Correlation between ε/Δ and the P factor

B. D. Foy,[1] R. F. Casten,[1] N. V. Zamfir,[1-4] and D. S. Brenner[2]
[1] *Brookhaven National Laboratory, Upton, New York 11973*
[2] *Clark University, Worcester, Massachusetts 01610*
[3] *Institute of Atomic Physics, Bucharest Magurele, Romania*
[4] *Institut fur Kernphysik, University of Köln, Köln, Germany*
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Empirical values of ε/Δ, the ratio of quadrupole deformation to the pairing gap parameter, are compared to the P factor for all even-even nuclei from Z = 42–98. The correlation is found to be remarkable and suggestive of predictive power for unknown nuclei.

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One of the fundamental goals of nuclear physics is to understand the structure of nuclei, the evolution of that structure across the nuclear chart, and the factors determining that evolution. This is ultimately a microscopic question, but macroscopic or phenomenological approaches can shed important light on essential ingredients in it. Studies (see, e.g., Refs. [1–5]) over the last years have shown that the valence nucleon product N_pN_n and the P factor, $P \equiv N_pN_n/(N_p + N_n)$, can be useful phenomenological guides since they are highly correlated with ("mean-field") observables such as $E(2_1^+)$, $E(4_1^+)/E(2_1^+)$, and $B(E2:0_1^+ \rightarrow 2_1^+)$ values and nuclear masses.

The correlations of N_pN_n or P with these collective observables are reasonable in light of the origins of collectivity in nuclei and the construction of the valence-nucleon-based quantities N_pN_n and P. The residual valence p-n interaction is widely accepted to be crucial in the onset and development of collectivity and deformation in nuclei. To the approximation that these interactions are orbit independent, N_pN_n is proportional to the integrated valence p-n interaction and hence should be correlated with quantities reflecting the mean-field shape and structure.

The P factor can be viewed as the number of p-n interactions per valence nucleon or, better, as proportional to the ratio of the valence p-n interaction to the pairing interaction. (The latter scales in first order as the number of valence protons and neutrons.) Thus we might expect P to be correlated with an empirical measure of this ratio such as ε/Δ, the ratio of quadrupole deformation to the pairing gap parameter, which has often been discussed [6] as an excellent measure of structure. A limited P-ε/Δ correlation is indeed implicit in recent studies [7,8] of variations of rotational spacings in the localized mass region from Dy to W. In Ref. [7] it was shown that fractional changes in empirical energy differences $E(4_1^+) - E(2_1^+)$ correlate very well with fractional changes in empirical values of ε/Δ and with P. In Ref. [8] it was shown that fractional changes in the rotational inertial parameter correlate very well with theoretical values of ε/Δ and with P. The implication is that ε/Δ itself should correlate with P.

The extension of these general types of correlation study to global dimensions was recently made [9] in an investigation of the relation of $B(E2{:}2_1^+ \rightarrow 0_1^+)$ and N_pN_n values for nuclei stretching from mass $A \sim 80$ to the actinides. The results of Refs. [7,8] hint that there might be a complementary global correlation of P and ε/Δ.

It is the purpose of the present paper to investigate such a correlation and to show that it is in fact remarkable. To do so we use ε values obtained from the $B(E2{:}0_1^+ \rightarrow 2_1^+)$ values tabulated by Raman *et al.* [10] using all values obtained from formally published work from Z=42 to 98. The Δ values were obtained as the average of Δ_p and Δ_n using the formula in Ref. [6] and tabulated values of nuclear binding energies [11].

Figure 1(a) shows the empirical ε/Δ values for all known even-even nuclei from Z=42 to 98. No nuclei, magic or otherwise, are omitted. Some of the binding energies [11] used in obtaining Δ were based on estimates from extrapolation or interpolation of systematic trends. Such values are indicated by a separate symbol in Fig. 1(a). A few values of ε/Δ have uncertainties >10% (either due to larger errors on measured values or from estimates in Ref. [11] of the accuracy of the systematics-based values). These nuclei are also indicated by a separate symbol in Fig. 1(a). Figure 1(b) shows values of P for those even-even nuclei for which empirical values of ε/Δ are available. Normal magic numbers are used in constructing P except that Z=40(64) is considered magic only for neutron numbers from N=50–58 (82–88), reflecting the Z=40 (64) subshell closures prior to the onset of deformation in the A=100 (150) region.

The correlation between the two plots is truly remarkable. Not only are the general trends the same but even many details. This includes the small kinks near Z=42 and 58–62 due to the Z=40 and 64 subshell closures which match closely in the empirical ε/Δ plot and the P plots. The correlation extends over 136 nuclei spanning masses from A ~80 to 250. The few small scale discrepancies are almost always cases where the experimental uncertainties ($\times$ symbols) are large.

Of course, a correlation of this quality suggests a direct comparison of ε/Δ and P. This is shown in Fig. 2. Again, the correlation is impressive and compact. Note that there is no regional normalization applied in Fig. 2. In fact, much of the breadth of the correlation that does exist is due to slightly different factors (an A dependence) relating ε/Δ and P in different mass regions.

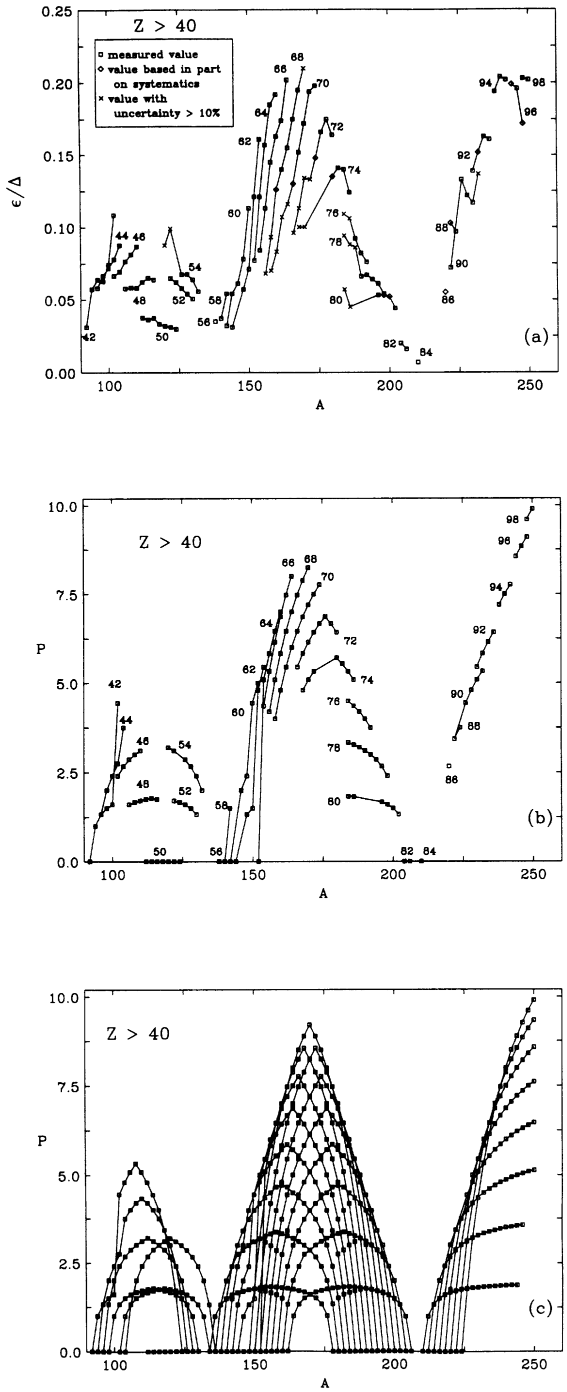

FIG. 1. Empirical ε/Δ and calculated P factors plotted against the mass number A. The labels on the curves are Z values. (a) ε/Δ vs A. The ε/Δ values (in MeV^{-1}) are obtained from Refs. [10,11] (see text). Squares represent measured values with combined errors less than 10%. Diamonds are values that use one or more binding energies (in obtaining Δ) based on systematics, with combined errors also less than 10%. $\times$ symbols represent points with errors larger than 10% for any reason. (b) P vs A for the same nuclei as in panel (a). See text. (c) P vs A for full shells in the A=80–250 region showing predictions for currently unknown nuclei and highlighting the dependence of the overall pattern on the set of accessible nuclei. See text.

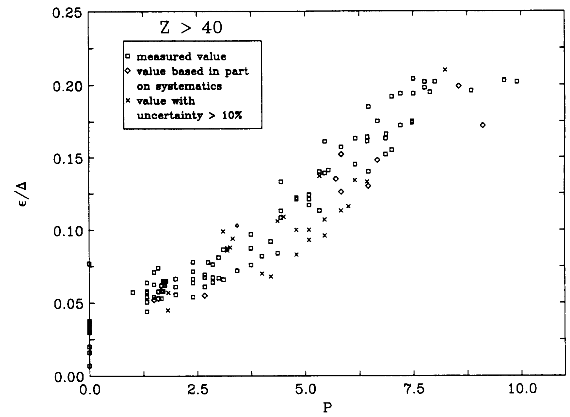

FIG. 2. Plot of ε/Δ vs P obtained by eliminating A in Figs. 1(a) and 1(b). The symbols have the same meaning as in Fig. 1(a).

These results suggest that the extremely simple quantity P is capable of correlating the evolution of structure over most of the nuclear chart. A microscopic understanding of this and similar global correlations [9,12] is a critical challenge to nuclear theory. In the meantime, empirical-phenomenological correlations such as in Fig. 1 can be put to practical use.

To see an example of this, note the difference in the overall pattern of ε/Δ values in the light rare earths (past ^{132}Sn) in Fig. 1(a) compared to the actinides. While there are similarities, the actinide pattern is clearly simpler. The question is why this difference occurs. While it might be thought to have origins in subtle details of shell structure and interactions, Fig. 1(a) discloses a much simpler explanation: The simpler actinide patterns results merely because the nuclei whose ε/Δ values would be expected to complicate it are unknown. This is clearly seen in Fig. 1(c), which shows P values for all nuclei with Z=42–50 and N=50–82, with Z=50–82, N=82–126, and with Z >82, $A \leq 250$. Assuming the predictive power of P—that is, that P remains correlated with ε/Δ for currently unknown nuclei—it is clear that new data on neutron-rich actinide nuclei should yield a quite different pattern. The nearly symmetrical pattern of P values for the full rare-earth region in Fig. 1(c) provides another set of predictions for ε/Δ values that may be obtainable with future radioactive beam facilities. Thus, on the one hand, Fig. 1(c) has predictive power that can be a guide to new experiments, while, on the other, future disagreements with those predictions can point to inadequacies in the counting of effective N_p and N_n values that relate to basic features of shell structure and nucleon residual interactions in newly accessible regions of the nuclear chart.

To summarize, a remarkable correlation of ε/Δ values with the P factor is found for all known nuclei from Z=42 to 98 ($A \sim$ 80–250), which highlights the ability of this parameter to account quantitatively for the evolution of nuclear structure. It continues to surprise that such an extraordinarily simple ansatz can reproduce the structural evolution of so many nuclei in such diverse regions so well. Extension of the ε/Δ-P correlation to new regions and unknown nuclei provides predictions for future experiments. Agreement with those predictions would further confirm the physical basis for the P factor and for the evolution of collectivity, while disagreements will signal new degrees of freedom and/or changes in shell structure far from stability.

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