Non-adiabatic contributions to static two-pion-exchange nuclear potentials

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Recent evidence of the importance of the pion in the dynamics of few-nucleon systems is reviewed. Calculations of the triton binding energy and the nucleon-nucleon phase shifts, and the application of chiral perturbation theory to few-nucleon systems, have highlighted the role of the pion. Qualitative aspects of chiral constraints in few-nucleon forces are reviewed, and the connection between the static Brueckner-Watson and Taketani-Machida-Ohnuma two-pion-exchange two-nucleon forces is shown to be the same as that between the Weinberg and Coon-Friar two-pion-exchange three-nucleon forces. Compact expressions for these forces are given in the Appendix.

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I. INTRODUCTION

Three recent developments have highlighted the role of the pion in intranuclear dynamics. As a consequence, in very light nuclei the influence of the pion is greater than had been previously supposed.

The first of these developments involves the three-nucleon problem [1]. Solving the Schrödinger (or Faddeev) equation for the triton is now rather routine. It is possible to perform these calculations with an error of less than 1%. From such solutions (called "complete," or "exact"), it is possible to determine the influence of individual parts of the nuclear potential. In this way we have learned [2,3] that the bulk of the triton potential energy ($\langle V\rangle \sim 50$ MeV) is due to the tensor force (i.e., $\langle V_T\rangle/\langle V\rangle \sim 2/3$), while the one-pion-exchange potential (OPEP) contributes 70–80 % of that energy. Similar results are obtained for other light nuclei [4]. This dominating pionic contribution is due to the tiny pion mass (which makes OPEP by far the longest-ranged component of the force), to the pion's negative parity (which is the origin of its strong tensor force), and to the near cancellation of the (strong) attractive and repulsive forces of shorter range.

All of these features can be attributed to the chiral symmetry that is concomitant with the underlying QCD [5], to the Goldstone-mode realization of that symmetry, and to the validity of "naive dimensional power counting" [6] in chiral perturbation theory treatments [7] of that symmetry.

The second development concerns the nucleon-nucleon (NN) force. Recently, the Nijmegen group [8] have performed a very comprehensive and sophisticated phase-shift analysis of the NN (pp and np) scattering data and have constructed new versions of NN forces [3] as well. This work has achieved a number of significant successes by explicitly treating the OPEP contribution, as well as the Coulomb, magnetic moment, and vacuum polarization forces. One success is a significantly better fit of potential model predictions to the data than had been previously obtained; the χ^2/per statistical degree of freedom is only slightly larger than 1, compared to a typical previous value of 2. Using their high-quality multienergy phase-shift analysis, it proved possible to determine accurately the various (charge-state) pion-nucleon coupling constants, which are consistent with being equal. Moreover, using the NN scattering data to fit [9] the pion masses, they find $m_{\pi^\pm} = 139.4(10)$ MeV and $m_{\pi^0} = 135.6(13)$ MeV, which are consistent with the free values. The small error bars on these masses are a reflection of the importance of OPEP in the nuclear force. Finally, the Nijmegen group [10] has recently taken the first steps toward incorporating the two-pion-exchange potentials into their scheme.

One of the qualitative results of the "exact" or "complete" triton calculations discussed earlier is that "realistic" potential models (viz., models that fit the NN scattering data reasonably well and contain OPEP) underbind the triton by up to one MeV. If this result still holds when relativistic corrections to the force (and other subtle improvements) are included, only three-nucleon forces remain to increase the binding to the experimental value. Three-body forces always arise when there are hidden degrees of freedom that are not explicitly treated. The two-pion-exchange three-nucleon force has the longest range. Typical of the mechanisms [11] leading to such forces are nucleon-antinucleon "pair" excitations, the virtual-Δ excitation (by pion emission and absorption), and pion rescattering via the (pion-nucleon) σ term.

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Alternative schemes exist for performing the calculation of these forces. The by-now-august Tucson-Melbourne three-nucleon force [11] was based on a pion-nucleon scattering amplitude derived using PCAC, current algebra, and phenomenological input. Much later, Coon and Friar [12] (denoted CF) used the Weinberg nonlinear (field-theoretic) model [13] to perform a consistent calculation of those components of the two-pion-exchange force that involve only pion and nucleon degrees of freedom. Roughly speaking, these components are characterized by nucleon-antinucleon "pair" excitations, and/or by "two pions in the air at the same time." Only the resulting three-nucleon forces were explicitly listed in CF, although parts of the two-pion-exchange potentials can be developed from their results.

The third development concerns the use of chiral perturbation theory to calculate nuclear forces. Recently, Weinberg [14] developed such three-nucleon forces using an effective chiral Lagrangian, together with a power-counting scheme that permits consistent expansions. Keeping only pion and nucleon degrees of freedom, he developed three-nucleon forces in the leading order (tree approximation) of the effective Lagrangian, and showed how higher-order potentials could be calculated. This application of chiral perturbation theory to nuclear forces was extended by Ordóñez and van Kolck [15] beyond leading order, and included both two-pion-exchange two- and three-nucleon forces. We note that the three-nucleon forces developed in Refs. [12] and [14] are nominally both quantitatively and qualitatively different, and the primary purpose of this work is to explain that difference. We will point out that the same difference originally arose long ago in the context of two-nucleon forces [16,17] and has its origin in different *definitions* of that force.

We further note that the dominance of pion degrees of freedom in certain nuclear processes is shown by one of the biggest success stories in nuclear physics in the past two decades: pion-exchange currents. Meson-exchange currents arise because charged mesons being exchanged between nucleons can and will interact with external electromagnetic fields. The pion-exchange parts of these currents are known to dominate, and in few-nucleon systems, where accurate calculations eliminate nuclear structure uncertainties, processes exist [18,19] that are dominated by those currents. A very rough (and subjective) estimate is that the non-OPE parts of these currents contribute $\sim$ 20–30 %. Typically, OPE currents are nearly sufficient to compensate for deficiencies in the impulse approximation, although sensitivity to the short-range parts of the former (and a lack of their understanding) makes a more quantitative statement problematic. Nevertheless, one- and two-pion-exchange processes appear to be dominant in many aspects of few-nucleon systems [20–22]. It would be extremely interesting if we could further refine our appreciation of their contribution.

In order to extract the most important parts of interaction operators in few-nucleon systems, we introduce a simple power-counting scheme [1]. If one uses the uncertainty principle in few-nucleon systems ($R \sim 1.5$–2.0 fm), one deduces an average momentum, $\bar{p}c \sim 100$–140 MeV. For convenience, we introduce a mnemonic and equate $\bar{p}c \sim m_\pi c^2$. This produces a triton kinetic energy ~ 30 MeV, which is somewhat lower than the more realistic $\langle T \rangle \sim 45$ MeV. Nevertheless, $\bar{p}c$ is significantly smaller than the nucleon mass, Mc^2, and counting powers of $1/M$ (i.e., $\bar{p}/Mc$) is significantly easier than counting powers of $1/c$. Because a nucleus is a weakly bound system, we count the potential and Hamiltonian as *implicitly* of order $(1/M)$, corresponding to the kinetic energy, which is explicitly of that order.

We make a slight distinction between non-adiabatic and non-static, appropriate to our subsequent discussion. Non-adiabatic terms in the potential are those which arise from retardation in the meson propagation (i.e., the finite time that it takes for a meson to traverse from one nucleon to another). Non-static terms are those which vanish when the nucleon mass, M, is taken very large. As we will see, perturbation theory allows non-adiabatic factors to be canceled, leaving a static residue, even though the non-adiabatic terms that we will examine are formally of order $(1/M)$.

In what follows, we will detail how choices (implicit or explicit) in the definition of the potential that arise naturally in Rayleigh-Schrödinger perturbation theory will also alter the definition of what constitutes a three-body force. These choices arise because the potential is an unphysical theoretical object, obtained from a *subamplitude* according to a set of prescriptions. None of these definitions is wrong, and consistency conditions relate each one to the others. Expediency and theoretical prejudice are usually our guides in selecting the prescriptions we use.

II. QUALITATIVE AND CHIRAL ASPECTS OF THE PION-EXCHANGE NUCLEAR FORCE

Historically, the construction of pion-exchange nuclear forces has been coupled to the question of which form of pion-nucleon coupling to use: pseudoscalar (PS) coupling or pseudovector (PV) coupling. Associated with the former is a dimensionless coupling constant, g [$= 13.02(5)$] [9], while associated with the latter is $f \equiv g/2M$ and, more frequently, a rationalized (squared) dimensionless constant, $f_0^2 = (\frac{g m_\pi}{2m})^2/4\pi = 0.0745(6)$. The former number is very large and is typical of strong-interaction couplings, while the second is quite small. Although the two types of pion-nucleon coupling between on-shell nucleons are identical, off-shell they are different. Naive PS coupling leads to very strong nucleon-antinucleon "pair" contributions to the two-pion-exchange two- and three-nucleon forces [23].

This strength can be easily determined by examining effective couplings of one and two pions to a nucleon. The effective nonrelativistic (single) pion-nucleon coupling has the well-known form: $-f\boldsymbol{\sigma}\cdot\boldsymbol{\nabla}\boldsymbol{\tau}\cdot\boldsymbol{\pi}$, where $\boldsymbol{\sigma}$ is the nucleon (Pauli) spin operator, $\boldsymbol{\tau}$ is the corresponding isospin operator, and $\boldsymbol{\pi}$ is the pion field operator. The leading-order effective two-pion seagull coupling from PS Born terms are generated by large (intermediate-state) nucleon-antinucleon Z graphs and can be obtained from the (negative of the) last term of Eq. (4) of CF: $2Mf^2\boldsymbol{\pi}^2$.

In chiral models, the large Born-term contribution is exactly canceled by a seagull [the last term in Eq. (4) of CF]. Historically, the *ad hoc* prescription of "pair" or "Z graph" suppression was invoked to eliminate this large vertex. Chiral symmetry [24] does this automatically, and the effective (chiral) Hamiltonian of Eq. (5) of CF does not contain such a large term. Finally, for technical reasons, the derivation of CF was accomplished by using a general chiral representation, neither pure PS nor PV coupling, but rotated by an arbitrary amount between them. This emphasizes an important point: the issue is not the form of the coupling (which is basically an unphysical choice of representation) but rather the constraints enforced by the underlying chiral symmetry.

The immediate consequences of that constraint can be estimated rather simply. We construct NN potentials and count powers of f_0^2, M, and m_π, and treat (and consequently ignore) the dimensionless radial factors in the potentials as order (1). Then, OPEP is of order $(f_0^2 m_\pi)$, or roughly 10 MeV (per nucleon pair), which for three nucleon pairs is close to the calculated 35–40 MeV contribution of OPEP to $\langle V\rangle$ in the triton. If one iterates OPEP in perturbation theory, the second-order process involves an intermediate Green's function $[(E-H)^{-1} \sim (\frac{1}{M})^{-1} \sim M]$ and produces a contribution of order $(f_0^4 M)$, which is not significantly smaller than f_0^2. This term and higher-order iterates are not small and they should be considered together, which occurs naturally when the Schrödinger equation is solved. The two-pion-exchange two-nucleon force (as we will see) is of leading order $(f_0^4 m_\pi)$, or ~ 1 MeV/pair. The unphysical PS seagull leads to terms of order $(f_0^4 M^2/m_\pi)$, or 35 MeV/NN pair.

The three-nucleon forces from "pure" two-pion exchange involving this seagull would be of order $(f_0^4 M)$, or 5 MeV/nucleon triplet, a very large contribution indeed. The result of CF was that this force is actually of order $(f_0^4 m_\pi^2/M)$ or ~ 0.1 MeV/triplet, and vanishes for static nucleons ($M \to \infty$). It is believed that other mechanisms generate the bulk of the complete two-pion-exchange three-nucleon force [25,26] (~ 0.5–1 MeV/nucleon triplet).

The net result of these considerations is that in the few-nucleon systems, OPEP dominates the two-, three-, ...pion-exchange two-nucleon forces, which get progressively smaller. Furthermore, the two-nucleon forces are much larger than the three-nucleon forces. Because the bulk of the potential energy comes from the part of the theory that is most believable and best understood, it is plausible that a credible case can be established that the triton binding energy can be understood from "first principles," although this is by no means certain.

III. TWO-PION-EXCHANGE NUCLEAR FORCES

In this section, we reprise the essential elements of the derivation of CF. In that work we derived forces of order (f_0^4) and (f_0^4/M); here we work to static order only ($M \to \infty$), which (greatly) simplifies the derivation to its essentials. At this level the representation dependence [12,27] on the unphysical chiral rotation angle (denoted by μ in CF) disappears. The method that we use is generic and avoids the unnecessary complications of quasipotential equations. To the order that we work there is also no dependence on the unphysical "quasipotential" parameter (denoted by ν in CF).

A Foldy-Wouthuysen reduction of the Lagrangian equations of motion for the Weinberg non-linear model leads to an effective static interaction [12] for one or two pions with each nucleon:

$$L_{\pi N} \simeq \bar{N}\left(-f\gamma_5\gamma^\mu\partial_\mu \boldsymbol{\tau}\cdot\boldsymbol{\pi} - \frac{f^2}{g_A^2}\gamma^\mu \boldsymbol{\tau}\cdot\boldsymbol{\pi}\times\partial_\mu\boldsymbol{\pi}\right)N, \quad (1)$$

where g_A is the axial-vector coupling constant (~ 1.26), and hence

$$H_{\pi N} \simeq -f\boldsymbol{\sigma}\cdot\boldsymbol{\nabla}(\boldsymbol{\tau}\cdot\boldsymbol{\pi}) + \frac{f^2}{g_A^2}\boldsymbol{\tau}\cdot\boldsymbol{\pi}\times\dot{\boldsymbol{\pi}} + \cdots \quad (2)$$

in the nucleon Hilbert space. Only the leading-order (one-pion) and Adler-Weisberger (two-pion-seagull) terms contribute to $H_{\pi N}$, which is the static limit of Eq. (5) of CF. Using superposition these can be extended to operators for the absorption of incoming pions by a nucleus:

$$J_\pi^\alpha(\mathbf{q}) = -if\sum_{i=1}^{A}\tau^\alpha(i)\boldsymbol{\sigma}(i)\cdot\mathbf{q}e^{i\mathbf{q}\cdot\mathbf{x}_i}, \quad (3)$$

$$\bar{S}_{2\pi}^{\alpha\beta}(\mathbf{q}_1+\mathbf{q}_2) = \frac{f^2}{g_A^2}\epsilon^{\alpha\beta\gamma}\sum_{i=1}^{A}\tau^\gamma(i)e^{i(\mathbf{q}_1+\mathbf{q}_2)\cdot\boldsymbol{x}_i}, \quad (4)$$

where $\epsilon^{\alpha\beta\gamma}$ is the antisymmetric tensor. The nuclear two-pion seagull in Eq. (4) will be ignored until the Appendix.

With this pion-nucleus (many-body) vertex operator, J_π, one can perform perturbation theory as illustrated in Fig. 1. The shaded line in Fig. 1(a) corresponds to the nucleus Green's function, $(E-H_0)^{-1}$, determined by a residual nuclear Hamiltonian, H_0, which does not contain pion exchanges. It is convenient and appropriate to add and subtract a "counter-term" potential, V_π^0, which for our static derivation is just OPEP. The rearranged perturbation theory [27] then uses an unperturbed Hamiltonian, H, which contains OPEP plus non-pionic contributions, and which leads to a "renormalized" Green's function, $(E-H)^{-1}$. For consistency's sake we must include the counter term (i.e., $-V_\pi^0$) in separate diagrams, such as Figs. 1(b) and 1(d)–1(g). The counter term, therefore, is nothing more than a device to insure that we do not double count. That is, it formally allows "subtraction" of OPEP from the perturbation calculation, since it is already included in H. Because OPEP is the static limit of Fig. 1(a), we are effectively performing perturbation theory in the retardation of the pion exchanges.

One can calculate Fig. 1(a) in any of a variety of ways, leading to the obvious result:

$$\hat{\Sigma}_\pi = V_\pi(E) = -\int\frac{d^3q}{(2\pi)^3(2E_\pi)}J_\pi^\alpha(\mathbf{q})\frac{1}{E_\pi+(H-E)} \times J_\pi^\alpha(-\mathbf{q}), \quad (5)$$

where $E_\pi = (\mathbf{q}^2 + m_\pi^2)^{1/2}$ and $(2E_\pi)^{-1}$ is the normalization factor for the pion fields. This result can be expanded using our power-counting scheme for $(H - E)/E_\pi \sim m_\pi/M \ll 1$, where only corrections of leading order (f_0^2/M) will be kept. We will consistently ignore contributions of order (f_0^4/M), (f_0^2/M^2), and (f_0^6). This produces

$$V_\pi(E) \simeq V_\pi^0 + V_{2\pi}^A + \frac{1}{2}\{E - H, \hat{Z}_\pi\} + \cdots, \tag{6}$$

where

$$V_\pi^0 = -\int \frac{d^3q}{(2\pi)^3} \frac{J_\pi^\alpha J_\pi^\alpha}{2E_\pi^2}, \tag{7}$$

$$V_{2\pi}^A = \frac{1}{2}\int \frac{d^3q}{(2\pi)^3} \frac{[J_\pi^\alpha, [H, J_\pi^\alpha]]}{2E_\pi^3}, \tag{8}$$

and

$$\hat{Z}_\pi = -\int \frac{d^3q}{(2\pi)^3} \frac{J_\pi^\alpha J_\pi^\alpha}{2E_\pi^3}. \tag{9}$$

Equations (5)–(9) correspond to the lowest-order terms of Eqs. (8) and (9) of CF, where higher-order terms in the expansion of $V_\pi(E)$ were kept.

There are several noteworthy features of the result in Eq. (6). The first is that because J_π is the sum of one-body operators, the double commutator involving $H = T + V$ in $V_{2\pi}^A$ produces a one-body operator from T (which we ignore) and a two-body operator from V. It contains no three-body forces. In addition, the leading-order term in Eq. (6) is the usual instantaneous OPEP. Thus, summing Fig. 1(a) and 1(b) leads to $V_{2\pi}^A + 1/2\{E - H, \hat{Z}_\pi\} + \cdots$, where each term arises from retardation. The most interesting feature is the explicit energy dependence of the potential, which has serious consequences. The Hamiltonian incorporating such a potential is state dependent, and this modifies the usual orthogonality relationship for wave functions with different energies, since $\frac{\partial}{\partial E}[E - H(E)]$ (which enforces orthogonality) is no longer the identity, but $(1 - \hat{Z}_\pi)$. (Skeptics should check the orthogonality of Klein-Gordon equation solutions [28] for the Coulomb potential case, or the well-studied Sturm-Liouville equation [29].) If charge and current operators are calculated in a similar formalism [27], one finds peculiar exchange-current terms of the form $-\frac{1}{2}\{\hat{Z}_\pi, J^\mu_{\rm imp}\}$, where $J^\mu_{\rm imp}$ is the impulse-approximation current operator. Another consequence is the presence of three- and even four-body operators contained in $-\frac{1}{2}\{V, \hat{Z}_\pi\}$, since both V and $\hat{Z}_\pi$ are two-body operators.

All of these consequences are well-known features of Rayleigh-Schrödinger perturbation theory and, although not incorrect, in our opinion they are highly undesirable. For energies below meson-production threshold it is possible to remove the energy-dependent term by means of a transformation. We write the Schrödinger equation incorporating our energy-dependent potential in the form

$$\frac{1}{2}\{E - H, 1 - \hat{Z}_\pi\}\Psi_E = 0, \tag{10}$$

where we ignore the (higher-order) retardation term: $V_{2\pi}^A$. Defining

$$\sqrt{1 - Z_\pi}\Psi_E \equiv \Psi, \tag{11}$$

multiplying Eq. (10) by $(1 - Z_\pi)^{-\frac{1}{2}}$ leads to

$$(H + \Delta H)\Psi = E\Psi, \tag{12}$$

where $\Delta H = -\frac{1}{8}[[\hat{Z}_\pi, H], \hat{Z}_\pi] + \cdots$ is of order (f_0^4/M) and can be ignored. The same transformation [27] eliminates the exchange-current terms of order $(\hat{Z}_\pi)$, as well as two-, three-, and four-body forces contained in $-\frac{1}{2}\{V, \hat{Z}_\pi\}$.

The problems discussed above can be analyzed in terms of the power counting we introduced earlier. The factor of $(E - H)$ is of order $(1/M)$ and is presumably small. It vanishes in first-order perturbation theory, but the Green's function, G, between the two bubbles in Fig. 1(c) is of order $(E - H)^{-1} \sim M$, and the two factors can abut and cancel in *second order*, leaving a static residue of order (f_0^4). Summing the bubbles [like Fig. 1(c)] to all orders leads to the inverse of Eq. (10):

$$G_E \sim (1 + \hat{Z}_\pi/2)G(1 + \hat{Z}_\pi/2), \tag{13}$$

where $G = (E - H)^{-1}$, which demonstrates that $\hat{Z}_\pi$ is

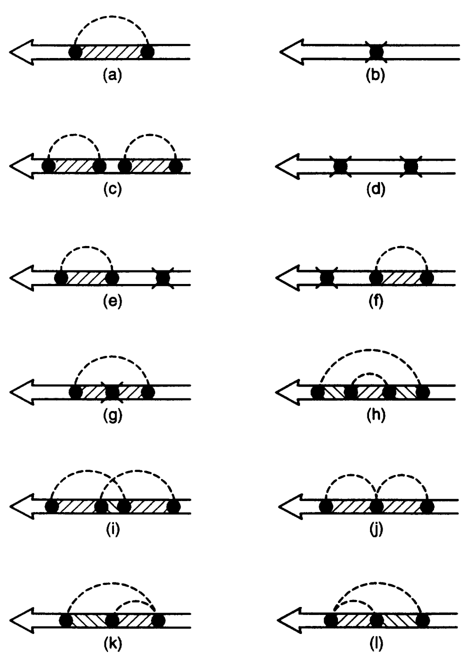

FIG. 1. Time-ordered pion-exchange graphs of order (f^2) are shown in panels (a) and (b), with the $\times$ representing the OPEP counter term. The remaining contributions of order (f^4) are the disconnected graphs (c)–(f), the overlapping graphs of (g)–(i), and the seagull graphs of (j)–(l). The cross hatching indicates virtual nuclear excitations driven by retarded pion exchanges.

the wave function renormalization "constant" (actually an operator) of this field-theory calculation. It is needed to extract a "clean" (i.e., with the usual residue at the pole) Green's function, G, from G_E.

We complete the perturbation theory by calculating the remaining potential diagrams to order (f_0^4) in an energy-independent (i.e., conventional) representation for the potential. Figures 1(a) plus 1(b) give $\langle V_{2\pi}^A\rangle$. Figures 1(c)–1(f) sum to zero in the order of terms kept here. Figures 1(g)–1(i) give $\langle V_{2\pi}^B\rangle$, where

$$V_{2\pi}^B = -\tfrac{1}{2}\int \frac{d^3q_1}{(2\pi)^3(2E_1^2)}\int \frac{d^3q_2}{(2\pi)^3(2E_2^2)}\frac{[J_\pi^\alpha(\mathbf{q}_1), J_\pi^\beta(\mathbf{q}_2)][J_\pi^\alpha(-\mathbf{q}_1), J_\pi^\beta(-\mathbf{q}_2)]}{(E_1+E_2)}, \tag{14}$$

using an obvious notation with $E_i = (q_i^2 + m_\pi^2)^{\frac{1}{2}}$. Note that this procedure includes V_π^0 to all orders in the basis Hamiltonian, H, and that $V_{2\pi}^B$ is a two-body potential *only* [mentioned above Eq. (18a) of CF]. In addition to OPEP (included in H), we have obtained $\langle V_{2\pi}^A\rangle + \langle V_{2\pi}^B\rangle$.

Alternatively, we can eliminate the counter term [Fig. 1(b)], which is the "mass renormalization" of our procedure, and perform the conventional calculation in the basis with Hamiltonian, H_0 (i.e., without OPEP). In that case, only Figs. 1(a), 1(c), 1(h), and 1(i) contribute. We find that Fig. 1(a) gives $\langle V_\pi^0\rangle$, while 1(c) gives $\langle V_\pi^0 G_0 V_\pi^0\rangle + \langle V_{2\pi}^{\rm TMO}\rangle$, where

$$V_{2\pi}^{\rm TMO} = \frac{1}{2}\{\hat{Z}_\pi, V_\pi^0\}, \tag{15}$$

and the notation (TMO) that we use is adopted from Ref. [10]. In addition, Figs. 1(h) plus 1(i) give $\langle V_{2\pi}^A\rangle + \langle V_{2\pi}^B\rangle - \langle V_{2\pi}^{\rm TMO}\rangle$. This sums to what we previously obtained, plus OPEP and its iterate $\langle V_\pi^0 G_0 V_\pi^0\rangle$; the two approaches are therefore consistent and give the same total result.

Finally, if we wish to define the entire $V_\pi(E)$ as the OPEP to be used in the Schrödinger equation, we need only calculate Figs. 1(h) and 1(i) to obtain the additional potential [to $V_\pi(E)$]. By not calculating 1(c) we are explicitly *defining* Fig. 1(a) (including the energy-dependent part) to be OPEP. The result is $\langle V_{2\pi}^A\rangle + \langle V_{2\pi}^B\rangle - \langle V_{2\pi}^{\rm TMO}\rangle$ and differs by $-\langle V_{2\pi}^{\rm TMO}\rangle$ from $\langle V_{2\pi}^A\rangle + \langle V_{2\pi}^B\rangle$, our energy-independent-potential result obtained below Eq. (14). The missing energy is contained *implicitly* in the energy-dependent part of $V_\pi(E)$. Clearly, the definition of our potential is subjective and subject to theoretical prejudice, although the total energy is not dependent on how we categorize it. We note again that $V_{2\pi}^{\rm TMO}$ contains two- and three-, and four-body terms.

IV. THE TMO POTENTIAL

Different implicit definitions of the potential have occurred historically. Brueckner and Watson [16] ignored the energy dependence of the non-overlapping (time-ordered) pion exchanges and obtained (in the static limit) the BW two-nucleon two-pion-exchange potential $\langle V_{2\pi}^A + V_{2\pi}^B\rangle - \langle V_{2\pi}^{\rm TMO}\rangle$. Taketani, Machida, and Ohnuma [17] (see also Sugawara and Okubo) treated the disconnected graphs and obtained $\langle V_{2\pi}^A + V_{2\pi}^B\rangle$. That is, the BW two-nucleon potential implicitly included the energy-dependent term in Eq. (6) in the definition of the potential, and the TMO two-nucleon potential did not [30]. These two-body potentials have been worked out many times before and recently by Rijken [10], both in momentum space and configuration space. For completeness, we present both forms in the Appendix (see also Ordóñez and van Kolck [15]). According to Rijken, the differences between the two potentials are substantial. Although our notation unfortunately implies that TMO calculated a correction to the BW potential, that work in fact preceded BW and calculated the complete potential.

Weinberg [14] recently calculated the static three-nucleon force in the leading order of chiral perturbation theory. His explicit choice of not calculating Fig. 1(c) leads to an implicit definition of OPEP that includes the energy-dependent term, and thus his result for the additional force can be shown (see the Appendix and the discussion in the previous section) to be the three-body parts of $-\langle V_{2\pi}^{\rm TMO}\rangle$, since $V_{2\pi}^A$ and $V_{2\pi}^B$ contain no three-body terms. If one insists on an energy-independent OPEP (which is the usual case) the transformation between the two forms (detailed above) leads to a vanishing static limit, in agreement with CF. We note that although CF and Weinberg used the same chiral Lagrangian (and hence must lead to the same result), CF was not performed within the conceptual framework of chiral perturbation theory (we were unaware of this interesting development). Nevertheless, after transforming to a conventional OPEP, the Weinberg (CF) result can be stated as a "theorem": *In the static limit* ($M \to \infty$) *this three-nucleon force vanishes in leading order in chiral perturbation theory.* What constitutes leading order and how one goes beyond this is discussed in Refs. [14,15]. Although we have not dealt with the topic, this result also applies to three-nucleon forces arising from the "contact" (i.e., δ-function) terms that appear as surrogates [1,14] for the sum of short-range two-nucleon potentials (e.g., from ρ exchange). We note that the two-pion-exchange Tucson-Melbourne three-nucleon force [11] contains higher-order terms in the chiral expansion which do not vanish in the static limit. The chiral perturbation theory approach to such higher-order terms is demonstrated in Ref. [15].

The qualitative reason for this result is that in the energy-independent representation for the potential, this two-pion-exchange three-nucleon force involves no loops, and has the generic form V_π^2/Λ, where Λ is a large-mass scale. For our simple case, $\Lambda \equiv M$, which guarantees a vanishing static limit. This generic form is mandated by chiral symmetry and is *not* what one obtains in non-chiral models [23]. The concomitant smallness of three-nucleon forces in few-nucleon systems seems to be confirmed by recent calculations [1–4,25,26].

V. SUMMARY AND CONCLUSIONS

Recent calculations have reemphasized the important role of the pion in the dynamics of few-nucleon systems. Chiral constraints enforce the dominance of two- over three-nucleon forces, and suggest [31] that many-pion exchanges between nucleons are less important than few-pion exchanges, with OPEP the largest contribution of all. Chiral perturbation theory calculations [14] demonstrate the vanishing of the leading-order component of the three-nucleon force in the static limit [12], if the usual energy-independent definition is chosen for the two-nucleon force. The latter problem is shown to be identical to one encountered many years ago in treatments of the two-nucleon force [16,17,30]. If one chooses an energy-independent OPEP, the two-pion-exchange TMO potential (as defined herein) should be included together with the BW potential in the Hamiltonian.

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APPENDIX

In this appendix, we present explicit forms for various potentials that were dealt with schematically in the body of the paper. We present results in both configuration space and in momentum space. Additional configuration-space forms (involving form factors) can be deduced using the tricks illustrated in Appendix B of Ref. [10]. Ignoring such form factors leads to the long-range behavior of the potential.

The one-pion-exchange potential has the form

$$V_\pi^0 = \frac{f_0^2}{2m_\pi}\sum_{i\neq j}\boldsymbol{\tau}(i)\cdot\boldsymbol{\tau}(j)(\boldsymbol{\sigma}(i)\cdot\boldsymbol{\nabla}_{ij})(\boldsymbol{\sigma}(j)\cdot\boldsymbol{\nabla}_{ij}) \times h_0(m_\pi x_{ij}), \tag{A1}$$

while

$$\hat{Z}_\pi = \frac{f_0^2}{2m_\pi^2}\sum_{i\neq j}\boldsymbol{\tau}(i)\cdot\boldsymbol{\tau}(j)(\boldsymbol{\sigma}(i)\cdot\boldsymbol{\nabla}_{ij})(\boldsymbol{\sigma}(j)\cdot\boldsymbol{\nabla}_{ij}) \times h_1(m_\pi x_{ij}), \tag{A2}$$

where in the limit of a pointlike pion-nucleon form factor ($F(\mathbf{q}^2)\to 1$) we have

$$h_0(z) \equiv 4\pi\int\frac{d^3q}{(2\pi)^3}\frac{F^2(\mathbf{q}^2m_\pi^2)e^{i\mathbf{q}\cdot\mathbf{z}}}{(\mathbf{q}^2+1)} \to \frac{e^{-z}}{z}, \tag{A3}$$

and

$$h_1(z) \equiv 4\pi\int\frac{d^3q}{(2\pi)^3}\frac{F^2(\mathbf{q}^2m_\pi^2)e^{i\mathbf{q}\cdot\mathbf{z}}}{(\mathbf{q}^2+1)^{3/2}} \to \frac{2}{\pi}K_0(z), \tag{A4}$$

where $K_0(z)$ is the modified Bessel function. Together with Eq. (3) these forms can be used in Eq. (8) to produce

$$V_{2\pi}^A = -\frac{f^4}{2}\sum_{i\neq j}\int\int\frac{d^3q_1}{(2\pi)^3}\frac{d^3q_2}{(2\pi)^3}e^{i(\mathbf{q}_1+\mathbf{q}_2)\cdot\mathbf{x}_{ij}}(a_{ij}+b_{ij})\left[\frac{1}{E_1^2E_2^3}+\frac{1}{E_1^3E_2^2}\right]F^2(\mathbf{q}_1^2)F^2(\mathbf{q}_2^2), \tag{A5}$$

where $a_{ij} = 3(\boldsymbol{\sigma}(i)\cdot\mathbf{q}_1\times\mathbf{q}_2)(\boldsymbol{\sigma}(j)\cdot\mathbf{q}_1\times\mathbf{q}_2)$ and $b_{ij} = 2\boldsymbol{\tau}(i)\cdot\boldsymbol{\tau}(j)(\mathbf{q}_1\cdot\mathbf{q}_2)^2$. This leads to

$$V_{2\pi}^A = -\frac{f_0^4}{m_\pi^3}\sum_{i\neq j}\hat{O}_{ij}^{2\pi}h_0(m_\pi x_{ij})h_1(m_\pi x'_{ij}), \tag{A6}$$

where

$$\hat{O}_{ij}^{2\pi} = 3\boldsymbol{\sigma}(i)\cdot(\boldsymbol{\nabla}_{ij}\times\boldsymbol{\nabla}'_{ij})\boldsymbol{\sigma}(j)\cdot(\boldsymbol{\nabla}_{ij}\times\boldsymbol{\nabla}'_{ij}) + 2\boldsymbol{\tau}(i)\cdot\boldsymbol{\tau}(j)(\boldsymbol{\nabla}_{ij}\cdot\boldsymbol{\nabla}'_{ij})^2, \tag{A7}$$

and the prime indicates which $\mathbf{x}_{ij}$ is to be differentiated (whereupon, set $\mathbf{x}'_{ij}$ to $\mathbf{x}_{ij}$). Similarly we can develop $V_{2\pi}^B$, which leads to

$$V_{2\pi}^B = \frac{f^4}{2}\sum_{i\neq j}\int\frac{d^3q_1}{(2\pi)^3}\frac{d^3q_2}{(2\pi)^3}\frac{e^{i\mathbf{q}_1\cdot\mathbf{x}_{ij}}e^{i\mathbf{q}_2\cdot\mathbf{x}'_{ij}}}{E_1^2E_2^2(E_1+E_2)}(a_{ij}+b_{ij})F^2(\mathbf{q}_1^2)F^2(\mathbf{q}_2^2) \tag{A8}$$

$$= \frac{f^4}{2}\sum_{i\neq j}\hat{O}_{ij}^{2\pi}g(\mathbf{x}_{ij};\mathbf{x}'_{ij}), \tag{A9}$$

where

$$g(z;z') = \int \frac{d^3q_1}{(2\pi)^3}\frac{d^3q_2}{(2\pi)^3}\frac{e^{i\mathbf{q}_1\cdot\mathbf{z}}e^{i\mathbf{q}_2\cdot\mathbf{z}'}F^2(\mathbf{q}_1^2)F^2(\mathbf{q}_2^2)}{E_1^2E_2^2(E_1+E_2)}. \tag{A10}$$

A convolution representation for g can be written using the identities [10]

$$\frac{1}{E_1E_2(E_1+E_2)} = \frac{2}{\pi}\int_0^\infty \frac{d\lambda}{(E_1^2+\lambda^2)(E_2^2+\lambda^2)} \tag{A11}$$

(which can be easily proven using partial fractions), and

$$\frac{1}{m_\pi}\frac{\partial}{\partial m_\pi}\frac{2}{\pi}\int_0^\infty \frac{d\lambda}{(E_1^2+\lambda^2)(E_2^2+\lambda^2)} = \frac{1}{E_1^2E_2^2(E_1+E_2)} - \frac{1}{E_1^2E_2^3} - \frac{1}{E_1^3E_2^2}, \tag{A12}$$

which leads to

$$g(z;z') = \left[\frac{1}{(4\pi)^2}\right]\frac{1}{m_\pi}\frac{\partial}{\partial m_\pi}\left[\frac{2}{\pi}\int_0^\infty d\lambda(m_\pi^2+\lambda^2)h_0(\sqrt{\lambda^2+m_\pi^2}z)h_0(\sqrt{\lambda^2+m_\pi^2}z')\right] + \frac{2h_0(z)h_1(z')m_\pi}{(4\pi)^2}. \tag{A13}$$

Adding $V_{2\pi}^A$ and $V_{2\pi}^B$ together to form $V_{2\pi}^0$ leads to a simple and compact form:

$$V_{2\pi}^0 = \sum_{i\neq j}\hat{O}_{ij}^{2\pi}h(x_{ij};x'_{ij}), \tag{A14}$$

with

$$h(z;z') = \frac{f_0^4}{2m_\pi^5}\frac{\partial}{\partial m_\pi}\cdot\frac{2}{\pi}\int_0^\infty d\lambda(m_\pi^2+\lambda^2)h_0(\sqrt{\lambda^2+m_\pi^2}z)h_0(\sqrt{\lambda^2+m_\pi^2}z'), \tag{A15}$$

where we should take $z'\to z$ after differentiation. For the case of pointlike pion-nucleon form factors, the integral in (A14) is easy to perform after differentiation:

$$h^{pt}(z;z') = -\frac{f_0^4}{\pi m_\pi^4}\left[\frac{z+z'}{zz'}\right]K_0[m_\pi(z+z')]. \tag{A16}$$

Note that $V_{2\pi}^0$ contains only central and tensor contributions and is local. Performing the derivatives in Eq. (A14) leads to the results of Ref. [17], and to those of Ref. [10] when the static TMO and BW potentials of the latter work are combined.

We can easily calculate the two- and three-nucleon parts of $V_{2\pi}^{\text{TMO}}$:

$$V_{2N}^{\text{TMO}} = \frac{f_0^4}{2m_\pi^3}\sum_{i\neq j}\hat{P}_{ij}^{\text{TMO}}h_0(x_{ij})h_1(x'_{ij}), \tag{A17}$$

where the combined spin-isospin operator is given by

$$\hat{P}_{ij}^{\text{TMO}} = (3-2\boldsymbol{\tau}(i)\cdot\boldsymbol{\tau}(j))[(\boldsymbol{\nabla}_{ij}\cdot\boldsymbol{\nabla}'_{ij})^2 - \boldsymbol{\sigma}(i)\cdot(\boldsymbol{\nabla}_{ij}\times\boldsymbol{\nabla}'_{ij})\boldsymbol{\sigma}(j)\cdot(\boldsymbol{\nabla}_{ij}\times\boldsymbol{\nabla}'_{ij})]. \tag{A18}$$

Similarly,

$$V_{3N}^{\text{TMO}} = \frac{f_0^4}{m_\pi^3}\sum_{i\neq j\neq k}\hat{Q}_{ijk}^{\text{TMO}}h_0(x_{ij})h_1(x_{ik}), \tag{A19}$$

where

$$\hat{Q}_{ijk}^{\text{TMO}} = (\boldsymbol{\sigma}(j)\cdot\boldsymbol{\nabla}_j\boldsymbol{\sigma}(k)\cdot\boldsymbol{\nabla}_k)[\boldsymbol{\nabla}_j\cdot\boldsymbol{\nabla}_k\boldsymbol{\tau}(j)\cdot\boldsymbol{\tau}(k) - \boldsymbol{\sigma}(i)\cdot\boldsymbol{\nabla}_j\times\boldsymbol{\nabla}_k\boldsymbol{\tau}(i)\cdot\boldsymbol{\tau}(j)\times\boldsymbol{\tau}(k)]. \tag{A20}$$

The three-nucleon potential derived by Weinberg [14] is the negative of V_{3N}^{TMO}. If one insists on an energy-independent OPEP, V_{3N}^{TMO} should be added to that result and the sum vanishes to this order.

Finally, we present the results for the seagulls depicted in Figs. 1(j)–1(l), plus the double seagull not considered in CF or Fig. 1. The latter involves two Adler-Weisberger interactions and generates only a two-nucleon potential. Ignoring the zero-range singularities is equivalent to renormalization in effective field theories such as the one we are using. The static part of Figs. 1(j)–1(l) gives a two-nucleon potential:

$$V_{2\pi}^{\rm AW} = -i\int\frac{d^3q_1}{(2\pi)^3}\int\frac{d^3q_2}{(2\pi)^3}\frac{\{\bar{S}_{2\pi}^{\alpha\beta}(\mathbf{q}_1+\mathbf{q}_2),[J_\pi^\alpha(-\mathbf{q}_1),J_\pi^\beta(-\mathbf{q}_2)]\}}{(2E_1)(2E_2)(E_1+E_2)}$$
$$= -\frac{2f^4}{g_A^2}\sum_{i\neq j}\boldsymbol{\tau}(i)\cdot\boldsymbol{\tau}(j)\int\int\frac{d^3q_1}{(2\pi)^3}\frac{d^3q_1}{(2\pi)^3}\frac{e^{i(\mathbf{q}_1+\mathbf{q}_2)\cdot\mathbf{x}_{ij}}\mathbf{q}_1\cdot\mathbf{q}_2}{E_1E_2(E_1+E_2)} \quad \text{(A21)}$$
$$= \frac{4f_0^4}{\pi g_A^2 m_\pi^3}\sum_{i\neq j}\boldsymbol{\tau}(i)\cdot\boldsymbol{\tau}(j)\boldsymbol{\nabla}_{ij}\cdot\boldsymbol{\nabla}'_{ij}i(x_{ij};x'_{ij}),$$

and Eq. (A11) can be used to produce

$$i(z;z') = \frac{1}{m_\pi}\int_0^\infty d\lambda(m_\pi^2+\lambda^2)h_0(\sqrt{\lambda^2+m_\pi^2}z)h_0(\sqrt{\lambda^2+m_\pi^2}z'),$$

where we have assumed that the introduction of form factors must have the product form given in Eq. (A10). Ignoring any such form factors then leads to

$$i^{pt}(z;z') = \frac{K_1[m_\pi(z+z')]}{zz'}, \quad \text{(A22)}$$

where, as before, we equate z and z' after differentiating.

The double seagull leads to the static two-nucleon result:

$$V_{2\pi}^{\rm AW'} = -\tfrac{1}{2}\int\frac{d^3q_1}{(2\pi)^3}\int\frac{d^3q_2}{(2\pi)^3}\frac{\bar{S}_{2\pi}^{\alpha\beta}(\mathbf{q}_1+\mathbf{q}_2)\bar{S}_{2\pi}^{\alpha\beta}(-\mathbf{q}_1-\mathbf{q}_2)(E_1-E_2)^2}{(2E_1)(2E_2)(E_1+E_2)}$$
$$= -\frac{f^4}{4g_A^4}\sum_{i\neq j}\boldsymbol{\tau}(i)\cdot\boldsymbol{\tau}(j)\int\int\frac{d^3q_1}{(2\pi)^3}\frac{d^3q_2}{(2\pi)^3}\frac{e^{i(\mathbf{q}_1+\mathbf{q}_2)\cdot\mathbf{x}_{ij}}(E_1-E_2)^2}{E_1E_2(E_1+E_2)} \quad \text{(A23)}$$
$$= -\frac{f_0^4}{2\pi g_A^4 m_\pi^3}\sum_{i\neq j}\boldsymbol{\tau}(i)\cdot\boldsymbol{\tau}(j)f(x_{ij};x'_{ij}).$$

Using $(E_1-E_2)^2/[E_1E_2(E_1+E_2)] = [E_1^{-1}+E_2^{-1}] - 4/(E_1+E_2)$, and the identity

$$\frac{1}{(E_1+E_2)} = \frac{2}{\pi}\int_0^\infty\frac{d\lambda\lambda^2}{(E_1^2+\lambda^2)(E_2^2+\lambda^2)}, \quad \text{(A24)}$$

we find that ignoring the bracketed terms above Eq. (A24) $[E_1^{-1}+E_2^{-1}]$ (because they have zero range) leads to

$$f(z;z') = -\frac{4}{m_\pi}\int_0^\infty d\lambda\lambda^2(m_\pi^2+\lambda^2)h_0(\sqrt{\lambda^2+m_\pi^2}z)h_0(\sqrt{\lambda^2+m_\pi^2}z'). \quad \text{(A25)}$$

Ignoring any form factors then produces

$$f^{pt}(z;z') = -\frac{4}{z^2}\left(\frac{\partial^2}{\partial z\partial z'}-m_\pi^2\right)K_1[m_\pi(z+z')]. \quad \text{(A26)}$$

Performing the derivatives in Eqs. (A21) and (A26) leads to the (static) results in (the second of) Ref. [17], if we make the identifications: $g = 4\pi f_0^2$ and $\lambda_2 = 4\pi f_0^2/g_A^2 \sim 0.6$, compared to their phenomenological value of $\lambda_2 \sim 0.4$. These results also agree with Ref. [15], who, to the best of our knowledge, performed the first chiral calculation. See also Ref. [32].

Our complete results for the static two-pion-exchange two-nucleon potential (to be used with an energy-independent OPEP) are given by the sum of the TMO and BW potentials in Eq. (A14) $[V_{2\pi}^0]$ plus Eq. (A21) $[V_{2\pi}^{\rm AW}]$ plus Eq. (A23) $[V_{2\pi}^{\rm AW'}]$. Although none of these results are new [15,17], we have presented in this appendix a succinct derivation with compact results in both momentum and configuration spaces. The static two-pion-exchange three-nucleon force in the approximation we have adopted in this work vanishes.

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