Properties of the rho meson in nuclear matter

C. M. Shakin and Wei-Dong Sun
Department of Physics and Center for Nuclear Theory, Brooklyn College of the City University of New York, Brooklyn, New York 11210
(Received 23 September 1993)

We calculate the vector-isovector hadronic current correlation function in the presence of nuclear matter. We find that the rho mass decreases by about 20% at nuclear matter density and the rho width is reduced by about 35%. These results are in general accord with the results of calculations made using QCD sum rules. In our calculations, and in the QCD sum-rule calculations, the main effect considered is reduction of the value of the quark condensate in the presence of matter. That reduction is here assumed to be 35% at nuclear matter density.

PACS number(s): 21.65.+f, 14.40.Cs

I. INTRODUCTION

Recently, we have seen several calculations made of the properties of the rho meson in nuclear matter [1–3]. We are here interested in the calculation of the vector-isovector hadronic current correlation function that was studied in Refs. [1,2] using QCD sum-rule techniques. The imaginary part of the correlator exhibits a large peak at $P^2 = m_\rho^2$, where m_ρ is the rho mass. In the sum-rule analysis, this peak can be represented by a delta function, $\lambda\delta(P^2 - m_\rho^2)$, as in [1], or a more complex form may be used that takes into account the rho width, as in [2]. The sum-rule analysis allows for a determination of the rho mass in terms of a number of vacuum condensates. Since the value of these condensates changes in the presence of nuclear matter, the value obtained for the rho mass becomes dependent upon the matter density. (We will denote the density-dependent rho mass as m_ρ^* in the following discussion.)

In the sum-rule calculations, the main effect, due to the presence of nuclear matter, is a reduction of the value of the quark condensate. Some correction terms, linear in the matter density, can be calculated in a model-independent manner, with the result depending upon the value chosen for the pion-nucleon sigma term, σ_N. For example, in the case of the up-quark condensate, we have

$$\langle \bar{u}u\rangle_\rho = \langle \bar{u}u\rangle_0 + \frac{\sigma_N}{2\hat{m}}\rho \ , \tag{1.1}$$

to first order in the matter density, ρ. Here $\hat{m}$ is the average of the current masses of the up and down quarks and $\langle \bar{u}u\rangle_0$ is the vacuum value of the up-quark condensate. The work of Refs. [1] and [2] takes into account a number of additional condensates, however, Eq. (1.1) describes the most important effect. There remains the question as to how Eq. (1.1) is to be used in the sum-rule calculations. For example, in Refs. [1,2] a very important role is played by the four-quark condensates. These are evaluated in a Hartree, or vacuum-saturation approximation. What then appears is the quantity $\langle \bar{q}q\rangle_\rho^2$, which is evaluated to first order in the density as [2]

$$\langle \bar{q}q\rangle_\rho^2 \simeq \langle \bar{q}q\rangle_0^2 + 2\langle \bar{q}q\rangle_0\langle N|\bar{q}q|N\rangle\rho \ . \tag{1.2}$$

Here $|N\rangle$ denotes the state of a nucleon, with normalization $\langle N'|N\rangle = (2\pi)^3\delta(\mathbf{p}-\mathbf{p}')$. Since one can always question the use of the factorization hypothesis for the four-quark condensates, we believe it is of value to calculate meson properties using other models and in this work we will present an alternative approach. Note that, in the case of the nucleon, the baryon density, $\langle \bar{q}\gamma^0 q\rangle_\rho$, plays a very important role. Because of cancellations between self-energy effects due to $\langle \bar{q}q\rangle_\rho$ and $\langle \bar{q}\gamma^0 q\rangle_\rho$, the nucleon stays close to its mass shell [4]. This feature has a direct analog in relativistic nuclear physics, where the σ and ω fields play a role analogous to that of the scalar and vector condensates. In contrast, meson properties are largely unaffected by the presence of the vector condensate, since the shift in the quark energy due to the vector condensate is cancelled by the corresponding shift in the antiquark energy.

Since the main effect being described in the QCD sum-rule calculations is a mean-field effect, it is natural to ask whether similar results may be obtained using quark models, such as that of Nambu–Jona-Lasinio (NJL), that describe mass generation through the breaking of chiral symmetry in the ground state of the system [5]. We have recently created a generalized version of the NJL model that contains a description of confinement [6]. We were then able to calculate hadronic current correlators for currents carrying various quantum numbers. Indeed, our studies of the vector-isovector correlator [7] and the axial-vector isovector correlator [8] gave excellent fits to the experimental data. Note that the imaginary part of the correlator may be related to the ratio of the cross section for $e^+ + e^- \to$ hadrons to the cross section for $e^+ + e^- \to \mu^+ + \mu^-$, in the case of the vector-isovector channel [9]. For the axial-vector correlator, an analysis of τ decay leads to an extraction of data that allows for the specification of the imaginary part of the correlator [10].

The organization of our work is as follows. In Sec. II we review our model of coupled-channel quark-hadron dynamics as it pertains to the calculation of hadronic current correlation functions. In Sec. III we consider the density dependence of various functions that are used to construct the correlation function. We also provide values for the rho mass and width at nuclear matter density. Finally, in Sec. IV, we present some further discussion of our results.

II. COUPLED-CHANNEL QUARK-HADRON MODEL

Our generalization of the NJL model to include a description of confinement has been extensively discussed in earlier work [6–8]. Therefore, we here limit ourselves to a summary of the equations that are necessary to understand the calculation we report upon.

We may define a vector-isovector current

$$j_a^\mu(x) = \bar{q}(x)\gamma^\mu \tau_a q(x) \tag{2.1}$$

and, in terms of this current, we define the correlation function in vacuum [7]

$$-iC^{\mu\nu}_{(\rho)}(P^2)_{ab} = \int d^4x e^{iP\cdot x}\langle 0|T[j_a^\mu(x) j_b^\nu(0)]|0\rangle \ , \tag{2.2}$$

and put

$$C^{\mu\nu}_{(\rho)}(P^2)_{ab} = -\hat{g}^{\mu\nu}(P) C_{(\rho)}(P^2)\delta_{ab} \ , \tag{2.3}$$

where

$$\hat{g}^{\mu\nu}(P) = g^{\mu\nu} - P^\mu P^\nu / P^2 \ . \tag{2.4}$$

At this point we drop reference to the isospin indices. It is useful to define tensors, $\hat{J}^{\mu\nu}_{(\rho)}(P)$ and $\tilde{C}^{\mu\nu}_{(\rho)}(P)$, with

$$\hat{J}^{\mu\nu}_{(\rho)}(P) = -\hat{g}^{\mu\nu}(P)\hat{J}_{(\rho)}(P^2) \tag{2.5}$$

and

$$\tilde{C}^{\mu\nu}_{(\rho)}(P) = -\hat{g}^{\mu\nu}(P)\tilde{C}_{(\rho)}(P^2) \ . \tag{2.6}$$

The tensor $\hat{J}^{\mu\nu}_{(\rho)}(P)$ is defined in Fig. 1. Thus, we write

$$C_{(\rho)}(P^2) = \hat{J}_{(\rho)}(P^2) + \tilde{C}_{(\rho)}(P^2) \ . \tag{2.7}$$

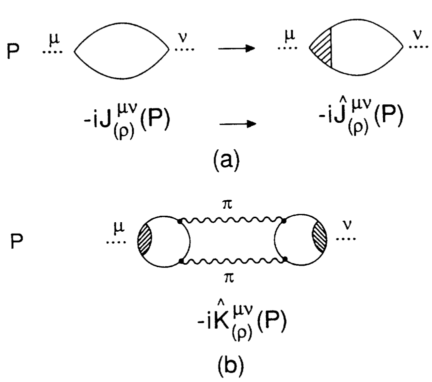

FIG. 1. (a) The basic quark-loop integral that appears in the study of the vector-isovector sector of the NJL model. (b) The calculation of the tensor $\hat{K}^{\mu\nu}_{(\rho)}(P)$ is indicated in a schematic fashion. The cross-hatched areas denote vertex functions for a confining poetntial, as calculated in Ref. [7].

Equation (2.7) serves to separate $C_{(\rho)}(P^2)$ into a "background" term, $\hat{J}_{(\rho)}(P^2)$, and a resonant term, $\tilde{C}_{(\rho)}(P^2)$ [7]. We have also developed methods to extend our model, so that Eq. (2.7) is valid for all P^2 [11]. In that case, we can write

$$C_{(\rho)}(P^2) = C^{(\mathrm{cont})}_{(\rho)}(P^2) + \tilde{C}_{(\rho)}(P^2) \ , \tag{2.8}$$

where $C^{(\mathrm{cont})}_{(\rho)}(P^2)$ is equal to $\hat{J}_{(\rho)}(P^2)$ in the low-energy region where the NJL model is applicable.

Now, again making reference to Fig. 1, we define a tensor

$$\hat{K}^{\mu\nu}_{(\rho)}(P) = -g^{\mu\nu}(P)\hat{K}_{(\rho)}(P^2) \ . \tag{2.9}$$

In Fig. 1, the wavy lines denote pions and the cross-hatched areas denote vertex functions that implement our model of confinement. Note that the $q\bar{q}$ cut that would appear in $\hat{K}_{(\rho)}(P^2)$, in the absence of a model of confinement, has been eliminated in our model. Therefore, we are able to obtain $\mathrm{Re}\hat{K}_{(\rho)}(P^2)$ from $\mathrm{Im}\hat{K}_{(\rho)}(P^2)$ by use of a dispersion relation [7].

We found that $C_{(\rho)}(P^2)$ could be expressed in terms of $\hat{J}_{(\rho)}(P^2)$ and $\hat{K}_{(\rho)}(P^2)$:

$$C_{(\rho)}(P^2) = \frac{\hat{J}_{(\rho)}(P^2) + \hat{K}_{(\rho)}(P^2)}{1 - G_V[\hat{J}_{(\rho)}(P^2) + \hat{K}_{(\rho)}(P^2)]} \ . \tag{2.10}$$

Here, G_V is a coupling constant of an extended NJL model, whose Lagrangian is [7]

$$\begin{aligned}\mathcal{L}(x) = {} & \bar{q}(x)[i\,\partial\!\!\!/ - m_q^0]q(x) \\ & + \frac{G_S}{2}\{[\bar{q}(x)q(x)]^2 + [\bar{q}(x)i\gamma_5 \boldsymbol{\tau} q(x)]^2\} \\ & - \frac{G_V}{2}\{[\bar{q}(x)\gamma^\mu \boldsymbol{\tau} q(x)]^2 + [\bar{q}(x)\gamma_5\gamma^\mu \boldsymbol{\tau} q(x)]^2\} \ . \end{aligned} \tag{2.11}$$

If we recall Eq. (2.7), we see that we can write

$$\tilde{C}_{(\rho)}(P^2) = \frac{G_V \hat{J}^2_{(\rho)}(P^2) + \hat{K}_{(\rho)}(P^2)[1 + G_V \hat{J}_{(\rho)}(P^2)]}{\hat{D}_{(\rho)}(P^2)} \ , \tag{2.12}$$

where

$$\hat{D}_{(\rho)}(P^2) = 1 - G_V[\hat{J}_{(\rho)}(P^2) + \hat{K}_{(\rho)}(P^2)] \ . \tag{2.13}$$

Finally, we note that we have

$$\mathrm{Im}\tilde{C}_{(\rho)}(P^2) = \frac{\mathrm{Im}\hat{K}_{(\rho)}(P^2)}{|\hat{D}_{(\rho)}(P^2)|^2} \ . \tag{2.14}$$

The values obtained for $\mathrm{Im}\tilde{C}_{(\rho)}(P^2)/4P^2$ in the absence of matter are shown in Fig. 2 [7]. In our earlier work, we used $m_q = 302$ MeV, $G_V = 7.58\ \mathrm{GeV}^{-2}$, $m_q^0 = 5$ MeV, and $\Lambda_3 = 702$ MeV. Here, Λ_3 is a cutoff used for all three-momenta that appear in loop integrals. (We also found that $g_{\pi qq} = 3.05$.)

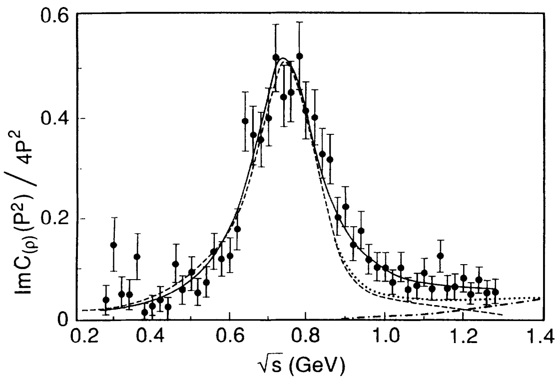

FIG. 2. The figure shows values of $\mathrm{Im}\tilde{C}_{(\rho)}(P^2)/4P^2$. The data points are taken from the compilation of Ref. [10]. The solid line is a (least-squares) phenomenological fit to the data made in Ref. [10]. The dash-dotted line is a continuum contribution suggested by the work of Shuryak [9]:

$$\mathrm{Im}C_{(\rho)}^{\mathrm{cont}}(P^2)/4P^2 = \left(\frac{1}{4\pi}\right)\left(1+\frac{\alpha_s}{\pi}\right)\left\{1+\exp\left[(\sqrt{s_0}-\sqrt{s})/\delta\right]\right\}^{-1}.$$

Here, we have taken $\sqrt{s_0} = 1.4$ GeV and $\delta = 0.2$ GeV. The dotted line represents $\mathrm{Im}C_{(\rho)}(P^2)/4P^2$, the sum of the resonant and continuum contributions.

Here we will concentrate on the modification of $\mathrm{Im}\tilde{C}_{(\rho)}(P^2)$ in the presence of nuclear matter. The presence of nuclear matter complicates the description of the rho propagator since there is an additional four-vector in the problem, η^μ. That four-vector describes the flow of the matter and in the matter rest frame we write $\eta^\mu = (1,0,0,0)$. Therefore, all the functions of the theory can depend upon two variables, P^2 and $P\cdot\eta$. A complete analysis of the rho propagator, with the rho self-energy developed as a function of two variables was given in Ref. [12]. In that work we calculated the modification of the rho propagator due to the excitation of nucleon particle-hole states. The results could be expressed as a function of $|\mathbf{P}|$ and P_0, for example, in the matter rest frame.

The problem under study here relates to the modification of the rho properties due to the (Lorentz) scalar and vector mean fields present in nuclear matter. These fields may be taken to couple to the quark and antiquark in the rho meson and a self-energy, $\Sigma = a+\gamma^0 b$, may be inserted in the quark propagator: $\bar{S}(p) = [\not p-(m_q+a)-\gamma^0 b]^{-1}$. It is easily seen that the calculation of the quark-loop integrals $J_{(\rho)}^{\mu\nu}(P^2, P\cdot\eta)$ with propagators such as $\bar{S}(p)$ yields a result independent of the vector self-energy term, $\gamma^0 b$. Therefore, we may take $J_{(\rho)}^{\mu\nu}(P^2, P\cdot\eta) \to J_{(\rho)}^{\mu\nu *}(P^2)$, where the asterisk is a reminder that m_q has been replaced by $m_q^* = m_q + a$ in the integrand. Similar remarks may be made for the quark-loop integrals that appear in the calculation of $K_{(\rho)}^{\mu\nu}(P^2, P\cdot\eta)$, so that we may also put $K_{(\rho)}^{\mu\nu}(P^2, P\cdot\eta) \to K_{(\rho)}^{\mu\nu *}(P^2)$.

One way to understand why the term $\gamma^0 b$ plays an important role when calculating properties of the nucleon in nuclear matter and drops out of the problem when calculating the properties of a meson is to consider the Dirac hole theory. The energy of a negative-energy state in the presence of the self-energy $\Sigma = a + \gamma^0 b$ is $\bar{\epsilon}(\mathbf{p}) = b - [\mathbf{p}^2 + (m^*)^2]^{1/2}$, while for a positive-energy state one has $\epsilon(\mathbf{p}) = b+[\mathbf{p}^2+(m^*)^2]^{1/2}$. If we excite a particle from a negative to a positive-energy state, b does not appear in the expression for the energy required, $\bar{\epsilon}(\mathbf{p}) - \epsilon(\mathbf{p})$. In the light of these remarks, we will consider the various functions defined here to be independent of $P\cdot\eta$, since such dependence would arise only from the $\gamma^0 b$ term of the quark self-energy. All studies of the modification of rho properties due to mean-field effects in nuclear matter have used the same approximation [5]. (For the general form of the rho propagator in matter, we again refer the reader to Ref. [12].)

Note that, in the NJL model, the value of the constituent quark mass, m_q, is proportional to the vacuum quark condensate in the chiral limit. In our previous analysis, we found that the condensate is reduced by about 35% at nuclear matter densities [13]. (This result depends directly on the value assumed for σ_N, the pion-nucleon sigma term. An analysis of experimental data yields $\sigma_N = 45 \pm 8$ MeV [14].) Therefore, we may consider

$$m_q^* = m_q\left(1 - 0.35\frac{\rho}{\rho_{NM}}\right), \tag{2.15}$$

where ρ is the baryon density of the medium and $\rho_{NM} = 0.17\ \mathrm{fm}^{-3}$ is the density of nuclear matter.

In our mean-field analysis, we need to determine the dependence of various quantities on m_q^*. For example, we may define $\hat{J}_{(\rho)}^*(P^2)$, $\hat{K}_{(\rho)}^*(P^2)$, f_π^*, f_ρ^*, $g_{\pi qq}^*$, $\tilde{C}_{(\rho)}^*(P^2)$, etc. In our earlier work [12] we have seen that the ratio (f_π^*/f_π) is equal to the square root of the ratio of the in-medium condensate to its vacuum value. The same is true for $(g_{\pi qq}^*/g_{\pi qq})$, since the Goldberger-Trieman relation, $m_q^* = g_{\pi qq}^* f_\pi^*$, is true in matter, in the chiral limit. Therefore, we have used $g_{\pi qq}^* = g_{\pi qq}(m_q^*/m_q)^{1/2}$ in our calculations. With $g_{\pi qq} = 3.05$, we have $g_{\pi qq}^* = 2.46$, if $(m_q^*/m_q) = 0.65$. [See Eq. (2.10.]

III. DENSITY DEPENDENCE OF VARIOUS AMPLITUDES

In this section we present the results of our calculations. In Fig. 3 we show the value of $\hat{J}_{(\rho)}(P^2)$, that we have obtained previously, as a solid line [7]. The value of this integral is increased when we replace m_q by m_q^*. Our

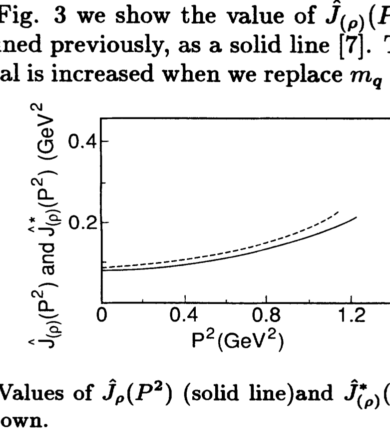

FIG. 3. Values of $\hat{J}_\rho(P^2)$ (solid line)and $\hat{J}_{(\rho)}^*(P^2)$ (dashed line) are shown.

result for $\hat{J}^*_{(\rho)}(P^2)$ is shown as a dashed line in Fig. 3. Note that the position of the resonance may be found by setting $\mathrm{Re}\hat{D}_{(\rho)}(P^2) = 1 - G_V[\mathrm{Re}\hat{K}_{(\rho)}(P^2) + \hat{J}_{(\rho)}(P^2)] = 0$, or $\mathrm{Re}\hat{D}^*_{(\rho)}(P^2) = 1 - G_V[\mathrm{Re}\hat{K}^*_{(\rho)}(P^2) + \hat{J}^*_{(\rho)}(P^2)] = 0$. Since $\mathrm{Re}\hat{K}_{(\rho)}(P^2)$ and $\mathrm{Re}\hat{K}^*_{(\rho)}(P^2)$ are positive, they provide a *downward* shift in the resonance position of about 120 MeV [7].

In Fig. 4, we exhibit $\mathrm{Re}\hat{K}_{(\rho)}(P^2)$, $\mathrm{Im}\hat{K}_{(\rho)}(P^2)$, $\mathrm{Re}\hat{K}^*_{(\rho)}(P^2)$, and $\mathrm{Im}\hat{K}^*_{(\rho)}(P^2)$. We can obtain some estimate of the modification of the resonance width by forming $R = \mathrm{Im}\hat{K}^*_{(\rho)}(m^{*2}_\rho)/\mathrm{Im}\hat{K}_{(\rho)}(m^2_\rho)$. Since we find $m^*_\rho = 620$ MeV, we have $R = 0.65$, which leads to $\Gamma^*_\rho/\Gamma_\rho \simeq 0.65$. We also find that $\Gamma_\rho = 150$ MeV and $\Gamma^*_\rho = 97$ MeV. [See Eqs. (3.1)–(3.5).]

Finally, in Fig. 5 we show values obtained for $\mathrm{Im}\tilde{C}_\rho(P^2)/4$ [solid line] and $\mathrm{Im}\tilde{C}^*_{(\rho)}(P^2)/4$ [dashed line]. Here, we clearly see the downward shift of the rho mass and reduction of the width. It is useful to note that, for $P^2 < 0$, we can approximate our results by the form [7]

$$\frac{\tilde{C}_{(\rho)}(P^2)}{4} = -\frac{f^2_\rho m^2_\rho}{P^2 - m^2_\rho} \tag{3.1}$$

and

$$\frac{\tilde{C}^*_{(\rho)}(P^2)}{4} = -\frac{(f^*_\rho)^2 (m^*_\rho)^2}{P^2 - (m^*_\rho)^2} . \tag{3.2}$$

Thus, we have $\tilde{C}_{(\rho)}(0)/4 = f^2_\rho$ and $\tilde{C}^*_{(\rho)}(0)/4 = (f^*_\rho)^2$. Values found for the various parameters are $f_\rho = 0.235$ GeV, $m_\rho = 0.770$ GeV, $f^*_\rho = 0.283$ GeV, and $m^*_\rho = 0.620$ GeV. (These values are collected in Table I.) More generally, we can write

$$\frac{\tilde{C}_{(\rho)}(P^2)}{4} \simeq -\frac{f^2_\rho m^2_\rho}{P^2 - m^2_\rho + i\theta(P^2 - 4m^2_\pi) m_\rho \left(\frac{P^2}{m^2_\rho}\right)\Gamma_\rho} \tag{3.3}$$

with a corresponding expression for $\tilde{C}^*_{(\rho)}(P^2)/4$. Here, we have assumed that the width depends upon P^2 linearly. A more precise characterization could be made by replacing $(P^2/m^2_\rho)\Gamma_\rho$ in Eq. (3.3) by

$$\Gamma_\rho(P^2) = \left(\frac{P^2}{m^2_\rho}\right)\left[1 - \frac{4m^2_\pi}{P^2}\right]^{3/2}\left[1 - \frac{4m^2_\pi}{m^2_\rho}\right]^{-3/2}\Gamma_\rho , \tag{3.4}$$

with Γ_ρ being a constant. One way to extract the value of Γ_ρ is to evaluate

$$\mathrm{Im}\frac{\tilde{C}_{(\rho)}(m^2_\rho)}{4} = \frac{f^2_\rho m_\rho}{\Gamma_\rho} \tag{3.5}$$

and recall that f^2_ρ is equal to the value of $\mathrm{Re}\tilde{C}_{(\rho)}(0)/4$. [See Eqs. (3.1), (3.2).] Using this procedure, we obtain $\Gamma_\rho = 150$ MeV and $\Gamma^*_\rho = 97$ MeV, as quoted above. We also see that $f^*_\rho/f_\rho = 1.2$. Our result for $(m^*_\rho/m_\rho) = 0.80$ is in accord with the result of Ref. [1] where the reduction in the rho mass from the vacuum value was found to be 18% (for $\rho = \rho_{NM}$).

It is also worth noting that, in the limit that $\Gamma_\rho \to 0$, we have

$$\frac{\tilde{C}_{(\rho)}(P^2)}{4\pi} \to f^2_\rho m^2_\rho \delta(P^2 - m^2_\rho) \tag{3.6}$$

and

$$\frac{\tilde{C}^*_{(\rho)}(P^2)}{4\pi} \to (f^*_\rho)^2 (m^*_\rho)^2 \delta[P^2 - (m^*_\rho)^2] . \tag{3.7}$$

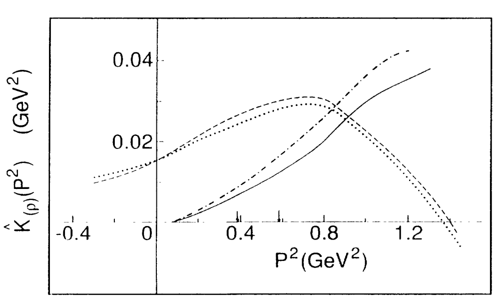

FIG. 4. The solid line shows $\mathrm{Im}\hat{K}_{(\rho)}(P^2)$ while the dash-dotted line shows $\mathrm{Im}\hat{K}^*_{(\rho)}(P^2)$. The dotted line shows $\mathrm{Re}\hat{K}_{(\rho)}(P^2)$, while the dashed line shows $\mathrm{Re}\hat{K}^*_{(\rho)}(P^2)$. The small arrows denote the values of $m^2_\rho = 0.59$ GeV^2 and $(m^*_\rho)^2 = 0.38$ GeV^2. [Here m^*_ρ is the rho mass at nuclear matter density predicted in this work ($m_\rho = 0.62$ GeV).]

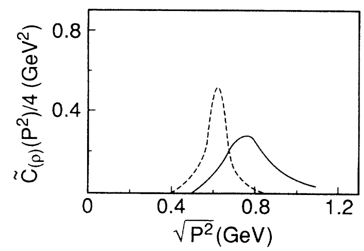

FIG. 5. The solid line shows $\mathrm{Im}\tilde{C}_{(\rho)}(P^2)/4$ and corresponds to the values of $\mathrm{Im}\tilde{C}_{(\rho)}(P^2)/4P^2$ shown in Fig. 2. The dashed line is $\mathrm{Im}\tilde{C}^*_{(\rho)}(P^2)/4$, as calculated in this work. [The value of m^*_ρ is 620 MeV, so that $(m^*_\rho/m_\rho) = 0.81$. We also find $\Gamma^*_\rho = 97$ MeV, so that $\Gamma^*_\rho/\Gamma_\rho) = 0.61$.]

TABLE I. Properties of the rho meson in vacuum and in nuclear matter. (The first set of values is taken from Ref. [7].)

	$\rho = 0$	$\rho = \rho_{NM}$	Expt.
m_ρ	0.770 GeV	0.620 GeV	0.770 GeV
f_ρ	0.235 GeV	0.282 GeV	
Γ_ρ	150 MeV	97 MeV	151.5±1.2 MeV
m_q	302 MeV	$m^*_q = 196$ MeV	

IV. DISCUSSION

We have presented a calculation of the modification of the rho mass and width using our generalized version of the NJL model. The results may be compared to those obtained in QCD sum-rule studies. For example, in Ref. [1] the result for the rho mass in matter is

$$m(\rho) = m_\rho \left(1 - C\frac{\rho}{\rho_{NM}}\right) \tag{4.1}$$

with $C = 0.18$. (The authors quote an uncertainty in the value of C of about 30% due to uncertainty in the value of $\hat{m}$ and of σ_N.) This reduction in the rho mass is in accord with our result. In Ref. [2], the rho mass at nuclear matter density is given as 530 MeV, which represents a larger reduction than that obtained in [1] and in our work.

Probably the greatest uncertainty in the sum-rule studies arises from the use of the finite-density factorization approximation. For example, one has relations of the form

$$\langle(\bar{q}\gamma_\mu\gamma_5\lambda^a q)(\bar{q}\gamma^\mu\gamma_5\lambda^a q)\rangle_\rho \simeq \frac{16}{9}\langle\bar{q}q\rangle_\rho^2 \tag{4.2}$$

whose validity is unknown [2]. The authors of Ref. [2] remark that if the four-quark condensates do not depend upon density, but are given by their vacuum values (obtained by use of the mean-field approximation), the rho meson mass does not decrease in matter. A similar problem arises in the work of Jin *et al.* [4], where the results for the scalar part of the *nucleon* self-energy are strongly dependent upon the treatment of the density dependence of the four-quark condensate. One disturbing feature of these results is that one makes closest contact with the self-energy of the nucleon used in relativistic nuclear physics in the case the four-quark condensates are replaced by their vacuum values (as obtained in the mean-field approximation). Given these uncertainties in the QCD sum-rule calculation of density-dependent quantities, we believe it is valuable to apply other models in the calculation of such quantities. One may hope that experimental data will provide some guidance in these matters [15].

This work was supported in part by a grant from the National Science Foundation and by the PSC-CUNY Faculty Award Program of the City University of New York.

[1] T. Hatsuda and S. H. Lee, Phys. Rev. C **46**, R34 (1992).
[2] M. Asakawa and C. M. Ko, Phys. Rev. C **48**, R526 (1993).
[3] M. Asakawa, C. M. Ko, P. Lávai, and X. J. Qui, Phys. Rev. C **46**, R1159 (1992); M. Lutz, S. Klimt, and W. Weise, Nucl. Phys. **A542**, 521 (1992).
[4] T. Hatsuda, H. Høgaasen, and M. Prakash, Phys. Rev. Lett. **566**, 2851 (1991); T. D. Cohen, R. J. Furnstahl, and D. K. Griegel, Phys. Rev. Lett. **67**, 961 (1991); R. J. Furnstahl, D. K. Griegel, and T. D. Cohen, Phys. Rev. C **46**, 1507 (1992); X. Jin, T. D. Cohen, R. J. Furnstahl, and D. K. Griegel, Phys. Rev. C **47**, 2882 (1993); X. Jin, M. Nielsen, T. D. Cohen, R. J. Furnstahl, and D. K. Griegel, University of Maryland Report No. UMPP 93-1997, 1993 (unpublished).
[5] For a review, see U. Vogl and W. Weise, Prog. Part. Nucl. Phys. **27**, 195 (1991).
[6] L. S. Celenza, C. M. Shakin, Wei-Dong Sun, J. Szweda, and Xiquan Zhu, Int. J. Mod. Phys. E (to be published).
[7] L. S. Celenza, C. M. Shakin, Wei-Dong Sun, J. Szweda, and Xiquan Zhu, Brooklyn College Report No. BCCNT 93/071/232, 1993 (unpublished).
[8] L. S. Celenza, C. M. Shakin, Wei-Dong Sun, and J. Szweda, Brooklyn College Report No. BCCNT 93/091/234, 1993 (unpublished).
[9] For a review of hadronic current correlators in QCD, see E. Shuryak, Rev. Mod. Phys. **65**, 1 (1993).
[10] R. D. Peccei and J. Sola, Nucl. Phys. **B281**, 1 (1987).
[11] C. M. Shakin, Wei-Dong Sun, and J. Szweda, Brooklyn College Report No. BCCNT 93/081/233, 1993 (unpublished).
[12] L. S. Celenza, A. Pantziris, and C. M. Shakin, Phys. Rev. C **45**, 205 (1992).
[13] L. S. Celenza, A. Pantziris, C. M. Shakin, and Wei-Dong Sun, Phys. Rev. C **45**, 2015 (1992); **46**, 571 (1992).
[14] J. Gasser, H. Leutwyler, and M. E. Sainio, Phys. Lett. **B253**, 252 (1991).
[15] G. E. Brown and M. Rho, Phys. Rev. Lett. **66**, 2720 (1991).