energy eigenstate $|J\rangle$ has an approximate good quantum number K which can be determined through the procedure proposed here. The triaxiality of the exact solutions of the model Hamiltonian and the characteristics of the energy spectra are discussed. Two new constrained Hartree-Fock (CHF) methods are proposed and solved. The comparisons of calculated CHF spectra with exact solutions show that the new CHF prescriptions proposed in this paper are the effective methods for producing states having good quasispin quantum numbers and the spectra with multiband structure.

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I. INTRODUCTION

The methods of researching the effectiveness of nuclear many-body theories in the exactly soluble model were first proposed by Lipkin and others [1–3]. But the original Lipkin model is not suitable for studying the projected Hartree-Fock (PHF) and the constrained Hartree-Fock (CHF) methods, which are important for restoring the broken symmetries of self-consistent solutions. In order to study the CHF and PHF methods Cambiaggio and Plastino introduced the spin-flip interactions into the Lipkin model [4], but they only took into account the "particle exciton" interactions. Therefore their model Hamiltonian has axial symmetry in a certain quasispin space. In view of the importance of the pairing correlation we added the "pairing exciton" interactions to the model Hamiltonian [5]. In this way triaxial symmetry has been introduced; in other words, it leads to additional symmetry destruction, so that our model is a more general SU(2)×SU(2) quasispin model and is more suitable for researching the methods of restoring broken symmetries. In Sec. II we will describe the model Hamiltonian briefly. In Sec. III the procedure for obtaining the coupling basis states $|JK\rangle$ and the eigenstates $|J\rangle$ will be established. In Sec. IV we will discuss how to determine the approximate band structure in the energy spectra of the model. In Sec. V we will list some of the new prescriptions of CHF calculations and compare their energy spectra with the exact ones. Some conclusions follow in Sec. VI.

II. TRIAXIAL QUASISPIN MODEL

In our model N fermions distribute between two degenerate levels separated by the single particle energy ε. In the particle number representation the model Hamiltonian is taken as

$$\mathcal{H} = \tfrac{1}{2}\varepsilon\sum_{p\sigma}\sigma a^{\dagger}_{p\sigma}a_{p\sigma} + \tfrac{1}{4}W\sum_{pq\sigma}a^{\dagger}_{p\sigma}a^{\dagger}_{q\sigma}(a_{q-\sigma}a_{p-\sigma} - a_{-q-\sigma}a_{-p-\sigma})$$
$$+\tfrac{1}{4}V\sum_{pq\sigma}\left[a^{\dagger}_{p\sigma}a^{\dagger}_{q-\sigma}(a_{q\sigma}a_{p-\sigma} - a_{-q\sigma}a_{-p-\sigma}) + \tfrac{1}{2}\sigma\sum_{\tau}\tau a^{\dagger}_{p\sigma}a^{\dagger}_{q\tau}(a_{q\tau}a_{p\sigma} - a_{-q\tau}a_{-p\sigma})\right], \quad (1)$$

where $a^{\dagger}_{p\sigma}$ and $a_{p\sigma}$ are fermion creation and annihilation operators in state p on level σ, respectively. $\sigma,\tau = +1$ or -1 denotes the particle in the upper level or lower level, and $p,q = \pm1,\pm2,\ldots,\pm\frac{1}{2}N$, for an even-$N$ system with "+" and "−" representing "spin up" and "spin down," respectively.

Now a particle-number operator N, two sets of operators $L_{\pm}, L_z$ and $I_{\pm}$, I_z, which completely commute each other, are introduced:

$$N = N_+ + N_- = \sum_p(a^{\dagger}_{p+}a_{p+} + a^{\dagger}_{p-}a_{p-}) = \sum_{p\sigma}a^{\dagger}_{p\sigma}a_{p\sigma},$$
$$L_+ = \tfrac{1}{2}\sum_p a^{\dagger}_{p+}(a_{p-} + a_{-p-}),\quad L_- = L^{\dagger}_+,\quad L_z = \tfrac{1}{4}\sum_{p\sigma}\sigma a^{\dagger}_{p\sigma}(a_{p\sigma} + a_{-p\sigma}), \quad (2)$$
$$I_+ = \tfrac{1}{2}\sum_p a^{\dagger}_{p+}(a_{p-} - a_{-p-}),\quad I_- = I^{\dagger}_+,\quad I_z = \tfrac{1}{4}\sum_{p\sigma}\sigma a^{\dagger}_{p\sigma}(a_{p\sigma} - a_{-p\sigma}).$$

It can be shown that $L_\pm$, L_z, and $I_\pm$, I_z are two sets of quasispin operators from the generators of group SU(2)×SU(2). From Eq. (2) we define another set of quasi-spin operators:

$$J_\pm = L_\pm + I_\pm, \quad J_z = L_z + I_z. \tag{3}$$

In terms of Eqs. (2) and (3) the Hamiltonian (1) can be rewritten as

$$H = J_z + v(J^2 - I^2 - L^2) + w(I_+L_+ + I_-L_-),$$
$$H = \mathcal{H}/\varepsilon, \quad v = V/\varepsilon, \quad w = W/\varepsilon, \tag{4}$$

where $J^2 = J_+J_- + J_z^2 - J_z$, and I^2 and L^2 have similar expressions. It can be shown that the Hamiltonian H may be exactly diagonalized in the common eigenstates $|IM_ILM_L\rangle$ of I^2, I_z and L^2, L_z, which are the basis functions of SU(2)×SU(2). For an even-particle system the unperturbed ground state is a state in which all particles are in the $\sigma = -1$ level and occupy the $\pm p$ states in pairs. We denote this state by $|\phi\rangle$:

$$|\phi\rangle = \prod_{p=1}^{N/2} a_{p-}^\dagger a_{-p-}^\dagger |0\rangle \equiv \prod_p a_{p-}^\dagger |0\rangle, \tag{5}$$

where $|0\rangle$ is the vacuum state. From Eqs. (2)–(5) we know that the state $|\phi\rangle$ belongs to the SU(2)×SU(2) basis and has the quantum numbers

$$J = \tfrac{1}{2}N, \quad J_z = -\tfrac{1}{2}N, \quad I = L = \tfrac{1}{4}N, \quad I_z = L_z = -\tfrac{1}{4}N.$$

All other basis functions can be obtained from $|\phi\rangle$ operated by the quasispin raising operators L_+ and I_+, such as $I_+|\phi\rangle$, $I_+L_+|\phi\rangle$, ...

III. COUPLING BASIS STATES $|JK\rangle$ AND GOOD QUASISPIN STATES $|J\rangle$

Generally speaking the common eigenstates $|IM_ILM_L\rangle$ of I^2, I_z and L^2, L_z are not the eigenstates of J^2, J_z, which are denoted by $|JK\rangle$ with $K \equiv M_J$. But we can expand the state $|JK\rangle$ on the basis $|IM_ILM_L\rangle$:

$$|JK\rangle = \sum_{M_IM_L} B_{M_IM_L}^{(JK)} |IM_ILM_L\rangle_K, \tag{6a}$$

$$\sum_{M_IM_I'} B_{M_I'M_L'}^{(JK)} B_{M_IM_L}^{(JK)} = \delta_{M_IM_I'}\delta_{M_LM_L'},$$
$$M_I + M_L = M_I' + M_L' = K, \tag{6b}$$

where the lower index K represents that only the basis with $M_I + M_L = K$ is contained in the expansion. Then after diagonalizing the operator J^2 in the basis space $|IM_ILM_L\rangle$, all J values which satisfy the conditions $|K| \le J \le \frac{1}{2}N$, and the corresponding coefficients $B_{M_IM_L}^{(JK)}$, i.e., all the states $|JK\rangle$ with definite K value, are obtained. This procedure is similar to that used for determining the total angular momentum of the state in the large-scale shell model calculation [6].

According to the Eqs. (2) and (3) and the definition $J^2 = J_+J_- + J_z^2 - J_z$ and taking into account the eigenvalues of the operators I_+ and L_+, it is not difficult to deduce the formula of the matrix elements of J^2:

$$\langle IM_I'LM_L'|J^2|IM_ILM_L\rangle = 2[I(I+1) + M_IM_L]\delta_{M_I'M_I}\delta_{M_L'M_L}$$
$$+[I(I+1) - M_I'M_I]^{1/2}[L(L+1) - M_L'M_L]^{1/2}(\delta_{M_I',M_I+1}\delta_{M_L',M_L-1} + \delta_{M_I',M_I-1}\delta_{M_L',M_L+1}). \tag{7}$$

It is evident from Eq.(4) that in the case of $w = 0$ the Hamiltonian H is diagonal in states $|JK\rangle$ and the corresponding energy is

$$E_{JK} \equiv \langle JK|H(w=0)|JK\rangle = \sum_{M_IM_L} (B_{M_IM_L}^{(JK)})^2 (K + 2vM_IM_L)\delta_{M_I'M_I}\delta_{M_L'M_L}$$
$$+v \sum_{M_LM_L'M_I'M_I} B_{M_I'M_L'}^{(JK)} B_{M_IM_L}^{(JK)} [I(I+1) - M_I'M_I]^{1/2}$$
$$\times[L(L+1) - M_L', M_L]^{1/2}(\delta_{M_I',M_I-1}\delta_{M_L',M_L+1} + \delta_{M_I',M_I+1}\delta_{M_L',M_L-1}). \tag{8}$$

In the general case of $w \neq 0$ the states $|JK\rangle$ are not the eigenstates of H, but they can be used as the basis vectors diagonalizing H. Then the good J eigenstates of H are expanded on the basis $|JK\rangle$:

$$|J\rangle = \sum_K A_K^J |JK\rangle, \quad K = \begin{cases} 0, \pm 2, \pm 4, \ldots & \text{for even } K, \\ \pm 1, \pm 3, \pm 5, \ldots & \text{for odd } K, \end{cases} \tag{9}$$

where summation index K takes either even integers or odd integers depending on state $|J\rangle$ belonging to an even K or odd K group [5]. This feature of the state $|J\rangle$ results from the fact that the w term of interaction only mixes with the states having K and $K \pm 2$, or $\Delta K = \pm 2$. It is easy to show that the matrix elements of H in basis functions $|JK\rangle$ are given by

$$\langle JK'|H|JK\rangle = E_{JK}\delta_{K'K} + w \sum_{M_I' M_L' M_I M_L} B^{(JK')}_{M_I' M_L'} B^{(JK)}_{M_I M_L} [I(I+1) - M_I' M_I]^{1/2} \times [L(L+1) - M_L' M_L]^{1/2} (\delta_{M_I', M_I - 1}\delta_{M_L', M_L - 1} + \delta_{M_I', M_I + 1}\delta_{M_L', M_L + 1}), \quad (10)$$

where E_{JK} is calculated in terms of Eq. (8). Then by diagonalizing H matrix the exact energy eigenvalues and the wave functions A^J_K of the good J states are obtained.

Before ending this section we should point out that the state $|JK\rangle$ is in fact the vector coupling state of two quasispins **I** and **L**. It is obvious from Eq. (3) that the quasispin **J** is just the vector sum of quasispin **I** and **L**: $\mathbf{J} = \mathbf{I} + \mathbf{L}$. Then the coupling state $|JK\rangle$ can be produced in terms of Clebsch-Gordan (CG) coefficients as follows:

$$|JK\rangle = \sum_{M_I M_L} (IM_I LM_L|JK)|IM_I\rangle|LM_L\rangle. \quad (11)$$

By comparing Eq.(6) with Eq. (11) we know that the coefficients $B^{(JK)}_{M_I M_L}$ are just the CG coefficients $(IM_I LM_L|JK)$. Consequently, the diagonalization of Eq. (7) is in itself a kind of numerical method for calculating the CG coefficients. Because we must establish the diagonalizing procedure to calculate the energy eigenvalues, it is very convenient to diagonalize the J^2 matrix using the same procedure. Then calculating the CG coefficients,which is required for use of Eq. (11), is not necessary at all.

IV. DETERMINATION OF BAND STRUCTURE

First of all, we work in the case of $w = 0$. In this case the energy spectra of an even-N system are given by Eq. (8). In order to more distinctly describe the characters of this energy spectrum, we rewrite Eq. (8) in terms of Eqs. (4) and (11) as follows:

$$E_{JK} \equiv \langle JK|H(w=0)|JK\rangle = K + vJ(J+1) - \tfrac{1}{2}vN(\tfrac{1}{4}N + 1), \quad (12)$$

where we have made use of the values $I = L = \frac{1}{4}N$. It is shown by Eq. (12) that when $w = 0$ the energy spectra of an even-N system are formally coincident with that of an axial symmetry rigid rotor. The axial symmetry of the Hamiltonian $H(w = 0)$ is fully exhibited. However, it must be pointed out that there is an important difference between Eq. (12) and the energy formula of the axial symmetry rigid rotor. The Hamiltonian of a rotor is invariant under the rotation of total spin by angle π about the intrinsic x axis. And so there are no odd J states in the $K = 0$ band for an even-N system. But in the SU(2)×SU(2) model the Hamiltonian $H(w = 0)$ does not have the mentioned invariance, which can be easily shown by using Eq. (4), so that the odd J states appear in the $K = 0$ band of the even-N system. In spite of the appearance of an odd J state, the research of the effectiveness of CHF methods does not appear to be affected. In the following we are only interested in the even J states and omit the odd J states in the band $K = 0$. In the case $w \neq 0$ the pairing interaction leads to K mixture of the states and results in triaxial symmetry of the Hamiltonian. In fact, the state $|J\rangle$ in Eq. (9) has the same form of the wave function of a triaxial rigid rotor (7).

As mentioned above, the good quasispin states $|J\rangle$ have no definite K values, therefore in the energy spectra no band structure should be exhibited. However, we can use the following procedure to distinguish an approximate band structure. We first calculate the overlapping

$$\langle J\underline{K}|J\rangle = \sum_K A^J_K \langle J\underline{K}|JK\rangle = A^J_{\underline{K}}, \quad (13)$$

and then take the state having maximum $A^J_{\underline{K}}$ among the states with same J value as a member of the $\underline{K}$ band. In this way a set of selected states possessing definite $\underline{K}$ constitute an approximate "rotational band." Here we use the approach quantum number $\underline{K}$ to denote this band in distinguishing the band quantum number K of the axial symmetry system. It must be noted that for this selected state $A^J_{\underline{K}}$ is not necessarily the maximum amplitude in the expansion of the state itself. By practical calculations we know that the procedure described above is effective in most cases, but not in all cases. We sometimes have no choice but to take the state which has small $A^J_{\underline{K}}$, even the minimum $A^J_{\underline{K}}$, as the member of the $\underline{K}$ band. This is related to the K structure of the state. As evidence an example is given here. In Fig. 1 the variation in the K structure of the states $\underline{K} = 0$, $J = 10$, for $N = 20$ system is exhibited with the varying strength parameters of the interactions. In the figure $a^2_K \equiv (A^J_K)^2$. It is shown that the single-peak K structure only exists in the region around $v = 0.1$ and $w = 0.05$. In the region around $v = 0.2$ there are double-peaks K structures, and when v increases beyond about 0.3 even triple-peaks

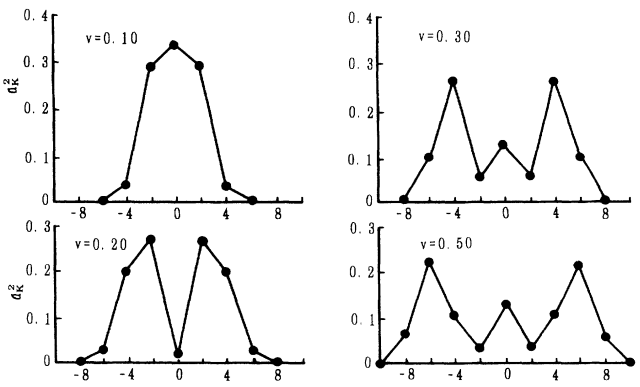

FIG. 1. The K structures of states $\underline{K} = 0$ and $J = 10$ of the system $N = 20$ at different values of v ($w = v/2$).

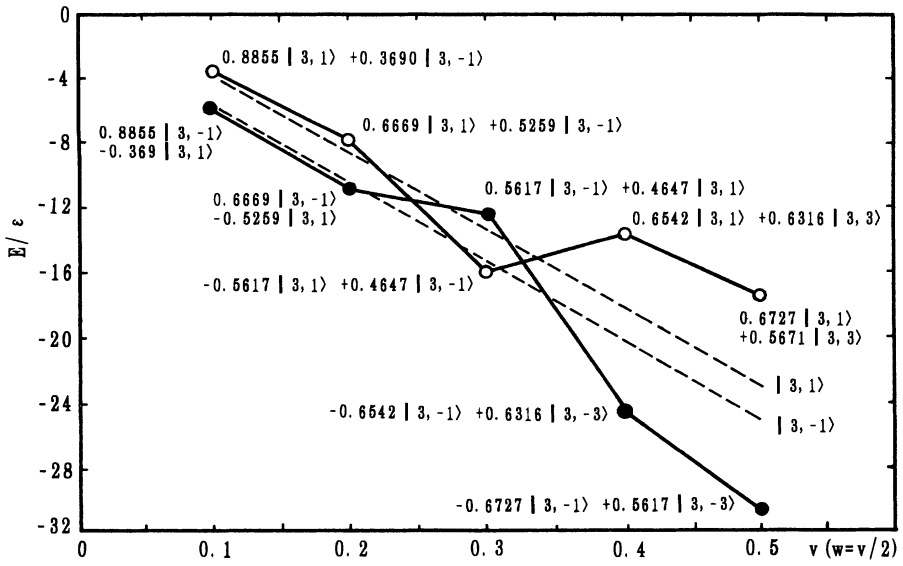

FIG. 2. The energies of states $J = 3$ in the bands $K = \pm 1$ of the system $N = 20$ and the first two maximum amplitudes in the K structures plotted versus the interaction strength. ○ for $K = 1$; • for $K = -1$.

K structures present. However, all the K structures are symmetrical about $K = 0$, and a kind of symmetrical "repulsion" in the strength distribution of the wave function appears. Because the amplitude for the peak value K is equal to that for the peak value $-K$, we cannot assign the $\underline{K}$ value according to the peak value for the state which has a symmetrical "repulsion" K structure. Only the value $\underline{K} = 0$ must be assigned to this state.

At the same time, with the increase of the interaction strength, the band-crossing phenomena are frequently observed. In Fig. 2 the energies of the states $J = 3$ in the bands $\underline{K} = \pm 1$ of the system $N = 20$ are shown. In the figure the numbers beside points and circles are the first two maximum amplitudes and the corresponding basis vectors $|JK\rangle$ in the K structures of the states $|J = 3\rangle$. The energies of the basis functions $|J = 3, K = \pm 1\rangle$ are also given by dashed lines for comparison. Two crossings between the band $\underline{K} = 1$ and the band $\underline{K} = -1$ are shown. The first crossing comes from the amplitude exchange between basis $K = \pm 1$, and the second crossing is due to the increase of amplitudes of basis $K = \pm 3$.

V. NEW CHF PRESCRIPTIONS AND GOOD J STATE ENERGY SPECTRA

In order to introduce the variational parameters, we perform two successive transformations with fermion operators $a^\dagger_{p\sigma}$ and $a_{p\sigma}$:

$$\begin{pmatrix} a_{-p-} \\ a_{p+} \end{pmatrix} = \begin{pmatrix} \cos\frac{1}{2}\beta & -i\sin\frac{1}{2}\beta \\ -i\sin\frac{1}{2}\beta & \cos\frac{1}{2}\beta \end{pmatrix} \begin{pmatrix} c_{-p-} \\ c_{p+} \end{pmatrix},$$

$$\begin{pmatrix} c_{p-} \\ c_{p+} \end{pmatrix} = \begin{pmatrix} \cos\frac{1}{2}\alpha & -i\sin\frac{1}{2}\alpha \\ -i\sin\frac{1}{2}\alpha & \cos\frac{1}{2}\alpha \end{pmatrix} \begin{pmatrix} b_{p-} \\ b_{p+} \end{pmatrix}. \tag{14}$$

Taking the unperturbed ground state $|\phi\rangle = \prod_p b^\dagger_p|0\rangle$ as trial function, we get in the b representation

$$\langle\phi|H|\phi\rangle = -\tfrac{1}{2}N\cos\alpha\cos\beta + \tfrac{1}{4}vN^2(\cos^2\beta - \tfrac{1}{2}) + \tfrac{1}{2}wN(\cos^2\alpha - \cos^2\beta), \tag{15a}$$

$$\langle\phi|J_z|\phi\rangle = -\tfrac{1}{2}N\cos\alpha\cos\beta, \tag{15b}$$

$$\langle\phi|J^2|\phi\rangle = \tfrac{1}{2}N(1 + \tfrac{1}{2}N\cos^2\beta), \tag{15c}$$

$$\langle\phi|J_y|\phi\rangle = \tfrac{1}{2}N\sin\alpha\cos\beta, \tag{15d}$$

$$\langle\phi|J_y^2|\phi\rangle = \tfrac{1}{4}N[1 - \cos^2\alpha + (N-1)\cos^2\beta - (N-2)\cos^2\alpha\cos^2\beta]. \tag{15e}$$

Let us consider now the J_y and J_y^2 simultaneous constrained HF theory. In this case the Hamiltonian and the constraint are

$$H' = H - \omega J_y - \theta J_y^2, \tag{16a}$$

$$\langle\phi|J_y|\phi\rangle = [J(J+1) - K^2]^{1/2}. \tag{16b}$$

Taking into account the variational conditions

$$\partial\langle\phi|H'|\phi\rangle/\partial\alpha = 0, \quad \partial\langle\phi|H'|\phi\rangle/\partial\beta = 0, \tag{17}$$

and Eqs. (15a), (15d), (15e), the solutions of Eqs. (16) can give

$$\begin{aligned} &\beta = 0, \quad \sin\alpha = (2/N)[J(J+1) - K^2]^{1/2}, \\ &\theta = 2w/(N+1), \\ &\omega = [J(J+1) - K^2]^{1/2}\{[\tfrac{1}{4}N^2 + K^2 - J(J+1)]^{-1/2} \\ &\qquad - 8w/(N+1)\}, \\ &E_{\mathrm{CHF}}(J_y, J_y^2) = -[\tfrac{1}{4}N^2 + K^2 - J(J+1)]^{1/2} + \tfrac{1}{8}vN^2 \\ &\qquad - 2w[J(J+1) - K^2 + \tfrac{1}{4}N]/(N+1), \end{aligned} \tag{18}$$

where $E_{\mathrm{CHF}}(J_y, J_y^2)$ are the energy eigenvalues of the J_y and J_y^2 simultaneous constrained Hamiltonian (16a). It is evident from the first two expressions of Eq. (18) that because of the condition $\sin^2\alpha \leq 1$ there is a restriction on J values: $J(J+1) \leq \frac{1}{4}N^2 + K^2$. Then if $K \geq (N/2)^{1/2}$, the maximum of J is $N/2$; if $K < (N/2)^{1/2}$, the maximum of J is $N/2$; if $K < (N/2)^{1/2}$, the maximum of J is restricted by the condition $J \leq \frac{1}{2}N - 1$.

Now let us turn our attention to another constrained HF method, that is, J_z and J^2 simultaneous constrained HF theory. The Hamiltonian and the constrained conditions are

$$H' = H - \lambda J^2 - \sigma J_z, \tag{19a}$$

$$\langle\phi|J^2|\phi\rangle = J(J+1), \quad \langle\phi|J_z|\phi\rangle = K. \tag{19b}$$

Taking into account Eqs. (15a)–(15c) and inserting Eq. (19) into Eq. (17), we can obtain the solutions of this problem:

$$\cos\beta = 2G_J/N, \quad \cos\alpha = -K/G_J, \qquad G_J \equiv [J(J+1) - \tfrac{1}{2}N]^{1/2}, \tag{20a}$$

$$\lambda = vN - \tfrac{1}{2}w(4 - NK^2/G_J^2), \quad \sigma = 1 + wNK/G_J^2, \tag{20b}$$

$$E_{\mathrm{CHF}}(J^2, J_z) = K + v(G_J^2 - \tfrac{1}{8}N^2) + \tfrac{1}{2}wN(K^2/G_J^2 - 4G_J^2/N^2), \tag{20c}$$

with the condition $J(J+1) \geq \frac{1}{2}N$. This condition means that lower J states near $J = 0$ are rejected in J^2 and J_z CHF theory. But because the eigenenergies only depend on G_J^2, we can get the energies of states for which $J(J+1) < N/2$ by extrapolating Eq. (20c) towards $J = 0$.

The eigenstates in the two CHF theories mentioned above possess good quantum numbers J and K. When $w = 0$ the states $|JK\rangle$ of the even-N system constitute the spectra of the rigid symmetrical rotor. When w is not equal to zero the band structure is destroyed by the K mixture. But we can distinguish the approximate band structure by using the method described in the last section. There are two examples, given in Figs. 3 and 4. It can be seen from Fig. 3 that if w is little, the band structure is only slightly affected. The orders of the states in the bands $\underline{K} = 0$, 1, and 2 are the same as that for $w = 0$, and the energy level spacings almost keep unchanged. With increasing w the band structure becomes more and more indistinct. The orders of the levels in the bands $\underline{K} \neq 0$ are strikingly perturbed, and the level spacing distributions are also different from that for $w = 0$, as shown in Fig. 4. When the value of w is very large, it is impossible to distinguish the band structure. However, it is worthwhile to point out that the energies of the states in the band $\underline{K} = 0$ are essentially not affected by the change of w. When the value of w becomes rather large, the band $\underline{K} = 0$ conserves fairly well in spite of the obvious changes in the K structures of the wave functions (see the examples shown in Fig. 1).

In Figs. 3 and 4 the energy spectra on the right-hand side are the yrast spectra for even K and odd K. Those spectra have no character of band structure at all: the level orders are thrown into confusion and the level spacings are irregular. In fact, the yrast states which are states possessing minimum energy for given quasispin J do not have the same value of $\underline{K}$, and so do not constitute a rotational band. This tells us that we should deal carefully with the "yrast band" for the actual nuclei and pay attention to the inner structure of the members of this band [8].

Also in Figs. 3 and 4 we compare the exact spectra with the constrained HF spectra; that is, CHF(J_y, J_y^2) and CHF(J^2, J_z) spectra, respectively. On the whole, the CHF(J_y, J_y^2) spectra are pressed too low relative to the exact $|JK\rangle$ spectra and $|J\rangle$ spectra, while CHF(J^2, J_z) spectra have energy level spacings which are comparable to that of $|JK\rangle$ and $|J\rangle$ spectra. At the same time, the bandhead levels of the bands $K = 1$ and $K = 2$ also approach the ground state $K = 0$, especially for CHF(J_y, J_y^2) spectra. Therefore, in practical applications of CHF theories it is necessary to add a K-dependent constant term to correct the energies of the bandhead levels.

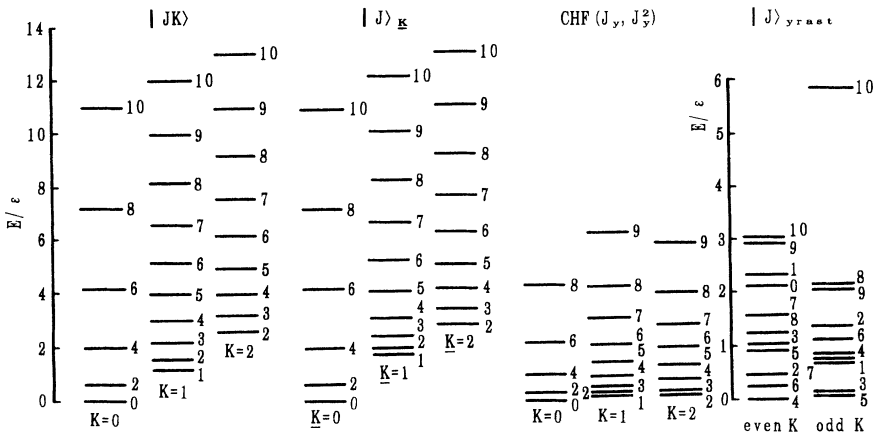

FIG. 3. The energy spectra of the system $N = 20$ for $v = 0.10$ and $w = 0.05$.

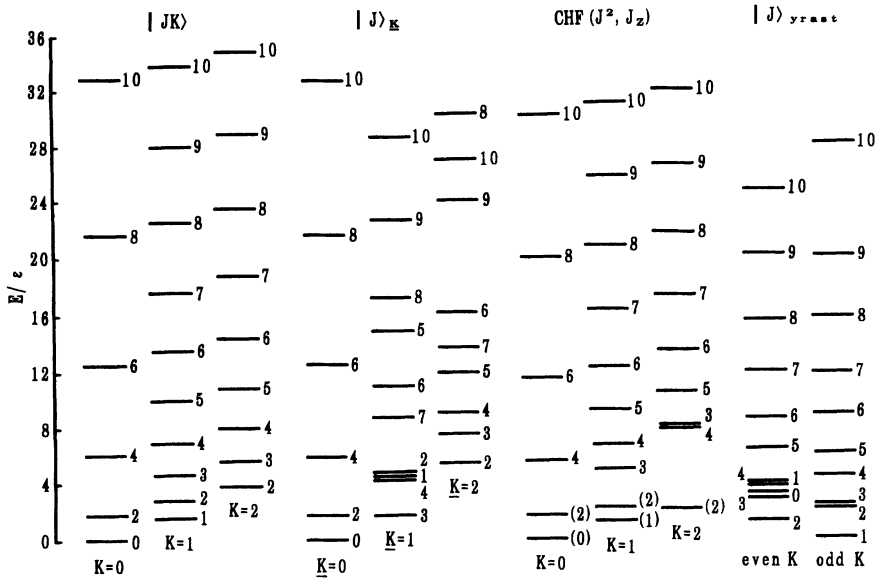

FIG. 4. The energy spectra of the system $N = 20$ for $v = 0.30$ and $w = 0.15$. The J values in brackets are extrapolated using Eq. (20c).

VI. CONCLUSIONS

(1) By means of diagonalizing the J^2 matrix, the coupling basis $|JK\rangle$ can be constructed from the SU(2)×SU(2) basis functions $|IM_I LM_L\rangle$. When $w = 0$ the states $|JK\rangle$ are the energy eigenstates of $H(w = 0)$, and the spectra possess the characteristics of the axial symmetry rigid rotor energy spectra.

(2) In the case $w \neq 0$ the Hamiltonian can be diagonalized in the coupling basis functions $|JK\rangle$. Then we obtain the eigenstates $|J\rangle$, which are of K mixture, and the corresponding energy spectra. If the interactions are not strong, the approximate band structure can be distinguished among the good J but K-mixed energy levels. In other words, it is possible to design an approximate quantum number K (denoted by $\underline{K}$) to the state $|J\rangle$. If the interactions are strong the band structure is destroyed by the striking K mixture, except for bands such that $\underline{K} = 0$. On the other hand, the yrast states do not constitute a good rotational band in either weak interactions or strong interactions mentioned here.

(3) From the comparisons of CHF(J_y, J_y^2) and CHF(J^2, J_z) spectra with the exact energy spectra, we know that CHF(J_y, J_y^2) spectra are too low in energy, while the CHF(J^2, J_z) spectra may be comparable to $|JK\rangle$ and $|J\rangle$ spectra, except for the states for which $K = 0$ and the condition $J(J+1) > \frac{1}{2}N$ is not satisfied. On the whole those two new CHF prescriptions are the effective methods to obtain the states having good quantum numbers J, K and the spectra with multiband structure, if the interactions are not very strong.

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